Bad and Good Ghosts
&
Superrenormalizable Theories

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Answer to Hawking Question

Who's Afraid of (Higher Derivative) Ghosts?

S W Hawking

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Families of Ghosts

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- Bad Ghosts
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- Super Good Ghosts
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- Ghosts in Love

High derivative theories

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- UV completion of Standard Model may involve high derivatives ($s = 0, s = 1/2, s = 1$)
- Problems with ghosts, causality and unitarity

Bad ghosts

High Derivative Scalar Theories

$$S(\phi) \;=\; \frac{1}{2}\int \left[\partial^{\mu}\phi^{\dagger}\mathcal{P}_n(\Delta/\Lambda^2)\partial_{\mu}\phi - m^2|\phi|^2 - \frac{\lambda}{12}|\phi|^4\right]$$

where $\Delta = d^*d = \partial^{\mu}\partial_{\mu}$

$$\mathcal{P}_n(\Delta) = \prod_{i}^{n}(\mu_i^2 + \Delta/\Lambda^2)$$

The theory is UV finite if $n > 1$ due to the UV regularity of the propagator

Speer [1969] Lowenstein [1972]

High Derivative Scalar Theories

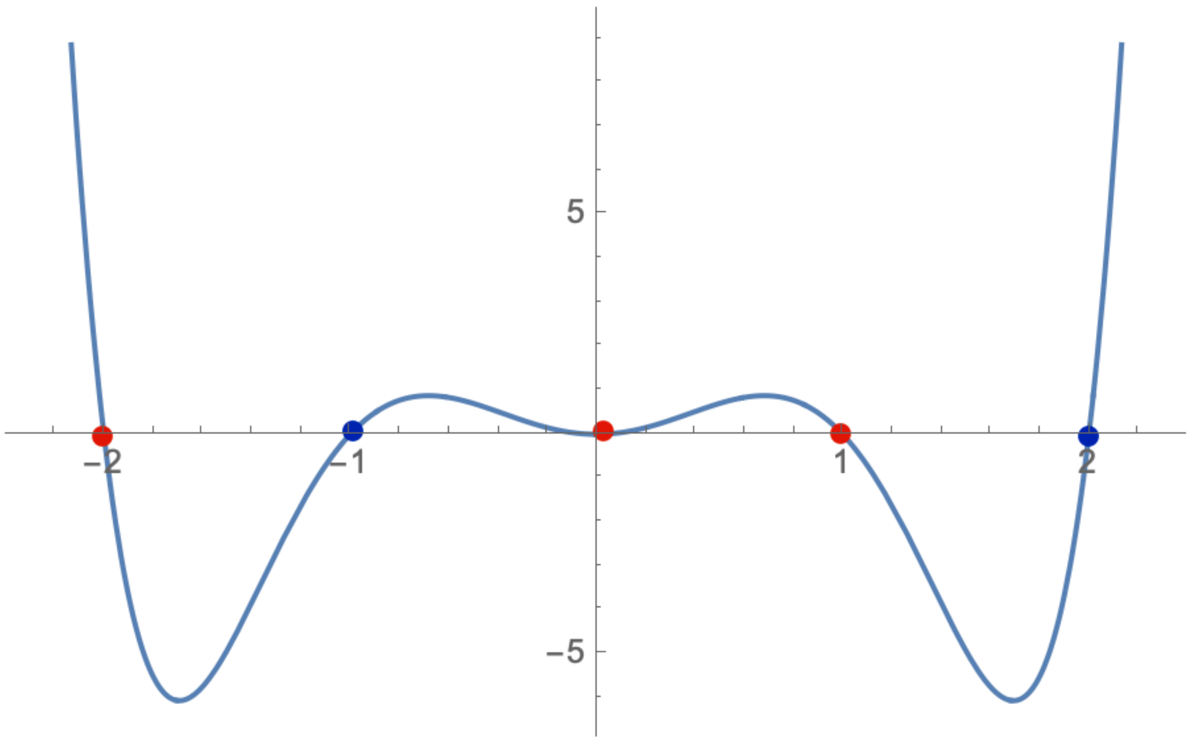
$$\Pi(p) = \frac{1}{p^2+m^2}\prod_i^n \frac{1}{\mu_i^2 + p^2/\Lambda^2}$$

only presents pairs $p_0 = \pm\sqrt{\mathbf{p}^2+\mu_i^2}$ of real poles, which guarantees the existence of analytic continuation in the complex p_0 plane till reaching the imaginary time of Euclidean formalism.

Problems: Unitarity, Causality, Ostrogradski instabilities, Bad ghosts

Bad Ghosts

$$\Pi_3(p) = \frac{1}{p^2 + m^2} \prod_i^2 \frac{1}{\mu_i^2 + p^2/\Lambda^2}$$



Osterwalder-Schrader positivity and Kallen-Lehmann representation

Källén-Lehmann Representation

Scalar Field Theories

Theorem: $S_2(x,y)$ is OS reflection positive iff the Fourier transform $S_2(k)$ has a Källén-Lehmann representation

$$S_2(k) = \int_0^\infty d\mu \frac{\rho(\mu)}{k^2+\mu^2} \quad \text{with } \rho(\mu) \geq 0$$

$S_2(k)$ is strongly positive:

$$\frac{d}{dk^2} k^2 S_2(k) \geq 0$$

$$\frac{(-k^2)^{n-1}}{n!(n-2)!} \left(\frac{d}{dk^2}\right)^{2n-1} k^{2n} S_2(k) \geq 0 \quad n > 1$$

[Widder, 1934]

Widder Inequalities

$S_2(k)$ is strongly positive:

$$\frac{d}{dk^2} k^2 S_2(k) = \frac{d}{dk^2} k^2 \int_0^\infty d\mu \frac{\rho(\mu)}{k^2 + \mu^2} = \int_0^\infty d\mu \frac{\mu^2 \rho(\mu)}{(k^2 + \mu^2)^2} > 0$$

$$\frac{2(-1)^{n-1}}{(n+1)!(n-1)!} \left(\frac{d}{dk^2} \right)^{2n-1} k^{2n} S_2(k) = c_n \int_0^\infty d\mu \frac{\mu^{2n} \rho(\mu)}{(k^2 + \mu^2)^{2n}} > 0$$

$$c_1 = 1,\ c_2 = 2,\ c_3 = 5,\ c_4 = 14,\ c_5 = 42,\ c_6 = 132 \quad c_n > 0$$

Holy Ghost

Holy Ghost

Good Ghosts
Faddeev-Popov Ghosts

Shy Ghosts

High Derivative Gauge Theories

$$S = \frac{1}{4g^2} \int d^4x \, F^a_{\mu\nu} F^{\mu\nu a} + \frac{1}{4g^2 \Lambda^{2n}} \int d^4x \, F^a_{\mu\nu} \, \Delta^n \, F^{\mu\nu a} \, ,$$

where

$$\Delta = d_A^* d_A + d_A d_A^*$$

is Hodge-covariant Laplacian operator

$$\Delta^{\nu\, b}_{\mu\, a} = -D^2 \delta^\nu_\mu \delta^b_a + 2 f^b{}_{ca} \, F_\mu{}^{\nu c}$$

Instantons are minima in each topological sector

L. Faddeev and A. Slavnov [1980]

Perturbation Theory

One-loop divergences [α-gauge]

$$\Gamma^{ab}_{\mu\nu}(p) = -c_n \frac{C_2(G)}{16\pi^2\varepsilon} i\,\delta^{ab}\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right)$$

with

$$c_n = \frac{29}{3} - 23n + 5n^2 \qquad n \geqslant 2,$$

$$c_1 = -\frac{43}{3}, \qquad c_0 = \alpha - \frac{13}{3}$$

Perturbation Theory

One-loop renormalization

$$S_{\mathrm{count}} = c_n \frac{C_2(G)}{128\pi^2} \left(\frac{2}{\varepsilon} + \log \frac{\Lambda^2_{\mathrm{QCD}}}{\Lambda^2} \right) F^a_{\mu\nu} F^{\mu\nu a},$$

β-function of the coupling constant

$$\beta_n = c_n \frac{g^3 C_2(G)}{32\pi^2} \quad n \geq 2$$

Asymptotic freedom only for $n = 0, 1, 2, 3, 4$

$$\widetilde{\mathbf{c}}_0 = -\frac{\mathbf{22}}{\mathbf{3}}, \mathbf{c}_1 = -\frac{\mathbf{43}}{\mathbf{3}}, \mathbf{c}_2 = -\frac{\mathbf{49}}{\mathbf{3}}, \mathbf{c}_3 = -\frac{\mathbf{43}}{\mathbf{3}}, \mathbf{c}_4 = -\frac{\mathbf{7}}{\mathbf{3}}, c_5 = \frac{59}{3}$$

Perturbation Theory

One-loop form factor

$$\Gamma^{ab}_{\mu\nu}(p) = -\frac{C_2(G)}{32\pi^2} i\delta^{ab}\left(p^2\eta_{\mu\nu} - p_\mu p_\nu\right)\Pi(p^2)$$

with

$$\Pi(p^2) = \left(b_n \log\frac{p^2+\Lambda^2}{\Lambda^2} + c_0 \log\frac{p^2}{\Lambda^2_{\mathrm{QCD}}}\right),$$

$$b_n = 14 - \alpha - 23n + 5n^2 \quad \text{for } n \geqslant 2$$

$$b_0 = 0, \quad b_1 = -10 - \alpha$$

Scaling Regimes

There are two different asymptotic regimes with two different beta functions:

- UV regime $p \gg \Lambda$

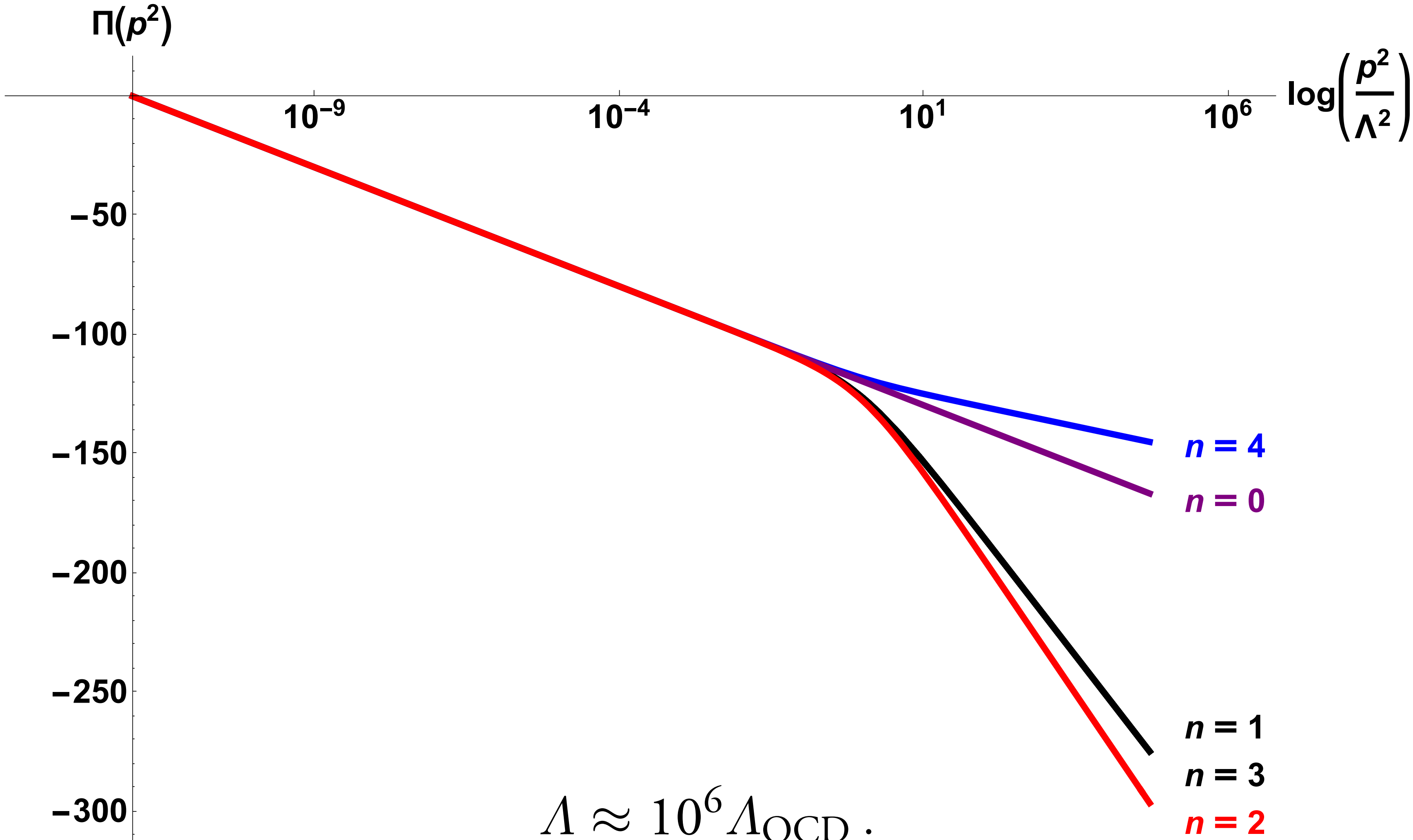
$$\beta_{\rm UV} = c_n \frac{g^3 C_2(G)}{32\pi^2}$$

- IR regime $\Lambda_{\rm QCD} < p \ll \Lambda$

$$\beta_{\rm IR} = -\frac{22}{3}\,\frac{g^3 C_2(G)}{32\pi^2}$$

M.A., F. Falceto and L. Rachwal [2021]

Two-point form factor



$$\Lambda \approx 10^6 \Lambda_{\mathrm{QCD}} \, .$$

Generalization

Replace the Hodge-covariant Laplacian operator by a generalized Laplacian

$$\Delta \quad \Rightarrow \quad {}^{\lambda}\Delta$$

$${}^{\lambda}\Delta_{\mu\,a}^{\nu\,b} = -\delta_a^b\,\delta_\mu^\nu\,D^2 + 2\,\lambda f^b{}_{ca}\,F_\mu{}^{\nu c}$$

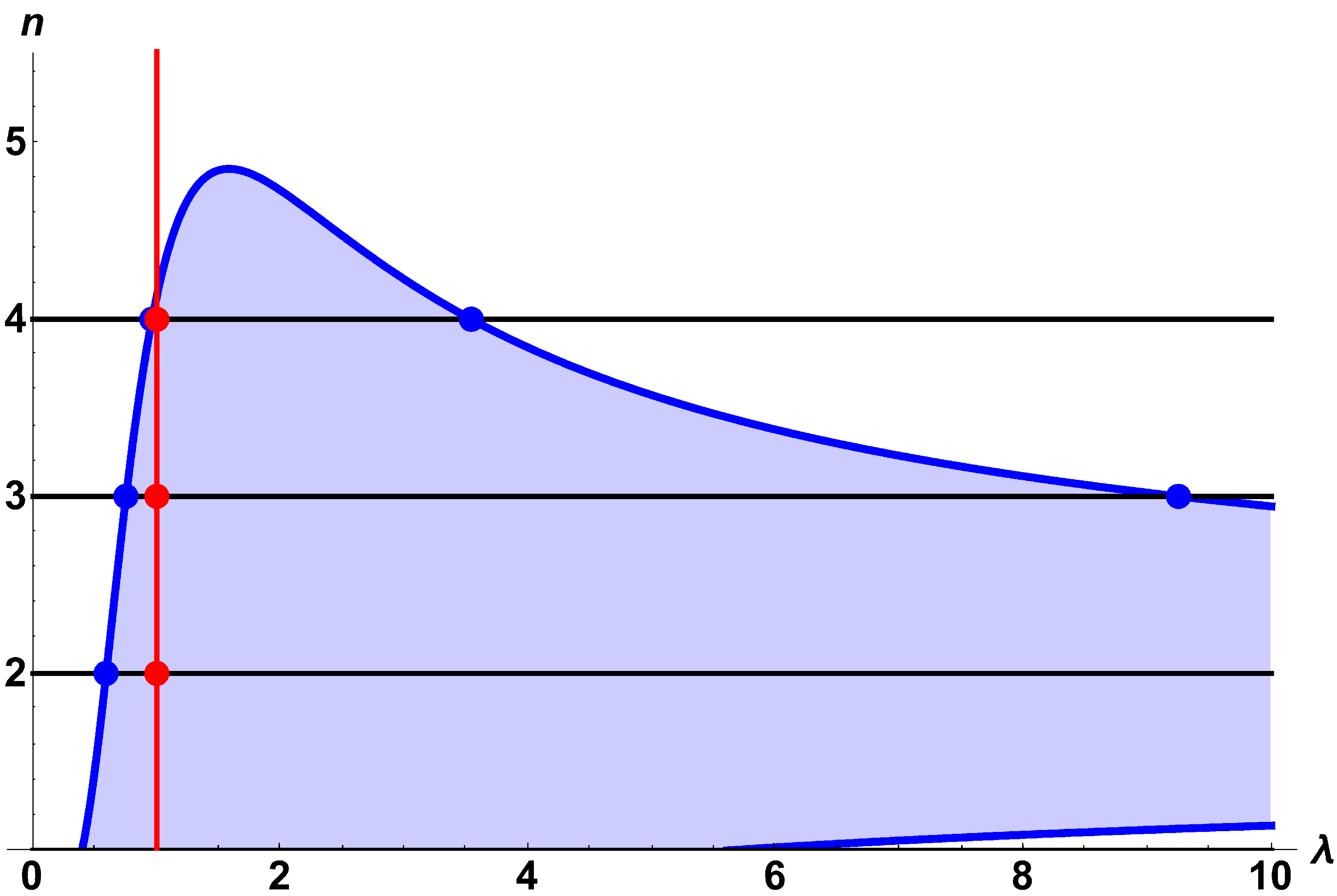
One-loop β-function coefficients

$$c_n = -\frac{7}{3} + 5n + 4n^2 - \left(4 + 10n + 4n^2\right)\lambda + \left(16 - 18n + 5n^2\right)\lambda^2$$

$$n \geq 2$$

$$c_1 = \frac{38}{3} - 18\lambda - 9\lambda^2$$

Two-point form factor



UV Finite theories

The range of asymptotically free theories is more restrictive for $\lambda \neq 1$

For some values of $\lambda \neq 1$ the theory is finite

$$\begin{array}{llll} n=1 & \lambda_1 = -2.55 & \lambda_2 = 0.55 \\ n=2 & \lambda \;\; = \;\;\; 0.59 & \\ n=3 & \lambda_1 = \;\;\; 0.75 & \lambda_2 = 9.25 \\ n=4 & \lambda_1 = \;\;\; 0.96 & \lambda_2 = 3.54 \end{array}$$

The theory is free of UV divergences

No instanton solutions $\Rightarrow$ new QCD potentials for axions

Higher Derivative Ghosts

- **BRST symmetry is preserved**
- **High derivative terms are not renormalized**
- $\beta_{\Lambda} = -\frac{1}{n}\beta_{g}$
- **Effective ghost masses run to infinite if** $\beta_{g} \neq 0$
- **Unitarity and Causality are fully recovered ?**

M.A., F. Falceto and L. Rachwal [2024]

Källén-Lehmann Representation

Higher Derivative Gauge Theories

The higher derivative kernel ($\alpha = 1$ gauge)

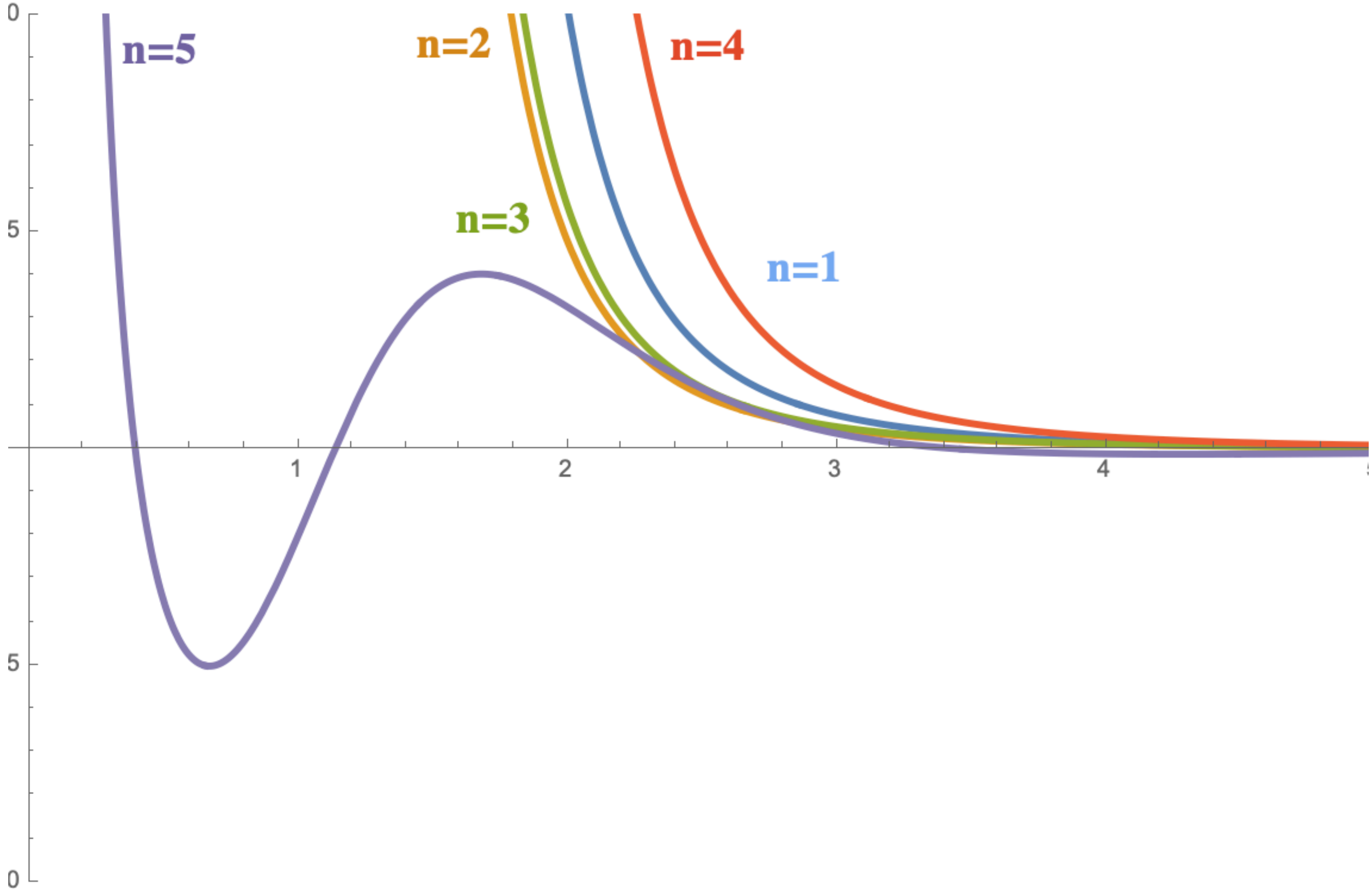
$$S_2(p^2)\delta_{\mu\nu} = \frac{1}{p^2\left(b_n \log\frac{p^2+\Lambda^2}{\Lambda^2} + c_0 \log\frac{p^2}{\Lambda^2_{\text{QCD}}}\right)}\delta_{\mu\nu}$$

is strongly positive for $n \leq 4$.

Exactly for the same cases that there is asymptotic freedom

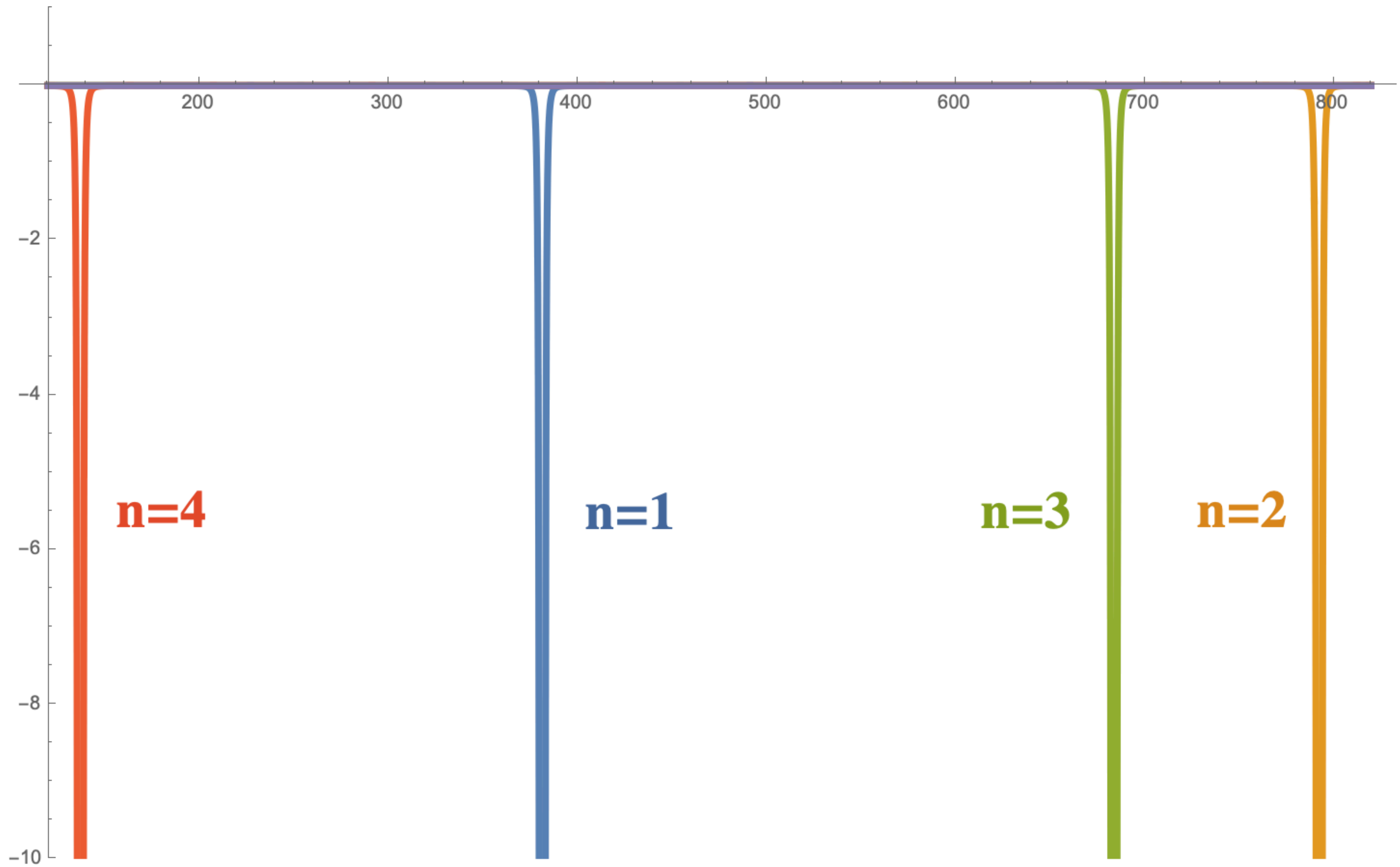
[M. A., F. Ezquerro and I. Shapiro, 2024]

Two-point form factor



For $n > 4$ violation of unitarity can appear at E $\approx \Lambda$

Two-point form factor



Violation of unitarity can appear at E $\approx 10^3 \Lambda$

Ghosts in Love

Ghosts in six-derivative quantum gravity

$$S_6 = \int d^4x\sqrt{g}\,\Big\{ -\frac{1}{\varkappa^2}\,(R+2\Lambda) + \frac{1}{2}\,C_{\mu\nu\alpha\beta}\,\Delta\,C_{\mu\nu\alpha\beta} + \frac{1}{2}\,R\,\Delta\,R + \mathcal{O}(R^3_{\dots})\Big\}\,, \qquad (1)$$

The last two terms only get renormalized at one loop The Einstein terms till two loop and the cosmological constant till three loops and the last term $\mathcal{O}(R^3_{\dots})$ get only finite corrections at any loop order of perturbation theory.

Ghosts in six-derivative quantum gravity

Gauge fixing:

$$S_{gf} = \frac{1}{2}\int d^4x\sqrt{g}\ \chi_\alpha Y^{\alpha\tau}\chi_\tau,$$

where

$$\chi^\alpha = \partial_\lambda h^\lambda_\alpha - \beta\partial_\alpha h \qquad Y^{\alpha\beta} = \gamma_1\eta^{\alpha\tau}\partial^2 + \gamma_2\partial^\alpha\partial^\tau$$

Physical Modes

$$h_{\mu\nu} = \bar{h}^{\perp\perp}_{\mu\nu} + \partial_\mu\varepsilon^\perp_\nu + \partial_\nu\varepsilon^\perp_\mu + \partial_\mu\partial_\nu\varepsilon + \frac{1}{4}h\,\eta_{\mu\nu}$$

Ghosts in six-derivative quantum theories

Complex conjugated ghosts

$$S_3(\Psi) \;=\; \frac{1}{2|m|^4}\int\Big[\Psi^\dagger(\Delta)(\Delta/\Lambda^2+m^2)(\Delta/\Lambda^2+m^{2*})\Psi-\frac{\lambda}{12}|\Psi|^4\Big]$$

Auxiliary fields

$$\Psi \;=\; \varphi_3+\alpha_1\varphi_1+\alpha_2\varphi_2, \qquad \alpha_1^2=\frac{-im^{*2}}{m^{*2}-m^2}, \qquad \alpha_2^2=\frac{im^2}{m^2-m^{*2}}$$

$$\mathcal{S}_3(\varphi_1,\varphi_2,\varphi_3)=\frac{i}{2}\int\varphi_1(\Delta+m^2)\varphi_1-\frac{i}{2}\int\varphi_2(\Delta+m^{*2})\varphi_2+\frac{1}{2}\int\varphi_3(\Delta)\varphi_3$$
$$+\frac{1}{4!}\lambda_1\int\varphi_1^4+\frac{1}{4!}\lambda_2\int\varphi_2^4-\frac{1}{4}\lambda_{12}\int\varphi_1^2\varphi_2^2$$

Ghosts Bound States

Condensate of complex conjugated ghosts

$$O_{\varphi_1\varphi_2}(x) = \varphi_1(x)\varphi_2(x)$$

$$C(x,y) = C(x-y) = \langle O_{\varphi_1\varphi_2}(x)\, O_{\varphi_1\varphi_2}(y)\rangle$$

at one loop

$$\widetilde{C}_1(p) = \Sigma(p) \;=\; \lim_{\varepsilon\to 0}\, \bar{\mu}^{\varepsilon}\left(\frac{e^{\gamma}}{4\pi}\right)^{\varepsilon/2}\int \frac{d^{4-\varepsilon}k}{(2\pi)^{4-\varepsilon}}\,\frac{i}{(p-k)^2+m^2}\,\frac{-i}{p^2+m^{*2}}$$

$$= \;-\frac{1}{(4\pi)^2}\int_0^1 dx\, \ln\frac{G(p^2,x)}{\bar{\mu}^2} \qquad (2)$$

$$G(p^2,x) = x(1-x)p^2 + (1-x)m^2 + xm^{*2}$$

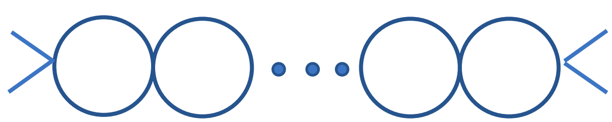
Arkani-Hamed et al, Mukohyama, Sorella et al, ...

Ghosts Bound States

Summing up a chain of loops

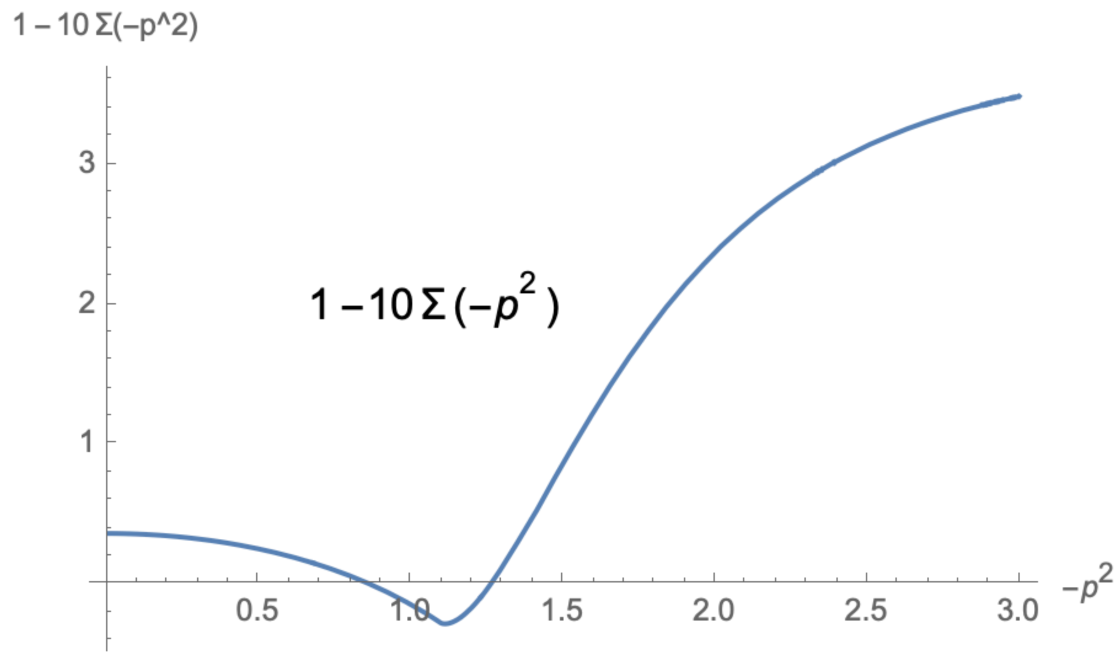
$$\widetilde{C}(p) = \frac{\frac{1}{(4\pi)^2}\Sigma(p)}{1 - \frac{\lambda_{12}}{(4\pi)^2}\Sigma(p)}$$

$$g = \frac{\lambda_{12}}{(4\pi)^2}$$



[M. A., G. Krein and I. Shapiro, 2024]

Ghosts Bound States

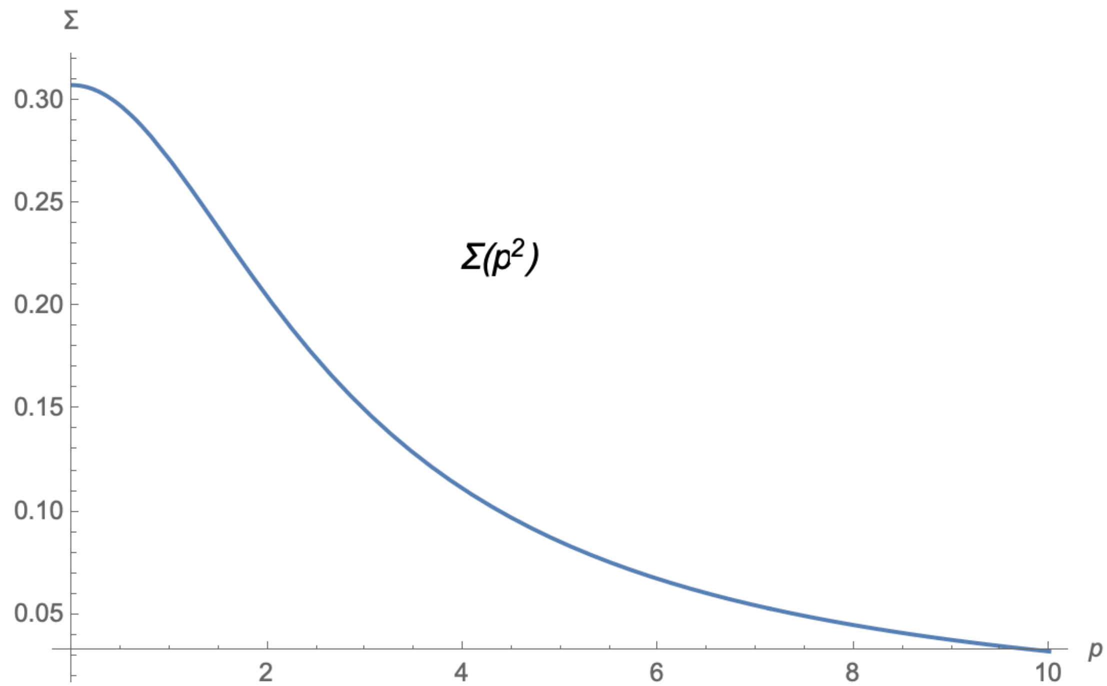


Bound state of ghosts with $\mathcal{M} = 0.88$ for $g = 10 \geq g_c$ and complex conjugated poles $m^2 = 1 + i$.

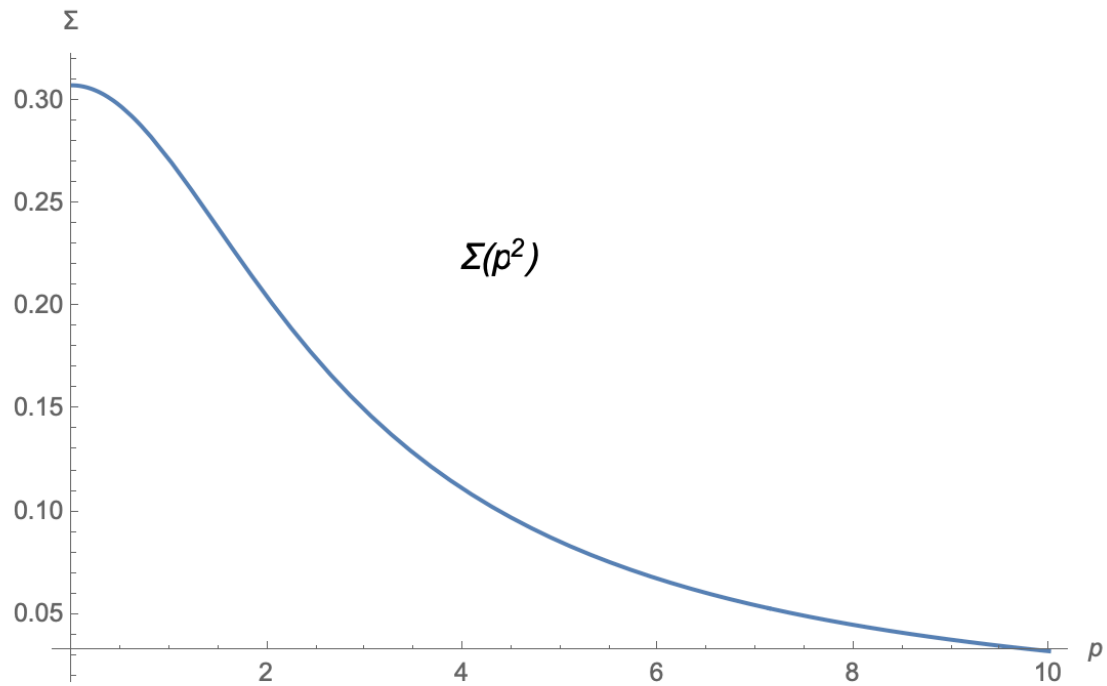
Ghosts in Love

Ghosts in Love

Kallen-Lehmann representation

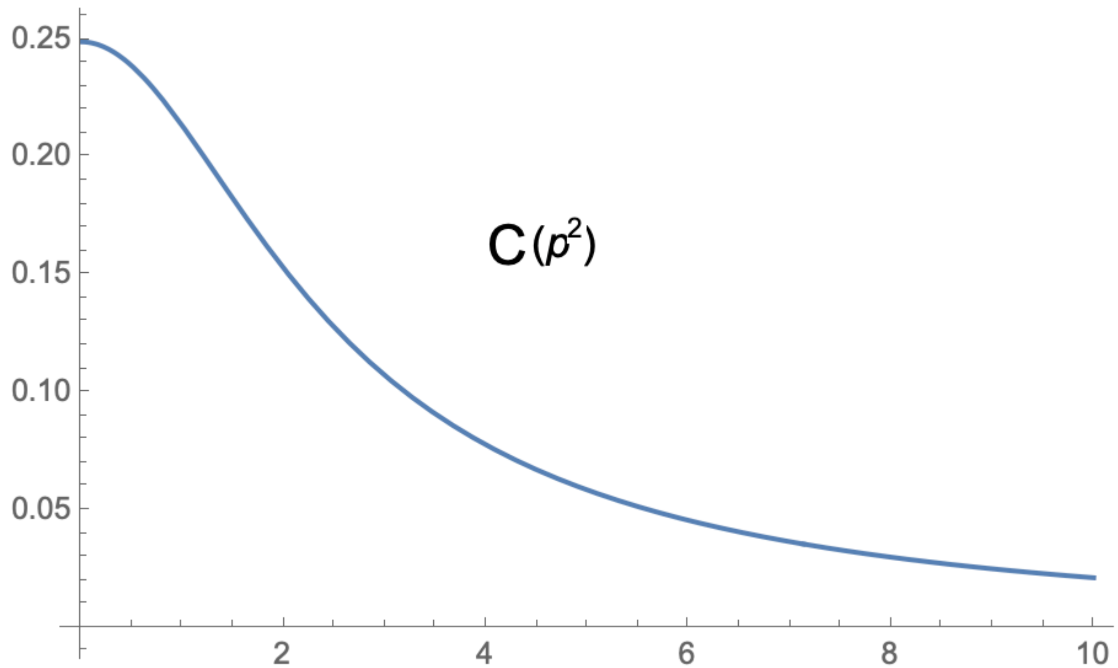


Kallen-Lehmann representation



$$\Sigma(p) = \Sigma(0) + 2\int_{(m+m^*)^2}^{\infty} ds\, \frac{\sqrt{s^2 - 4(m_I^2)^2 - 4m_R^2 s}}{s}\left(\frac{1}{p^2+s} - \frac{1}{s}\right)$$

Kallen-Lehmann representation



same effect holds for six derivatives quantum gravity.

CONCLUSIONS

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- In six derivatives theories with complex conjugated ghosts there is ghost condensation
- The possible contribution to dark matter of ghost condensates is negligeble

$$\rho^{BS}(\mathcal{E}_{end}) \;\propto\; \rho_c(\mathcal{E}_{end}) \times 10^{-78}$$