



Quantum Gravity and Cosmology 2024



Primordial Black Holes seeded during Inflation

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2024. 7. 2

Based on: X. Wang, YZ, M. Sasaki, [arXiv: 2404.02492]
S. Pi, YZ, Q. Huang, M. Sasaki, JCAP 05 (2018) 042 [arXiv: 1712.09896]
X. Wang, YZ, Kimura and M. Yamaguchi, SCPMA 6 (2023) 66 [arXiv: 2209.12911]
R. Kimura, T. Suyama, M. Yamaguchi and YZ, JCAP 04 (2021) 031 [arXiv: 2102.05280]
H. Wang, YZ, T. Suyama, in preparation

Why Primordial Black Hole (PBH)?

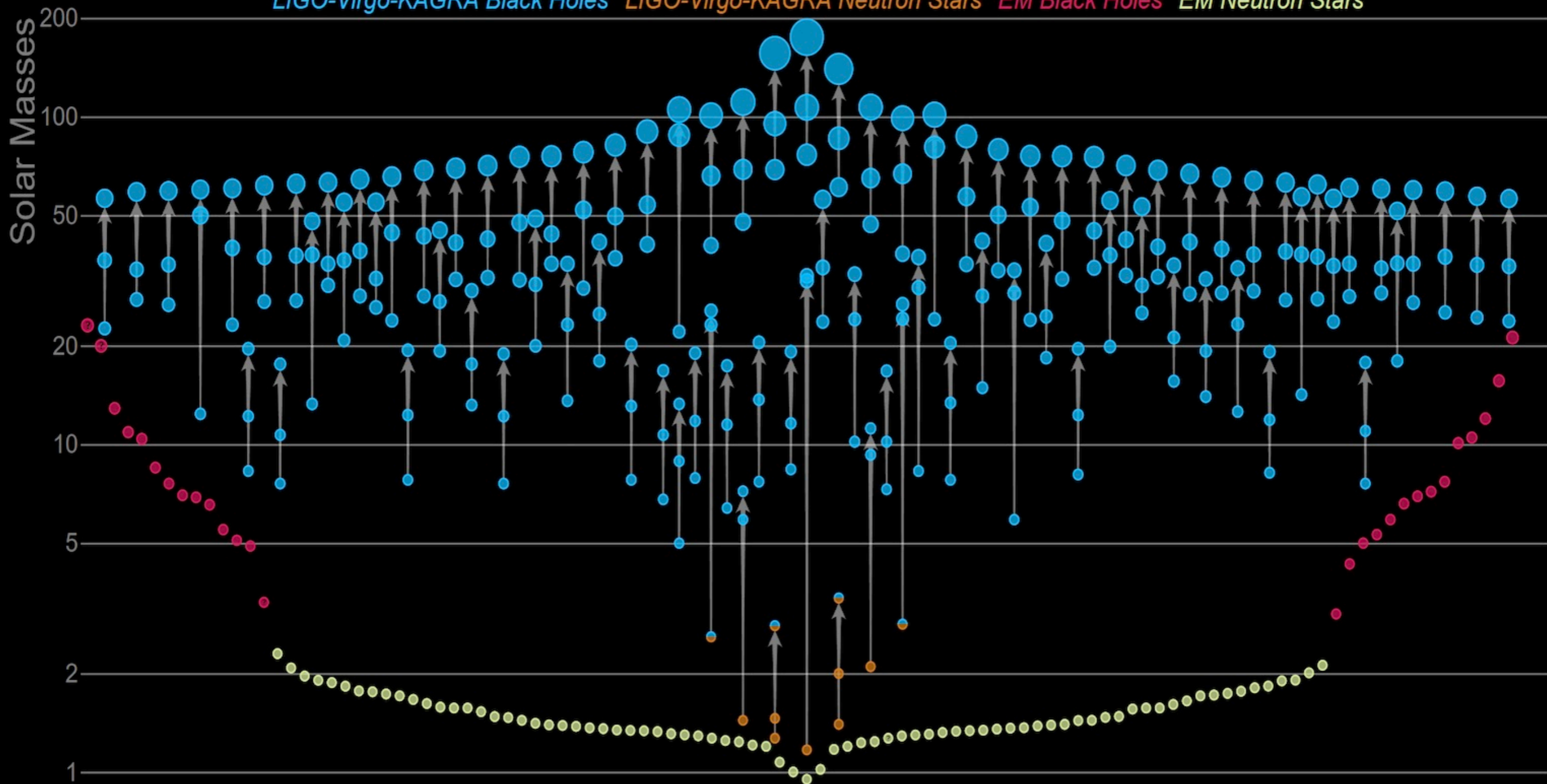
- BHs exist in the universe
- No need for new physics
- PBHs may dominate Dark Matter
- Detected GW events from LIGO may originate from the merger of PBH binaries

M. Sasaki, T. Suyama, T. Tanaka, S. Yokoyama, PRL 117, no. 6, 061101 (2016)

- A possible way to probe the primordial power spectrum of curvature perturbation on small scales
- ...

Masses in the Stellar Graveyard

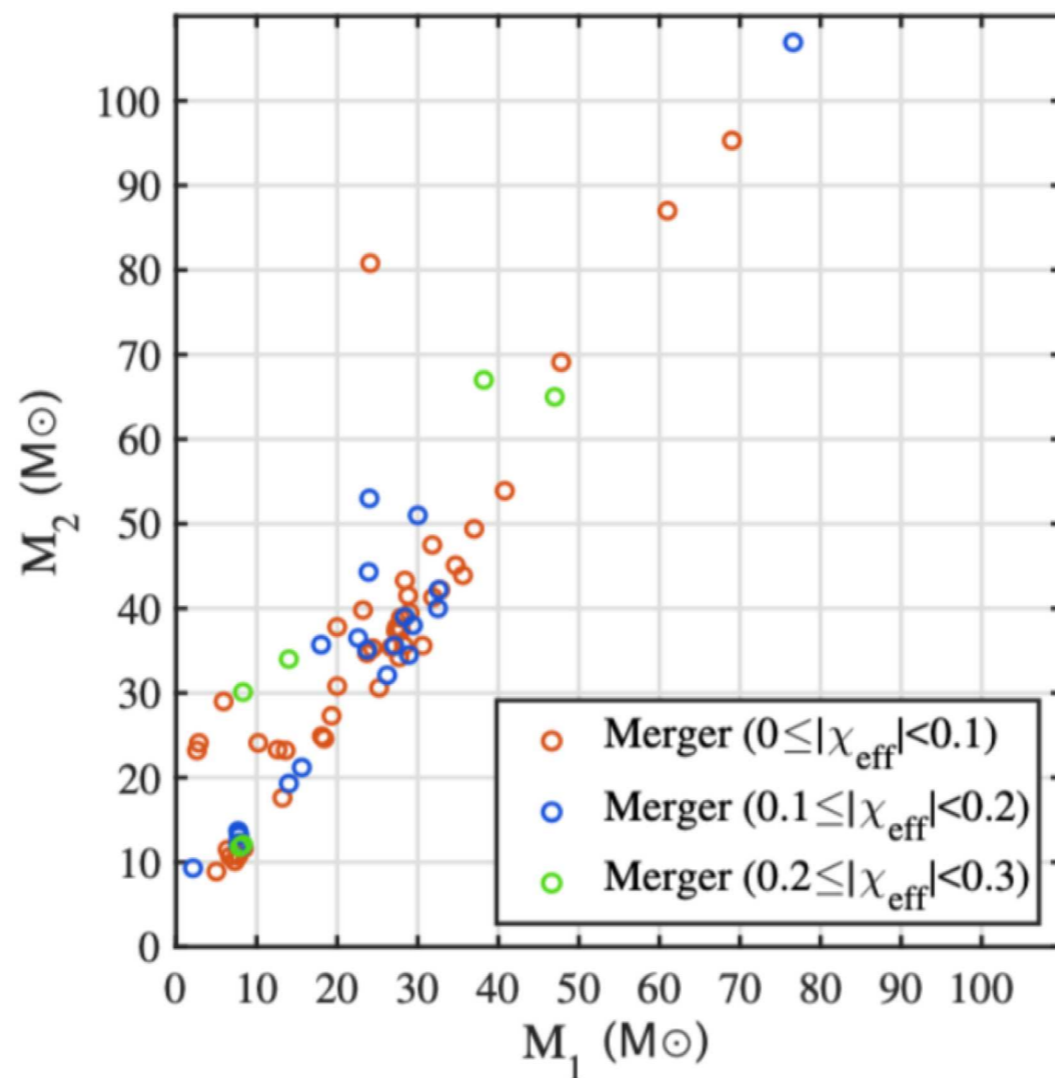
LIGO-Virgo-KAGRA Black Holes *LIGO-Virgo-KAGRA Neutron Stars* *EM Black Holes* *EM Neutron Stars*



LIGO-Virgo-KAGRA | Aaron Geller | Northwestern

<https://media.ligo.northwestern.edu>

According to LIGO/Virgo O3 data



X. Wang, YZ, Kimura and M.
Yamaguchi, SCPMA 6 (2023) 66

91 BBH candidates, 76 of them satisfy $m \gtrsim 1.5 M_\odot$ and $|\chi_{\text{eff}}| \lesssim 0.3$

- 1. Merging BHs are Massive
- 2. Spin of BH is small

Contradiction

BHs from collapse of Stars

satisfy

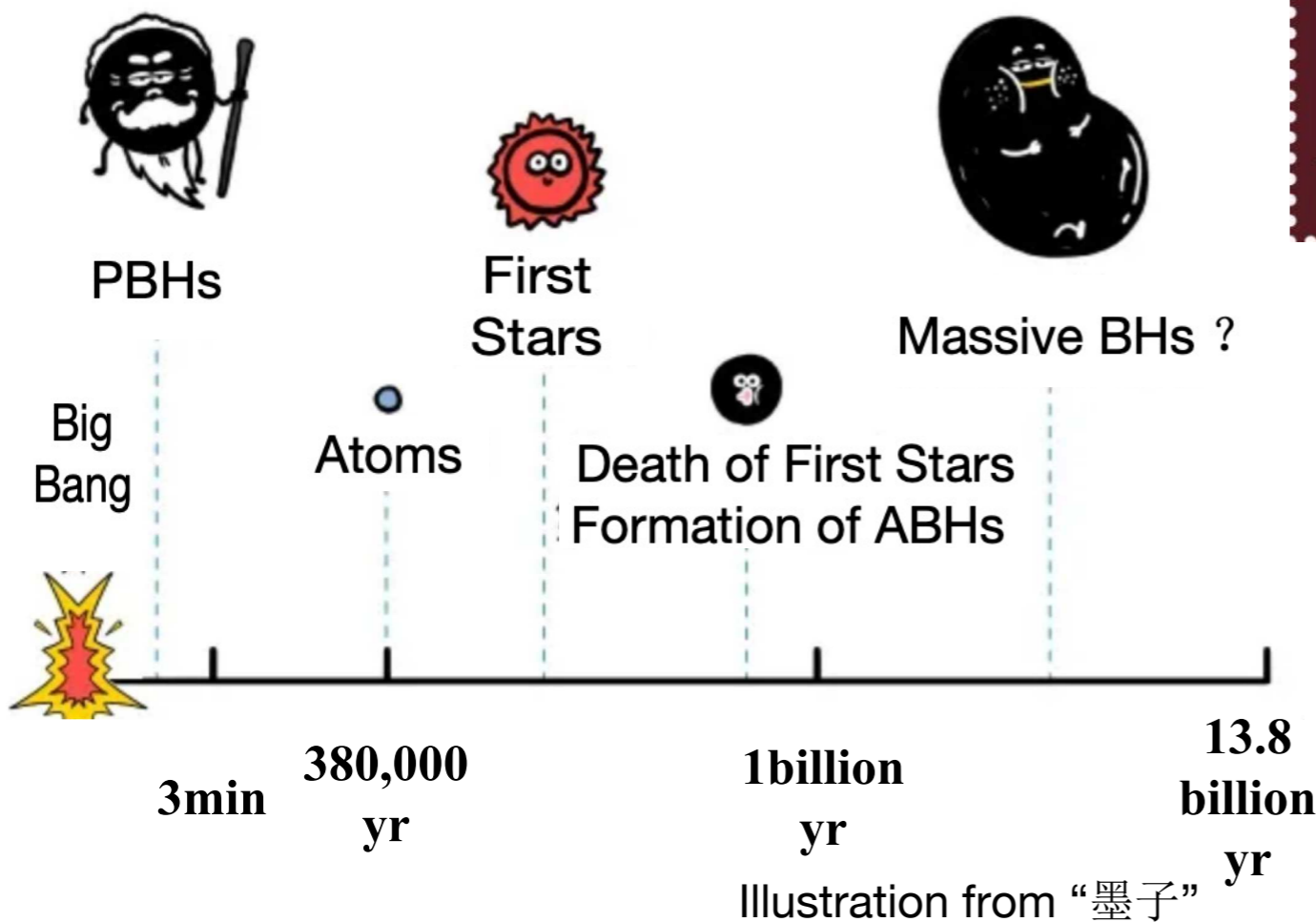
Primordial BHs

Massive BHs ?

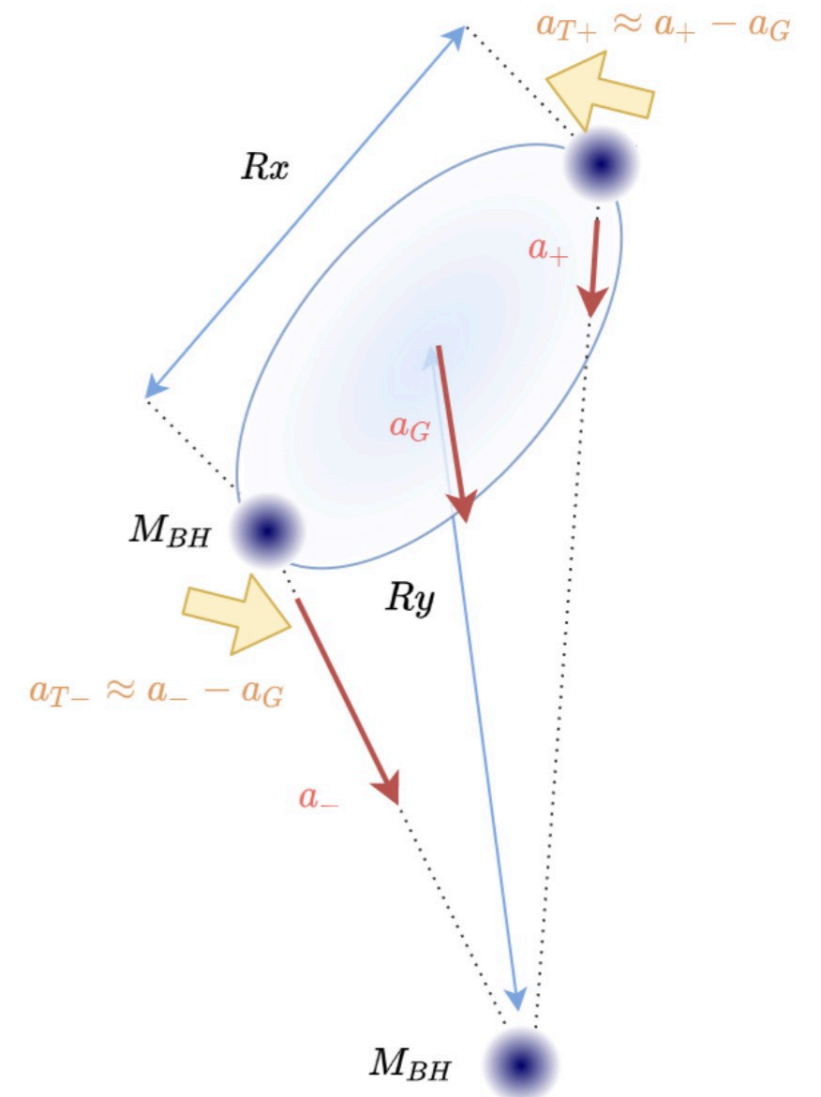
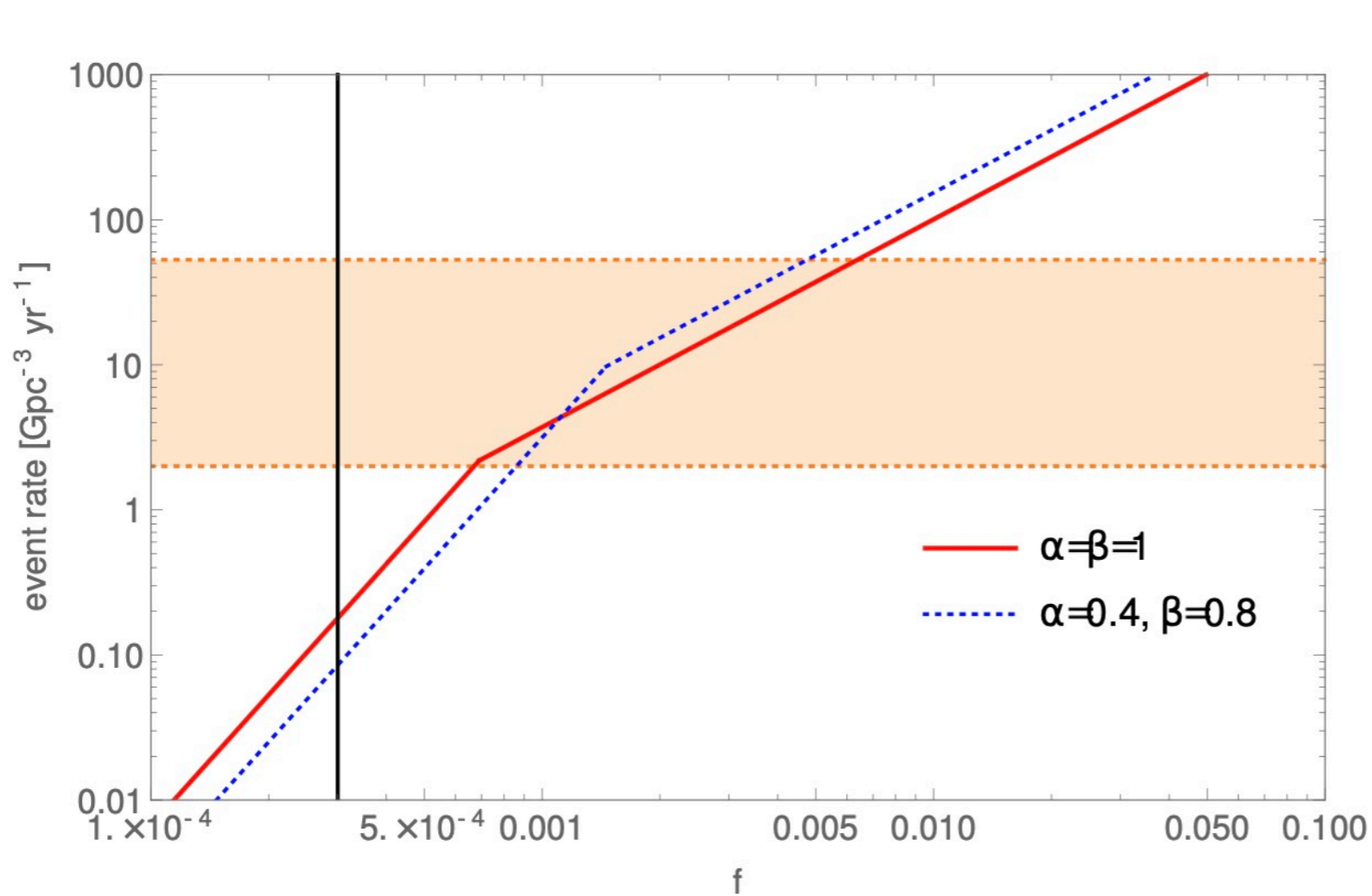


Y. B. Zel'dovich, I. D. Novikov, Soviet Astronomy 10, 602 (1967)

S. Hawking, Mon. Not. Roy. Astron. Soc. 152, 75 (1971)



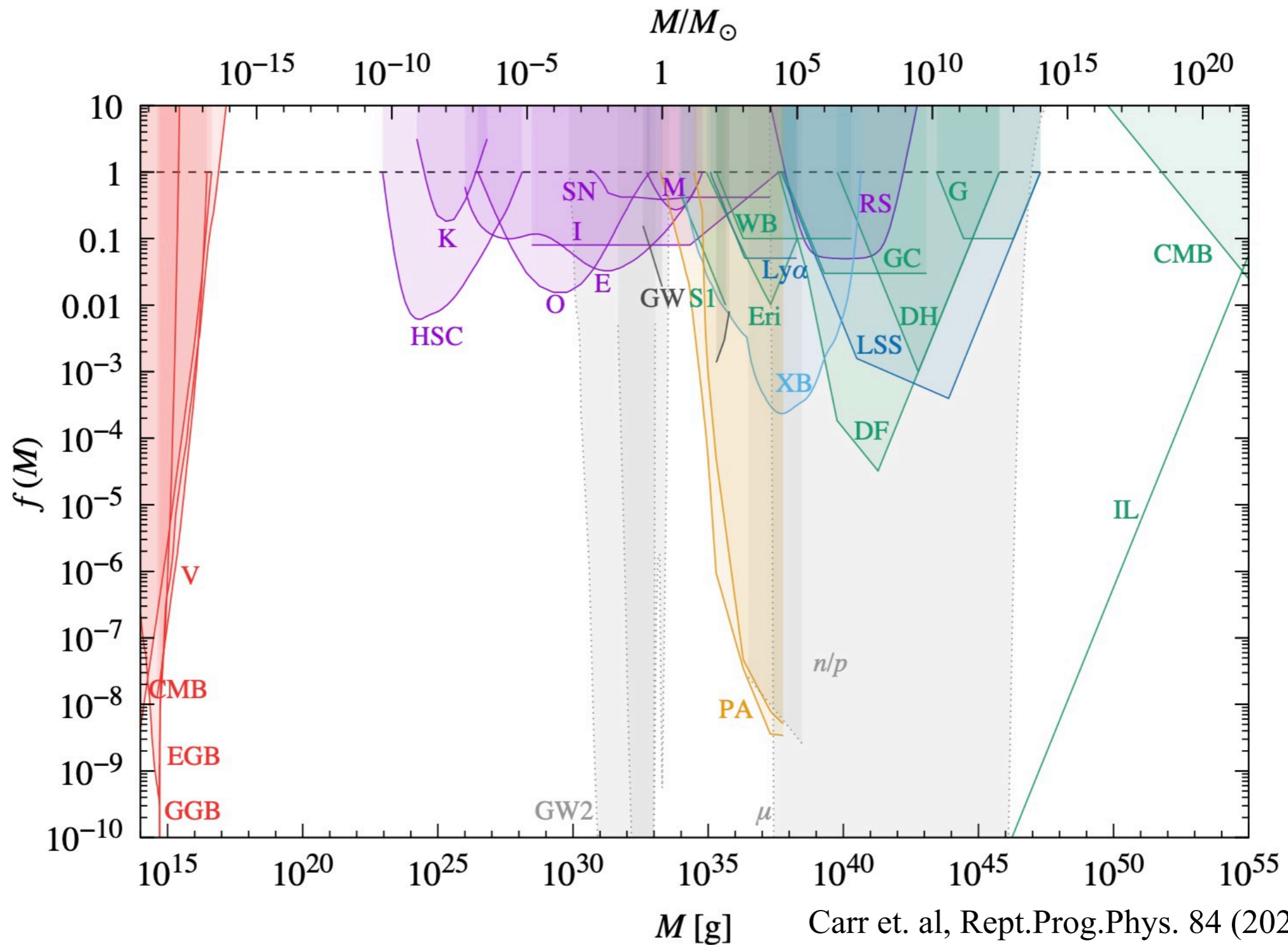
Massive PBHs for explanation of LIGO events



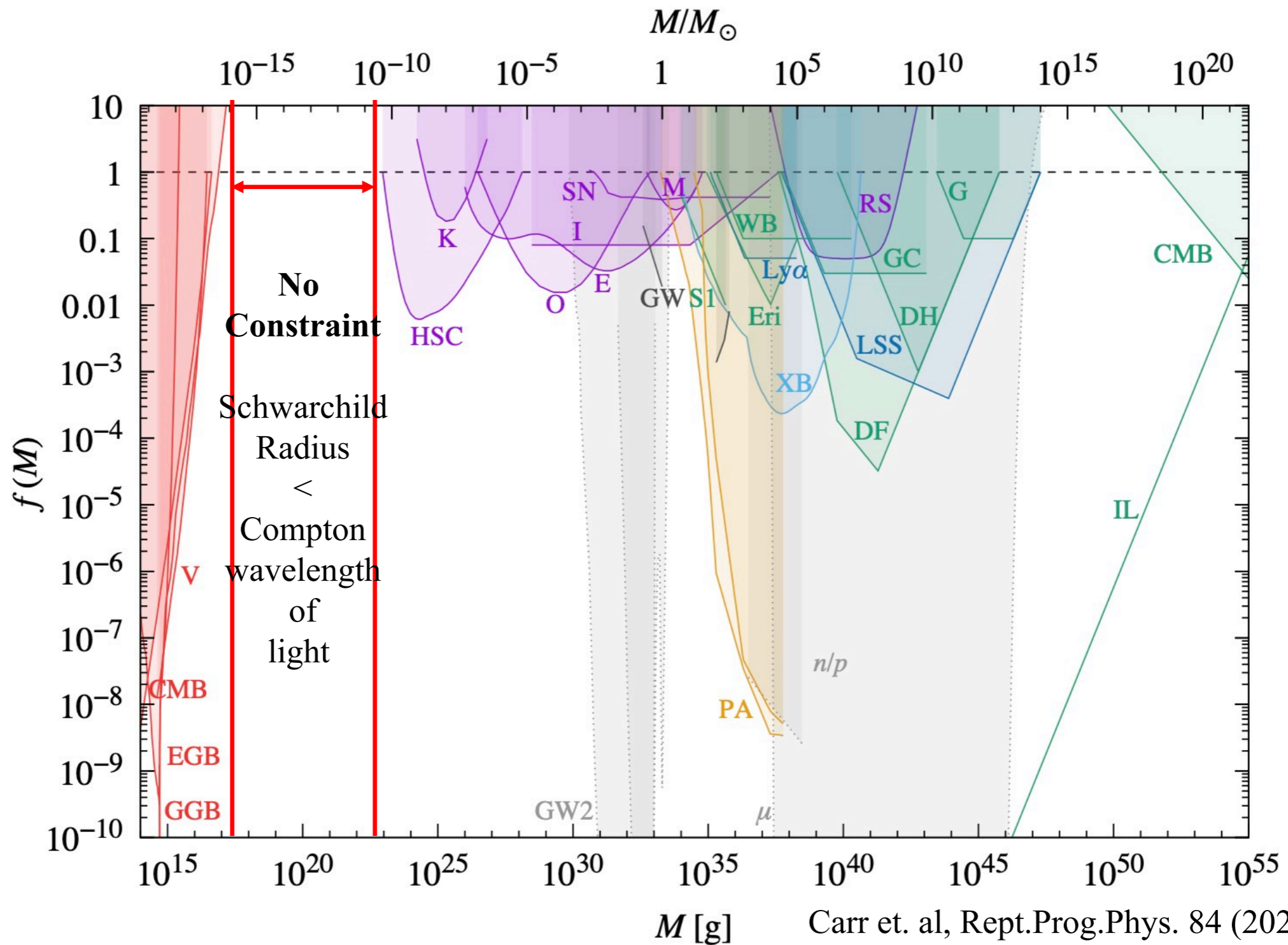
Sasaki, Suyama, Tanaka, Yokoyama, PRL 117 (2016) 6, 061101

Illustrated by Xinpeng

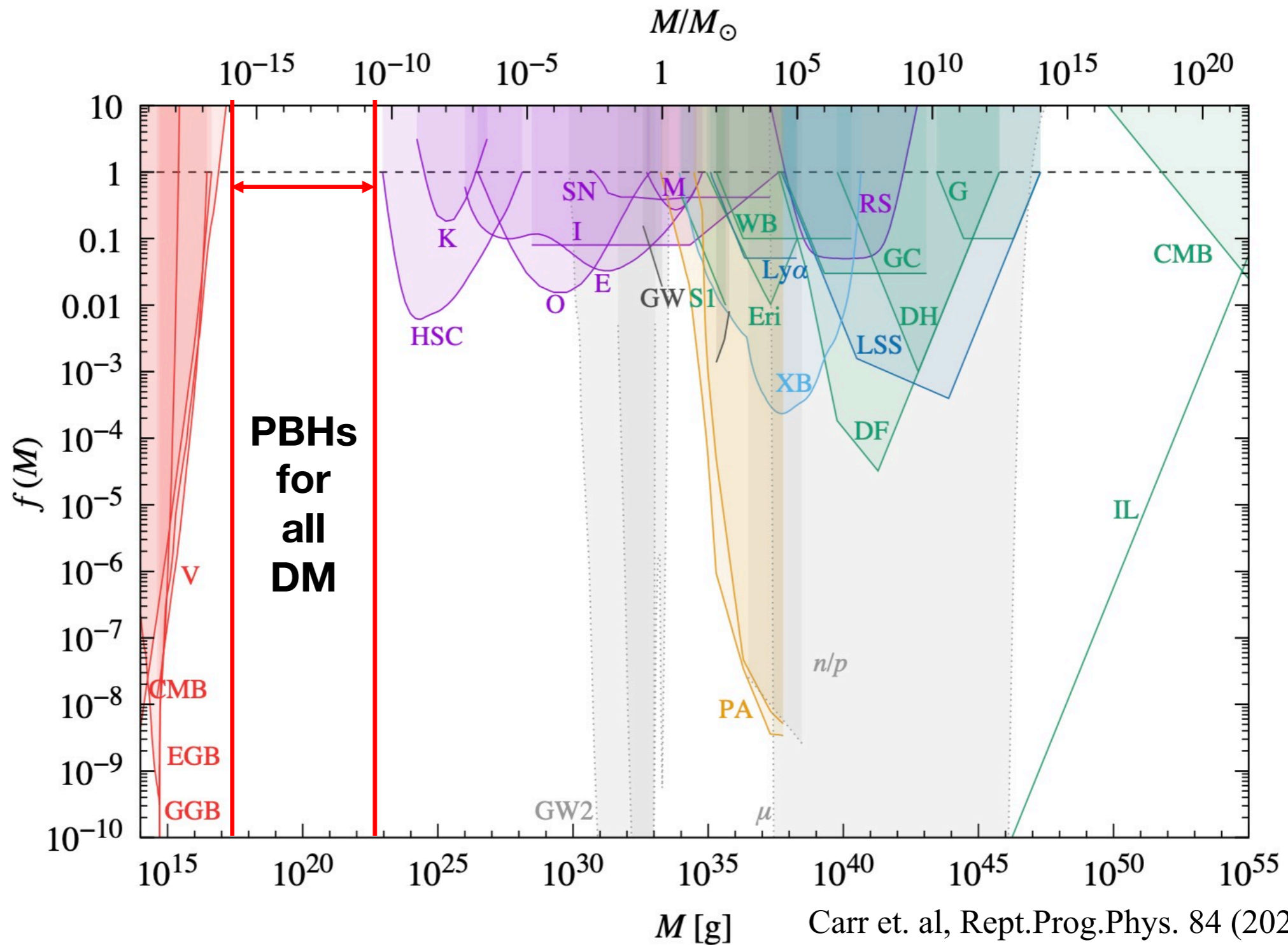
Moreover, **small PBHs** are candidate for **DM**



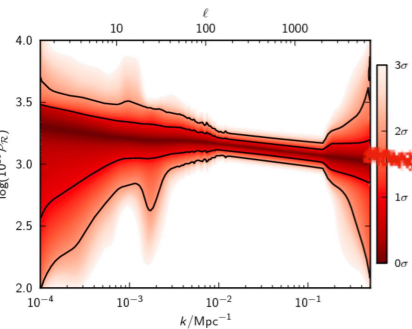
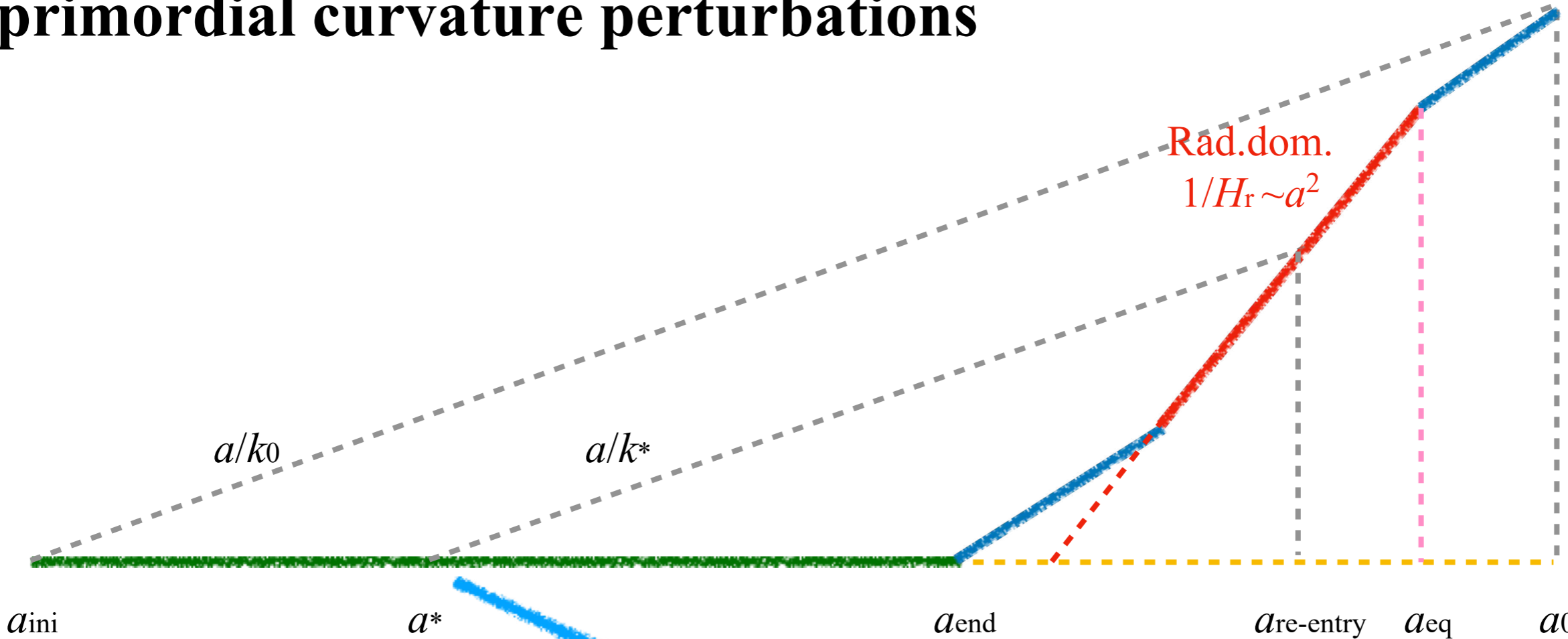
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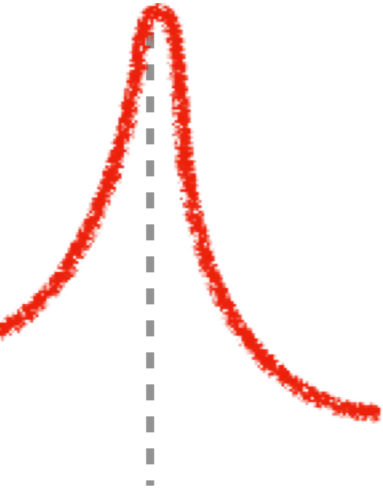
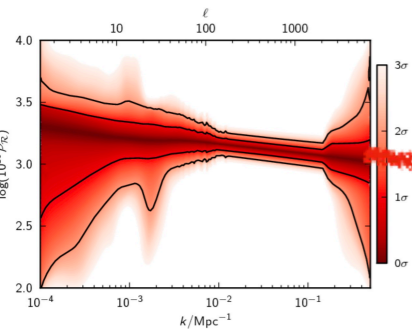
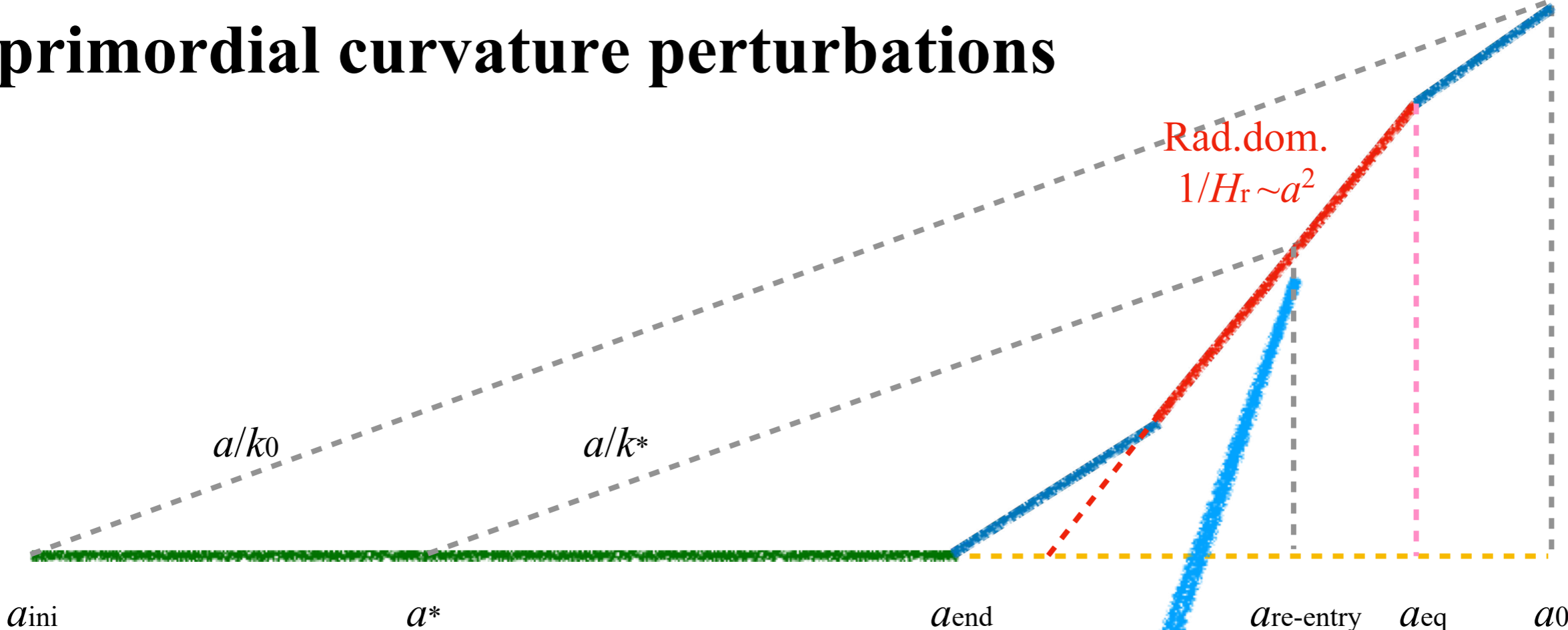
Formation of PBHs from enhancement of primordial curvature perturbations



$k^* = Ha^*$

There is a peak on the primordial density perturbation, which leaves horizon and gets frozen at a^* .

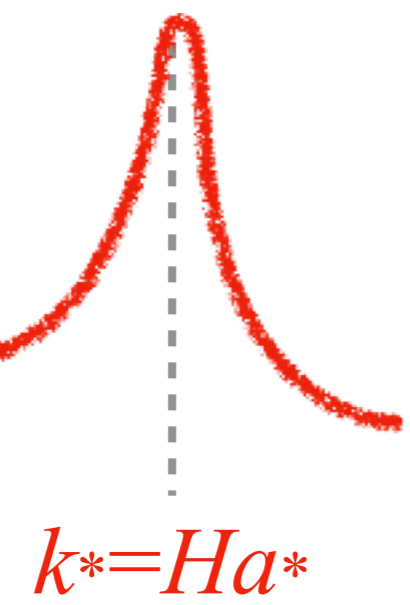
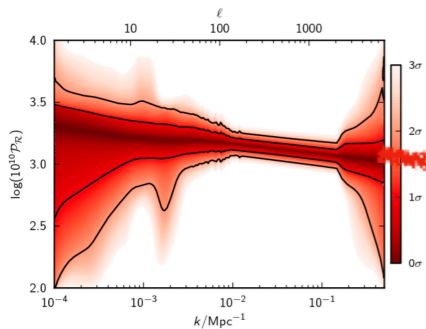
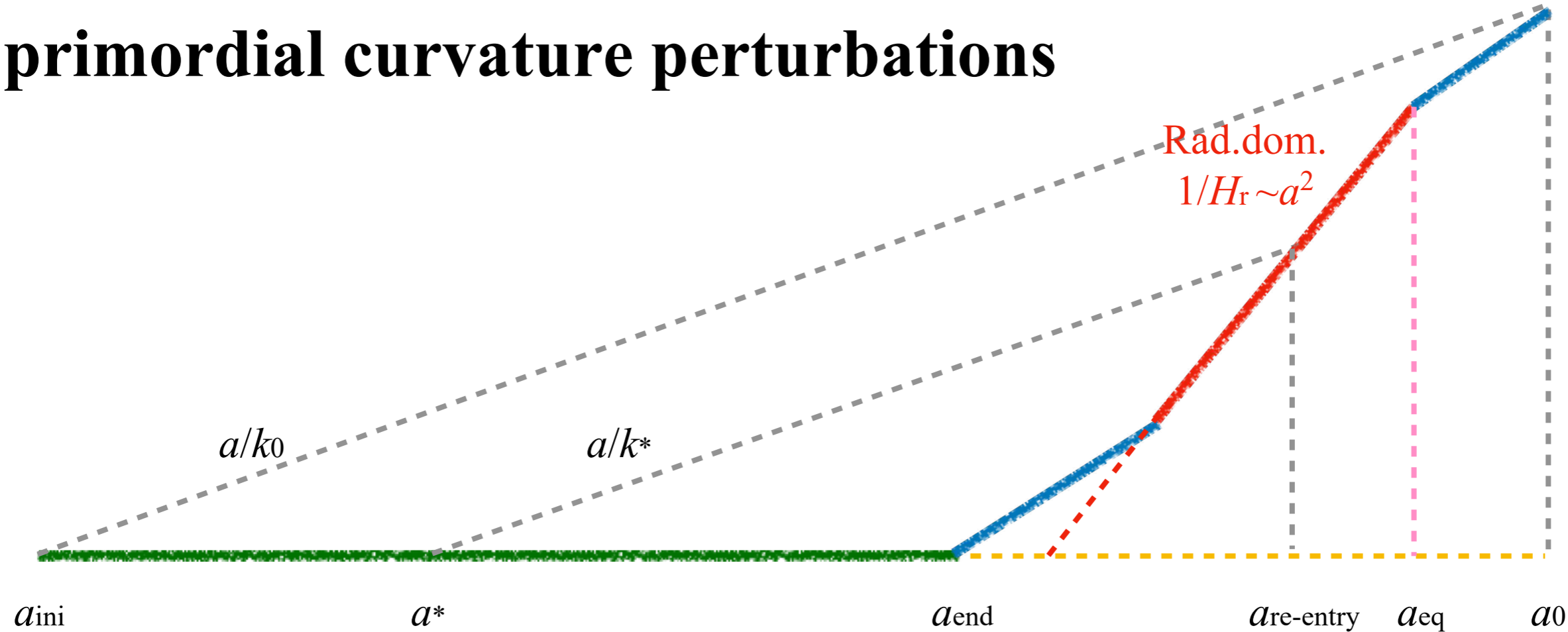
Formation of PBHs from enhancement of primordial curvature perturbations



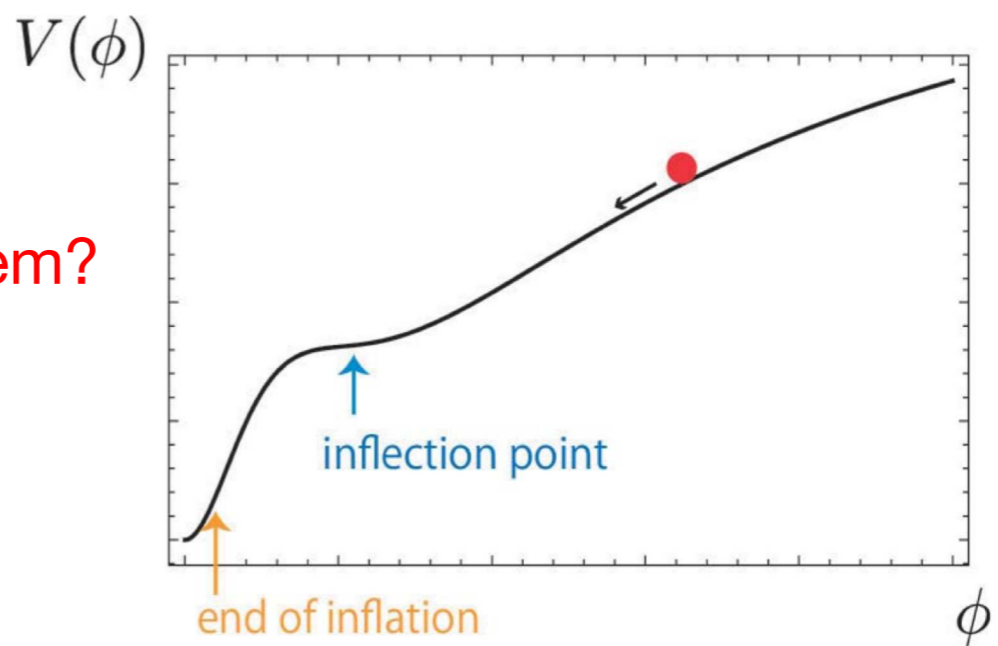
$k^* = H a^*$

The peak scale re-enters the horizon at radiation dominated era. If it exceeded some critical value $O(0.1)$, PBH will form. Its mass is $O(M_H)$.

Formation of PBHs from enhancement of primordial curvature perturbations



Fine tuning problem?



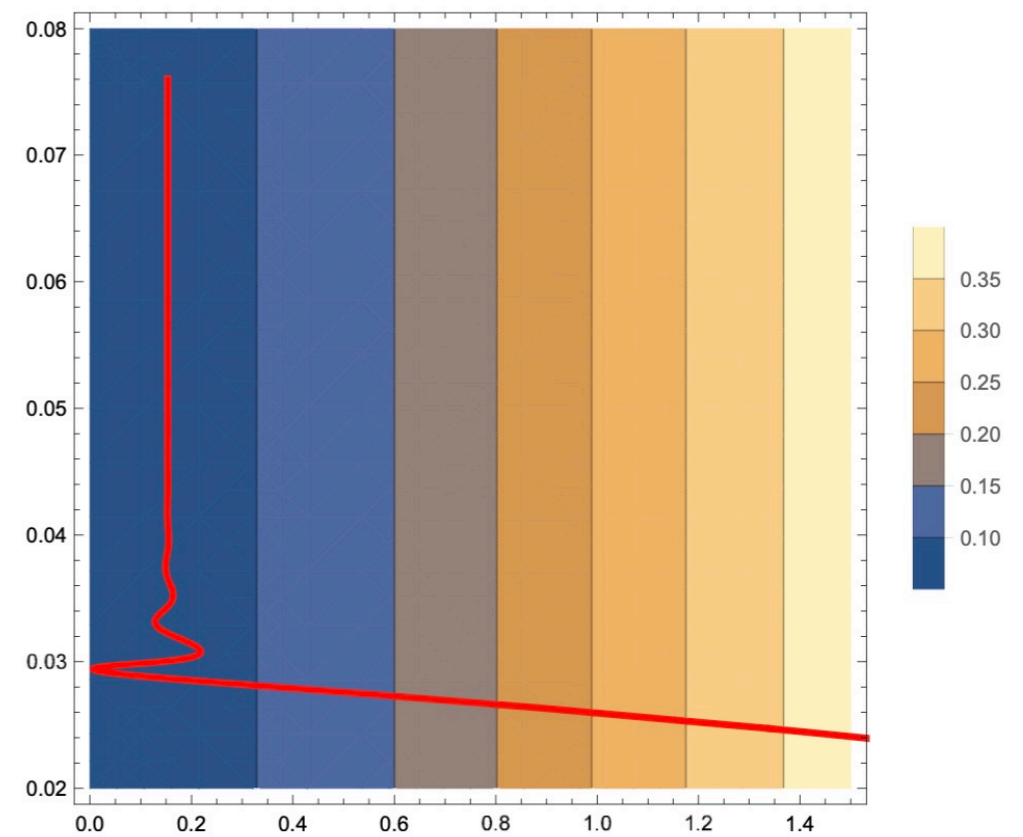
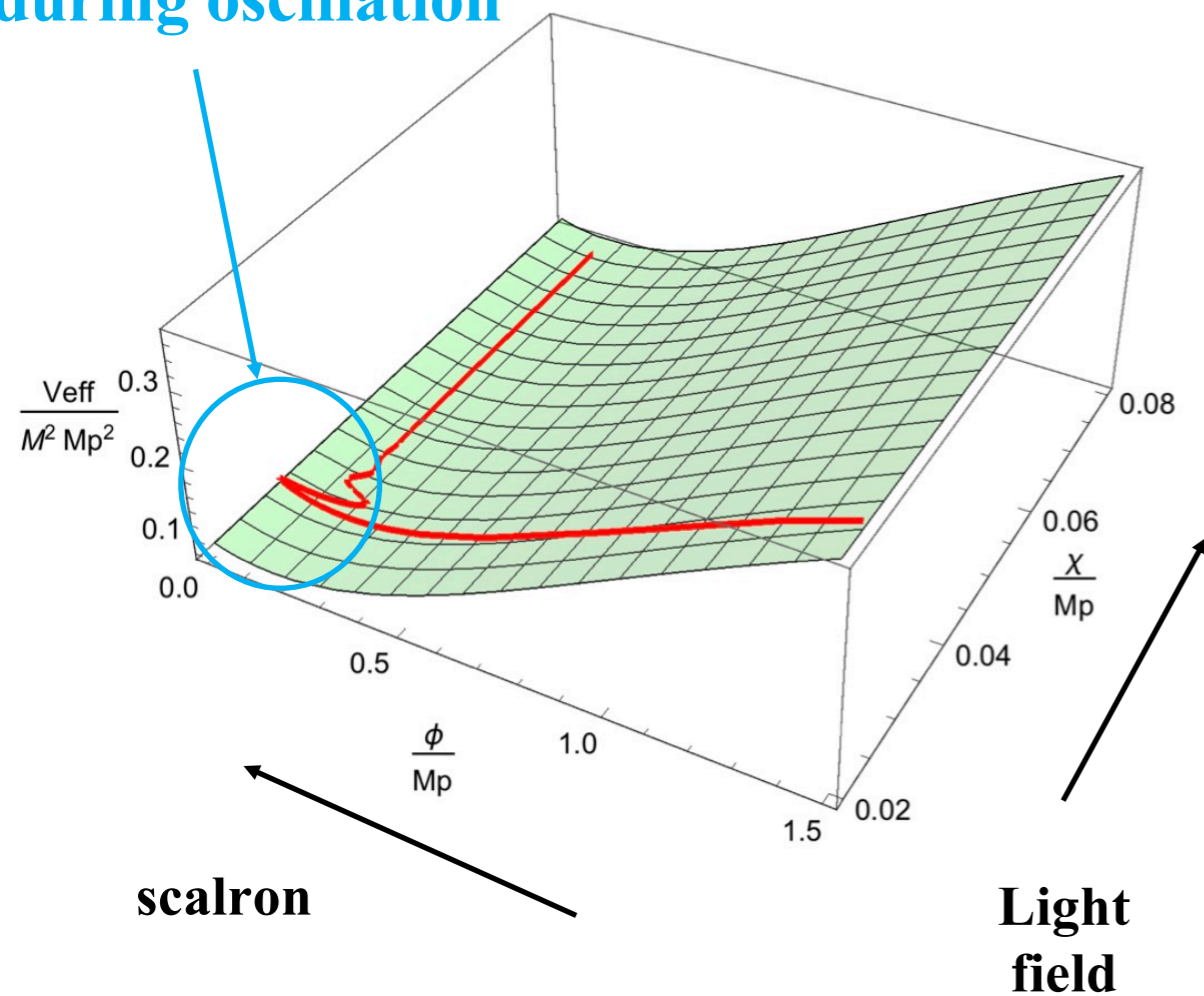
We proposed a model driven by Starobinsky model with a non-minimally coupled field

$$S_J = \int d^4x \sqrt{-g} \left\{ \frac{M_{\text{Pl}}^2}{2} \left(R + \frac{R^2}{6M^2} \right) - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - V(\chi) - \frac{1}{2} \xi R \chi^2 \right\}$$

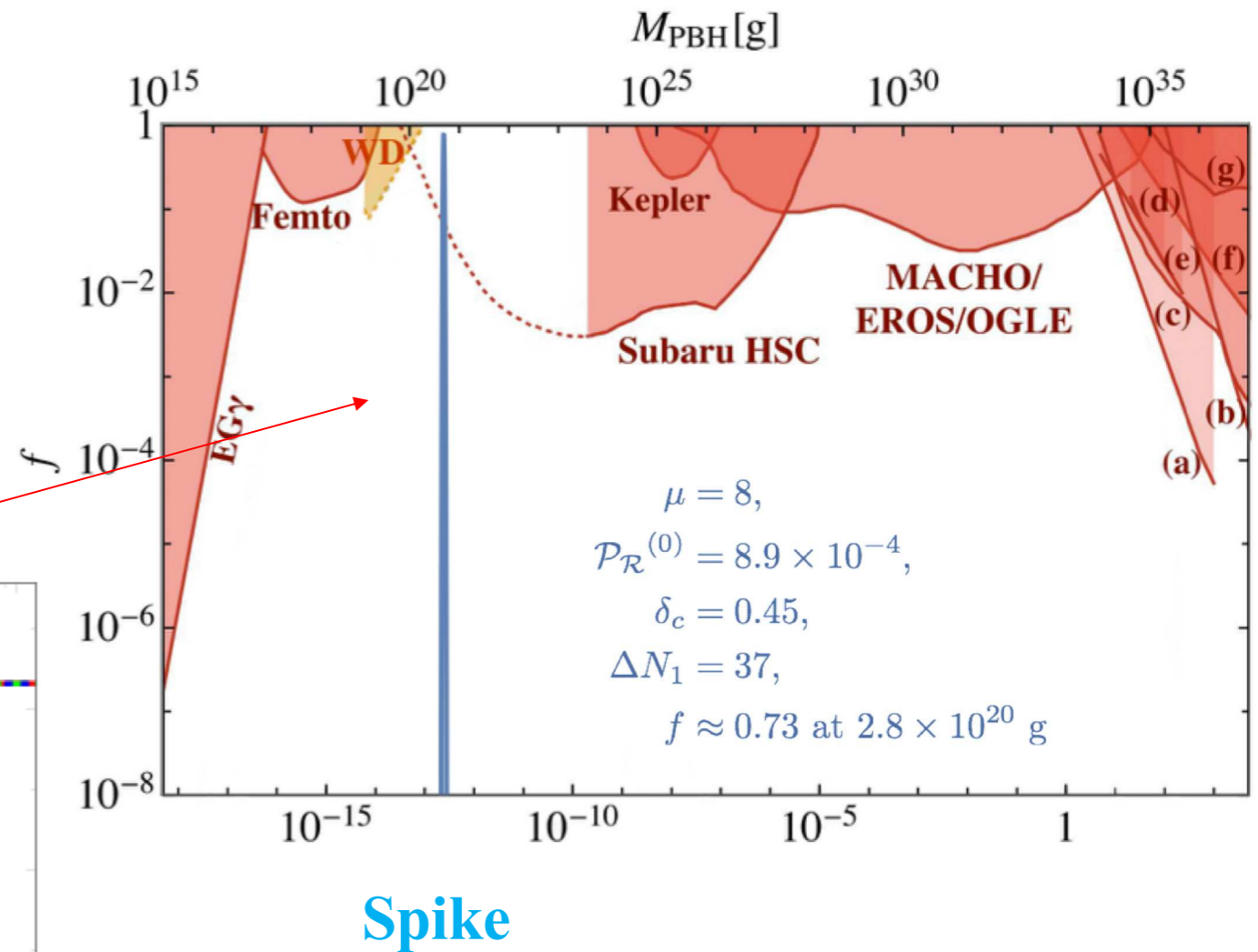
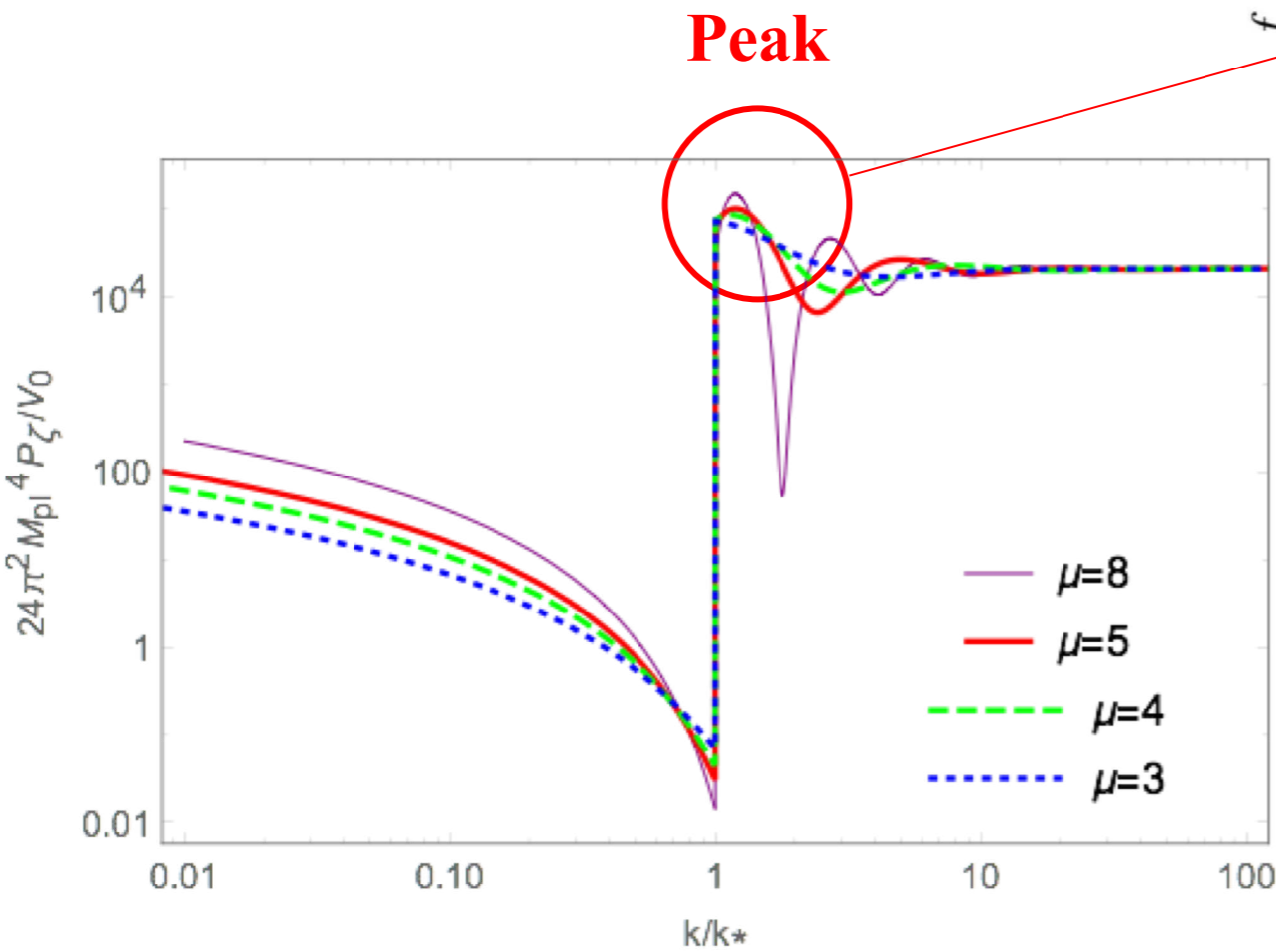
$$V(\chi) = V_0 - \frac{1}{2} m^2 \chi^2$$

Pi, YZ, Huang, Sasaki, JCAP 05 (2018) 042 [arxiv:1712.09896]

**Peak created
during oscillation**



$$f \equiv \frac{\Omega_{\text{PBH}}}{\Omega_{\text{CDM}}} \propto \exp \left[-\frac{1}{\mathcal{P}(k)} \right]$$



Defects: (1) No exit from inflation

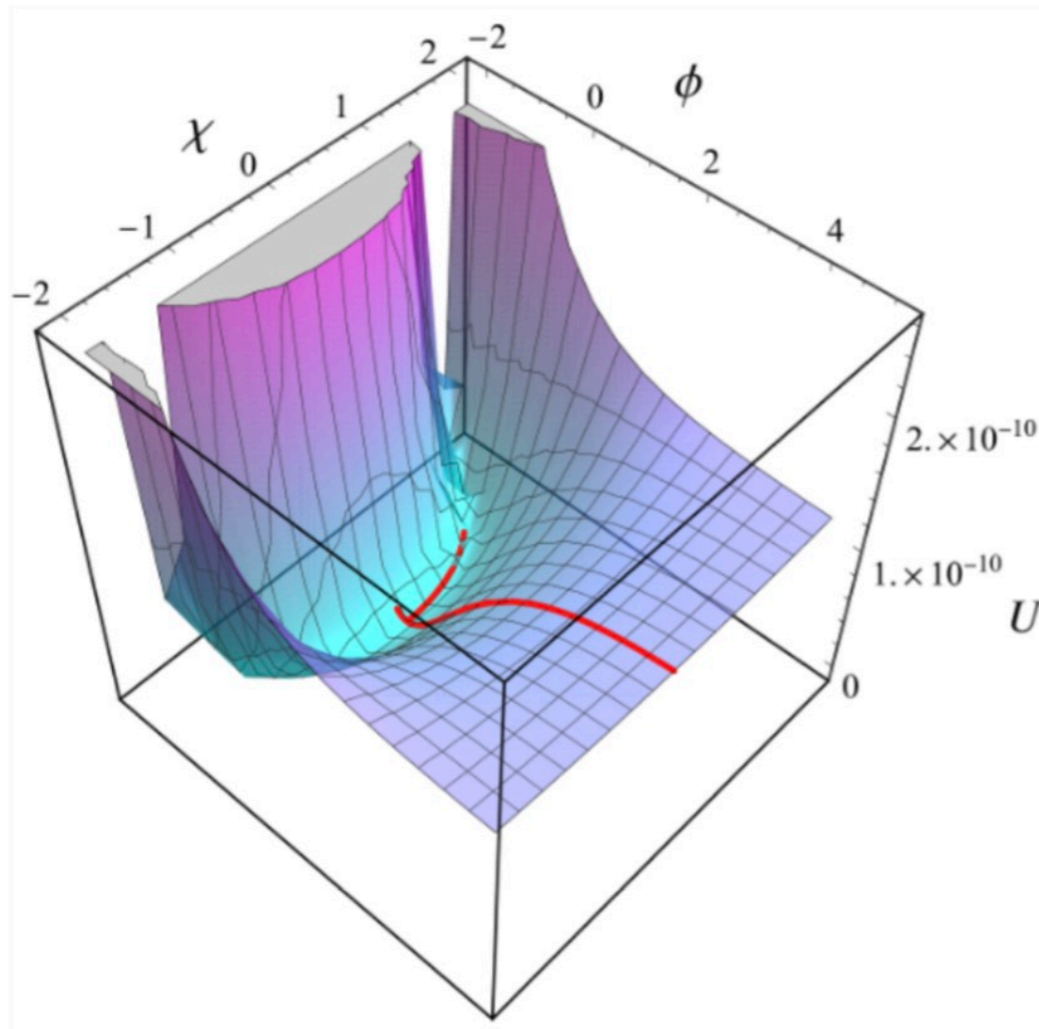
(2) Incomplete interpretation on enhancement mechanism

Avoid huge perturbations:

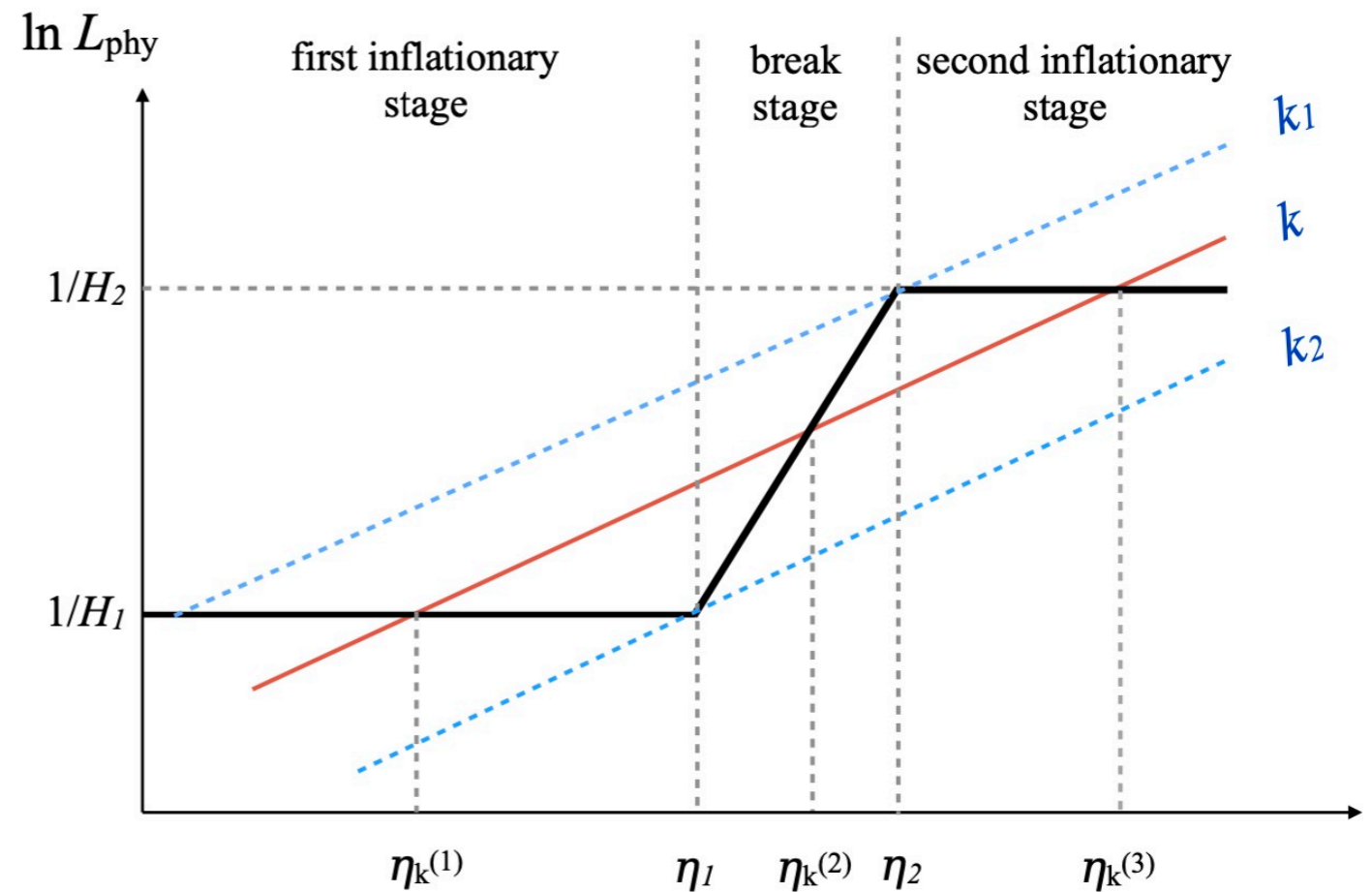
$$f(R) = R + \frac{R^2}{6M^2} - \frac{\xi R}{M_{\text{pl}}^2} (\chi - \chi_0)^2$$

Expand the potential to 4-th order:

$$V(\chi) \equiv V_0 - \frac{1}{2}m^2\chi^2 + \frac{1}{4}\lambda\chi^4$$



X. Wang, YZ, M. Sasaki, [2404.02492]



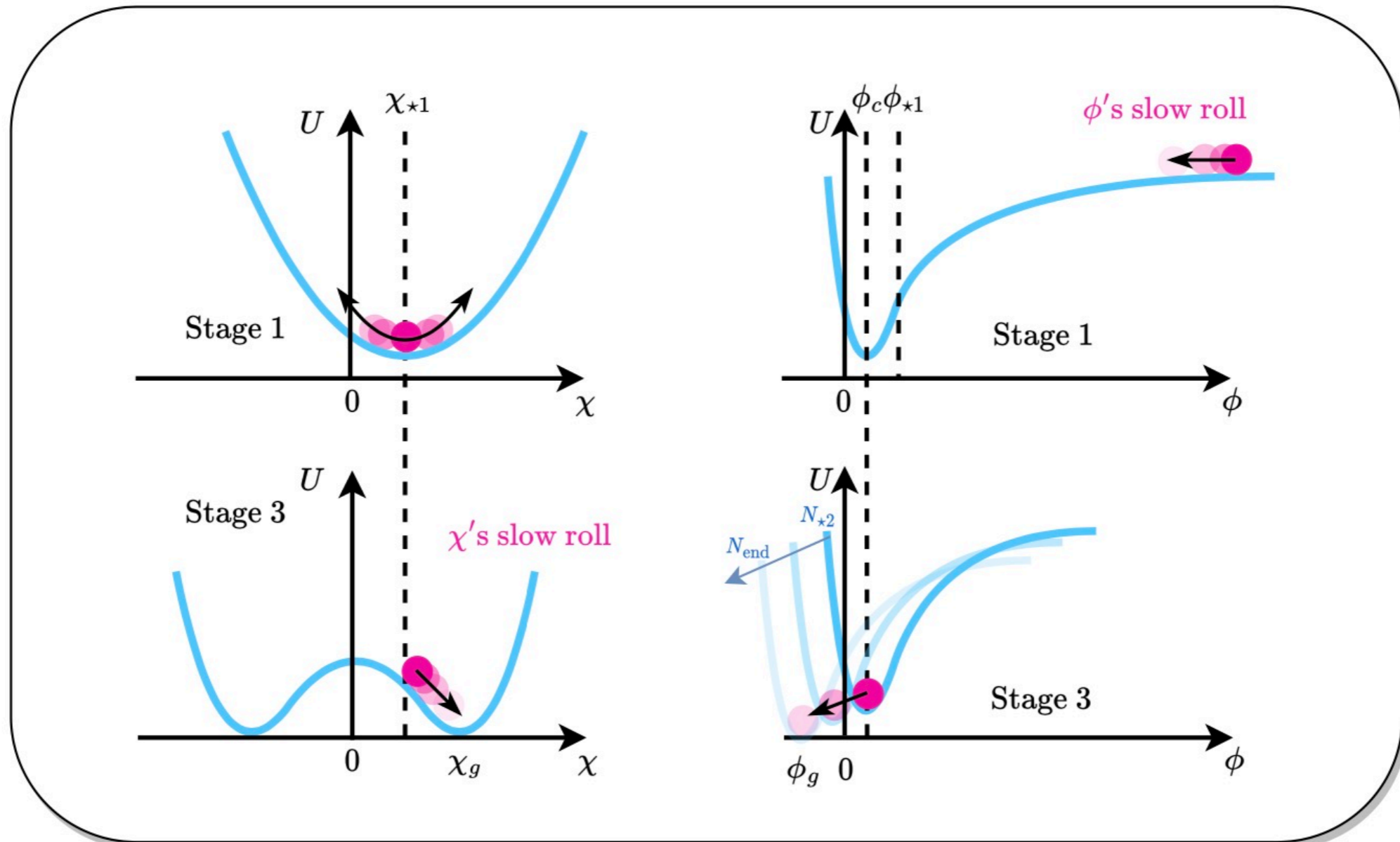
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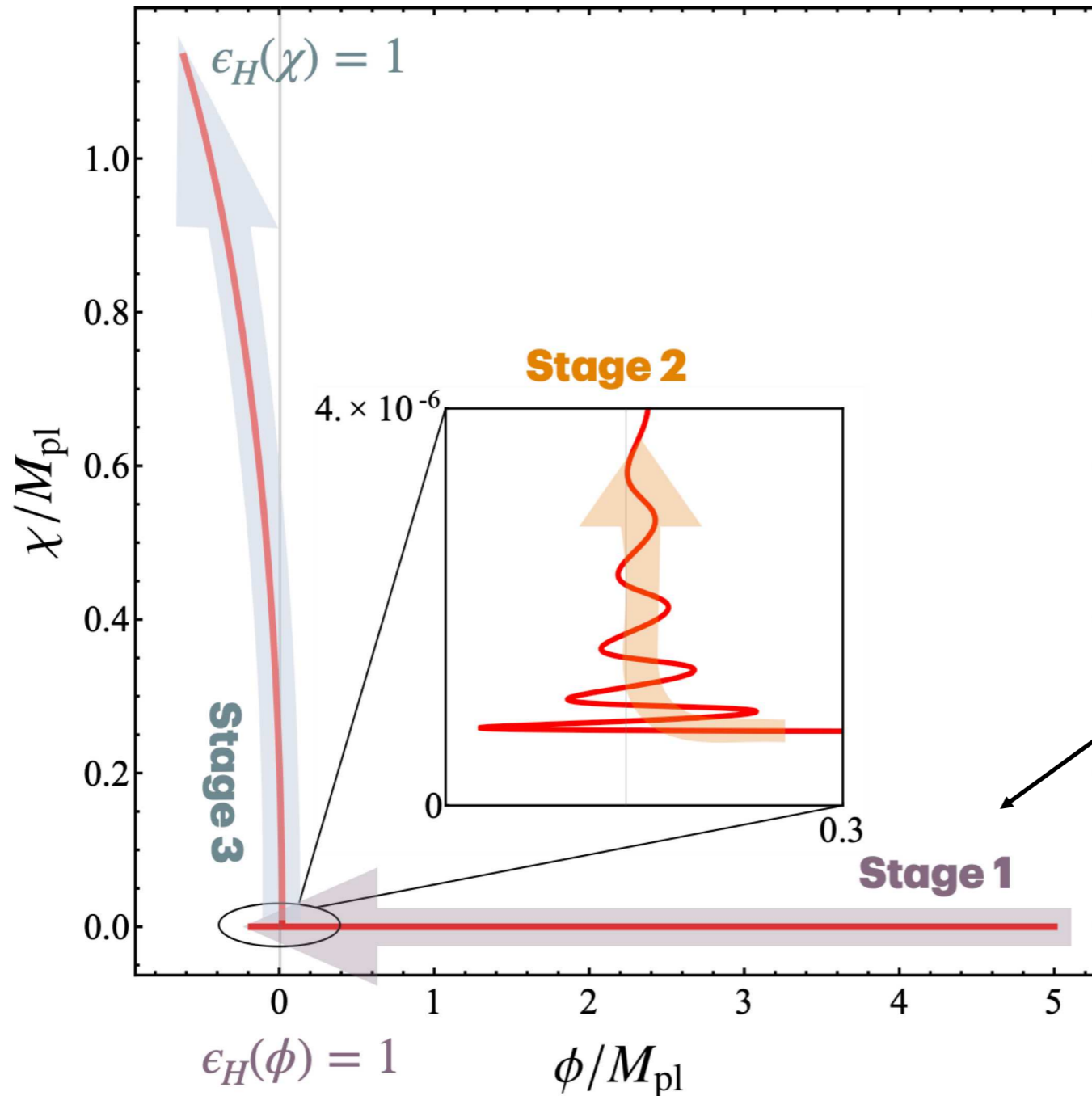


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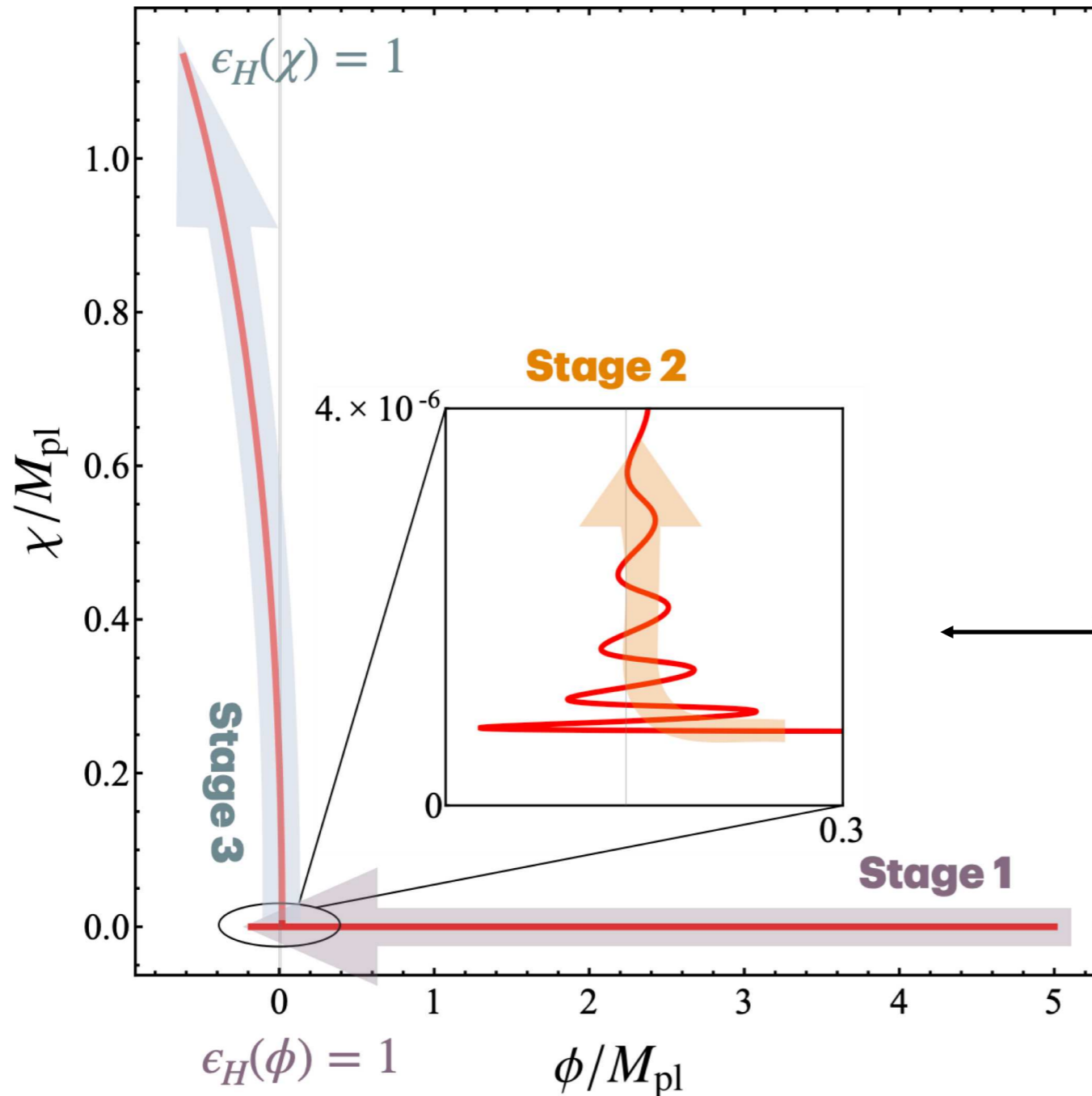
Inflaton rolls along ϕ direction (R^2 inflation), χ behaves like a damped oscillator around χ_0 .

Avoid huge perturbations:

$$f(R) = R + \frac{R^2}{6M^2} - \frac{\xi R}{M_{\text{pl}}^2} (\chi - \chi_0)^2$$

Expand the potential to 4-th order:

$$V(\chi) \equiv V_0 - \frac{1}{2}m^2\chi^2 + \frac{1}{4}\lambda\chi^4$$



The slow roll is shortly violated by inflaton's oscillation along ϕ direction.

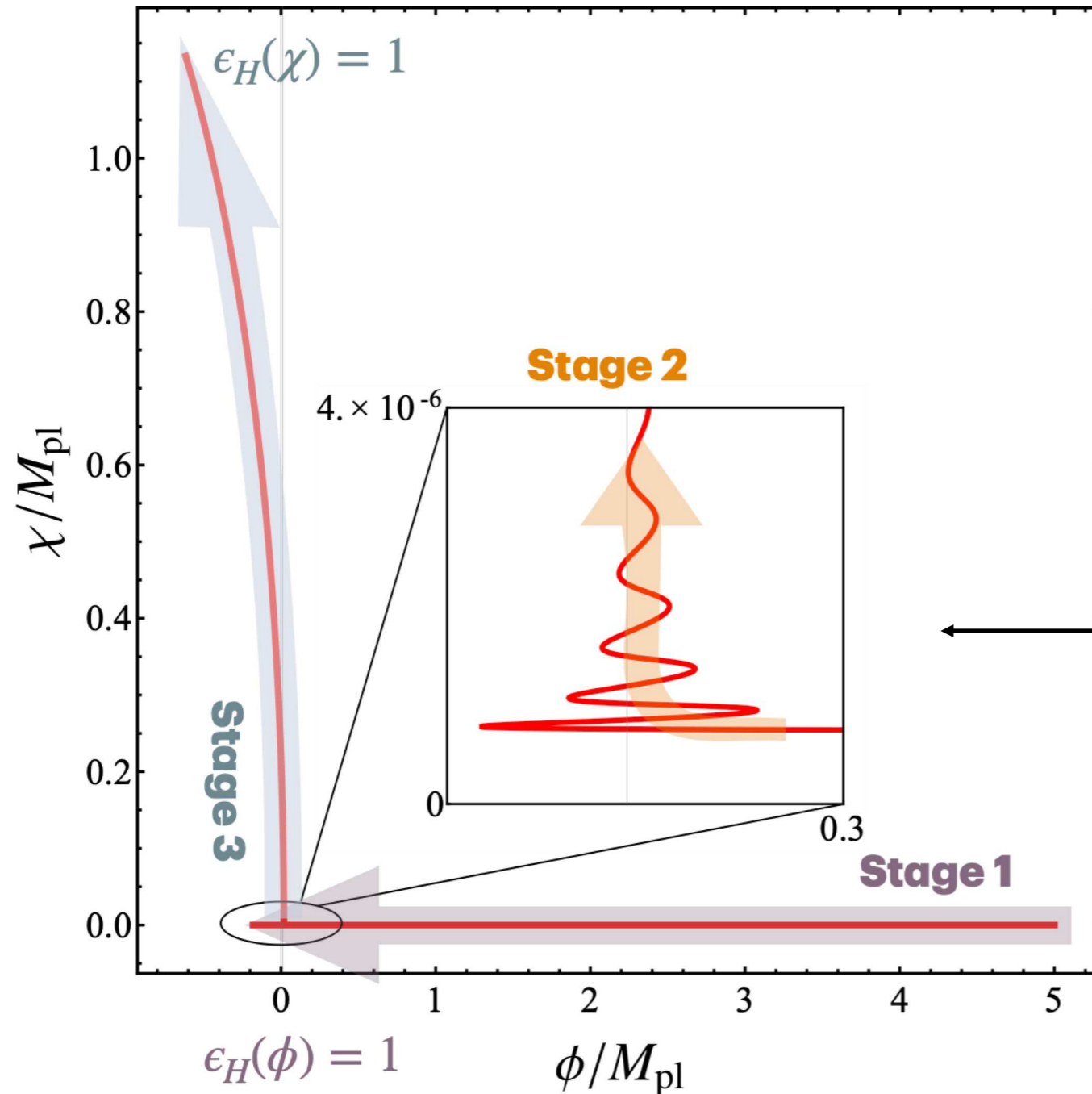
χ accelerates to attractor phase.

Avoid huge perturbations:

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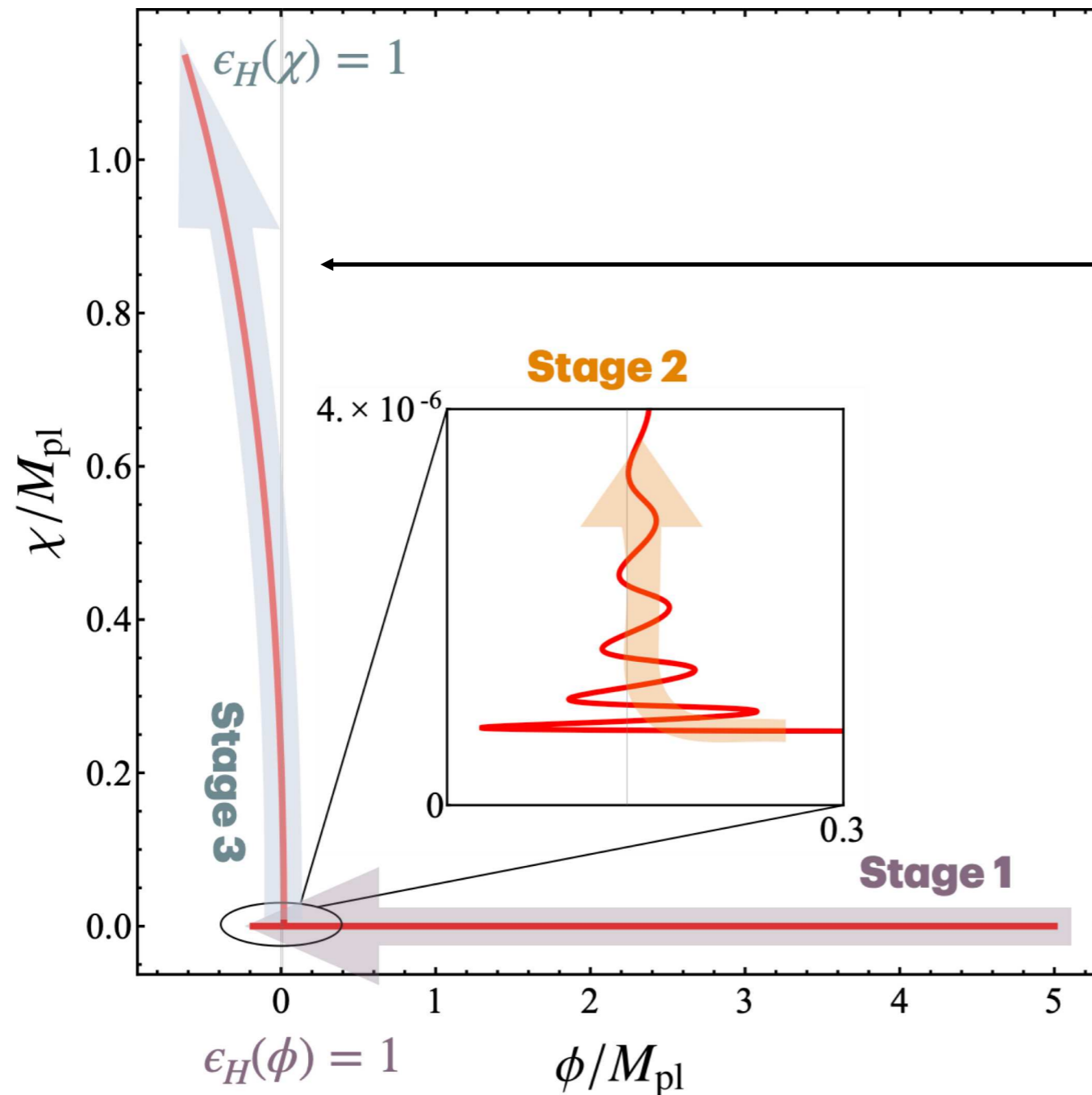
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Expand the potential to 4-th order:

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Inflaton rolls along χ direction,
 ϕ behaves like a under-damped oscillator around its potential valley. The inflation ends when $\epsilon_H(\chi) = 1$.

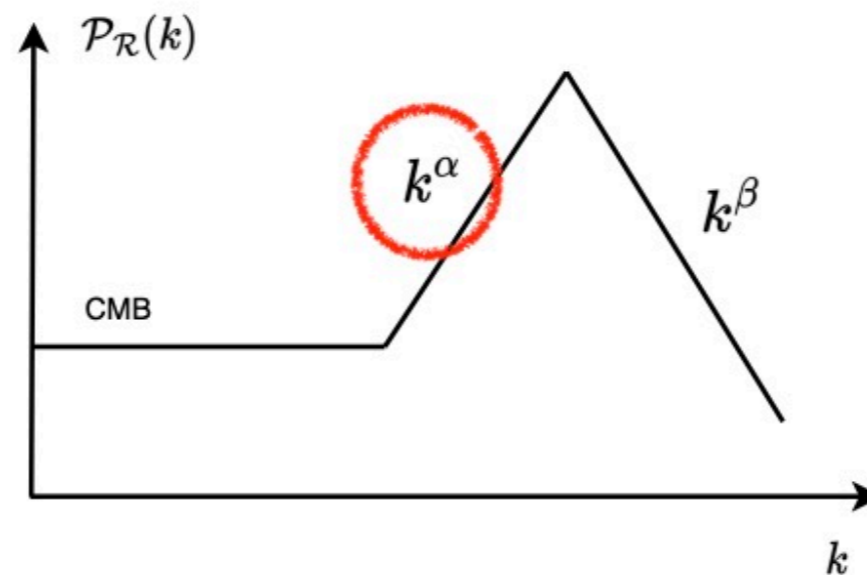
The Primordial Power Spectrum can be analytically approximated as

$$P_{\mathcal{R}}(k) \approx \frac{M^2}{16\pi^2} \times \begin{cases} \left[\frac{2}{3} \left(\ln \frac{k_1}{k} + \frac{3}{4} F_{\star} \right)^2 + \left(\frac{g_1 h}{\chi_0} \right)^2 \left(\frac{k}{k_1} \right)^{\alpha} \right] \left[1 + \left(\ln \frac{k_1}{k} + \frac{3}{4} F_{\star} \right)^{-1} \right], & \text{for } k < k_1 \\ \frac{g_2^2 h^2}{\chi_0^2 \mu^2} \left(\frac{k}{k_1} \right)^{\beta}, & \text{for } k \geq k_1 \end{cases}$$

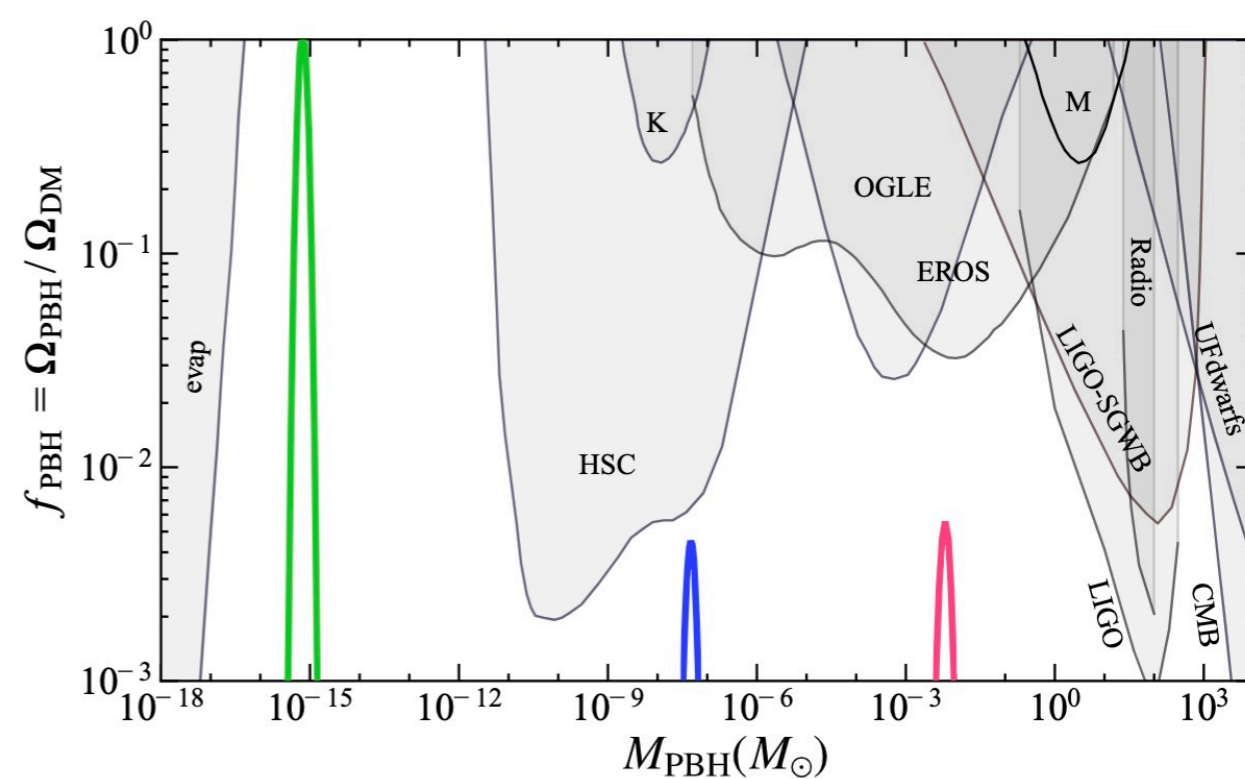
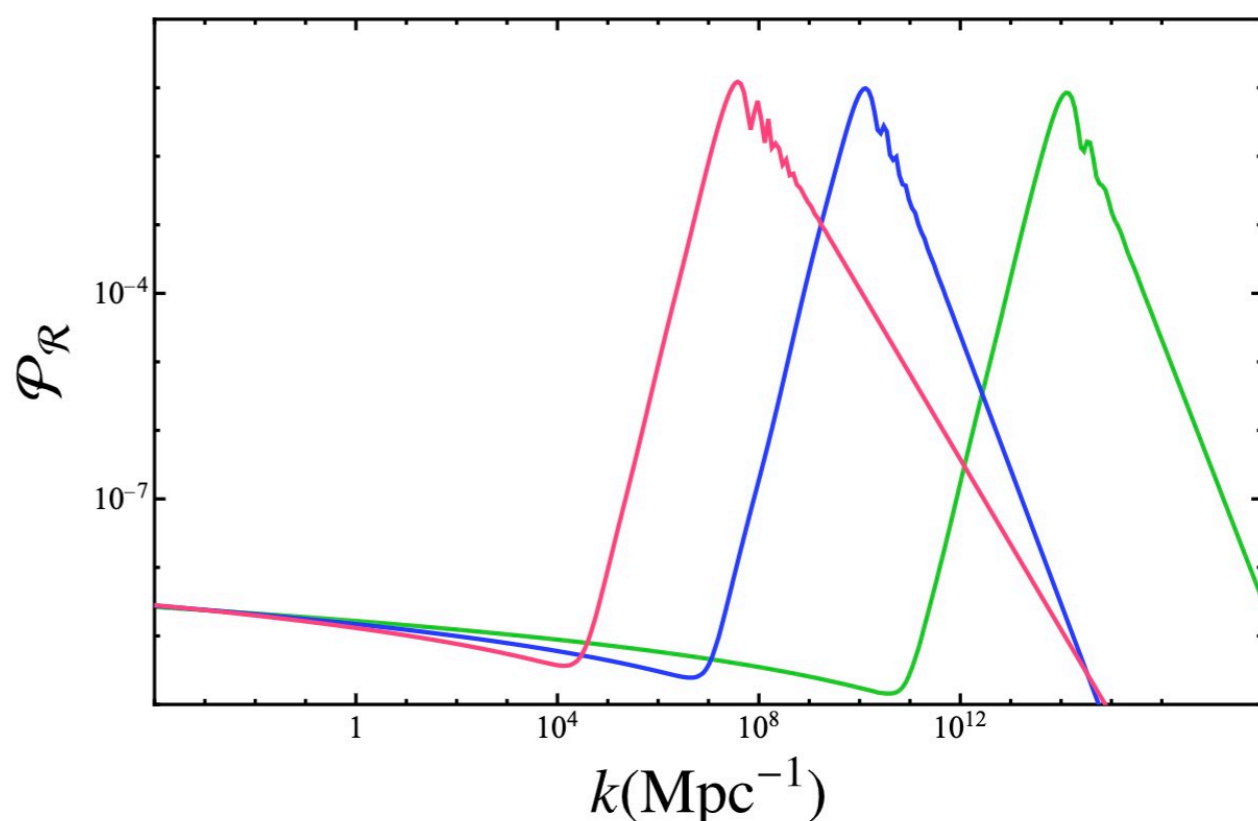
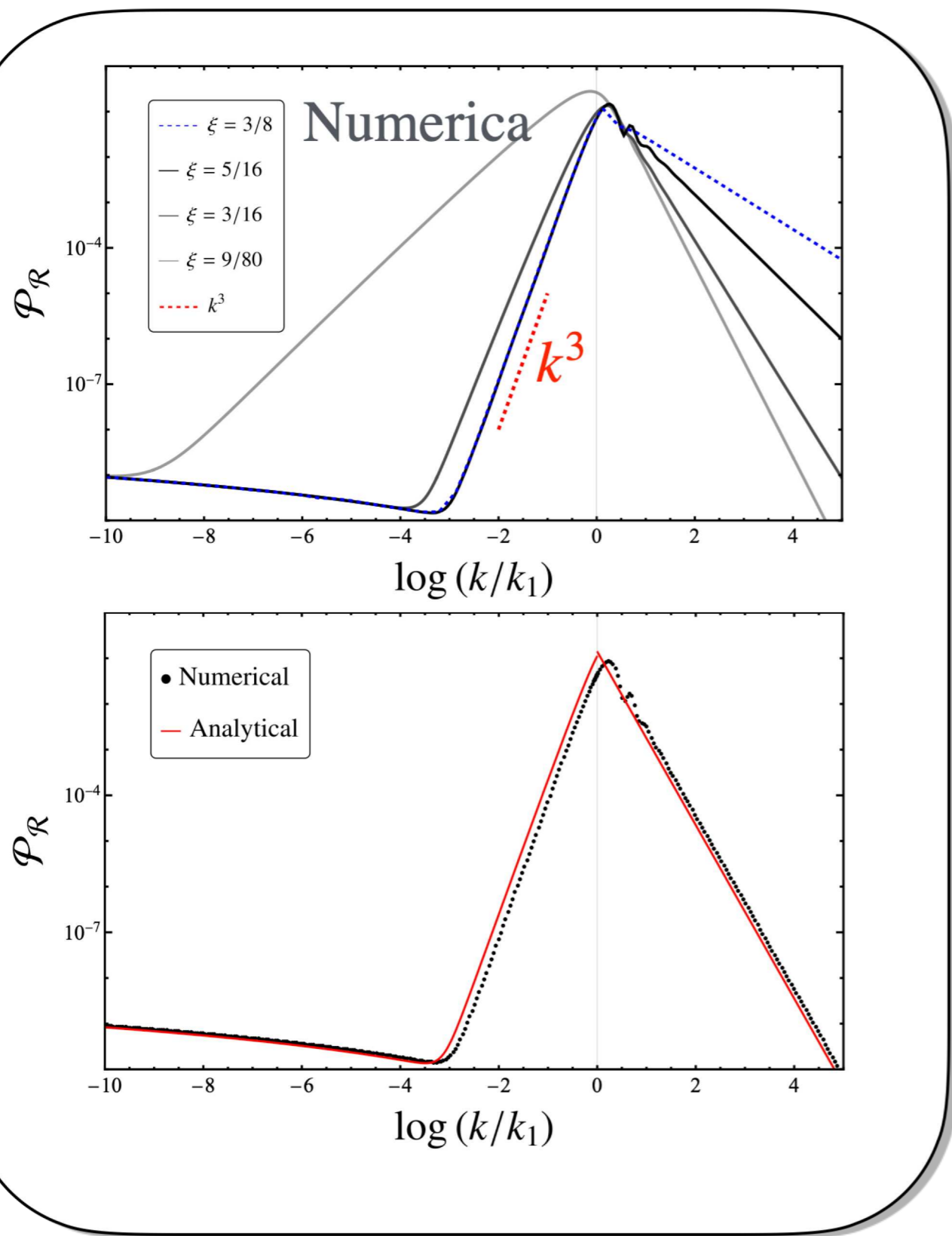
Broken power-law!

$$\alpha \equiv \text{Re} \left(3 - 3 \sqrt{1 - \frac{16}{3} \xi} \right) \leq 3$$

$$\beta \equiv 3 - \sqrt{3 + \frac{48\xi(A-1)}{1 + 1/\mu^2}},$$



**α is determined by
the behavior of χ
during late 1st stage**

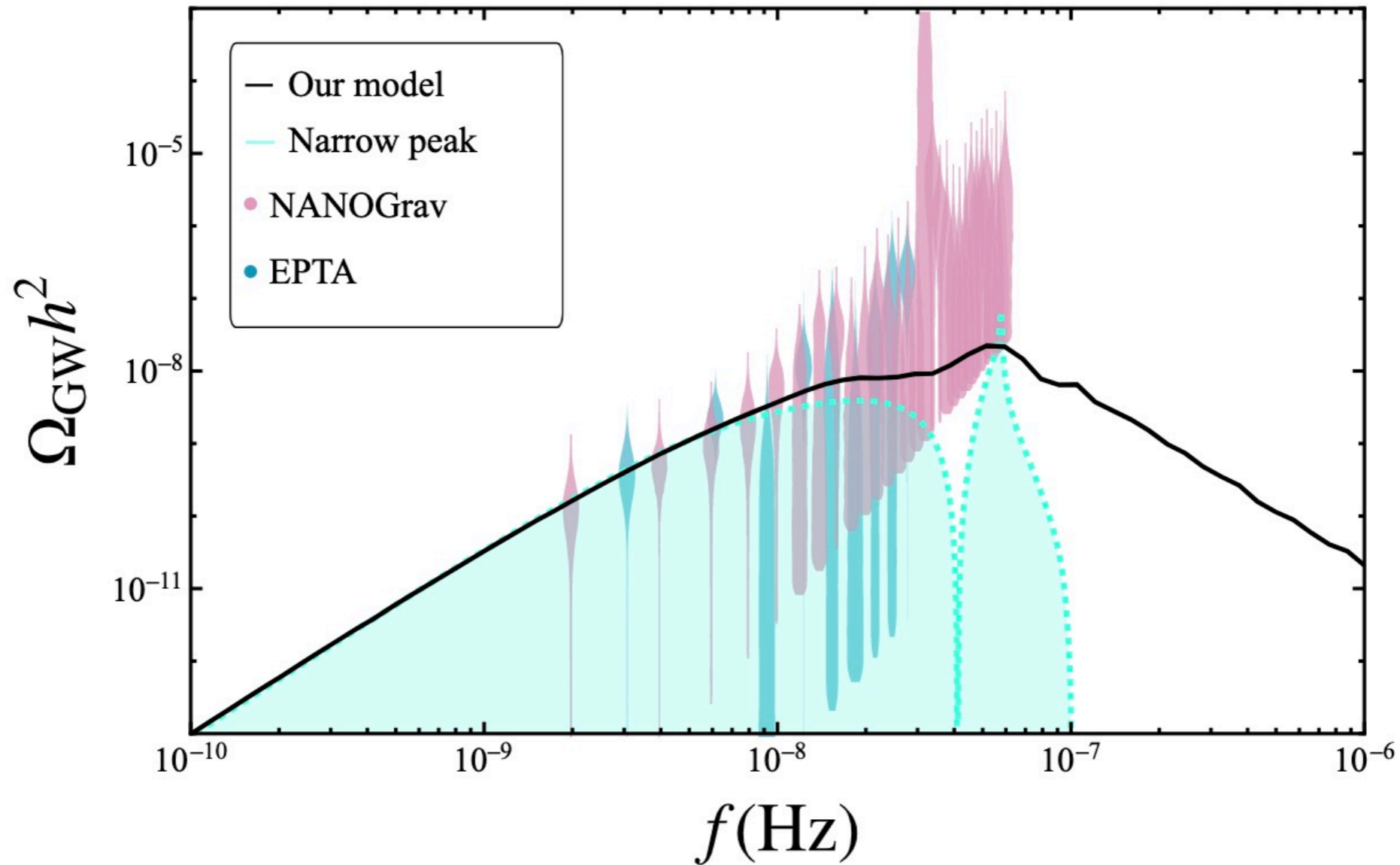


Induced Gravitational Wave (IGW)

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\hat{T}_{ij}{}^{lm} \mathcal{S}_{lm}$$

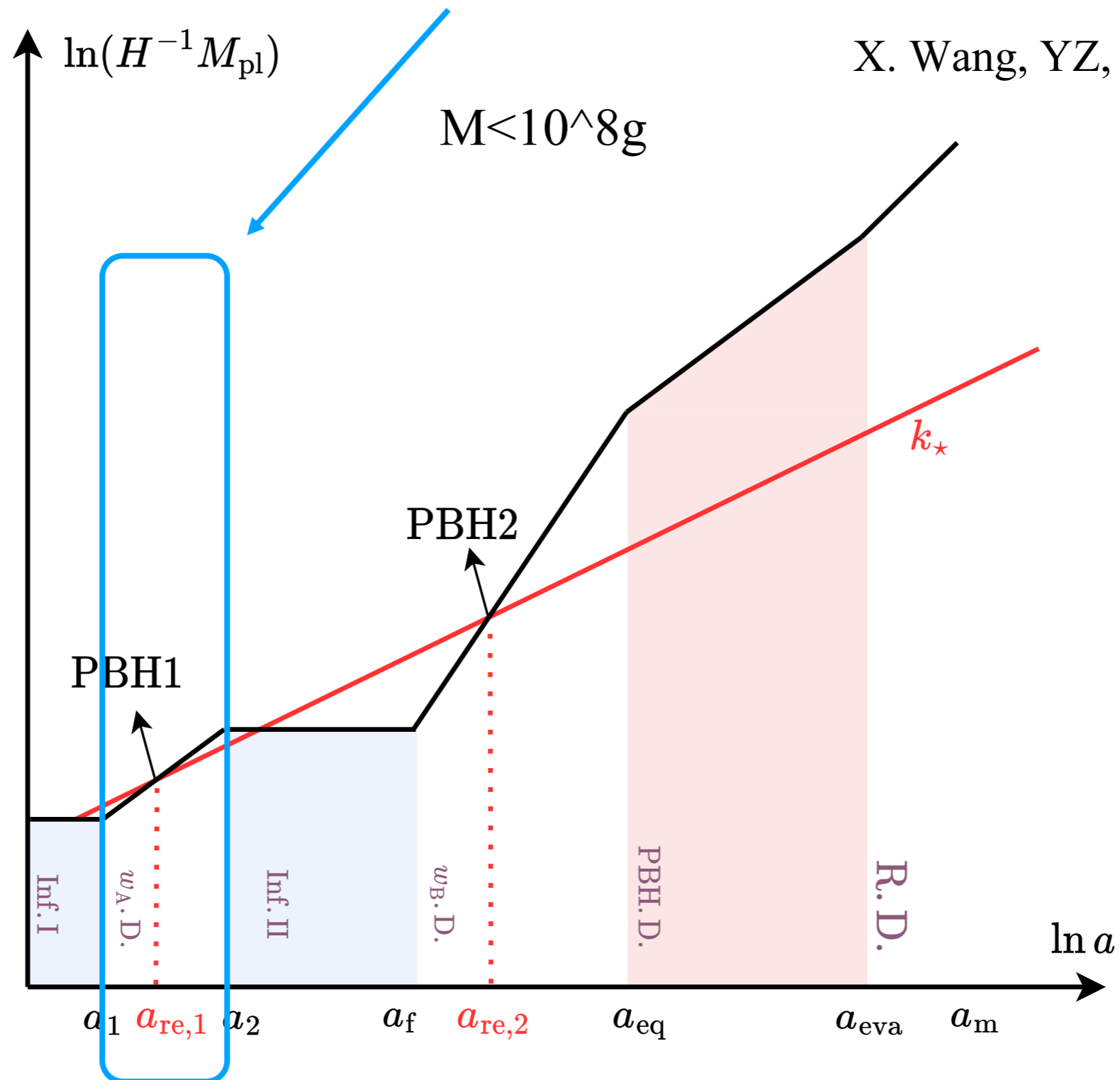
$$\mathcal{S}_{ij} = 4\Phi\Phi_{|ij} + 2\Phi_{|i}\Phi_{|j} - \frac{3}{\kappa^2 a^2 \rho} \left[\mathcal{H}^2 \Phi_{|i}\Phi_{|j} + 2\mathcal{H}\Phi_{|i}\Phi'_{|j} + \Phi'_{|i}\Phi'_{|j} \right]$$

K. Ananda,
C. Clarkson,
D. Wands,
Phys.Rev.D 75
(2007) 123518



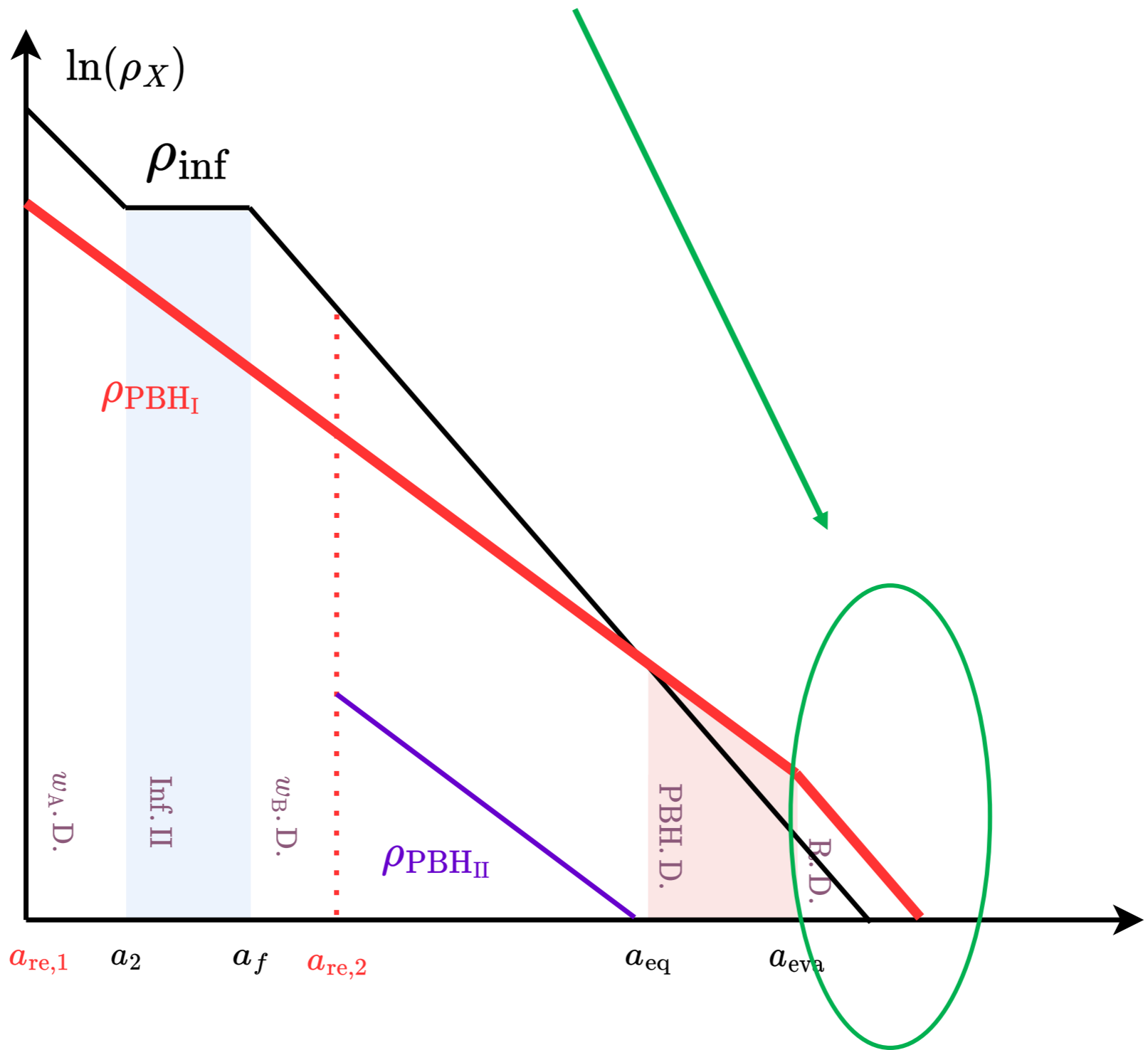
Data from: NANOGrav, 2023 (2306.16219), EPTA, 2023 [2306.16227]

An interesting scenario: **Light** PBH formation during **break stage**



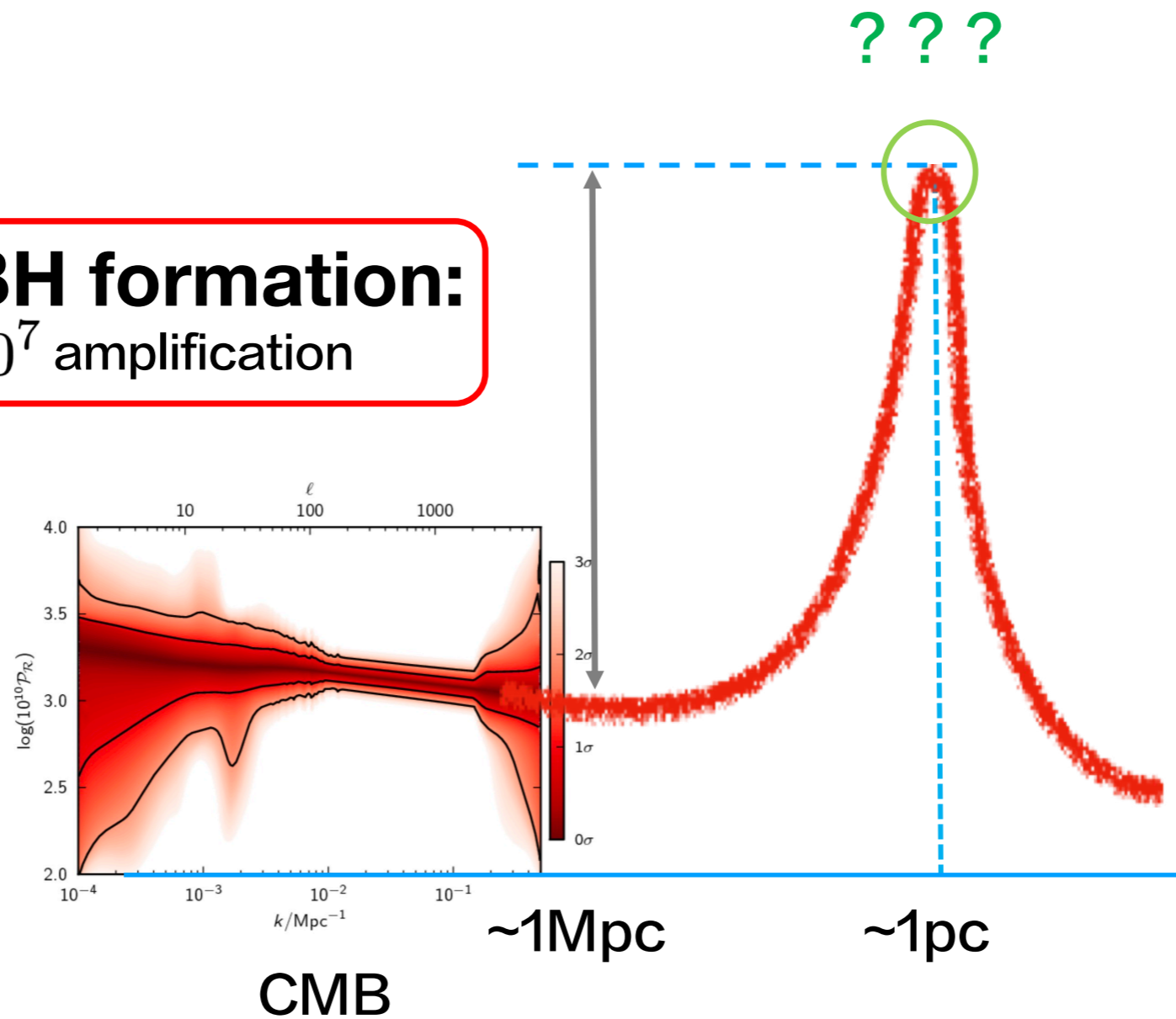
X. Wang, YZ, M. Sasaki, in prep

Radiation-dominated Era from Evaporation of PBHs



Question: Any hints on such huge amplification?

PBH formation:
 10^7 amplification



Idea

PBHs from large curvature perturbations

+ LIGO data from mergers of PBH binaries



Primordial Power Spectrum of
curvature perturbations?
(on small scales)

Assumptions

0. At least some of the BBH LIGO events are PBHs

merger rate from observations

1. PBHs formed out of high peaks of curvature perturbations

the simplest case

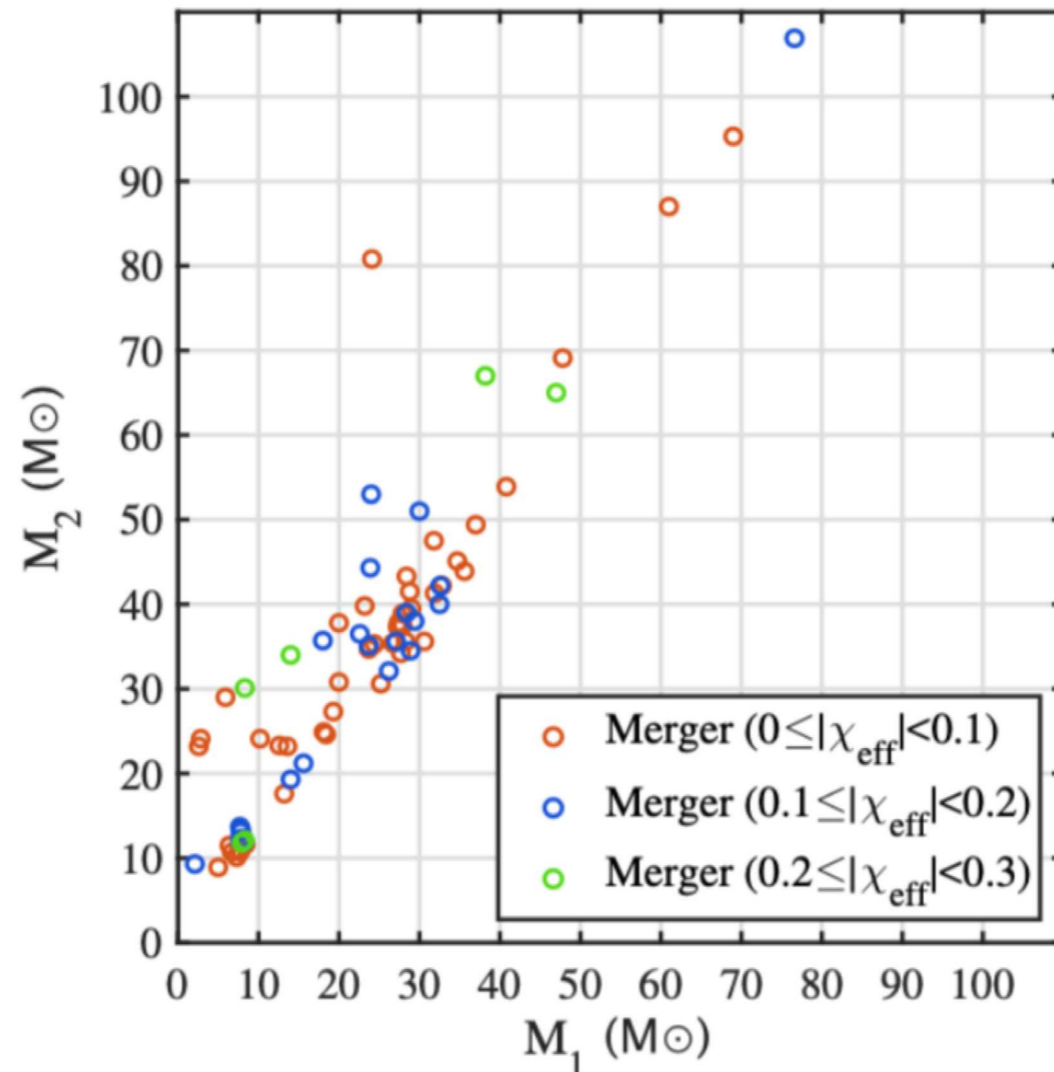
2. Window function: top-hat form

semi-analytic expression for calculation of merger rate

3. Gaussian distribution of density perturbation

simple relation between between power spectrum and the variance

The LIGO/Virgo released data $\mathcal{R}(m_i, m_j) = \mathcal{R}_0 N(m_i, m_j)$, $\mathcal{R}_0 \equiv \frac{R_{\text{total}}}{N_{\text{total}}}$



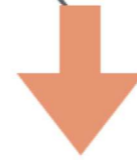
91 BBH candidates, 76 of them satisfy $m \gtrsim 1.5M_{\odot}$ and $|\chi_{\text{eff}}| \lesssim 0.3$

We simply identify these events as PBH mergers

WARN: The largest Uncertainty comes from here!

Merger rate is given by

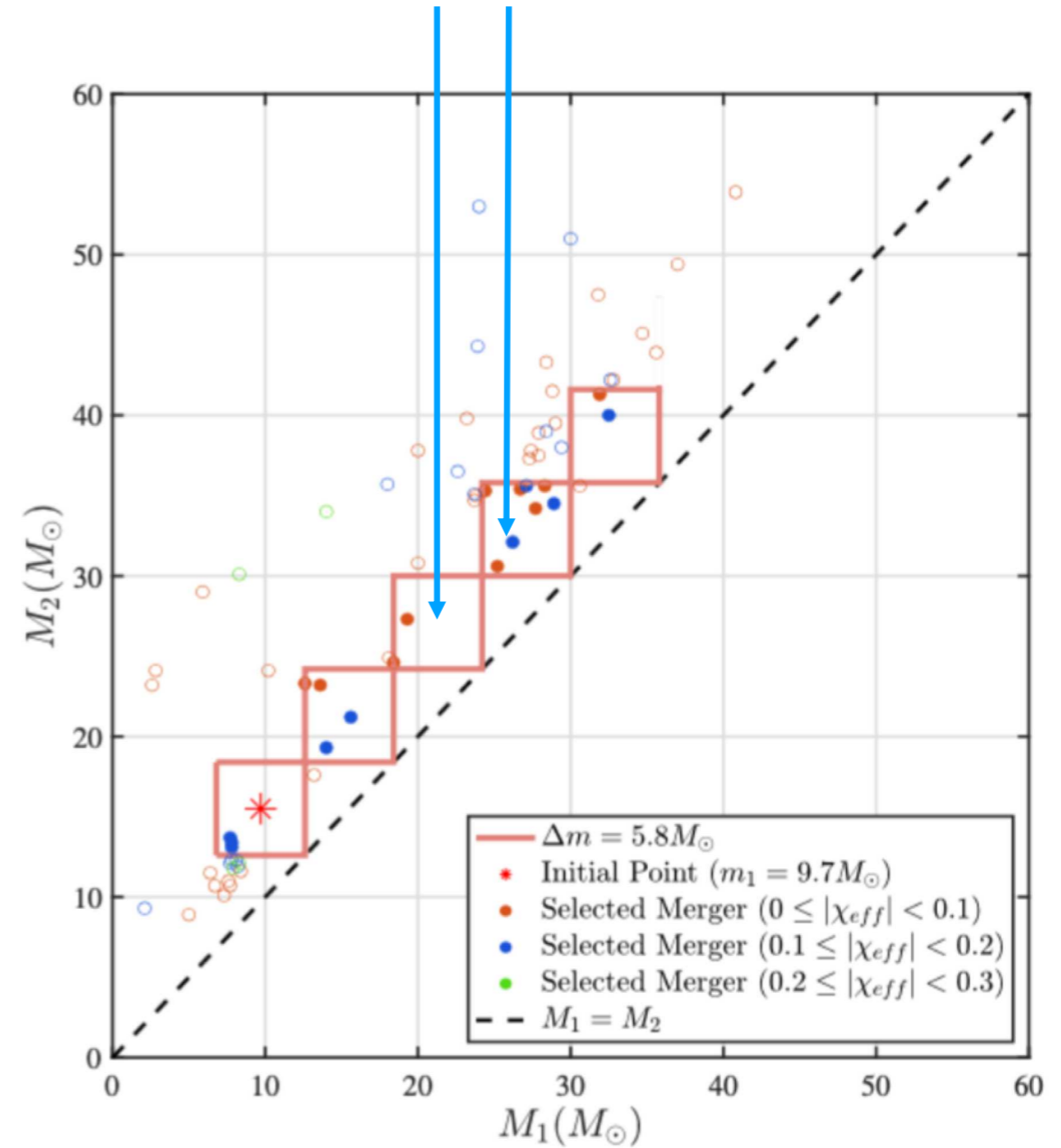
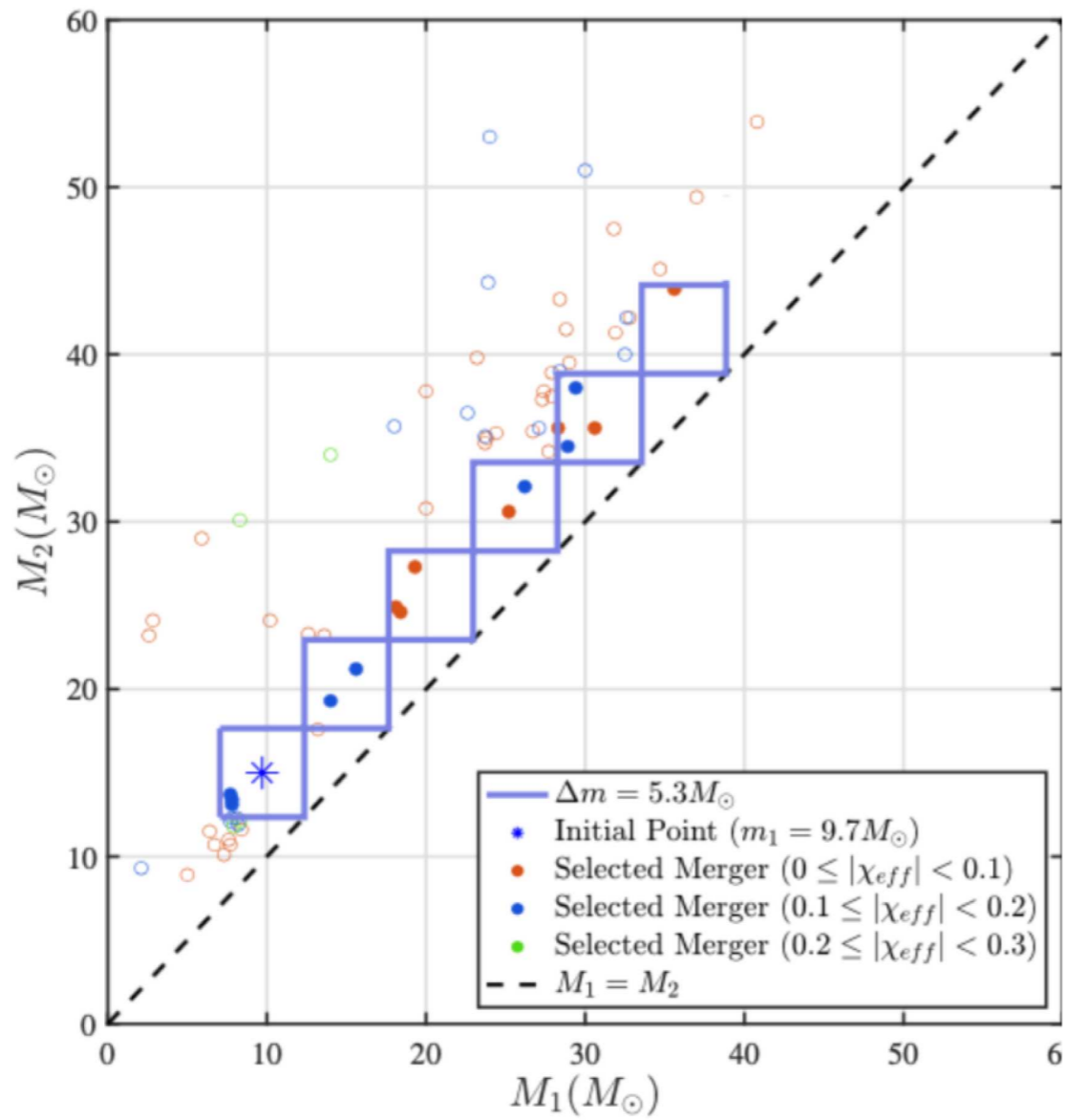
$$\mathcal{R}(m_1, m_2, t) = \frac{1.6 \times 10^6}{\text{Gpc}^3 \text{yr}} S \left[\cancel{f} \mid \cancel{f_{\text{PBH}}}, \cancel{m_t} \right] f_{\text{PBH}}^{\frac{53}{37}} \left(\frac{t}{t_0} \right)^{-\frac{34}{37}} \left(\frac{m_1 + m_2}{M_\odot} \right)^{-\frac{32}{37}} \left(\frac{(m_1 + m_2)^2}{m_1 m_2} \right)^{\frac{34}{37}} m_1 m_2 f(m_1) f(m_2)$$



$$\left\{ \begin{array}{l} \frac{f(m_1)}{f(m_3)} = \frac{f(m_1)f(m_2)}{f(m_2)f(m_3)} = \frac{\mathcal{R}(m_1, m_2, t)}{\mathcal{R}(m_2, m_3, t)} \left(\frac{m_2 + m_3}{m_1 + m_2} \right)^{36/37} \left(\frac{m_3}{m_1} \right)^{3/37} \\ \frac{f(m_1)}{f(m_2)} = \frac{f(m_1)f(m_1)}{f(m_1)f(m_2)} = \frac{\mathcal{R}(m_1, m_1, t)}{\mathcal{R}(m_1, m_2, t)} \left(\frac{m_1 + m_2}{m_1 + m_1} \right)^{36/37} \left(\frac{m_2}{m_1} \right)^{3/37} \end{array} \right.$$

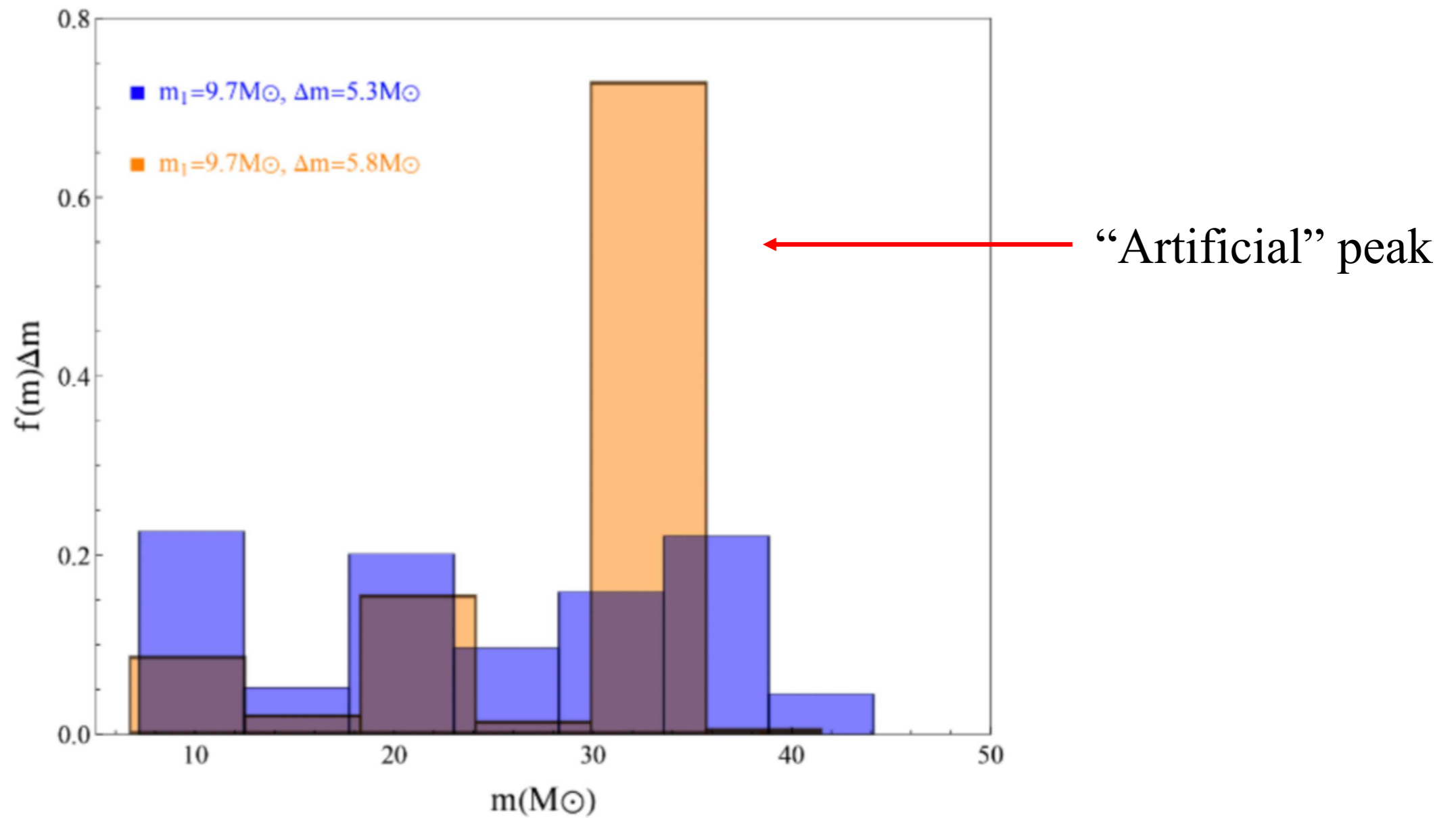
If $f(m_1)$ is fitted, to obtain $f(m_n)$ ($n = 2, 3, 4, \dots$),
we need to get $\mathcal{R}(m_1, m_1, t)$, $\mathcal{R}(m_n, m_{n+1}, t)$ ($n = 1, 2, 3, \dots$)

Large difference of # of points included



Two typical ways of division

Then it is straightforward to obtain the **mass function**



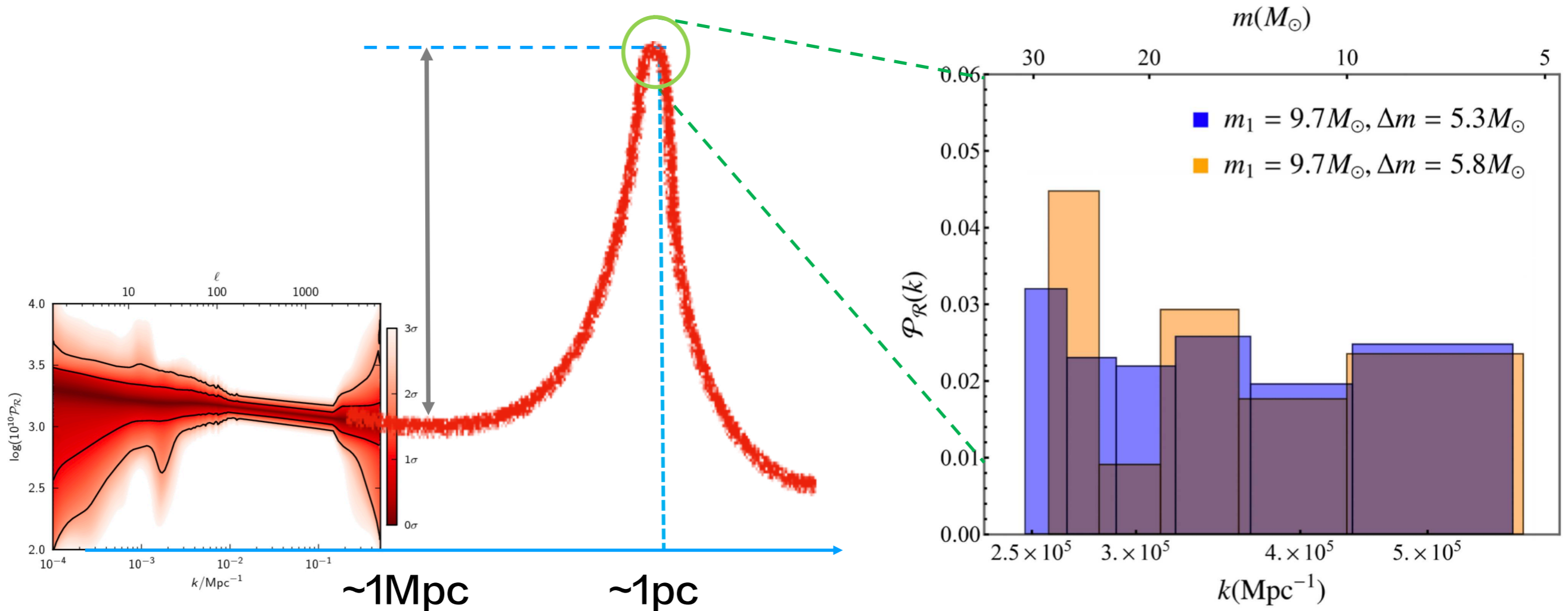


Common feature:

Amplitude of order **0.01**

Consistent with PBH scenario

10^7 amplification !



CMB

$\sim 1 \text{Mpc}$

$\sim 1 \text{pc}$

R. Kimura, T. Suyama, M. Yamaguchi and YZ,
JCAP 04 (2021) 031 [arXiv: 2102.05280]

X. Wang, YZ, Kimura and M. Yamaguchi,
SCPMA 6 (2023) 66 [arXiv: 2209.12911]

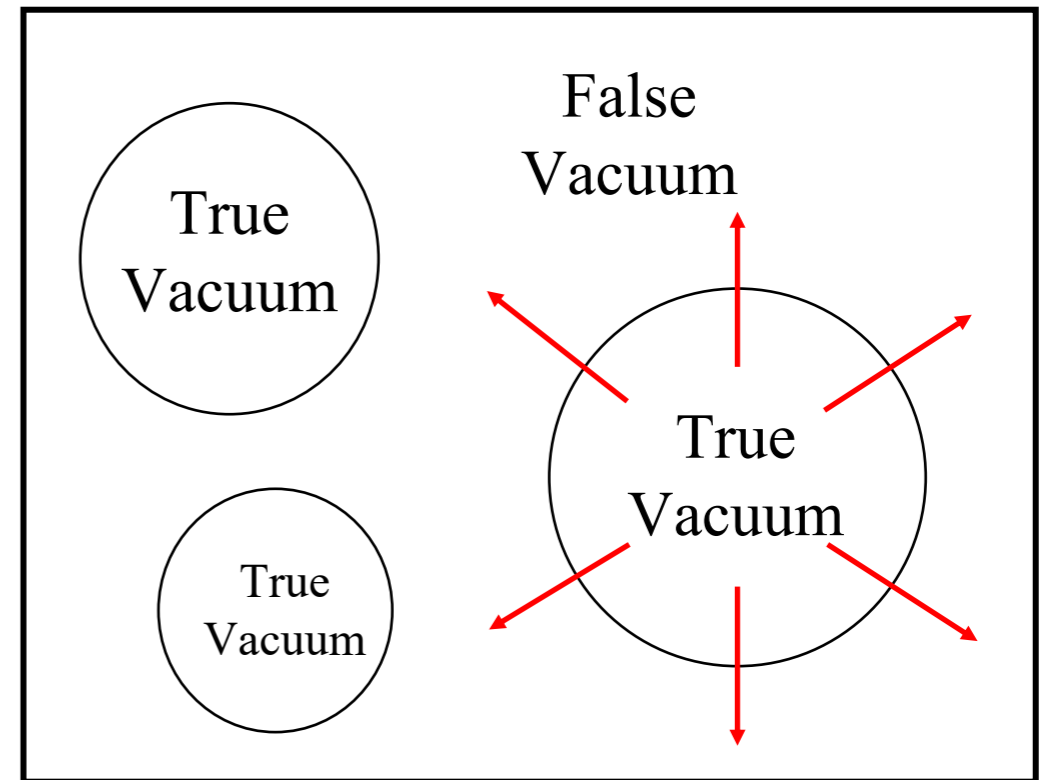
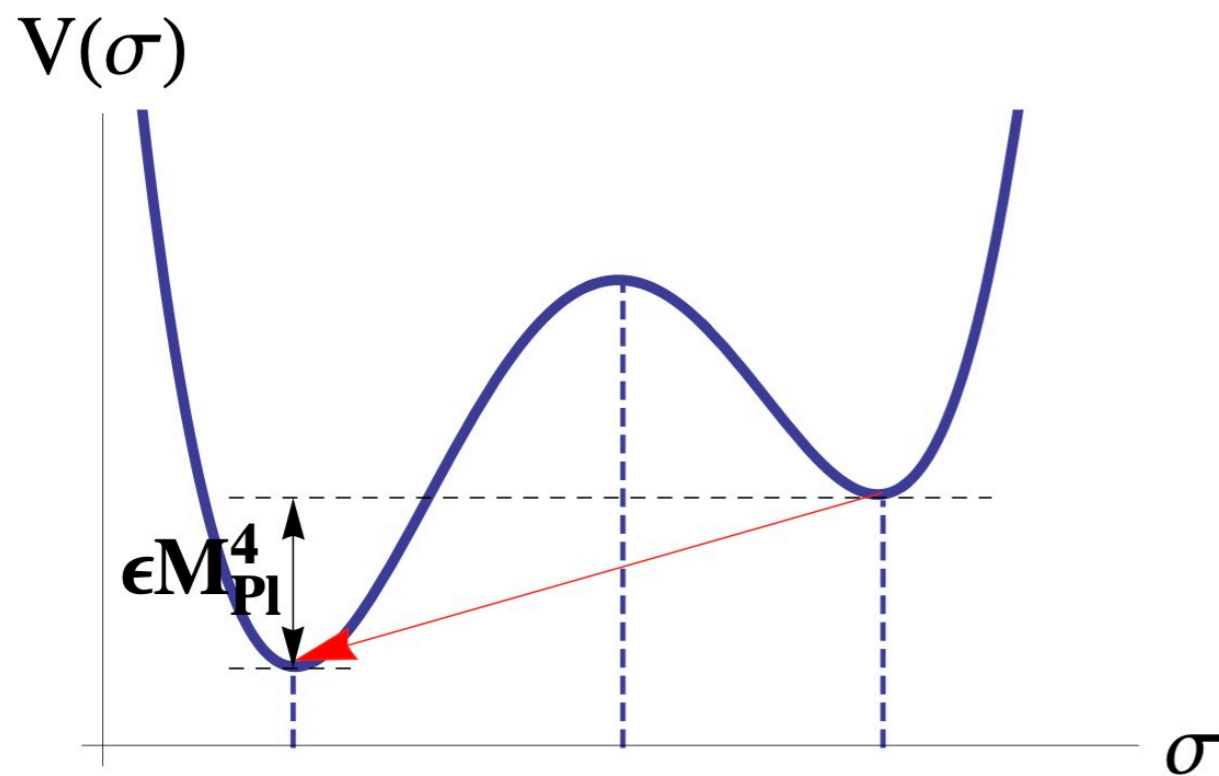
Question: If **no IGW observed, can we **rule out** PBH scenario?**



Question: Is there any PBH scenario **without amplification
of primordial power spectrum?**

Another mechanism: The Bubble Nucleation

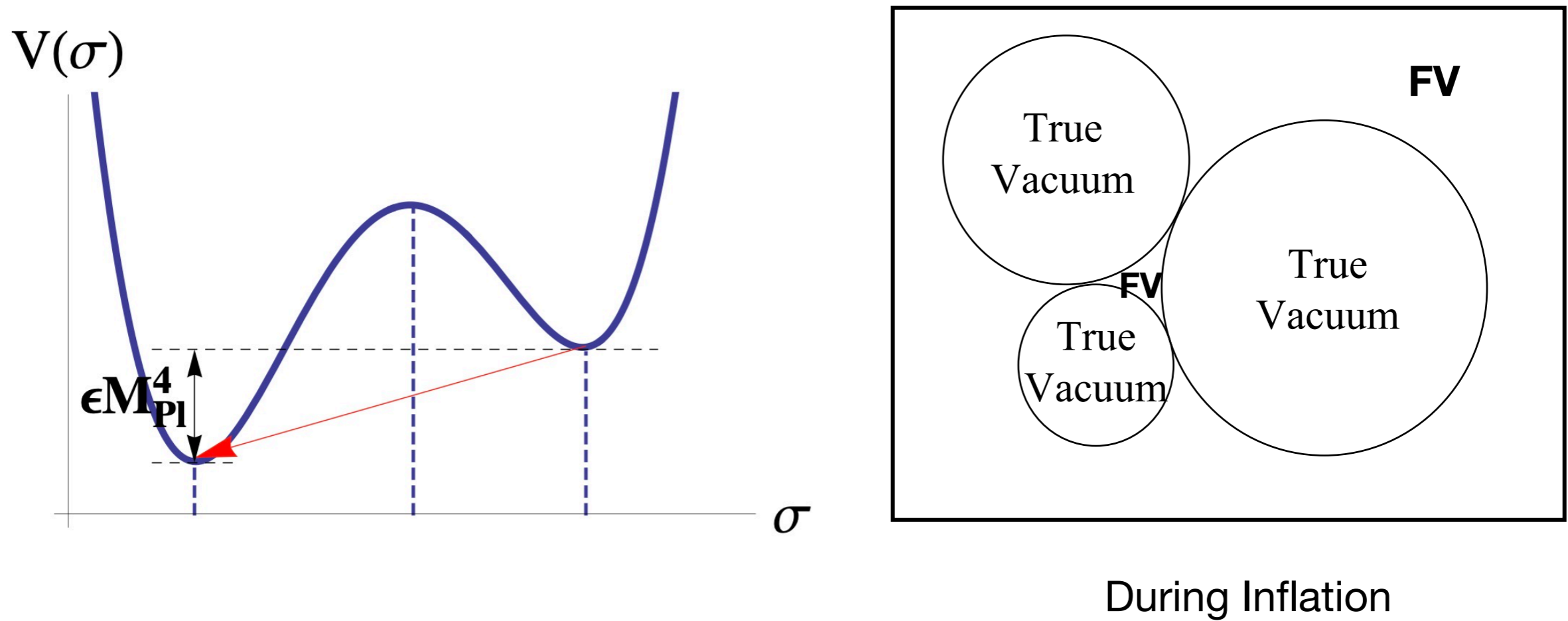
Bubble will be nucleated via tunneling process in early universe



During Inflation

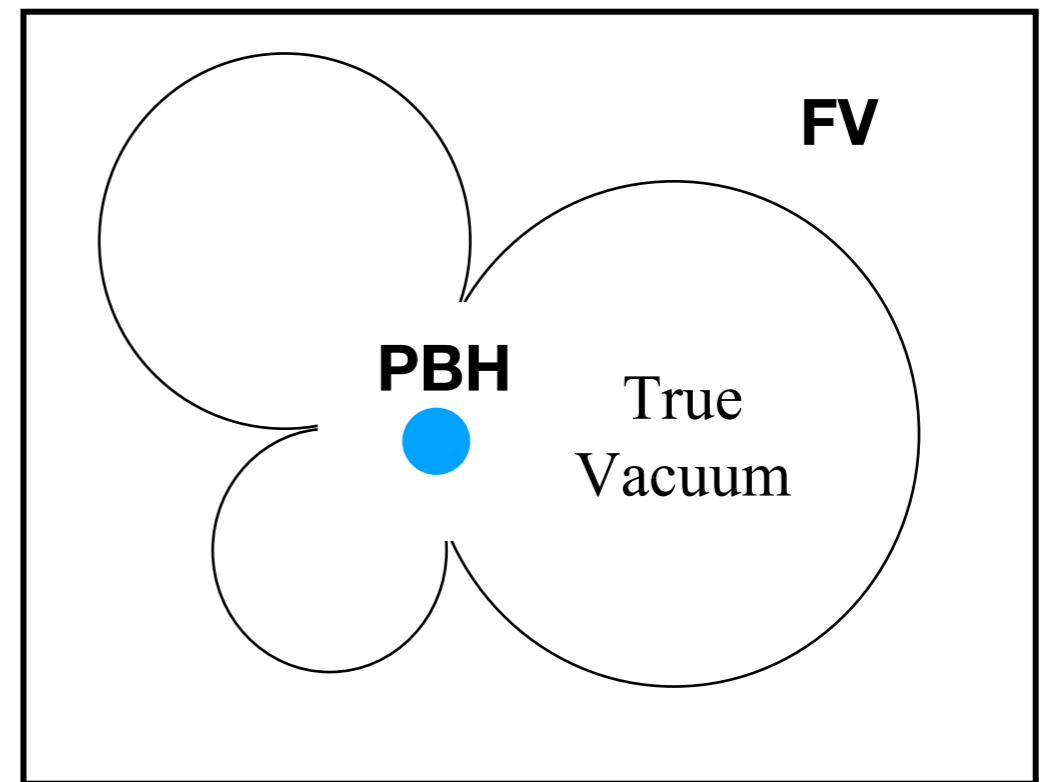
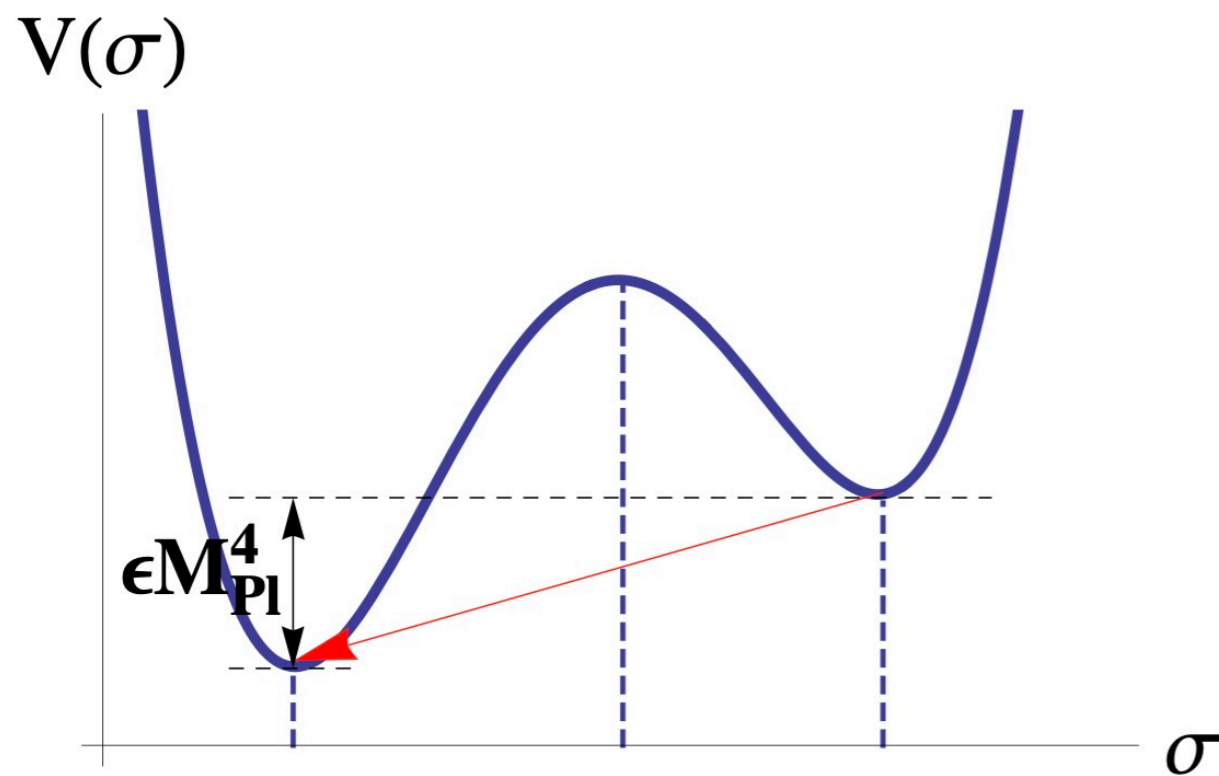
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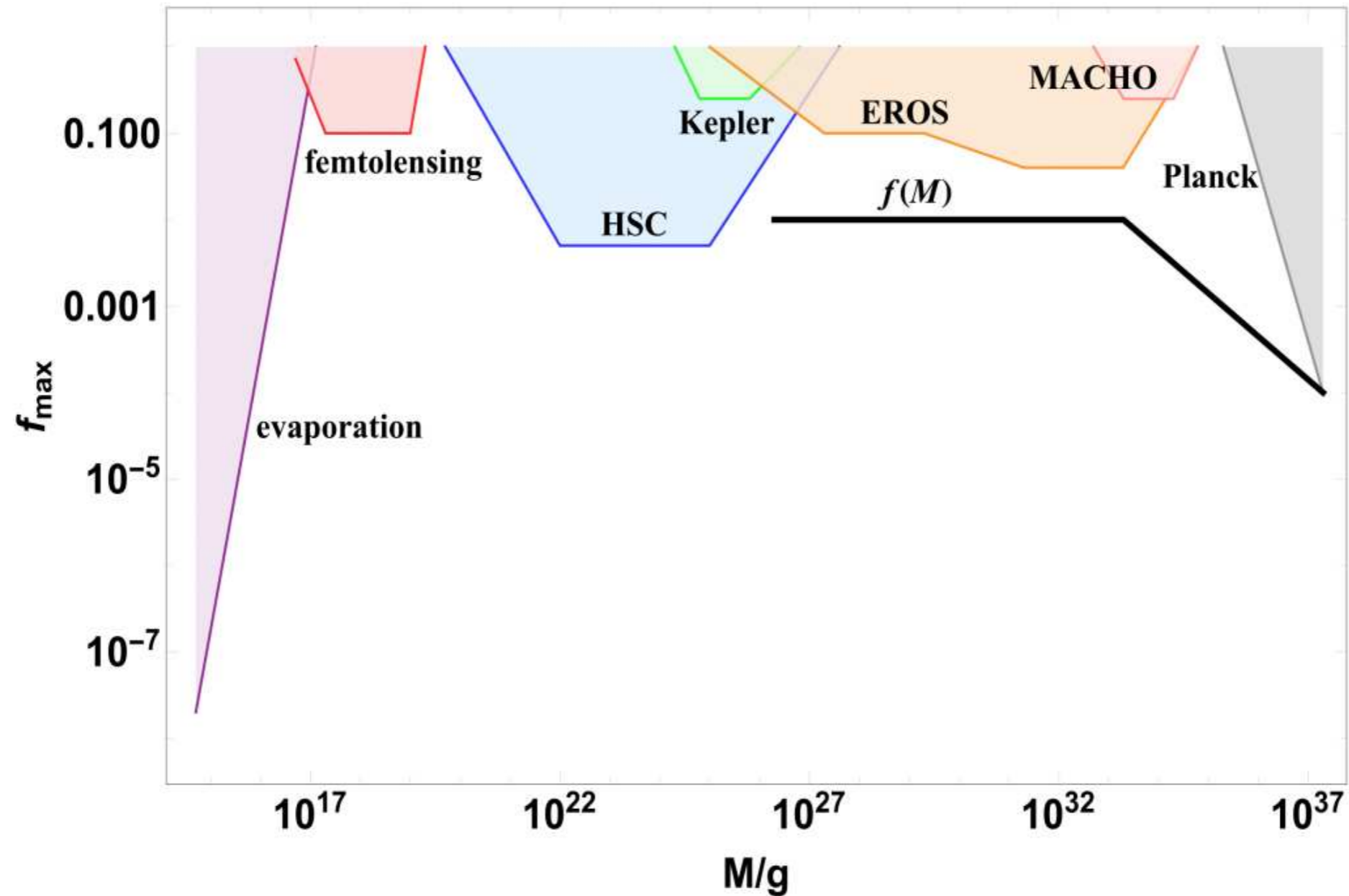
Bubble will be nucleated via tunneling process in early universe



During Inflation

Case of a constant tunneling rate

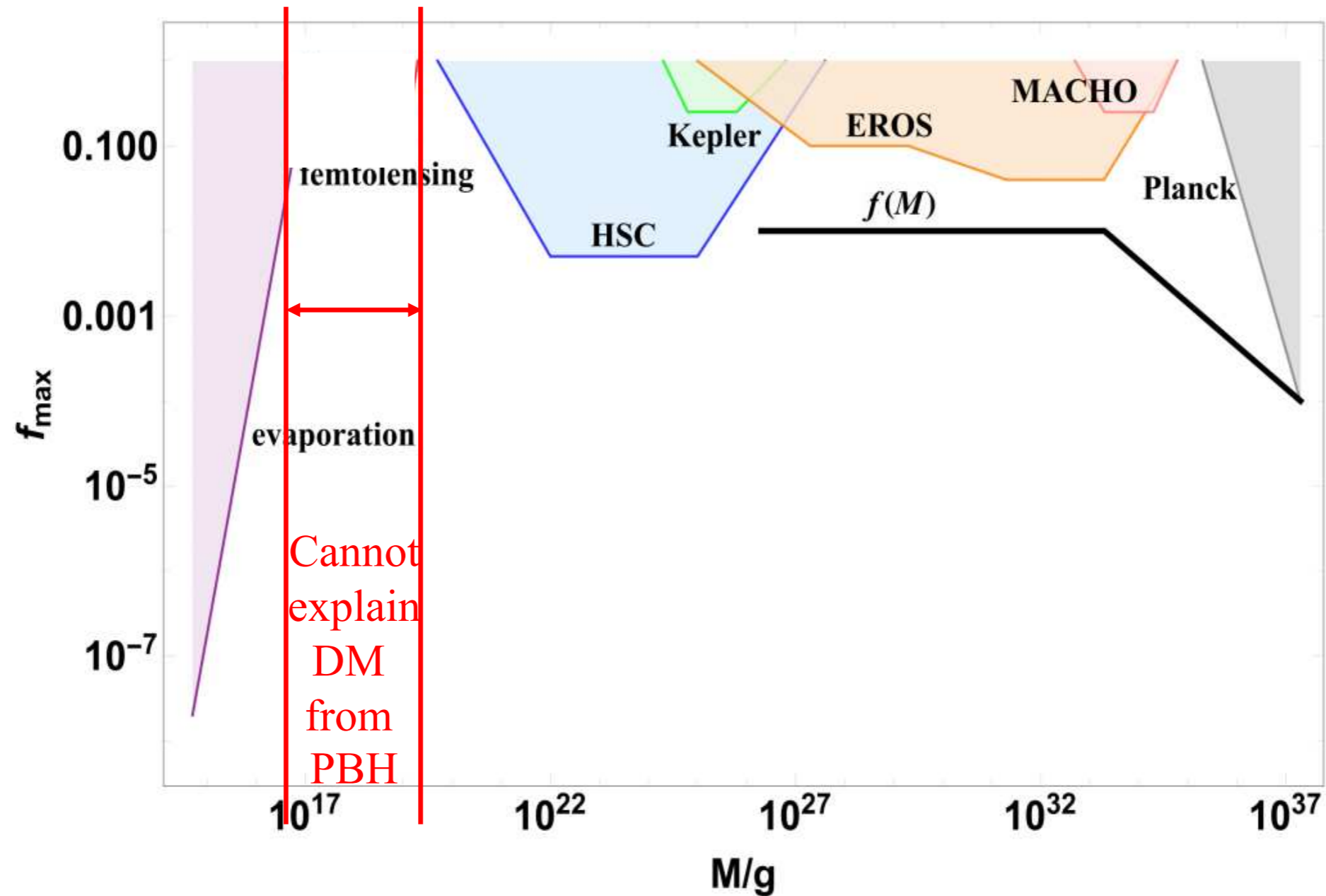
H. Deng, A. Vilenkin, JCAP 2017(12): 044.



We need a **varying tunneling rate** to explain DM

Case of a constant tunneling rate

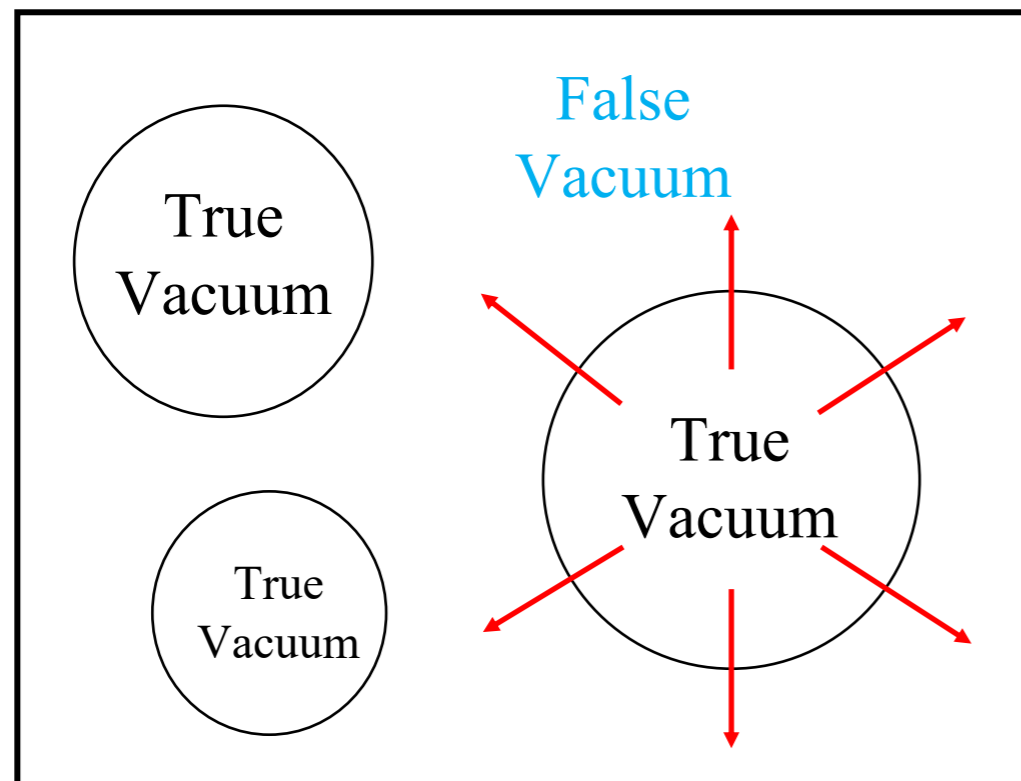
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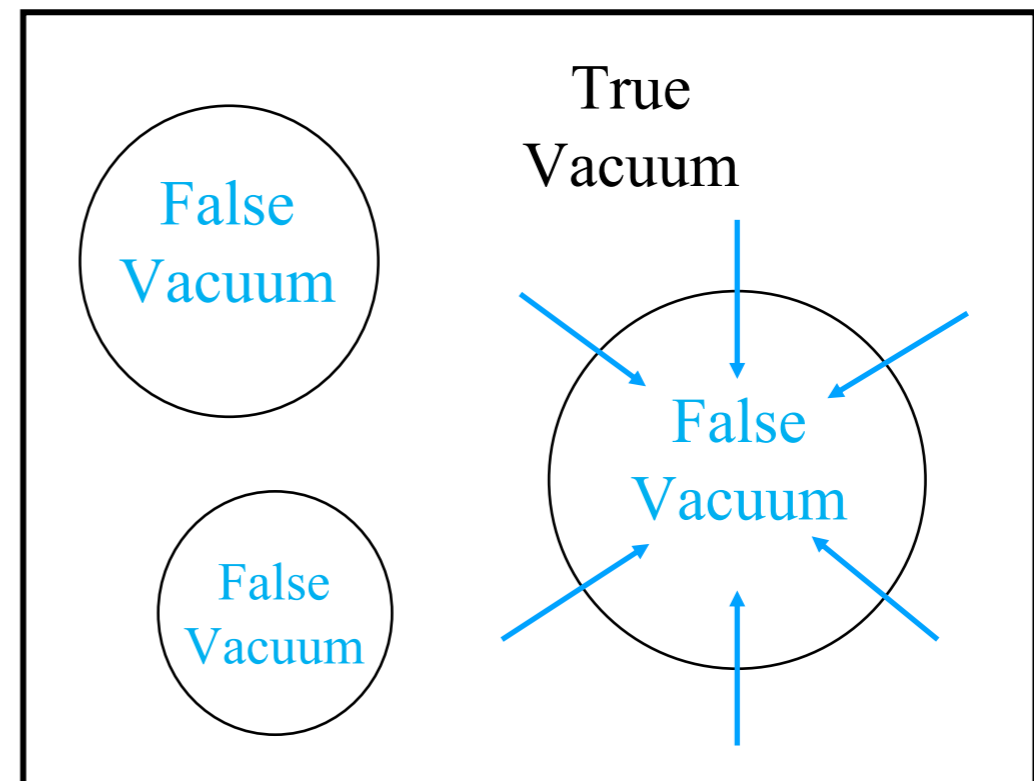
We need a **varying tunneling rate** to explain DM

The Bubble Nucleation

Bubble will be nucleated via tunneling process in early universe



During Inflation



After Inflation

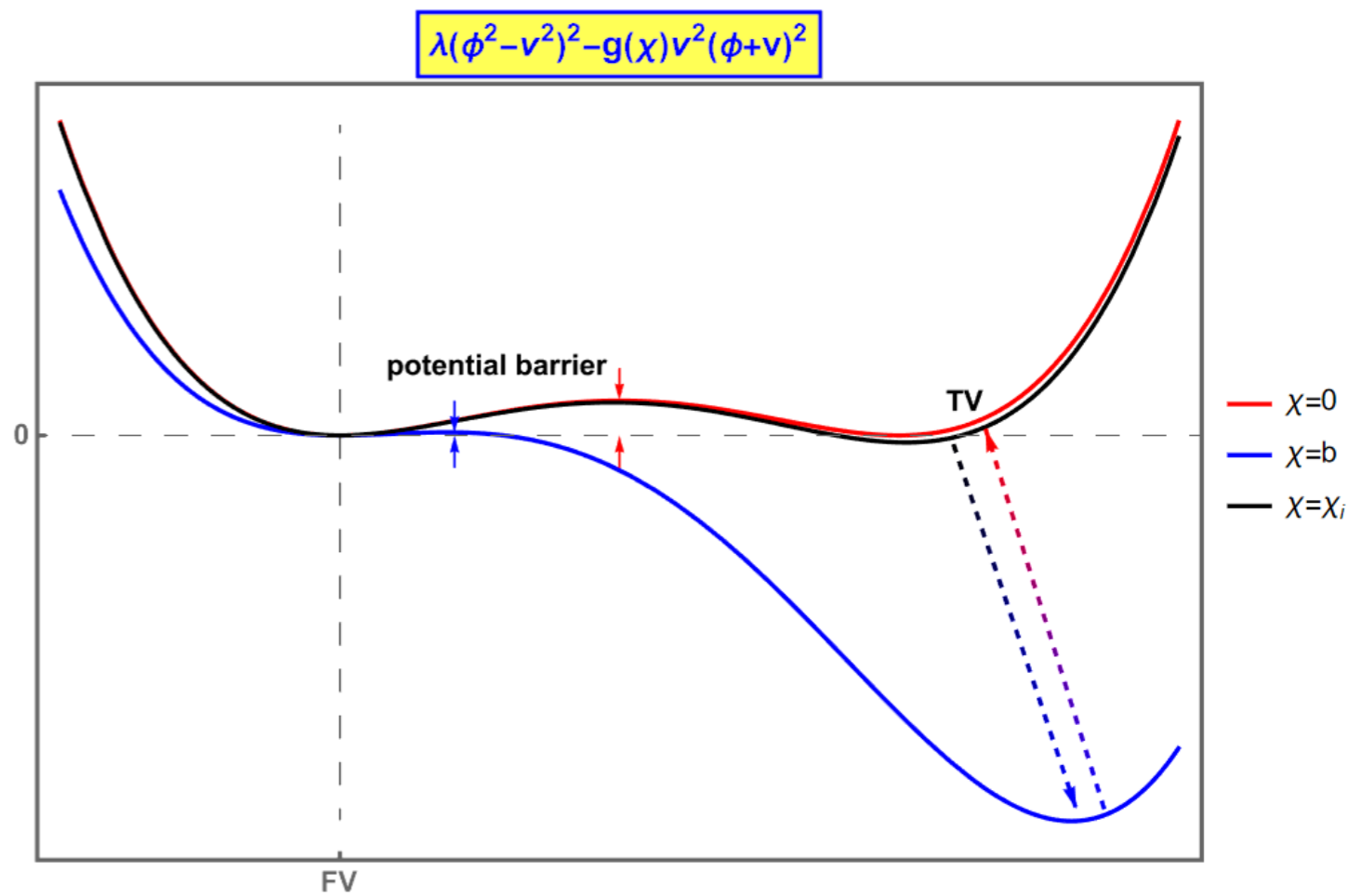
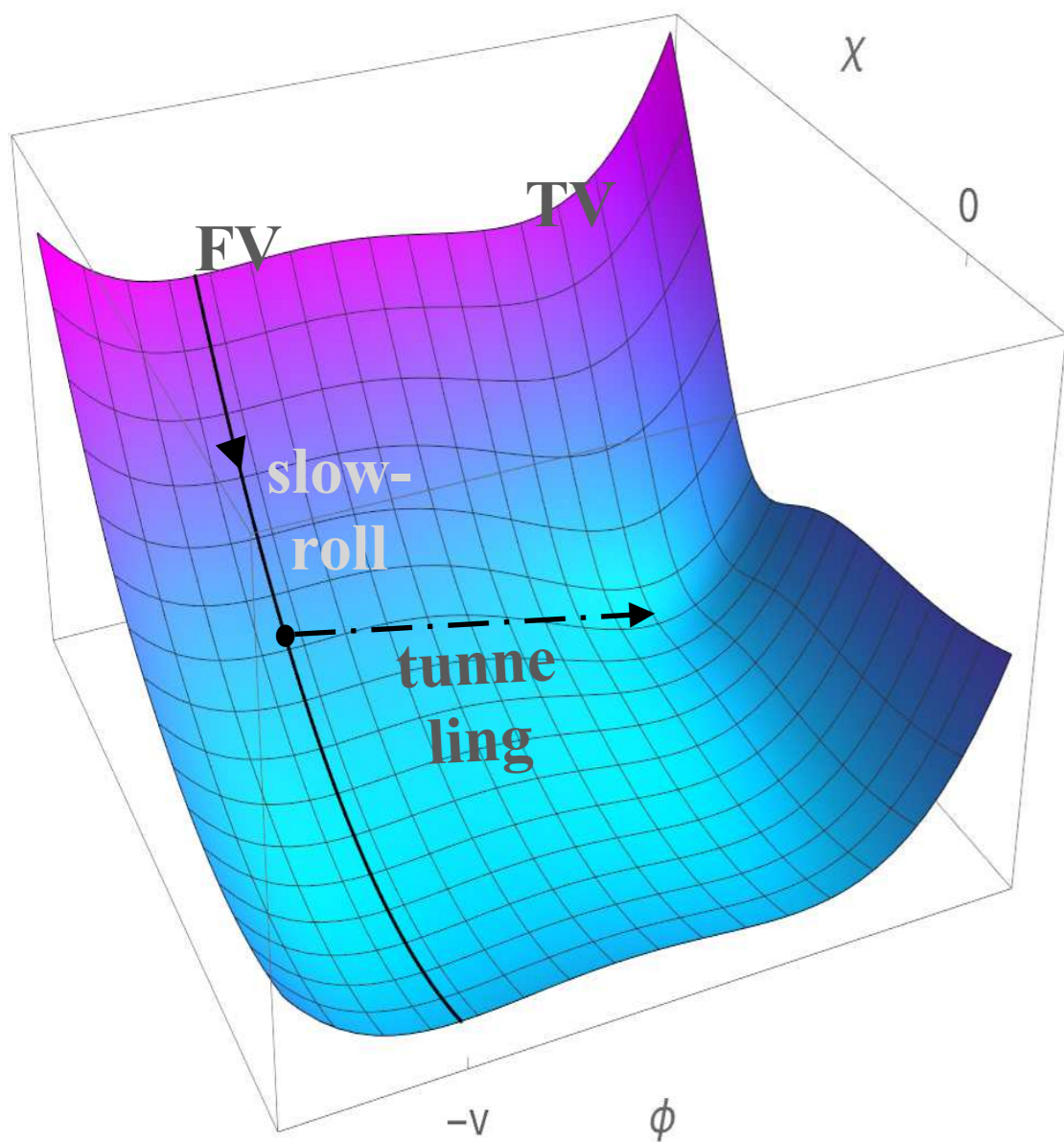
Question: How to realize this scenario?

Running of local minima

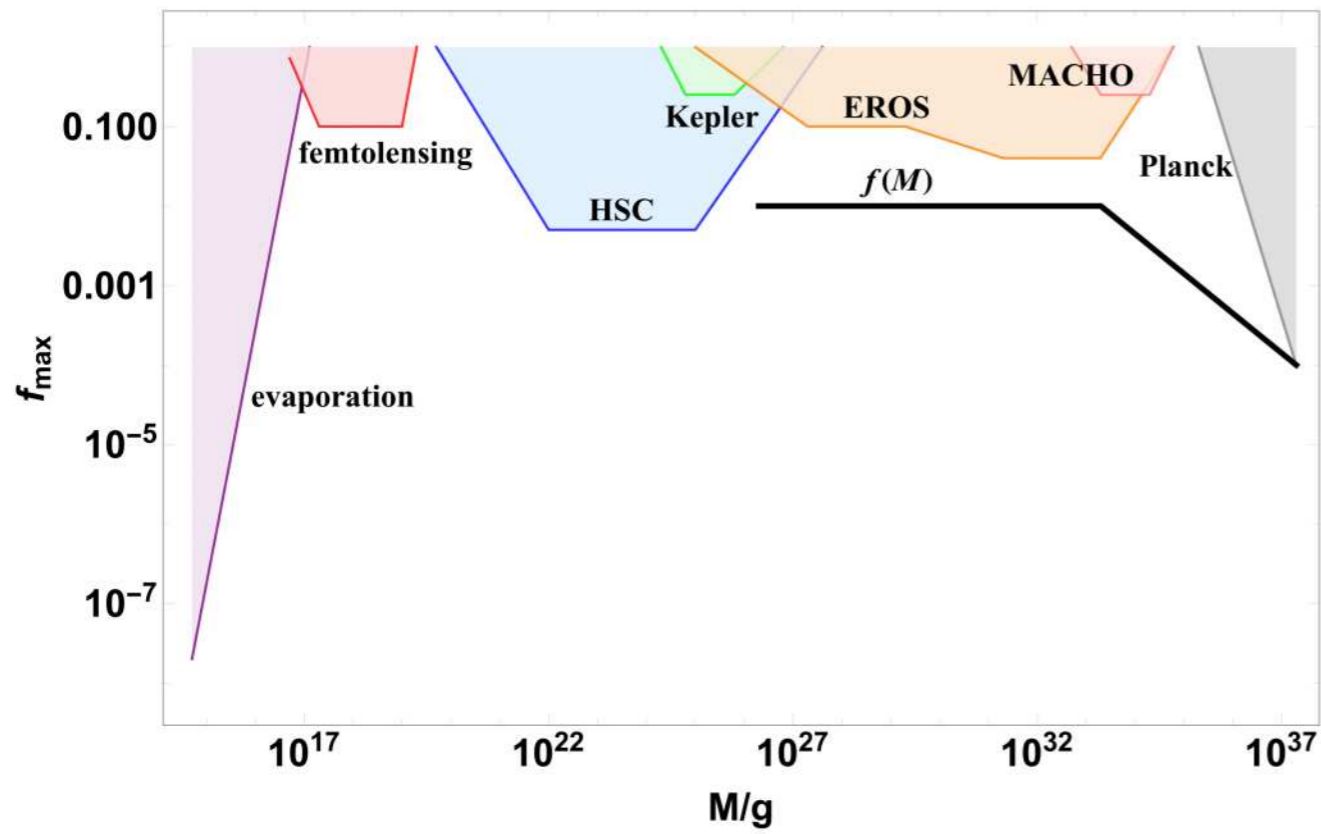
$$V(\chi, \phi) = \frac{1}{2} m^2 \chi^2 + \lambda(\phi^2 - v^2)^2 - g(\chi)v^2(\phi + v)^2$$

inflaton

instanton

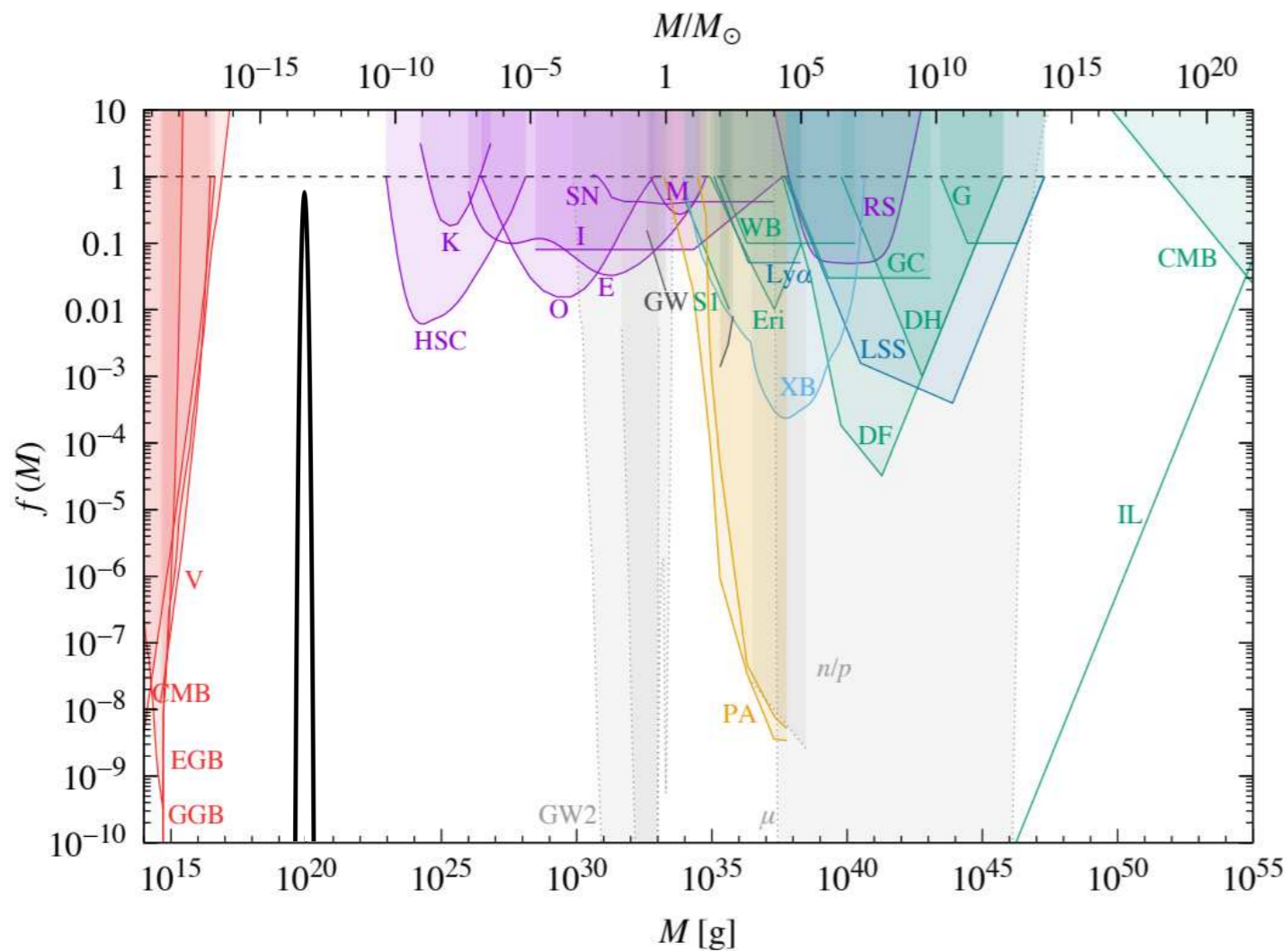


H. Wang, YZ, T. Suyama, in preparation



Single Field case

H. Deng, A. Vilenkin, JCAP 2017(12): 044.



Two field with nonminimally coupling

Summary

