

On the origins on CMB anomalies, direct-sum inflation and quantum gravity

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(Based on the recent work in collaboration with Enrique Gaztanaga

[arXiv: 2401.08288](https://arxiv.org/abs/2401.08288) (*JCAP* 06 (2024) 001) and

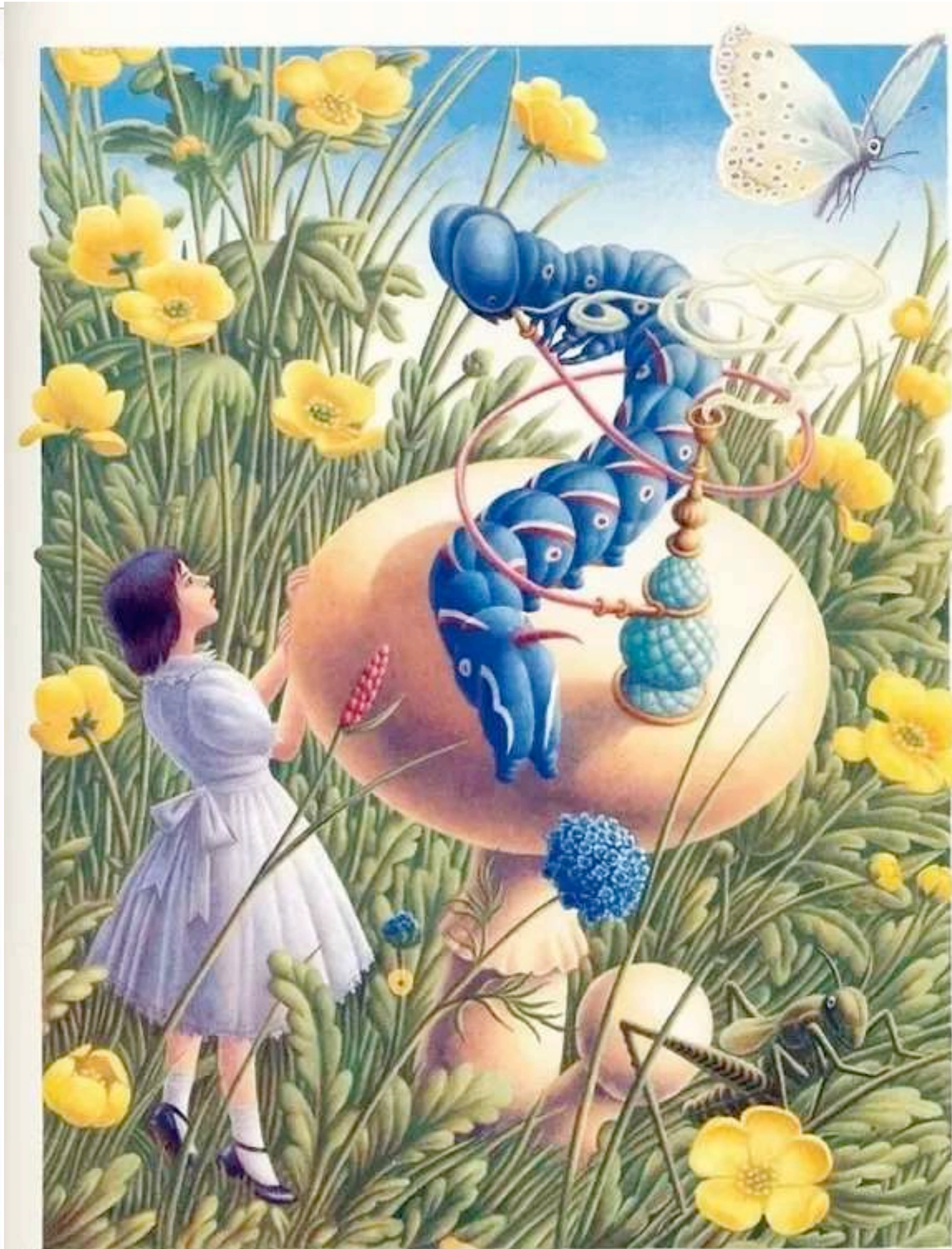
[arXiv:2405.20995](https://arxiv.org/abs/2405.20995), [2305.06046](https://arxiv.org/abs/2305.06046), [2307.10345](https://arxiv.org/abs/2307.10345), [2209.03928](https://arxiv.org/abs/2209.03928) with J. Marto

[arXiv: 2408.XXXX](https://arxiv.org/abs/2408.XXXX) (with EG and JM)

**Quantum gravity and Cosmology 2024 SPST,
ShanghaiTech U., China**

Take away message-1 Always question assumptions and pay attention to small

And simple things.



This talk is about
wonderland created by
gravity and
quantum mechanics

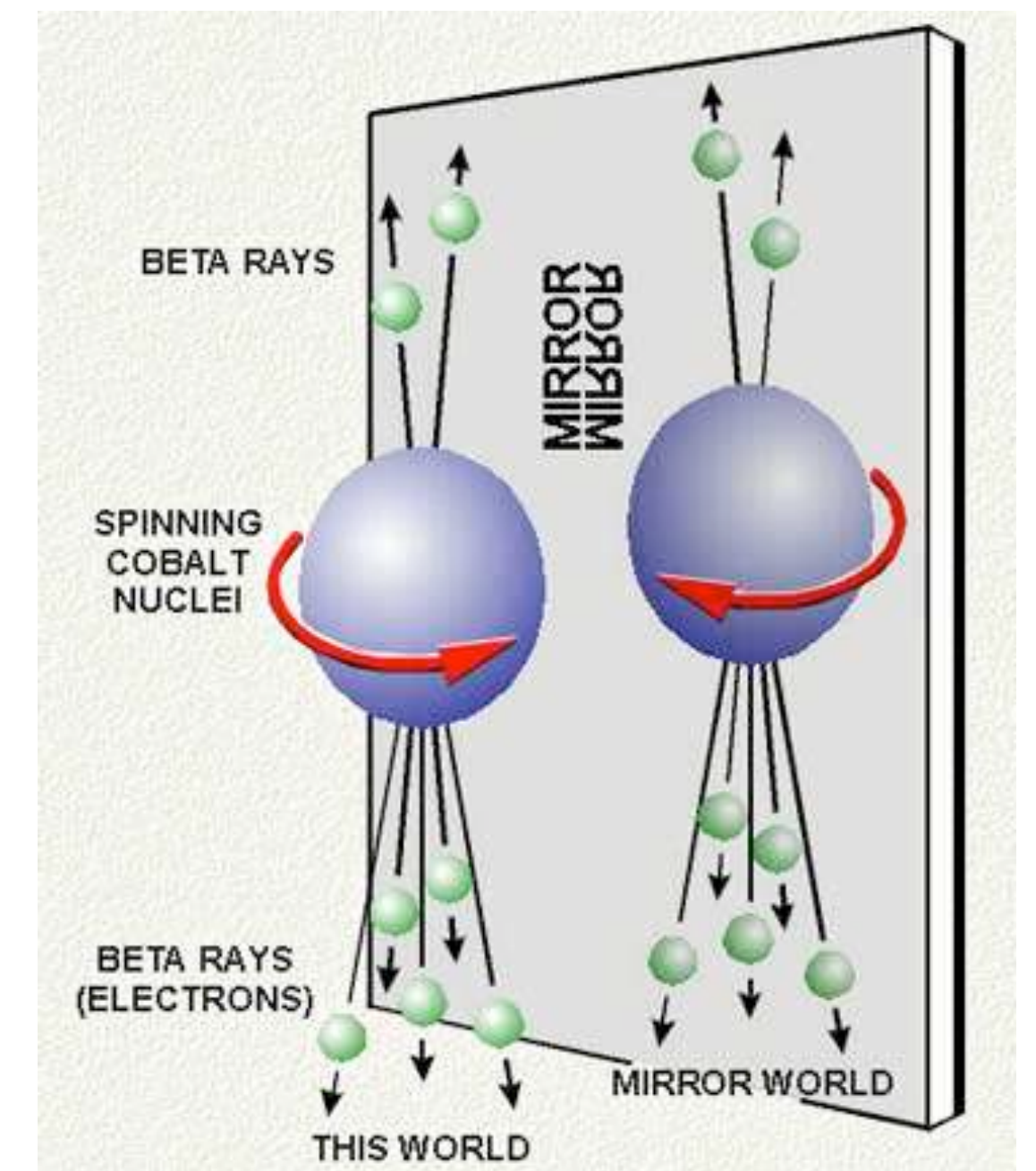
Fundamental questions on fundamental interactions

- Out of all fundamental forces of nature gravity is the weakest force
- The standard model of particle physics is built on understanding discrete symmetries or asymmetries. Parity, Time reversal and Charge conjugation.
- In building standard model of particle physics, the discrete symmetry played an important role.

Wu experiment: “The parity violation in beta decay played an important role in building SM of particle physics”

Pauli rejected outcome of the experiment.

Abdus Salam placed it on par with Michelson-Morley experiment.



Hidden features in the Planck CMB: Parity Asymmetry

$$\mathcal{T}(\hat{n}) \equiv \frac{\Delta T(\hat{n})}{T_0} = \sum a_{\ell m} Y_{\ell m}(\hat{n})$$

$$a_{\ell m} = \int d\Omega Y_{\ell m}^*(\hat{n}) \mathcal{T}(\hat{n})$$

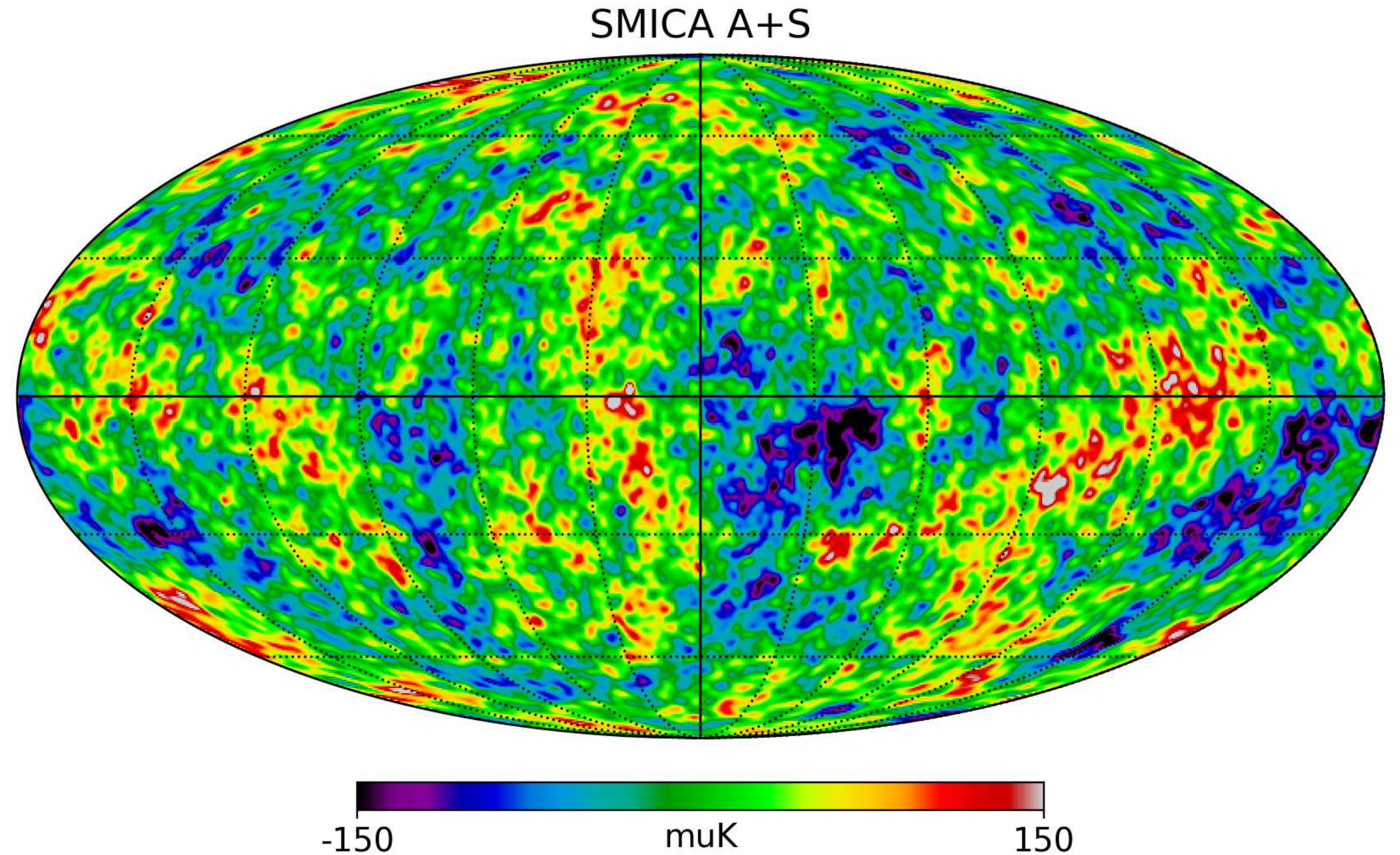
$$\mathcal{T}(\hat{n}) = S(\hat{n}) + A(\hat{n})$$

$$S(\hat{n}) \equiv \frac{1}{2} [\mathcal{T}(\hat{n}) + \mathcal{T}(-\hat{n})] = S(-\hat{n})$$

$$A(\hat{n}) \equiv \frac{1}{2} [\mathcal{T}(\hat{n}) - \mathcal{T}(-\hat{n})] = -A(-\hat{n})$$

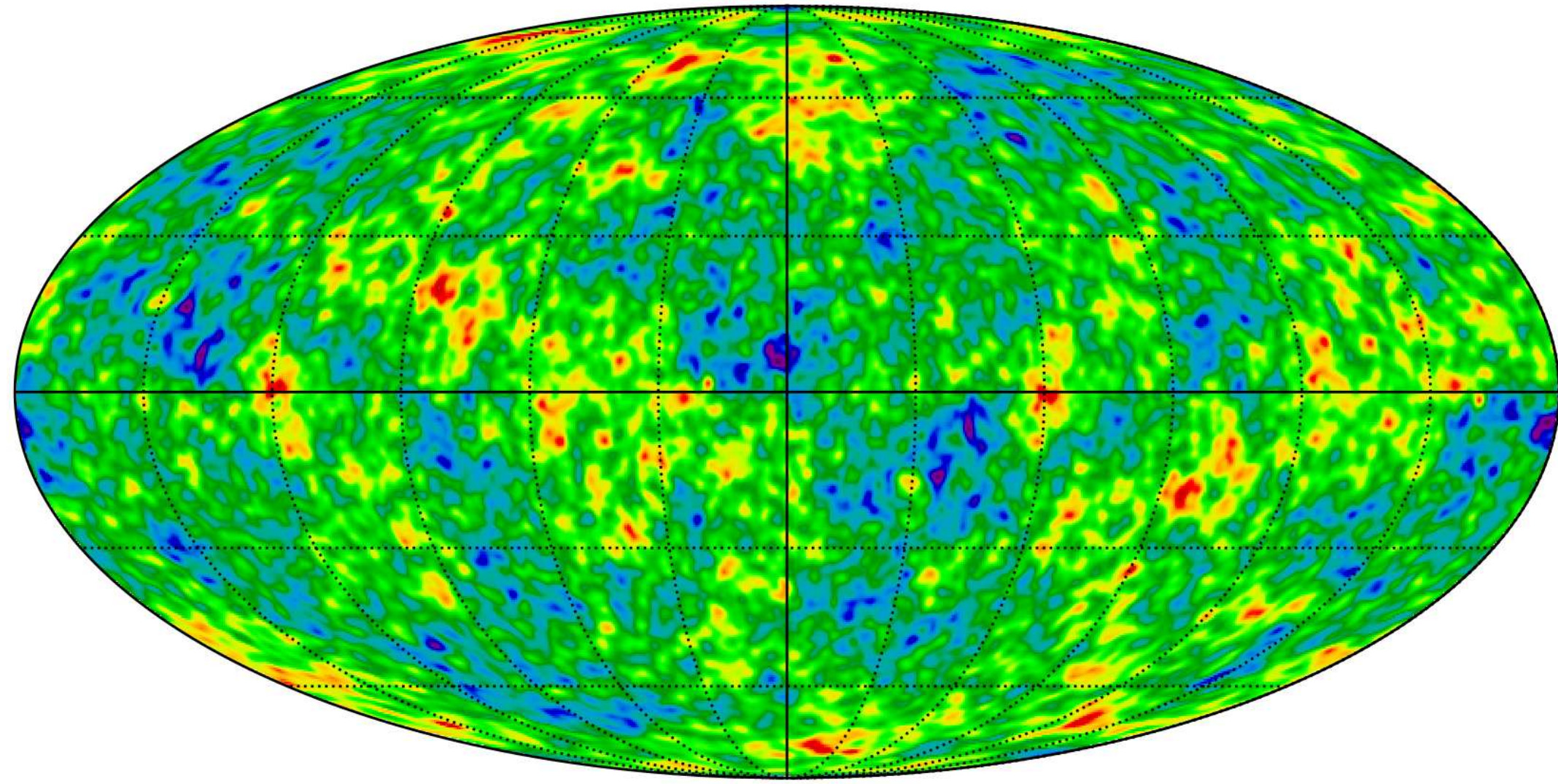
The angular $\mathbb{T}\mathbb{T}$ power spectrum is

$$C_\ell = \frac{1}{2\ell + 1} \sum_m |a_{\ell m}|^2$$

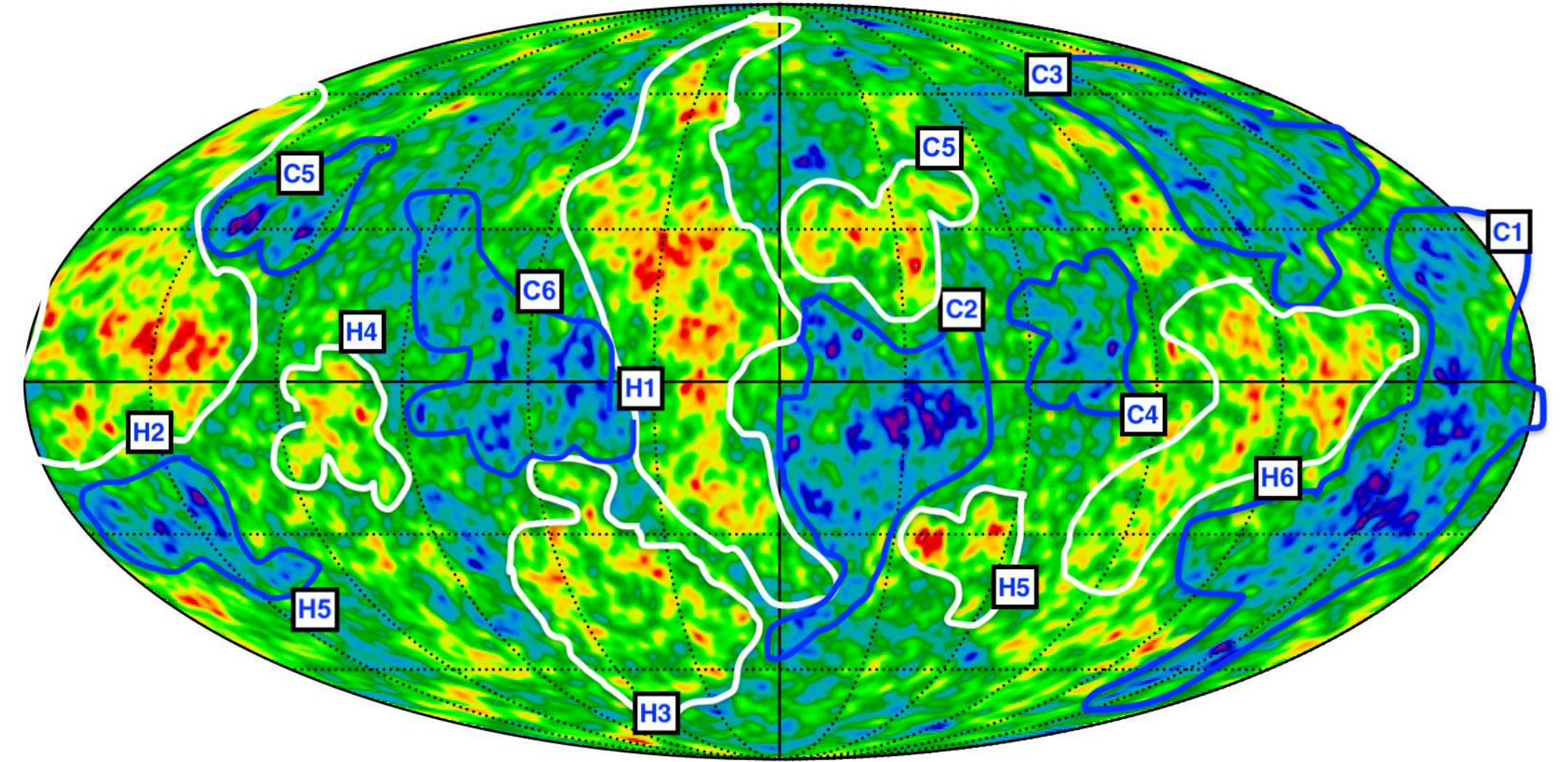


Wonderland of gravity and
and quantum mechanics

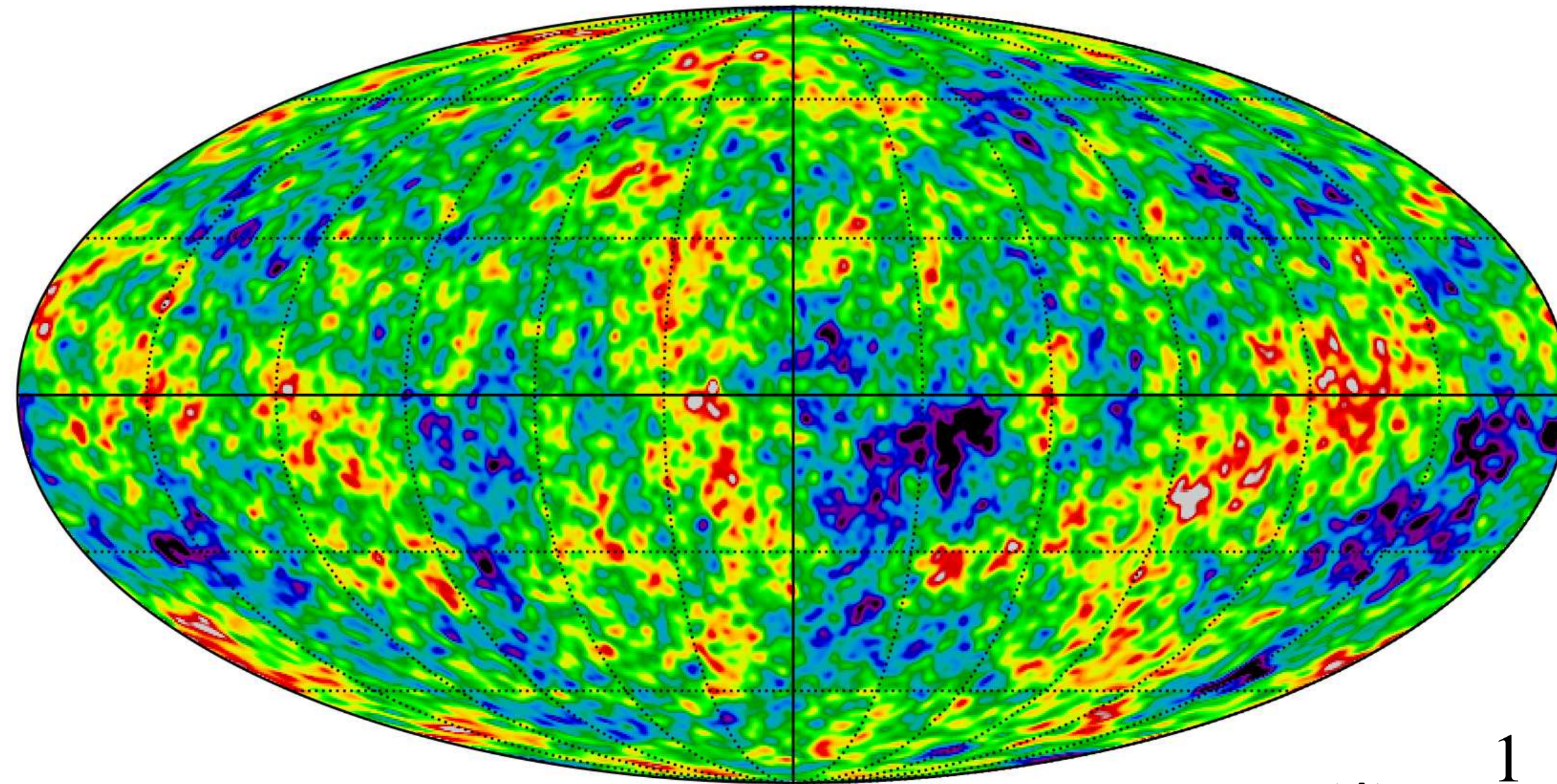
SMICA S



SMICA A



SMICA A+S

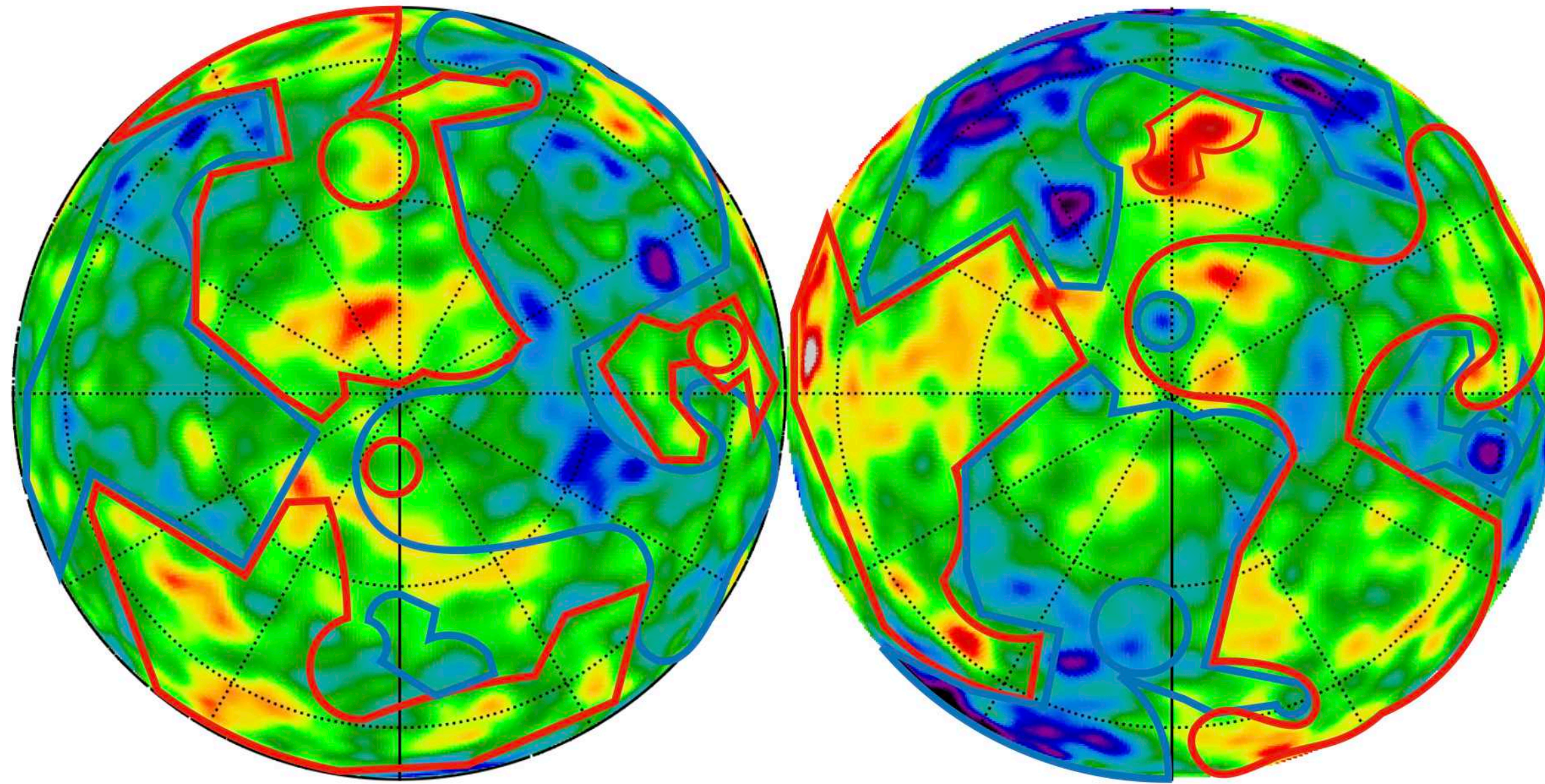


$$S(\hat{n}) \equiv \frac{1}{2} [\mathcal{T}(\hat{n}) + \mathcal{T}(-\hat{n})] = S(-\hat{n})$$

$$A(\hat{n}) \equiv \frac{1}{2} [\mathcal{T}(\hat{n}) - \mathcal{T}(-\hat{n})] = -A(-\hat{n})$$

The parity asymmetry and the quantum fluctuations

SMICA North/South

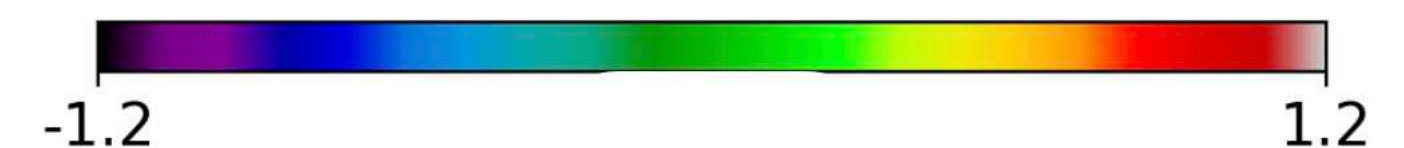
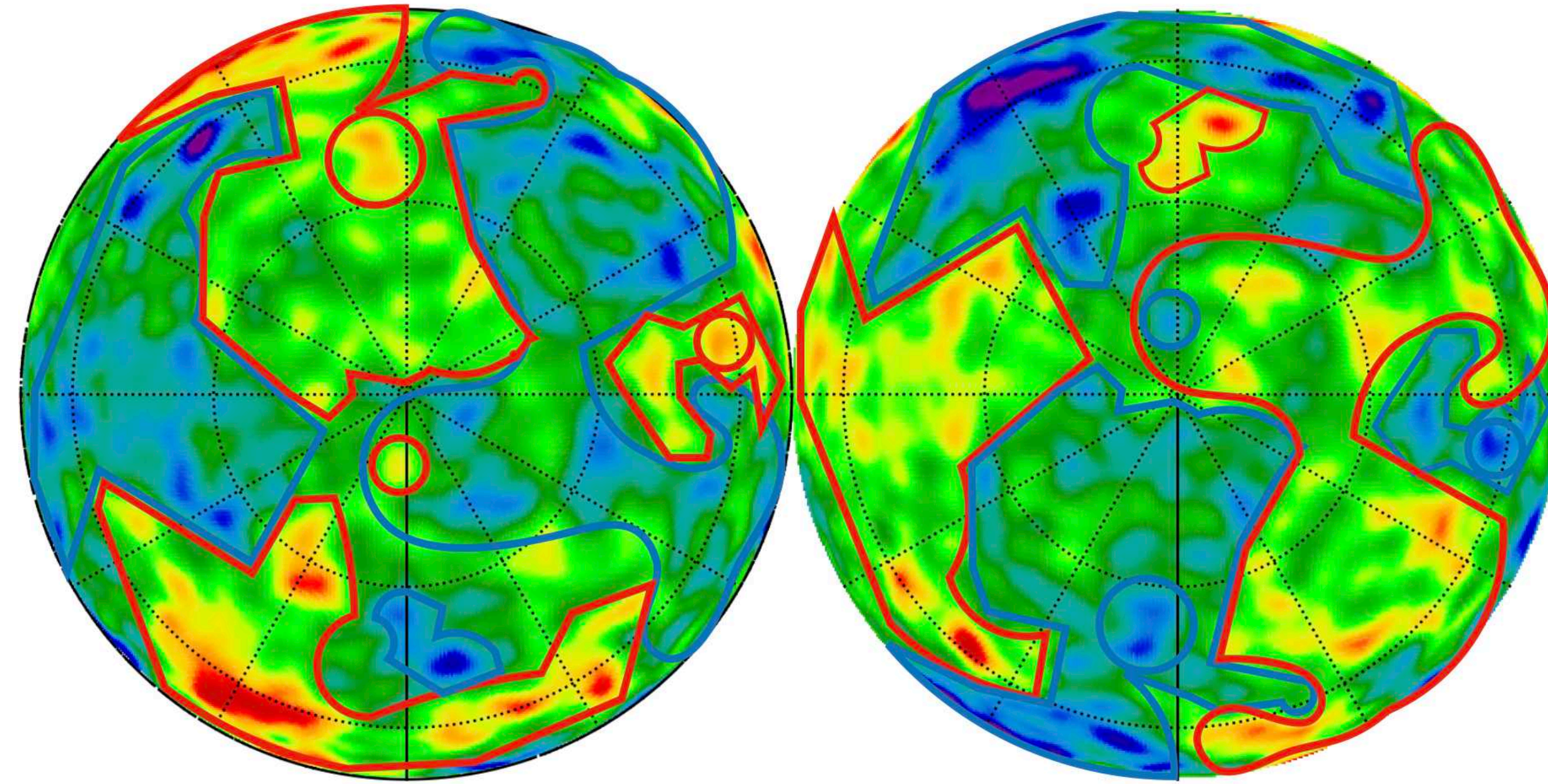


Parity conjugate positions can be found by doing a horizontal flip into the opposite Pole

$$\mathcal{T}(\hat{n}) = \tilde{\mathcal{T}}(\hat{n}) + \Delta\mathcal{T}(\hat{n})$$

$$\Delta\mathcal{T}(\hat{n}) = -\Delta\mathcal{T}(-\hat{n})$$

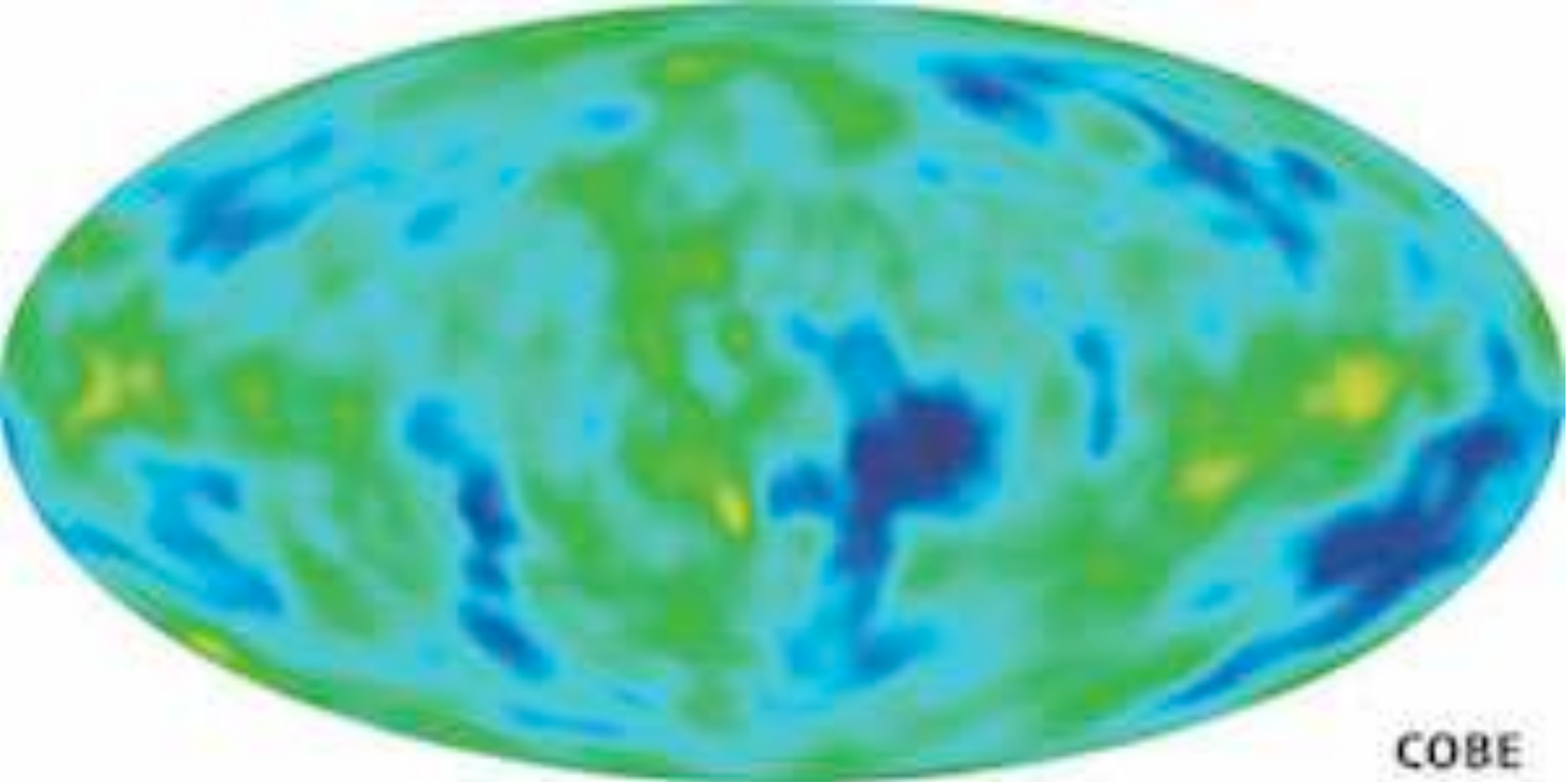
SMICA A



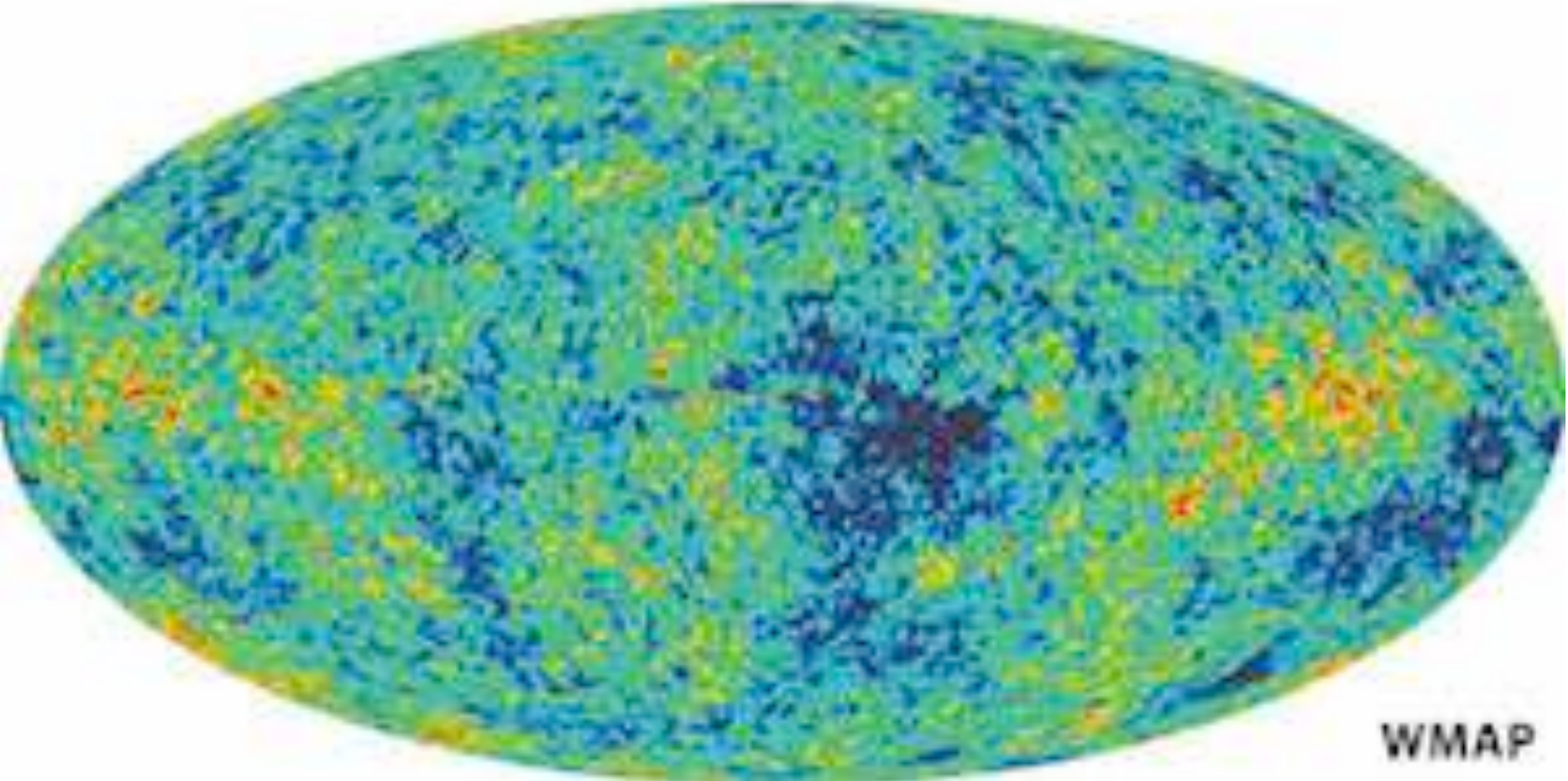
Parity conjugate positions can be found by doing a horizontal flip into the opposite Pole

$$a_{lm} = \begin{cases} \tilde{a}_{lm} - \Delta a_{lm} \equiv a_{lm}^S & \text{for } l = \text{even} \\ \tilde{a}_{lm} + \Delta a_{lm} \equiv a_{lm}^A & \text{for } l = \text{odd} \end{cases}$$

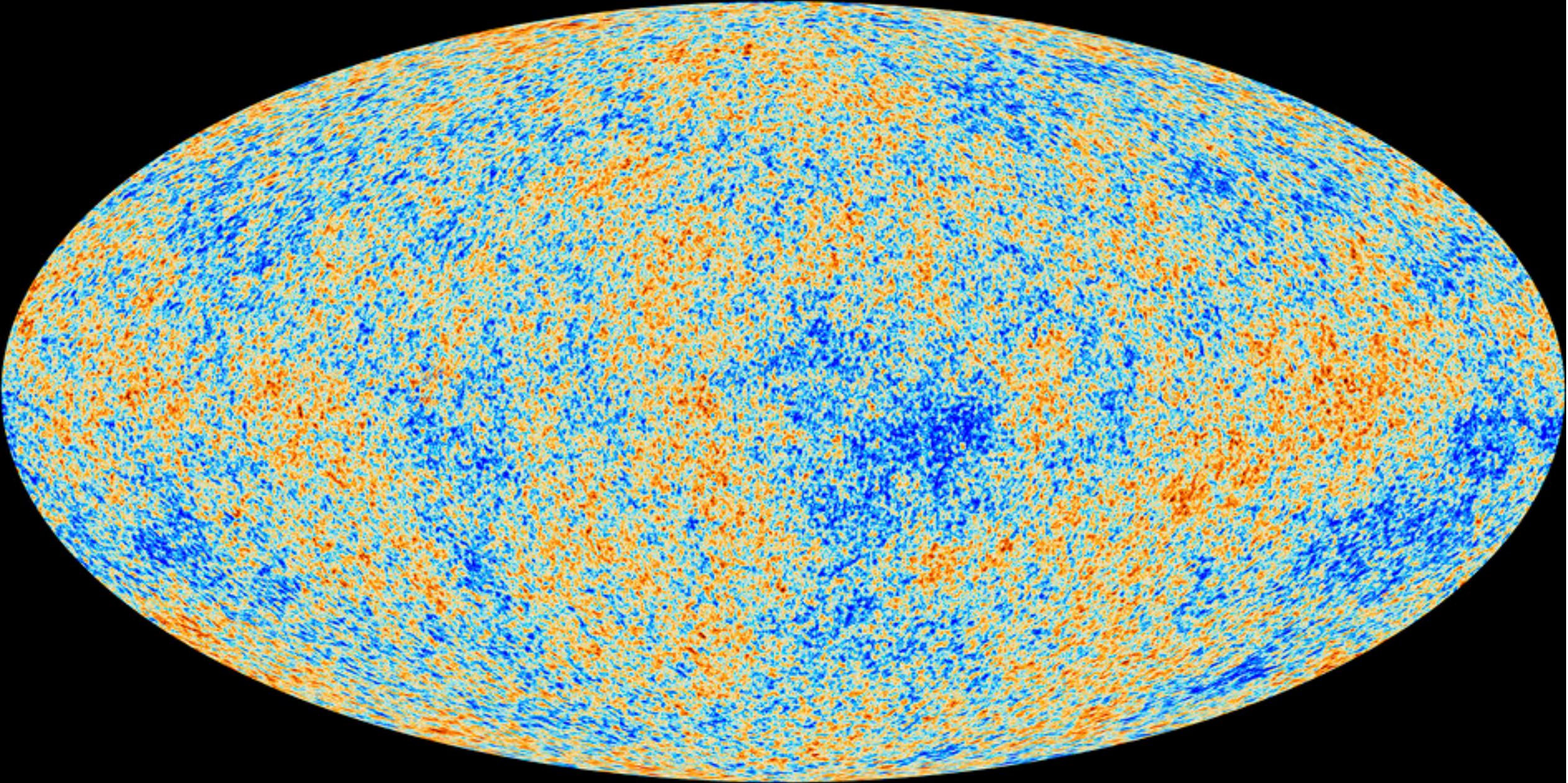
From COBE to WMAP to Planck (1989-2019)



COBE

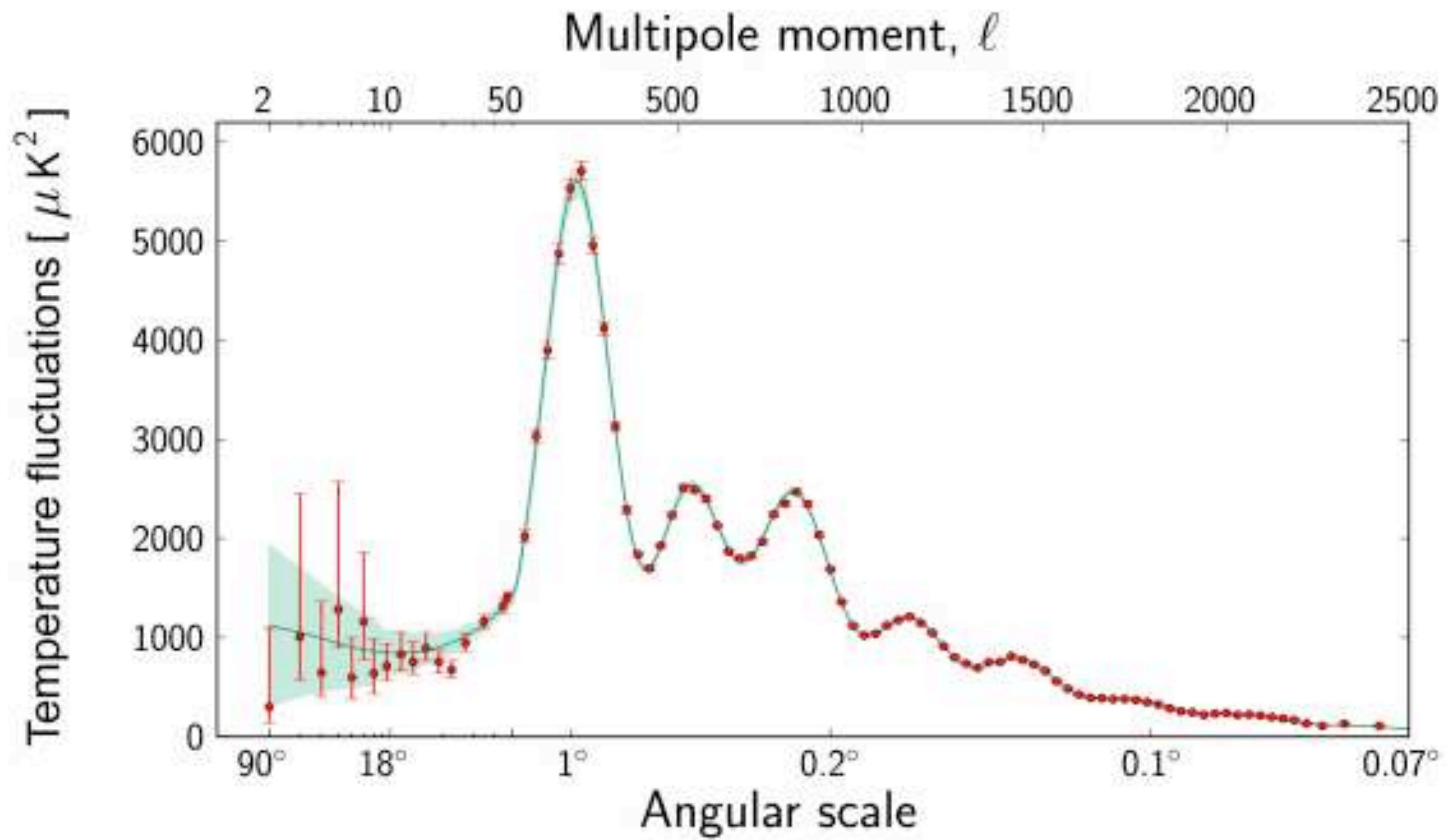


WMAP



Observe large scales

Notice the oscillations in C_ℓ



Planck data A&A 641, A10 (2020)

CMB angular power spectrum

$$C_\ell = \frac{2}{9\pi} \int \frac{dk}{k} \mathcal{P}_\zeta(k) j_\ell^2(k/k_s)$$

The near scale invariant power spectrum

$$\mathcal{P}_\zeta = A_s \left(\frac{k}{k_*} \right)^{n_s - 1}$$

But its only good for $\ell \gtrsim 200$ or $\theta < 1^\circ$

CMB is consistent with inflation?

Yes, because $n_s = 0.964 \approx 1$

No, because its SI power spectrum is

Not good with $\theta > 7^\circ$ or $\ell \lesssim 30$

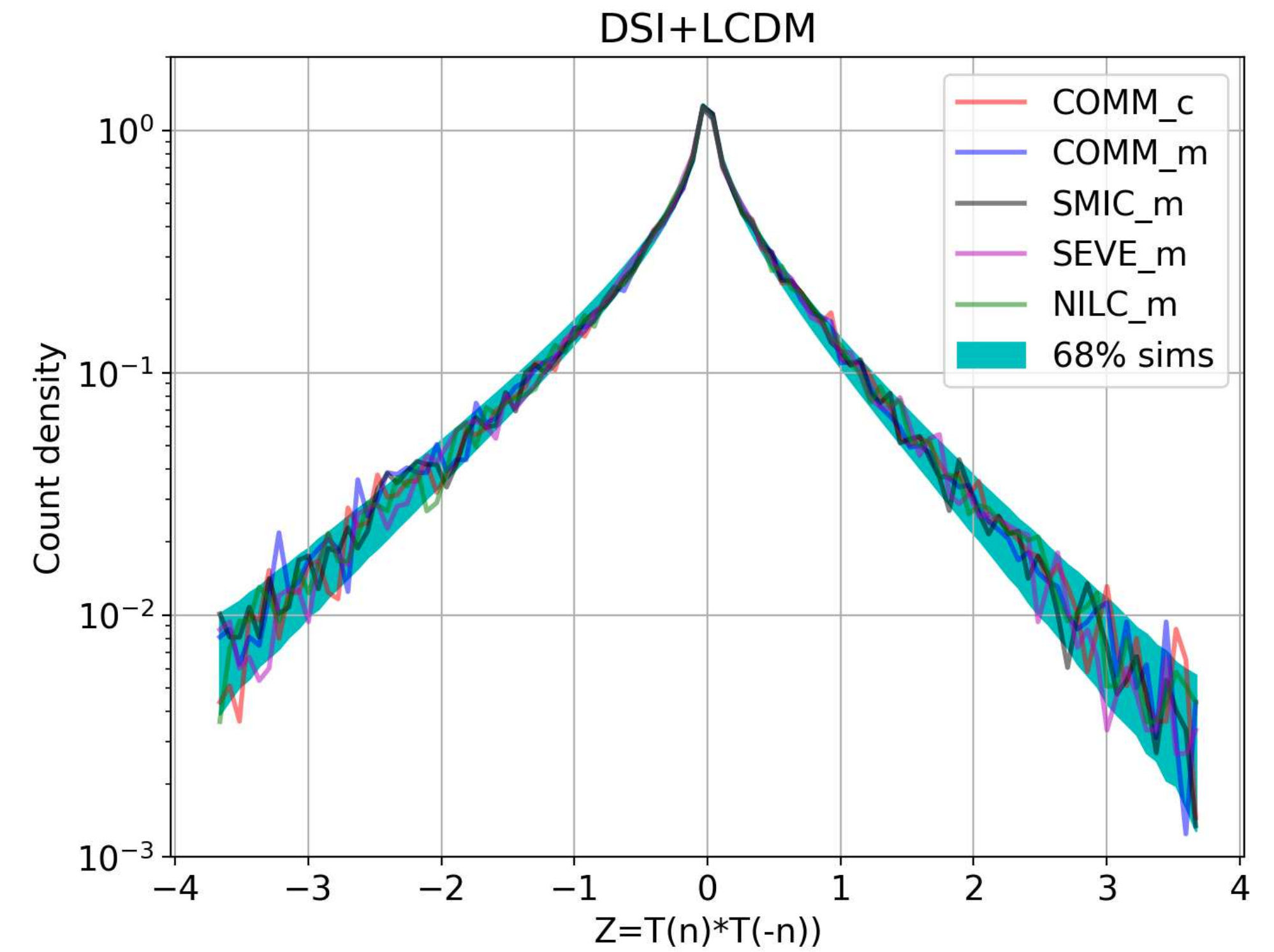
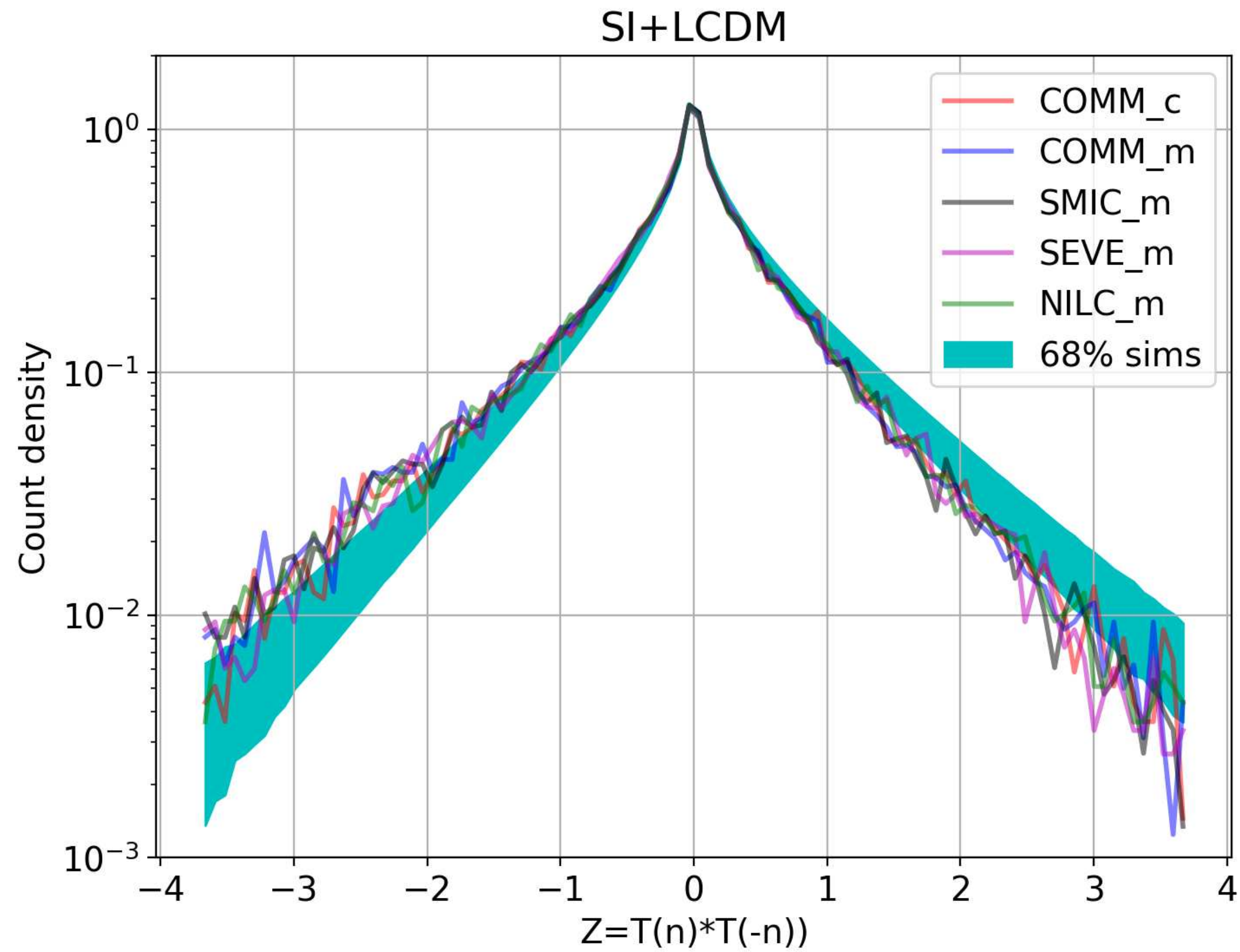
**Its okay nothing to worry
Its all cosmic variance,**

$$\Delta C_\ell = \frac{C_\ell}{\sqrt{(2\ell + 1)f_{sky}}}$$



don't do that please

We measure correlations in configuration space: there is an issue at 180 degrees



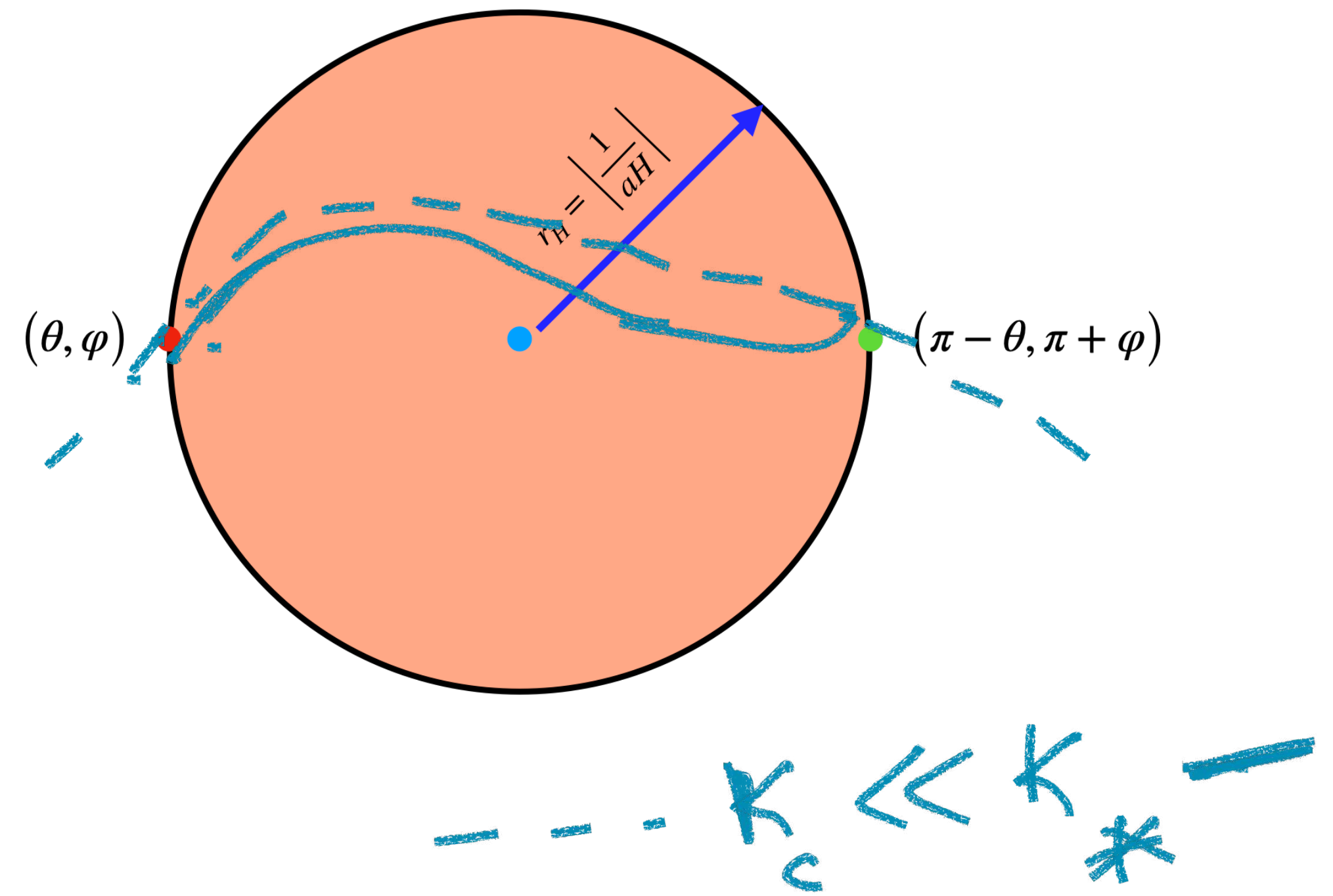
Direct-sum inflation

DSI predictions

$$C_{\ell}^{\text{odd}} = \frac{2}{9\pi} \int_0^{k_c} \frac{dk}{k} j_{\ell}^2 \left(\frac{k}{k_s} \right) \mathcal{P}_{\zeta}(k) (1 + \Delta \mathcal{P}_v)$$

$$C_{\ell}^{\text{even}} = \frac{2}{9\pi} \int_0^{k_c} \frac{dk}{k} j_{\ell}^2 \left(\frac{k}{k_s} \right) \mathcal{P}_{\zeta}(k) (1 - \Delta \mathcal{P}_v)$$

$$\Delta \mathcal{P}_v = (1 - n_s) \operatorname{Re} \left[\frac{2}{H_{3/2}^{(1)} \left(\frac{k}{k_*} \right)} \frac{\partial H_{\nu_s}^{(1)} \left(\frac{k}{k_*} \right)}{\partial \nu_s} \right]_{\nu_s = \frac{3}{2}}$$

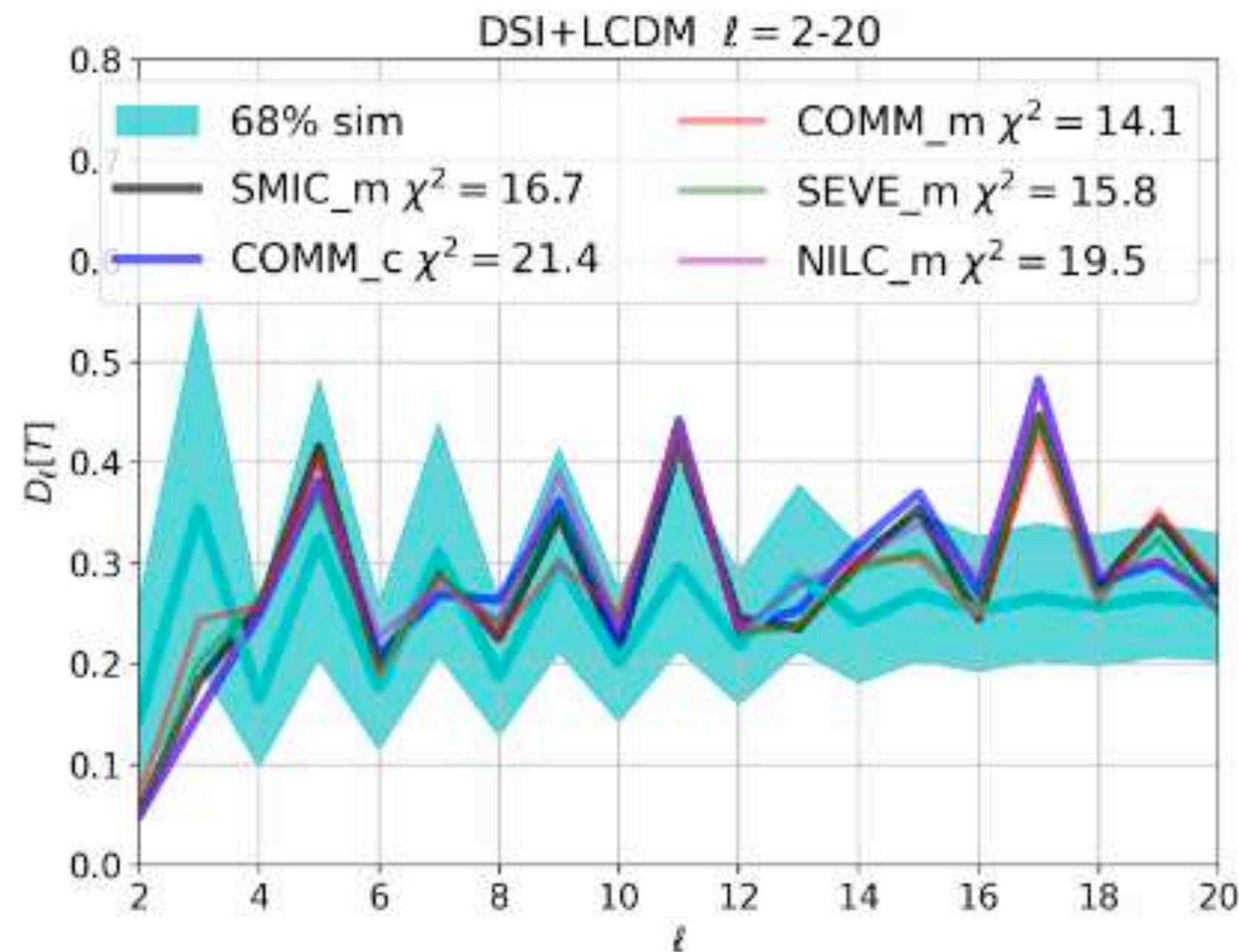
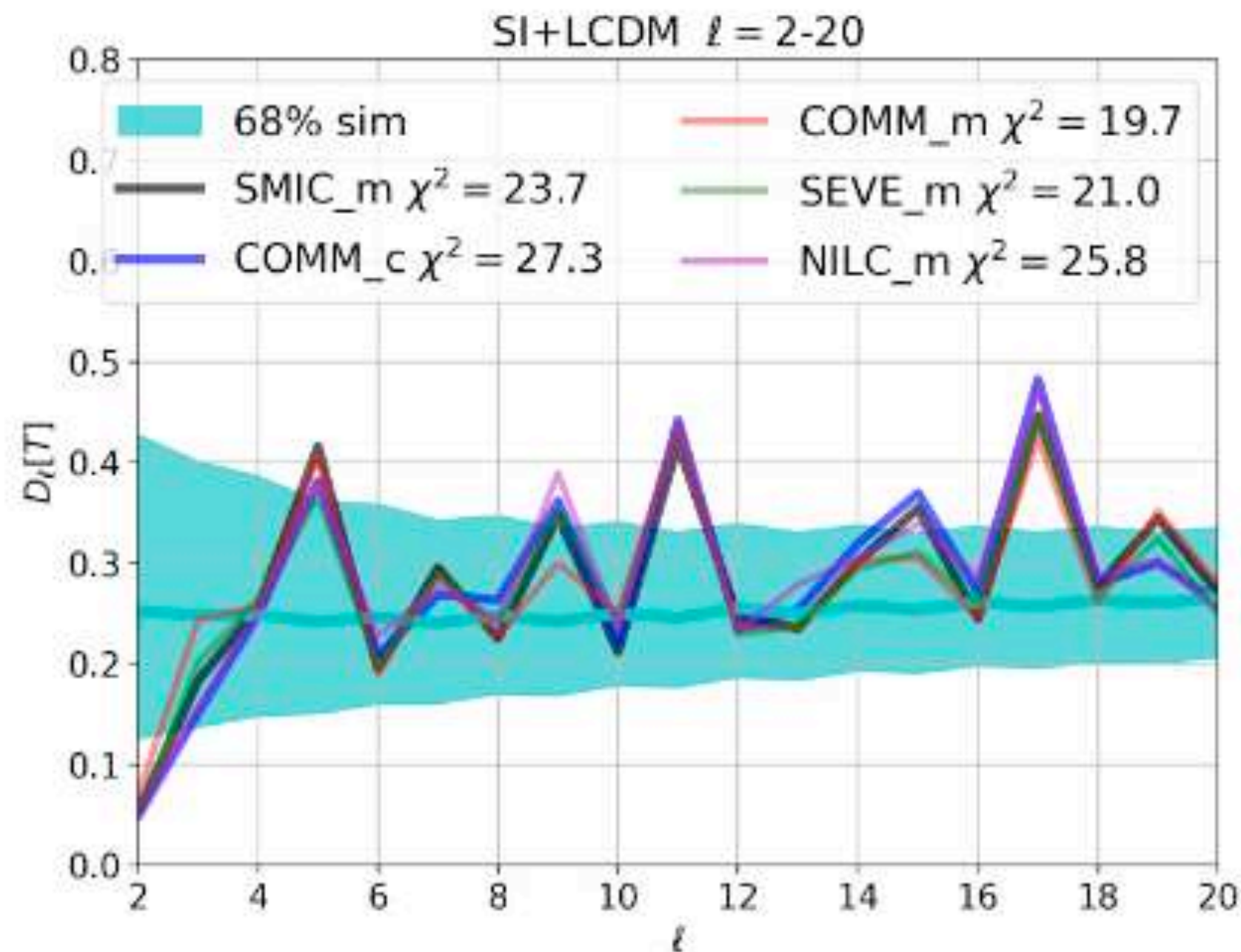
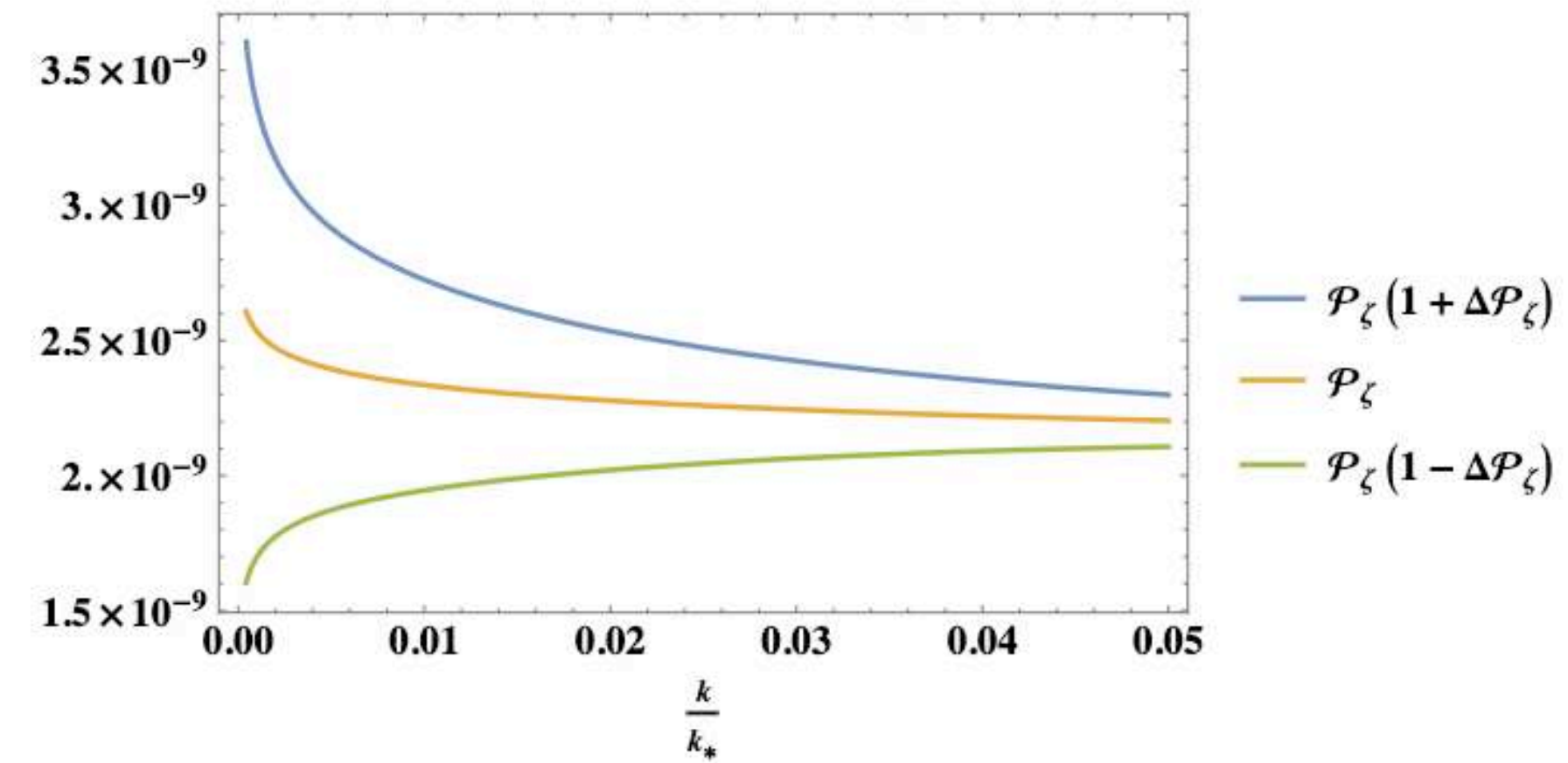


$n_s \approx 0.964$ at $k_* = 0.05 \text{ Mpc}^{-1}$ $k_c = 0.02k_*$ Coarse graining scale $k_s = 7 \times 10^{-5} \text{ Mpc}^{-1}$

DSI and CMB data

$$\mathcal{P}_\zeta = \frac{k^3}{2\pi^2} \frac{1}{2a^2\epsilon} \Bigg|_{\text{classical}} \mathcal{P}_\nu \Bigg|_{\tau=\mp\frac{1}{a_*H_*}}$$

$$\approx \frac{H_*^2}{8\pi\epsilon_*} \left(\frac{k}{k_*}\right)^{n_s-1} \frac{1}{2} \left[2 + \Theta(\tau)\Theta(x)\Delta\mathcal{P}_\nu\left(\frac{k}{k_*}\right) - \Theta(-\tau)\Theta(-x)\Delta\mathcal{P}_\nu\left(\frac{k}{k_*}\right) \right]$$



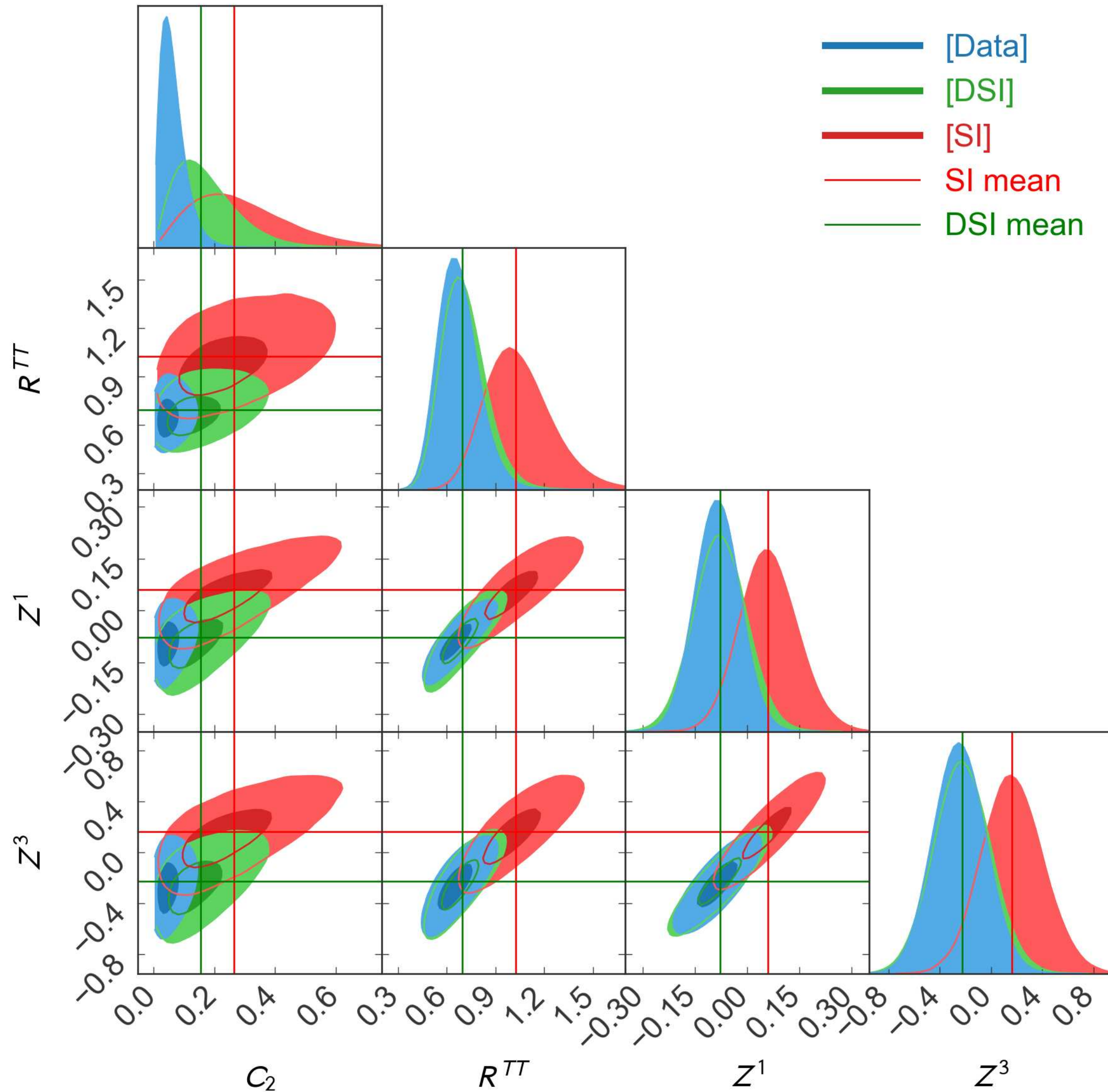
See our paper

arXiv:2401.08288

for more details
especially

For discussion on
Stochastic inflation and
non-Markovian nature of
inflationary fluctuations.

Direct-sum inflation vs Standard inflation



$$R^{TT} = \frac{D_+(\ell_{max})}{D_-(\ell_{max})} = \frac{\sum_{\ell=even}^{\ell_{max}} \ell(\ell+1)C_\ell}{\sum_{\ell=odd}^{\ell_{max}} \ell(\ell+1)C_\ell}$$

$$w(180^\circ) = \langle Z \rangle = \langle \mathcal{T}(\hat{n})\mathcal{T}(-\hat{n}) \rangle$$

$$= \sum_{\ell}^{\ell_{max}} \frac{2\ell+1}{4\pi} [C_{\ell=even} - C_{\ell=odd}]$$

$$Z^1 \equiv \langle Z(\bar{n}) \rangle = \sum_{i=1}^{12N_{side}^2} P(Z_i)Z_i$$

$$Z^3 \equiv \langle Z^3 \rangle = \sum_{i=1}^{12N_{side}^2} P(Z_i)(Z_i - \langle Z \rangle)^3$$

Testing models with data and vice versa: Standard Inflation versus Direct-sum Inflation

$$R^{TT} \approx 0.79$$

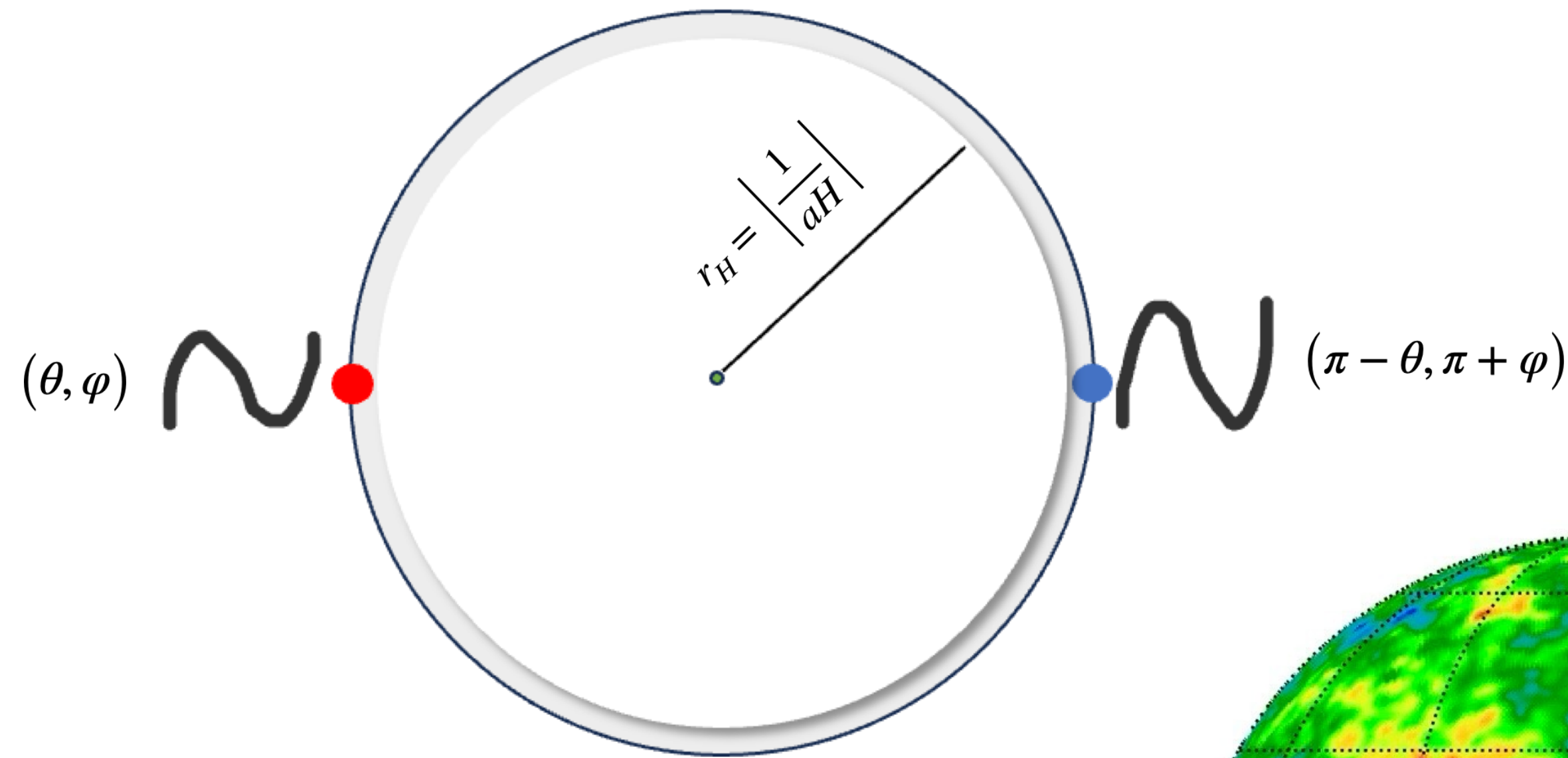
p-value: $p[M D] \times 10^2$	SI	DSI	ratio
HARMONIC space:			DSI/SI
C_2	0.09	3.3	37
$R^{TT} (\ell \leq 20)$	0.7	39.5	56
$C_2, R^{TT} (\ell \leq 20)$	0.003	1.96	653
CONFIGURATION space:			
$Z^1 = w[\pi]$	1.12	45.3	40
Z^3	2.10	45.6	22
Z^1, Z^3	0.67	36.3	54
COMBINED:			
$Z^1, R^{TT} (\ell \leq 20)$	0.45	34.6	77
Z^1, C_2	0.016	2.65	166

p-value: $p[D M] \times 10^2$	SI	DSI	ratio
PARITY:			DSI/SI
C_2	2.62	8.88	3.4
$R^{TT} (\ell \leq 20)$	1.00	39.7	40
$Z^1 = w[\pi]$	3.89	46.3	12
Z^3	1.78	37.1	21
Z^3, R^{TT}	0.58	32	55
C_2, R^{TT}	0.12	5.59	47

DSI is 650 times more favourable than the standard inflation:

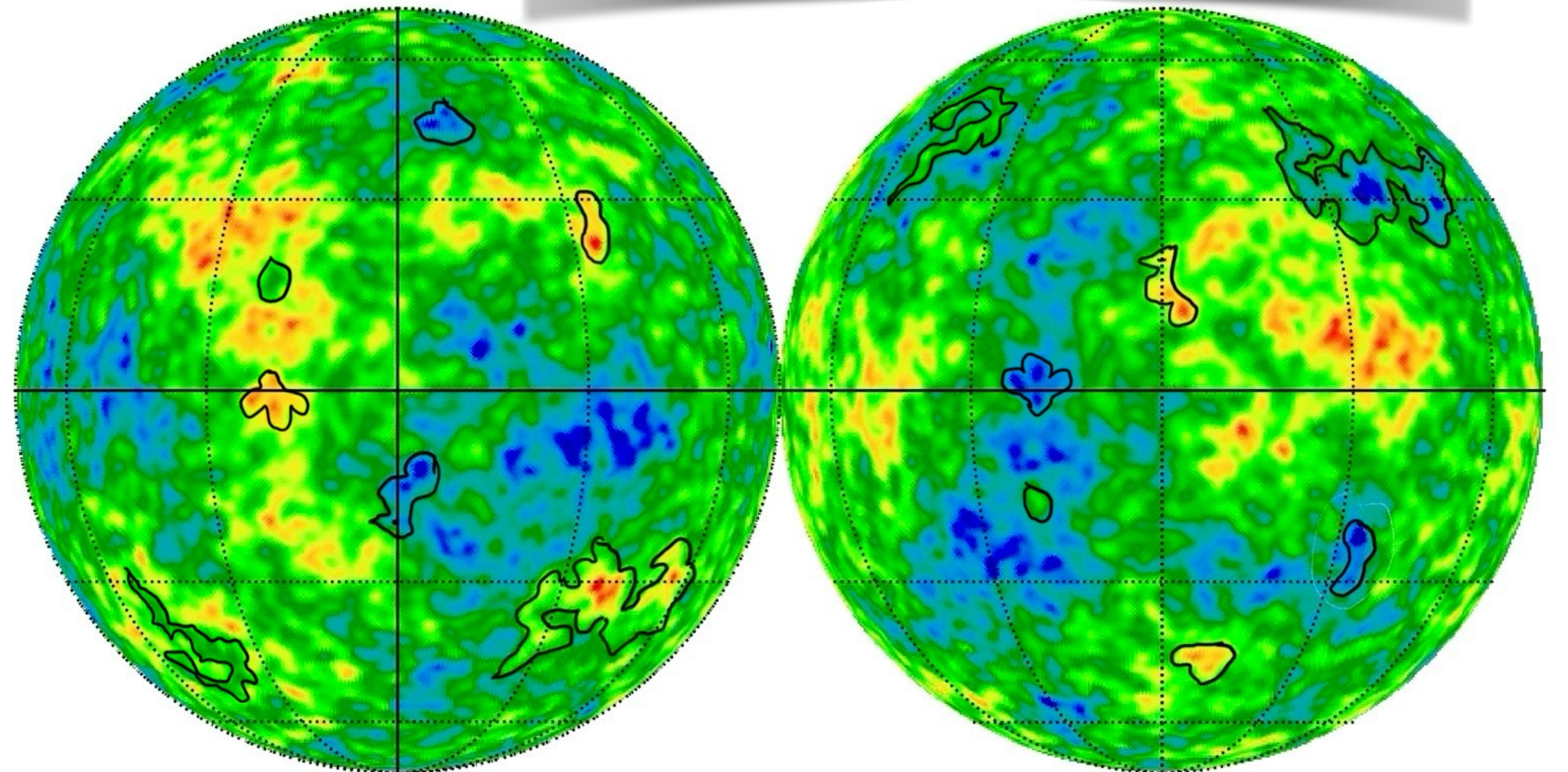
A compelling evidence for DQFT in curved spacetime

The physics of direct-sum inflation

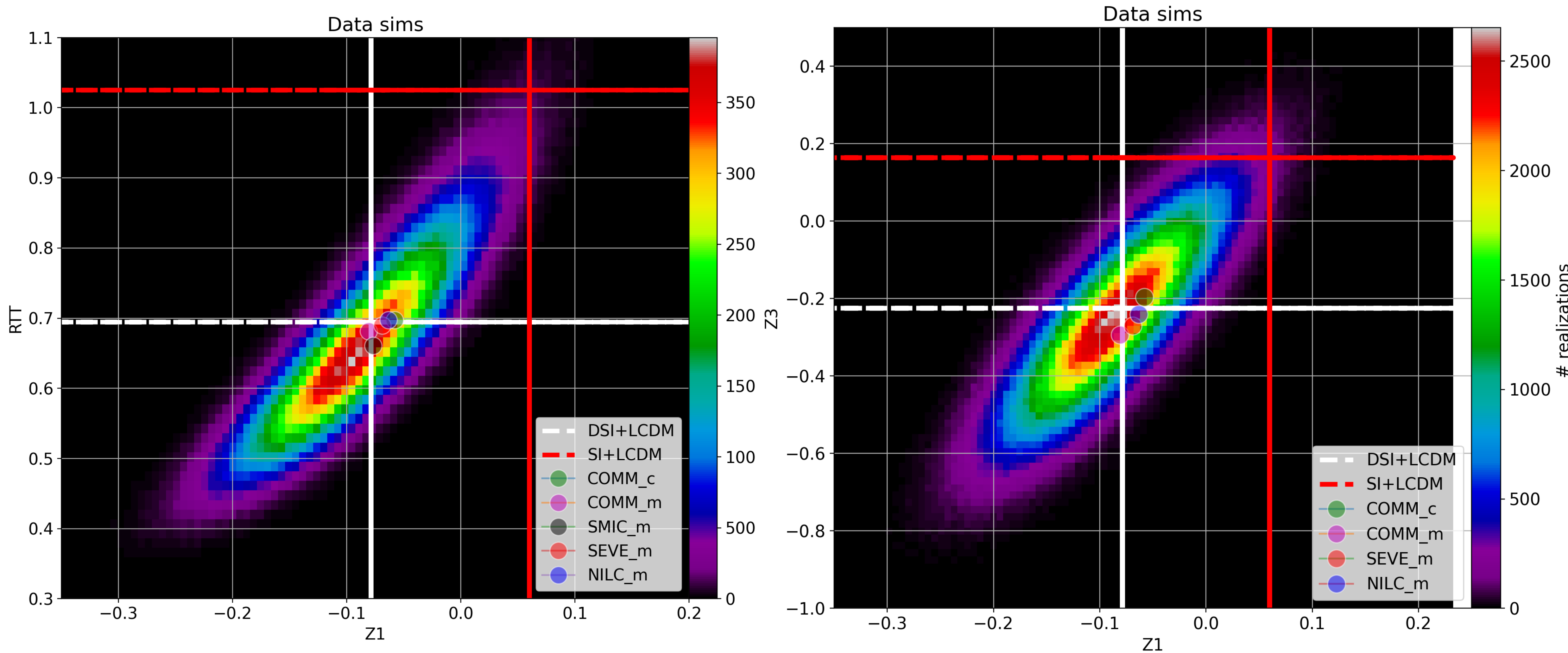


Our direct-sum inflation (DSI) is a framework in which a quantum fluctuation evolves forward and backward in time at parity conjugate points.

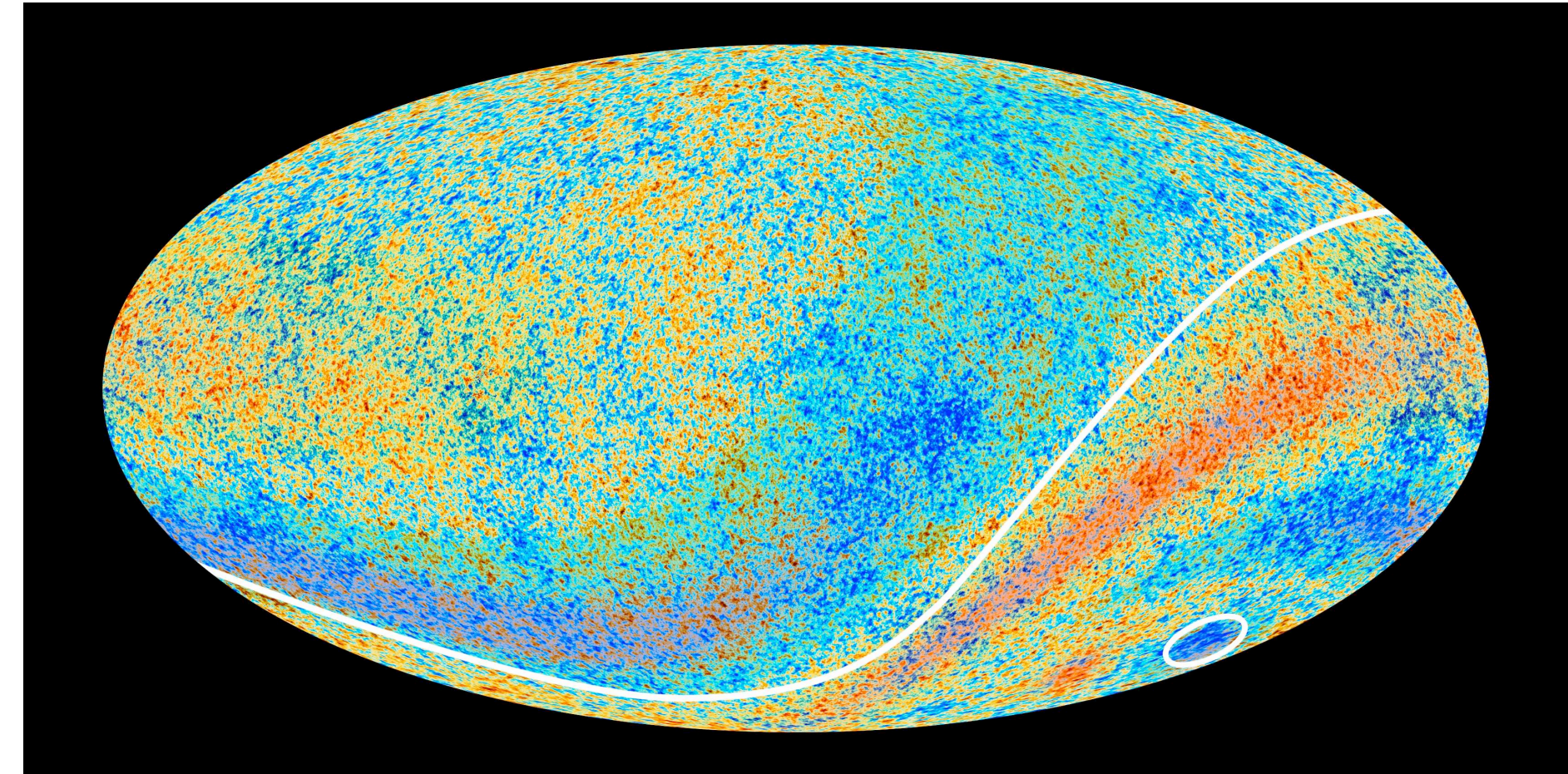
Inflation violates time reversal symmetry which implies P-violation in the framework of DSI.



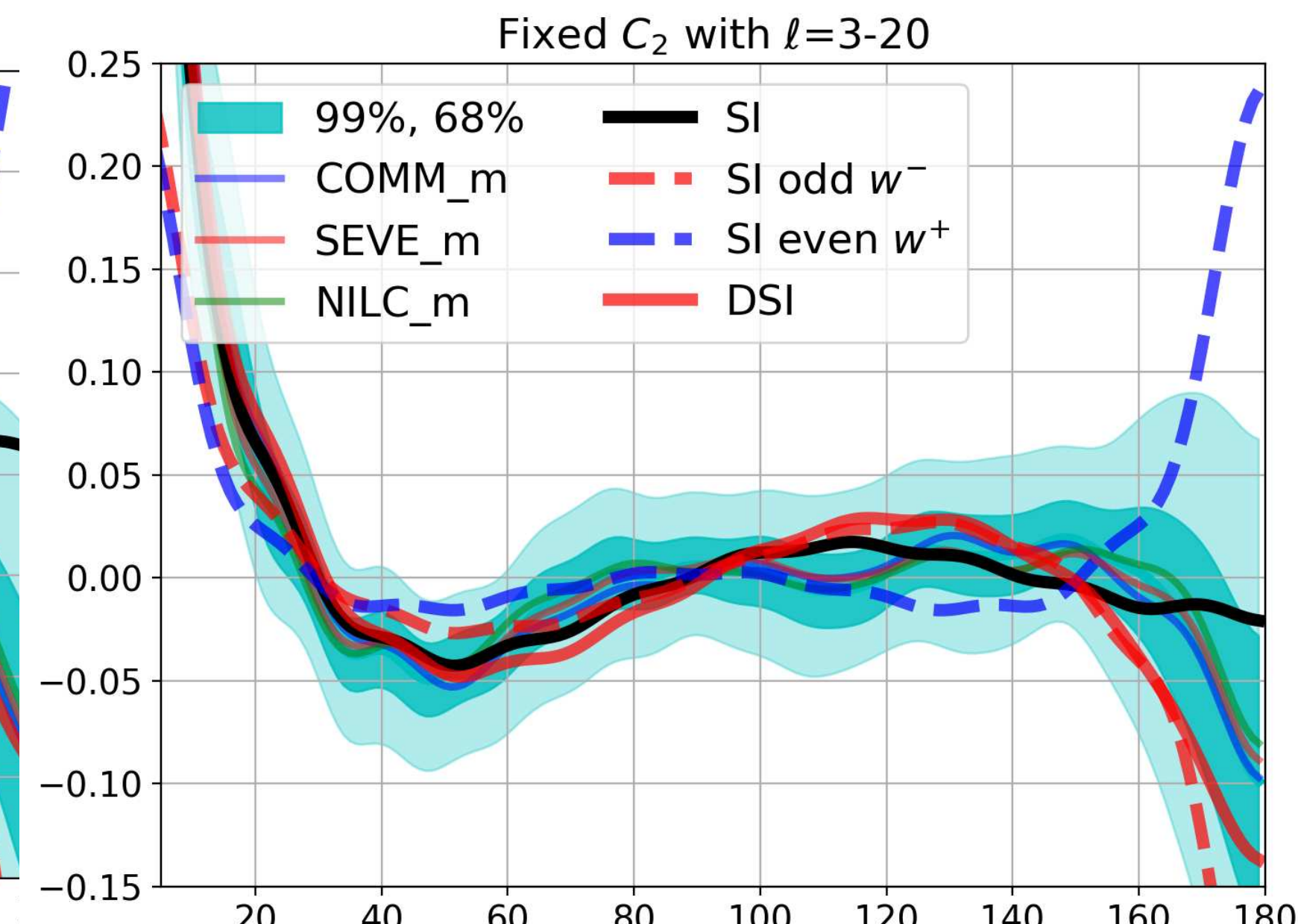
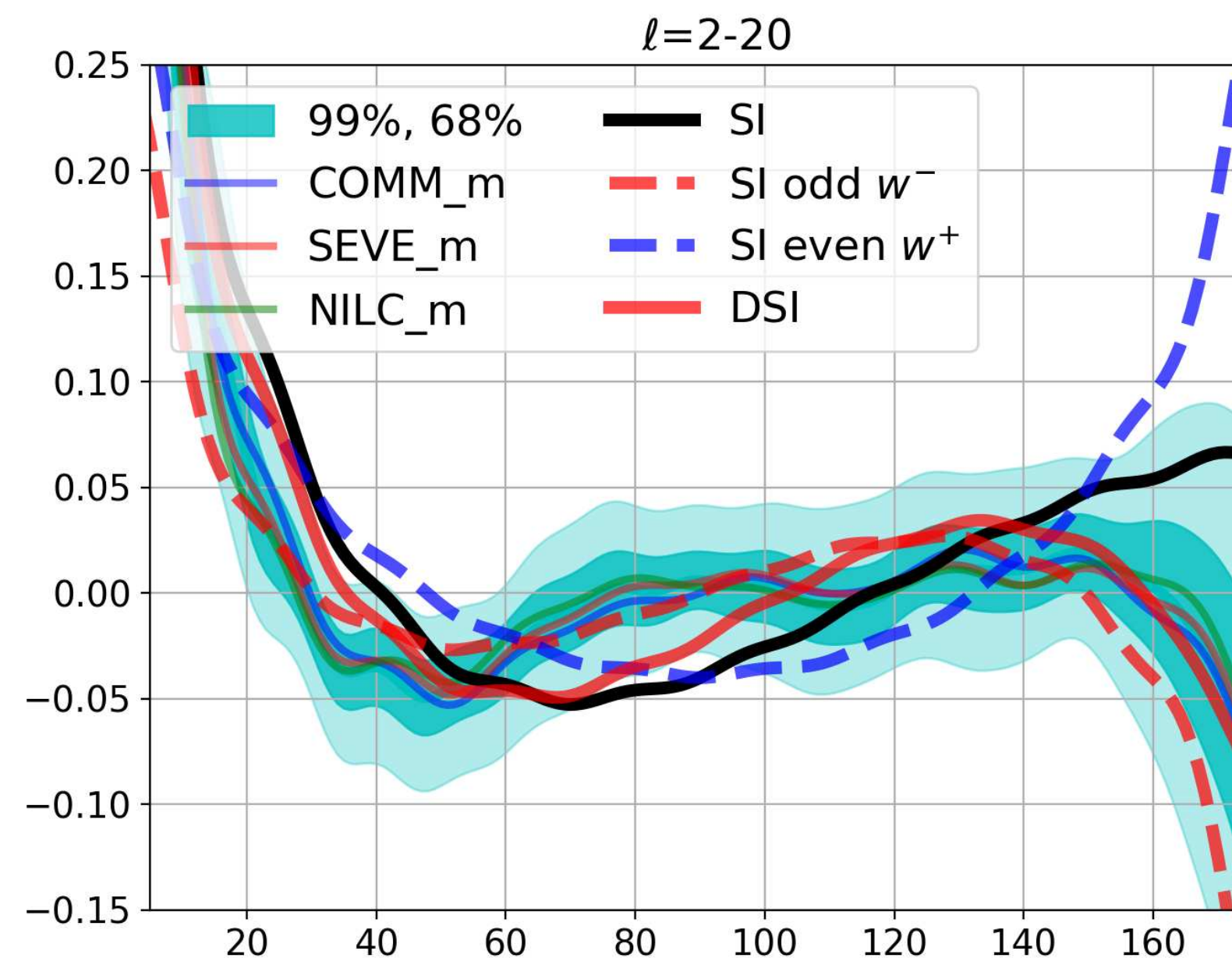
Compelling proof of parity asymmetry and DSI



There are no other anomalies: Ruling out hemispherical power asymmetry and axis of evil

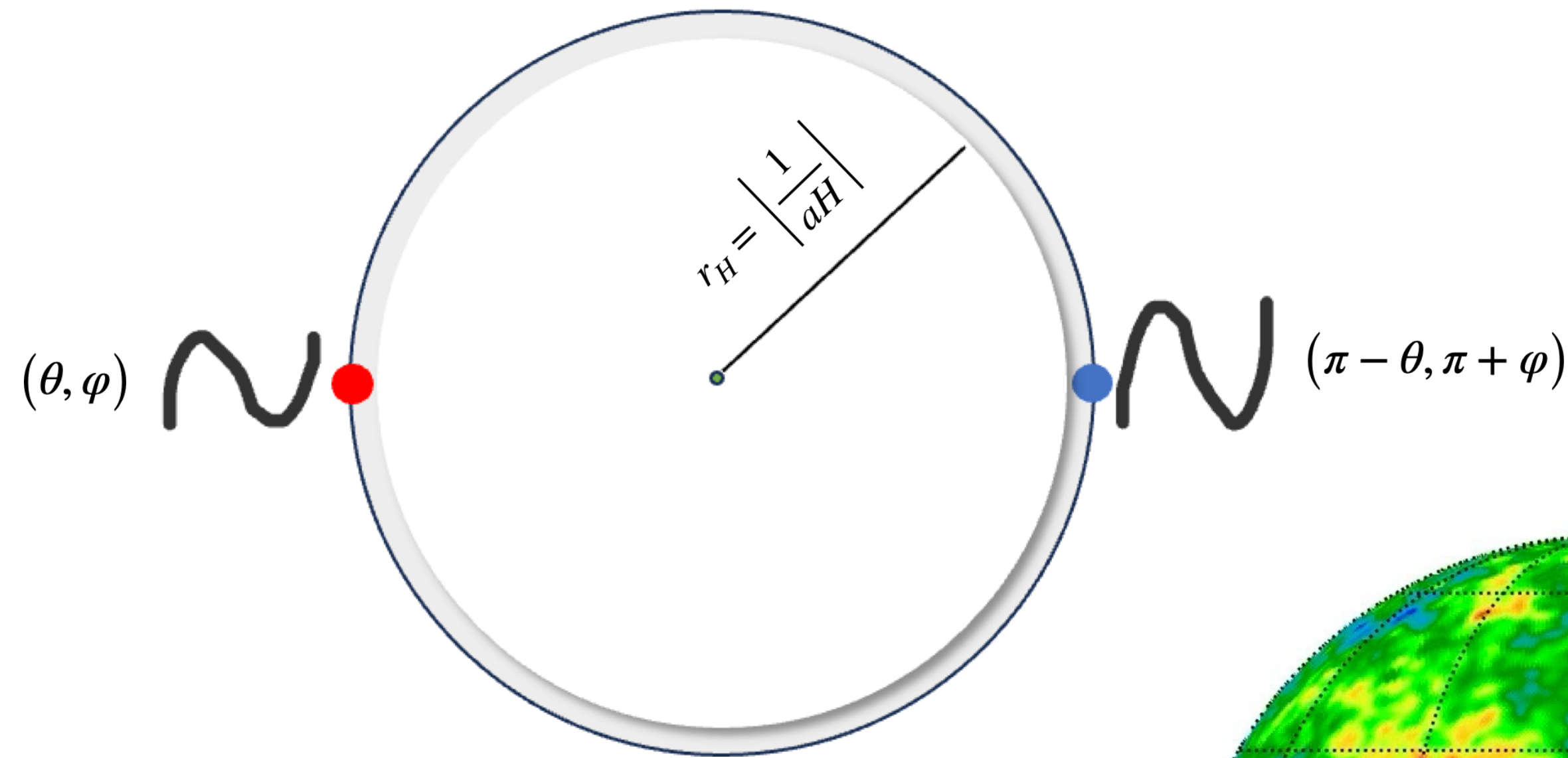


p-value: $p[D M] \times 10^2$	SI	SI- C_2	DPM
PARITY:			A=0.07 (0.5)
C_2	2.62	47.8	2.6 (2.1)
$R^{TT} (\ell \leq 20)$	1.00	4.07	1.0 (0.4)
$Z^1 = w[\pi]$	3.89	19.3	4.0 (3.9)
Z^3	1.78	9.0	1.8 (1.8)
Z^3, R^{TT}	0.58	3.01	0.7 (0.3)
C_2, R^{TT}	0.12	1.31	0.1 (0.03)
OTHER ANOMALIES:			
2-PT $w(\theta): S_{1/2}$	0.08	9.8	
HPA: σ_{16}^2	0.49	4.6	
HPA: $\sigma_{16}^2[db]$	2.50	14.9	



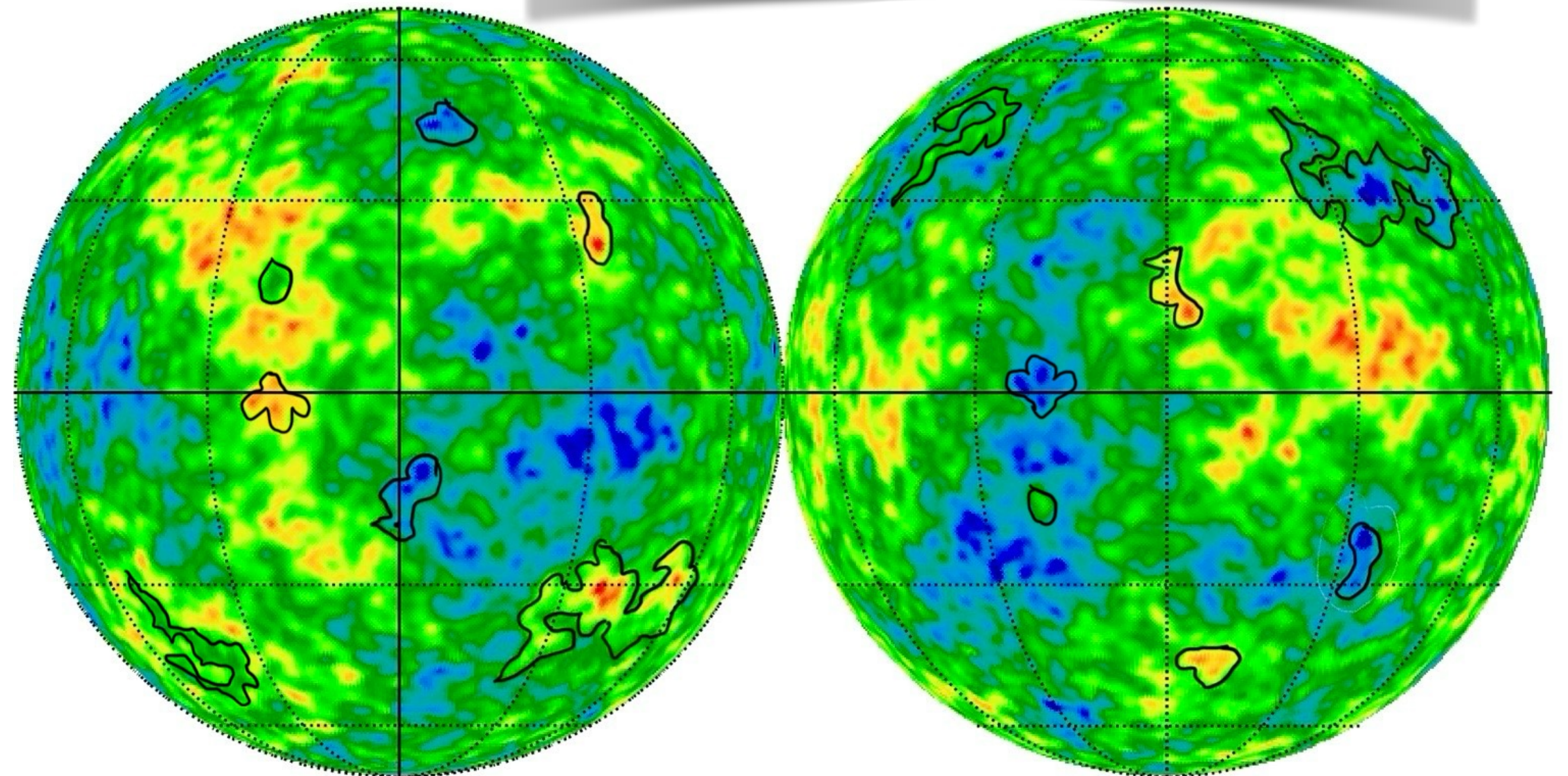
**These anomalies which
are not real resulted in
theory speculations in
100s of papers.**

The physics of direct-sum inflation



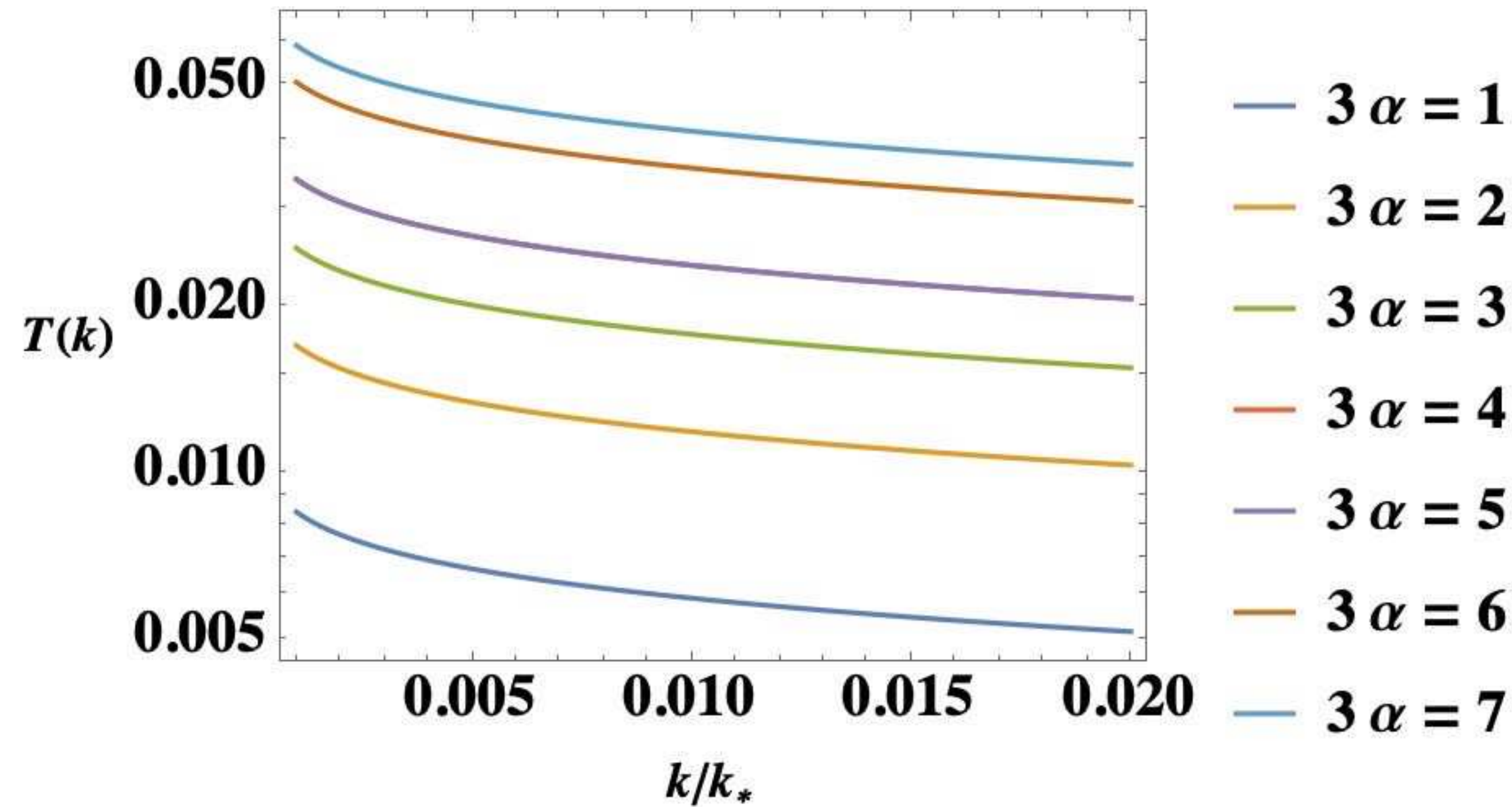
Our direct-sum inflation (DSI) is a framework in which a quantum fluctuation evolves forward and backward in time at parity conjugate points.

Inflation violates time reversal symmetry which implies P-violation in the framework of DSI.



Parity Asymmetry in Primordial Gravitational Wave Spectra

arXiv: 2209.03928v4 KSK, J. Marto



$$V(\phi) = V_0 \left(1 - e^{-\sqrt{\frac{2}{3\alpha}}\phi} \right)^2$$

$$T(k) = \frac{\mathcal{P}_{h+}(k, \hat{\mathbf{x}}) - \mathcal{P}_{h-}(k, -\hat{\mathbf{x}})}{4\mathcal{P}_h}$$

$$\hat{u}_{ij} = \frac{1}{\sqrt{2}} \hat{u}_{ij}^+(\tau, \mathbf{x}) \oplus \frac{1}{\sqrt{2}} \hat{u}_{ij}^(-\tau, -\mathbf{x}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \hat{u}_{ij}^+(\tau, \mathbf{x}) & 0 \\ 0 & \hat{u}_{ij}^-(\tau, \mathbf{x}) \end{pmatrix}$$

Einstein-Rosen's conjecture (1935) to solve the problems of GR+QM

A particle in the physical world should be mathematically described by a bridge between two sheets of spacetime.



Featured in Physics Free to Read Access by

The Particle Problem in the General Theory of Relativity

A. Einstein and N. Rosen
Phys. Rev. **48**, 73 – Published 1 July 1935

Physics

Article References Citing Articles (894) PDF Export Citation

ABSTRACT

The writers investigate the possibility of an atomistic theory of matter and electricity which, while excluding singularities of the field, makes use of no other variables than the $g_{\mu\nu}$ of the general relativity theory and the ϕ_μ of the Maxwell theory. By the consideration of a simple example they are led to modify slightly the gravitational equations which then admit regular solutions for the static spherically symmetric case. These solutions involve the mathematical representation of physical space by a space of two identical sheets, a particle being represented by a "bridge" connecting these sheets. One is able to understand why no neutral particles of negative mass are to be found. The combined system of gravitational and electromagnetic equations are treated similarly and lead to a similar interpretation. The most natural elementary charged particle is found to be one of zero mass. The many-particle system is expected to be represented by a regular solution of the field equations corresponding to a space of two identical sheets joined by many bridges. In this case, because of the absence of singularities, the field equations determine both the field and the motion of the particles. The many-particle problem, which would decide the value of the theory, has not yet been treated.

Received 8 May 1935

Origin of ER conjecture

A particle in the physical world should be mathematically described by a bridge between two sheets of spacetime.

- ER focussed on understanding **quantum** fields at $r > 2GM$
- There are two realizations to represent

$$r > 2GM \implies \begin{cases} U < 0, V > 0 \\ U > 0, V < 0 \end{cases}$$

$$T \rightarrow -T \implies U \rightarrow -U, V \rightarrow -V$$

$$UV = \left(1 - \frac{r}{2GM}\right) e^{r/2GM}, \quad U = \pm \kappa e^{-\kappa u}, \quad V = \mp \kappa e^{-\kappa u}$$

$$ds^2 = \frac{2GM}{r} e^{1-\frac{r}{2GM}} dUdV + r^2 d\Omega^2, \quad X = \frac{V-U}{2}, \quad T = \frac{U+V}{2}$$

ER ignored angular coordinates (θ, φ)

Two time realizations for one physical Universe

Let us take De sitter Universe $R = 12H^2$

$$ds^2 = -dt^2 + e^{2Ht} d\mathbf{x}^2, \quad a = e^{Ht}$$

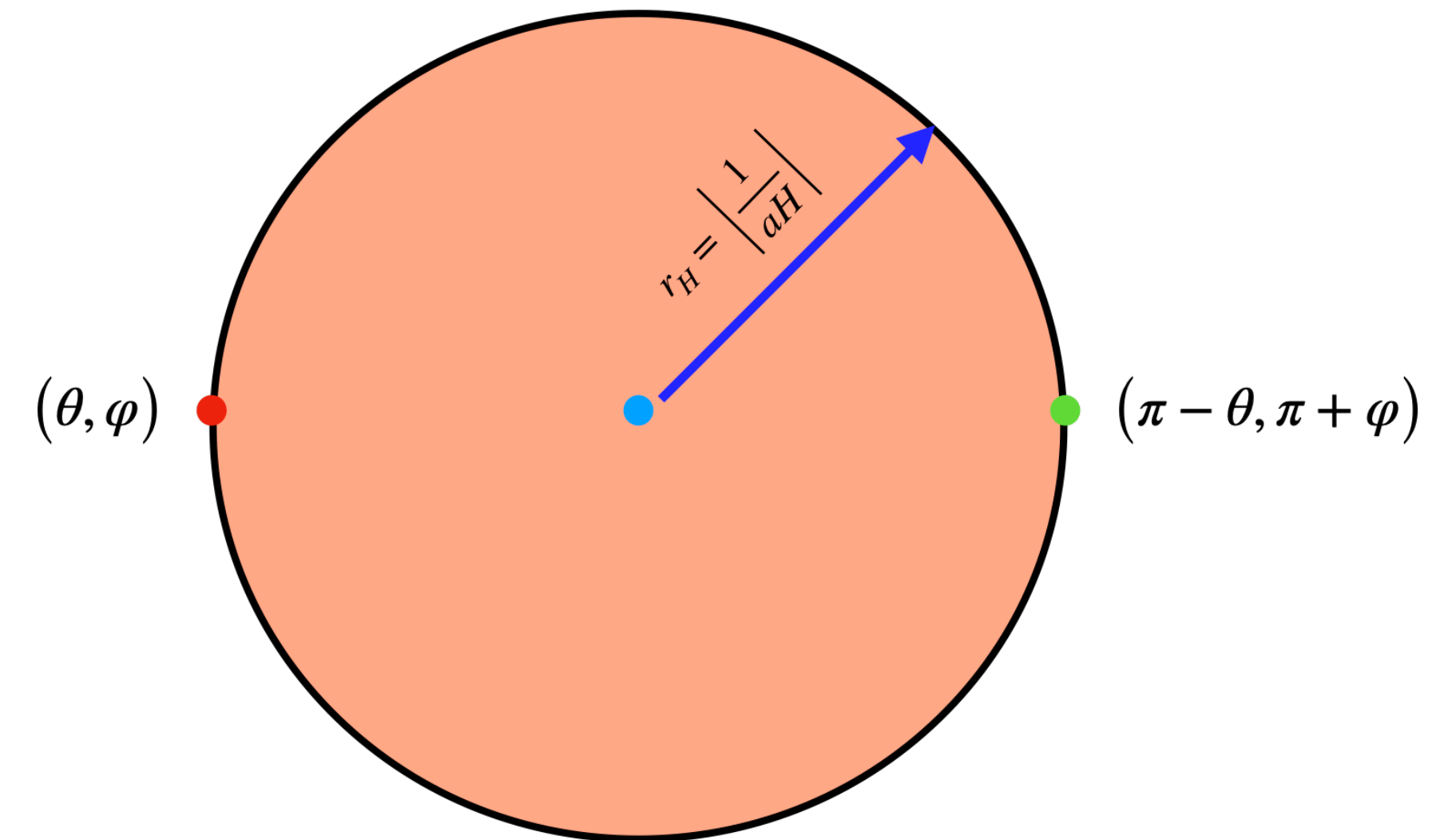
$$ds^2 = \frac{1}{H^2\tau^2} (-d\tau^2 + dx^2), \quad H = \frac{1}{a} \frac{da}{dt}$$

Doing QFT with $\tau < 0$ is the origin of unitarity problem

Expanding Universe $\implies \begin{cases} H > 0; t : -\infty \rightarrow \infty \\ H < 0; t : \infty \rightarrow -\infty \end{cases}$

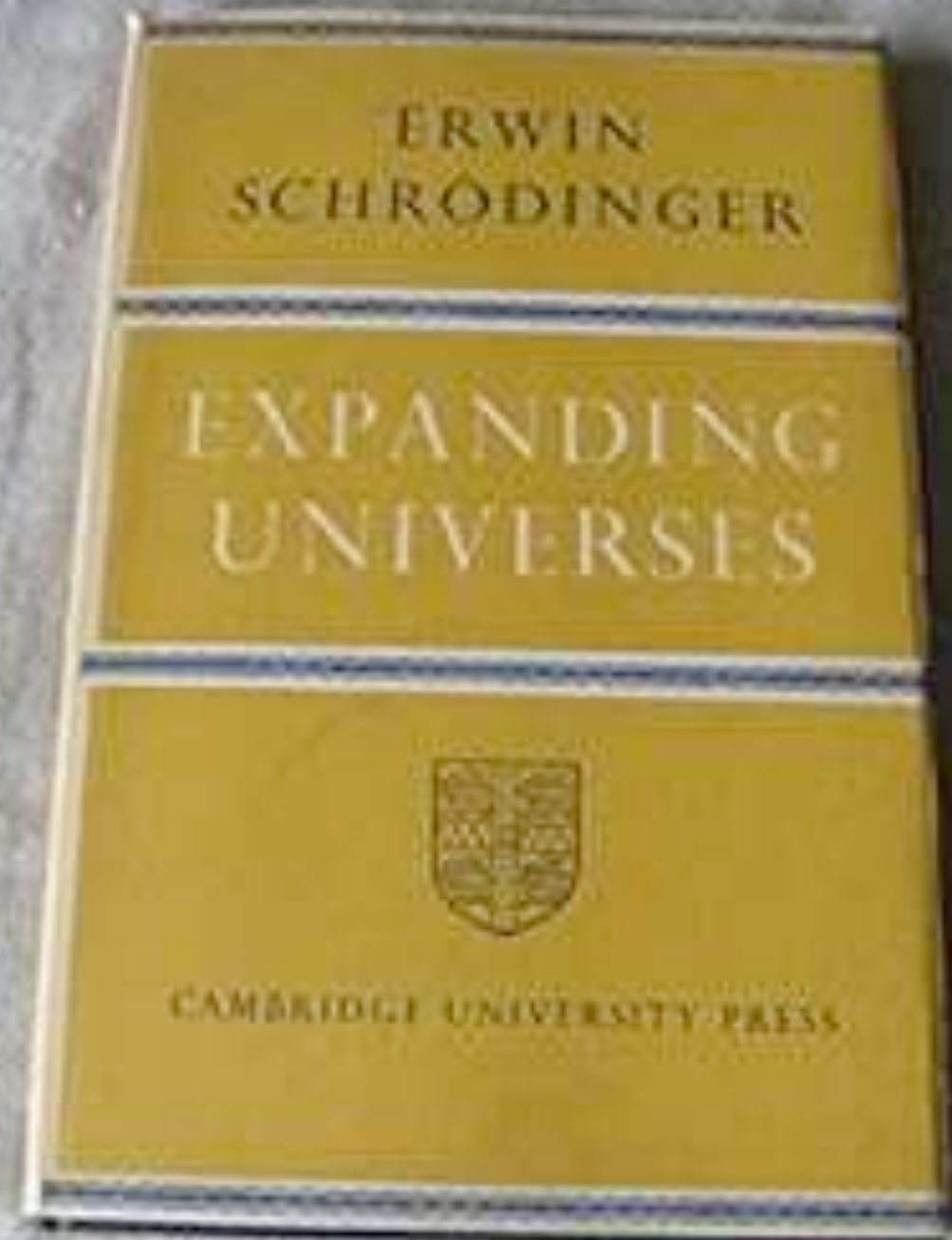
The PT symmetry

$$\tau \rightarrow -\tau \implies \left\{ t \rightarrow -t, H \rightarrow -H, \mathbf{x} \rightarrow -\mathbf{x} \right\}$$



The fundamental question of unitarity in curved space-time

1956

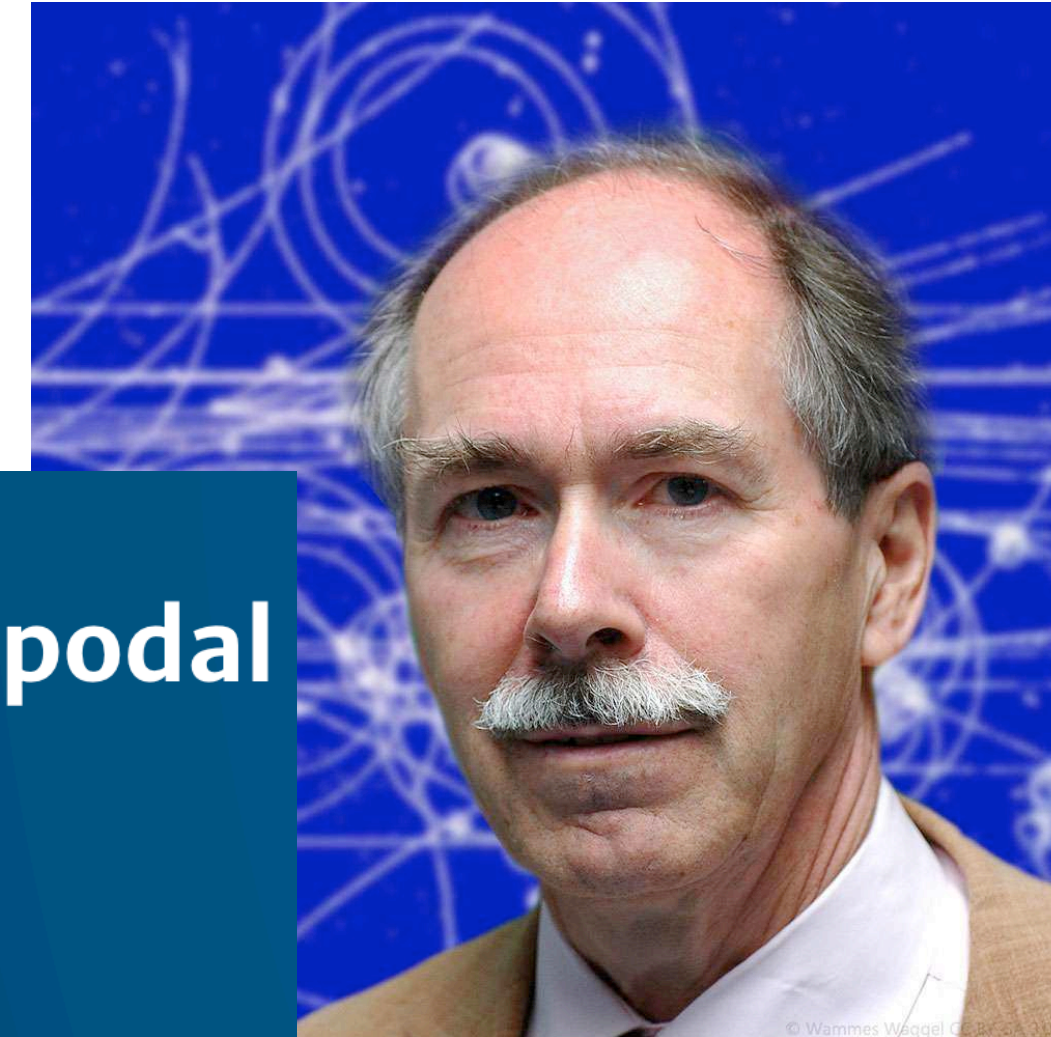


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Black Hole Unitarity and Antipodal Entanglement

[Open access](#) | Published: 05 May 2016

Volume 46, pages 1185–1198, (2016) [Cite this article](#)



In standard QFT in curved spacetime unitarity is lost because pure states evolve into mixed states. This is because part of a pure state can disappear beyond the horizon then we end up with mixed states within the horizon.

Anti-unitary nature of time and definition of positive energy



Space and time are not on equal footing in quantum theory

Time is parameter and position is an operator: Wigner (1926)

$$i \frac{\partial |\Psi\rangle}{\partial t_p} = \hat{H} |\Psi\rangle = E |\Psi\rangle \implies |\Psi\rangle = e^{-iEt_p} |\Psi_0\rangle, \quad t_p : -\infty \rightarrow \infty$$

$$-i \frac{\partial |\Psi\rangle}{\partial t_p} = \hat{H} |\Psi\rangle = E |\Psi\rangle \implies |\Psi\rangle = e^{iEt_p} |\Psi_0\rangle, \quad t_p : \infty \rightarrow -\infty$$

J. Donogue, G. Menezes (2019), 'tHooft (2018)

A quantum state in the physical world should be mathematically described by a bridge between two sheets of spacetime (ER 1935)

- We divide the physical space (spatial) into two parity conjugate regions
- A state is direct-sum of two components
$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|\Psi_+\rangle \oplus |\Psi_-\rangle) = \frac{1}{\sqrt{2}} \begin{pmatrix} |\Psi_+\rangle \\ |\Psi_-\rangle \end{pmatrix}$$
in the pair of parity conjugate regions corresponding to two superselection sector Hilbert spaces
$$\mathcal{H} = \mathcal{H}_+ \oplus \mathcal{H}_-$$
- A positive energy state in \mathcal{H}_+ is defined according to arrow of time $t : -\infty \rightarrow \infty$ where as a positive energy state in \mathcal{H}_- is defined according to arrow of time $t : \infty \rightarrow -\infty$

The direct-sum is the mathematical bridge

Quantum Harmonic Oscillator & Schrödinger Equation

$$i\frac{\partial}{\partial t}|\Psi\rangle = \hat{H}|\Psi\rangle$$

Where \hat{H} is Hamiltonian $[\hat{H}, \mathcal{PT}] = 0$

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}k^2\hat{x}^2, \quad \hat{p} = -i\frac{\partial}{\partial x}, \quad [\hat{x}, \hat{p}] = i \quad (\hbar = 1)$$

PT symmetry of Schrodinger Equation

$$\mathcal{P} : x \rightarrow -x, \quad \mathcal{T} : t \rightarrow -t \implies$$

$$i\frac{\partial}{\partial t}|\Psi\rangle = \hat{H}|\Psi\rangle \rightarrow -i\frac{\partial}{\partial t}|\Psi\rangle = \hat{H}|\Psi\rangle$$

PT should preserve \hat{p} invariant

$$\implies \mathcal{PT}\hat{p} = -i\frac{\partial}{\partial x}$$

$$(x \rightarrow -x, i \rightarrow -i)$$

Time reversal is anti-unitary operation, Wigner (1926)

Positive Energy State Definition in QM

$$i\frac{\partial}{\partial t}|\Psi\rangle = \hat{H}|\Psi\rangle \implies |\Psi\rangle_t = e^{-i\hat{H}t}|\Psi\rangle_0$$

$$i\frac{\partial}{\partial \bar{t}}|\Psi\rangle = \hat{H}|\Psi\rangle \implies |\Psi\rangle_t = e^{-i\hat{H}\bar{t}}|\Psi\rangle_0$$

Irrespective of our labeling of position and time (Especially time which is special in quantum theory because we time is not an operator, it is a parameter)

Since $\bar{t} = -t$, if the first line is positive energy state with arrow of time

$t : -\infty \rightarrow \infty$, the second line is positive energy state with arrow of time

$t : \infty \rightarrow -\infty$ **ARROW OF TIME IS DIFFERENT !!!!!!!** J. Donoghue, G. Menezes, JHEP 11 (2021) 010,

Phys.Rev.Lett. 123 (2019) 17, 171601

QFT in Minkowski spacetime: Quantum Mechanics+Special Relativity

We decompose field operator following definition of positive energy state fixing the arrow of time $t : -\infty \rightarrow \infty$

$$\hat{\phi}(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2|k_0|}} \left[\hat{a}_{\mathbf{k}} e^{ik \cdot x} + \hat{a}_{\mathbf{k}}^\dagger e^{-ik \cdot x} \right] \quad k \cdot x = -k_0 t + \mathbf{k} \cdot \mathbf{x}$$

The field operators commute for space-like distances (causality condition)

$$[\hat{\phi}(x), \hat{\phi}(y)] = 0, \quad (x - y)^2 > 0$$

If we instead fix our arrow of time $t : \infty \rightarrow -\infty$, we decompose field operator following way

$$\hat{\phi}(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2|k_0|}} \left[\hat{a}_{\mathbf{k}} e^{-ik \cdot x} + \hat{a}_{\mathbf{k}}^\dagger e^{ik \cdot x} \right]$$

Superselection rules

Direct-sum split of Hilbert space is a way out

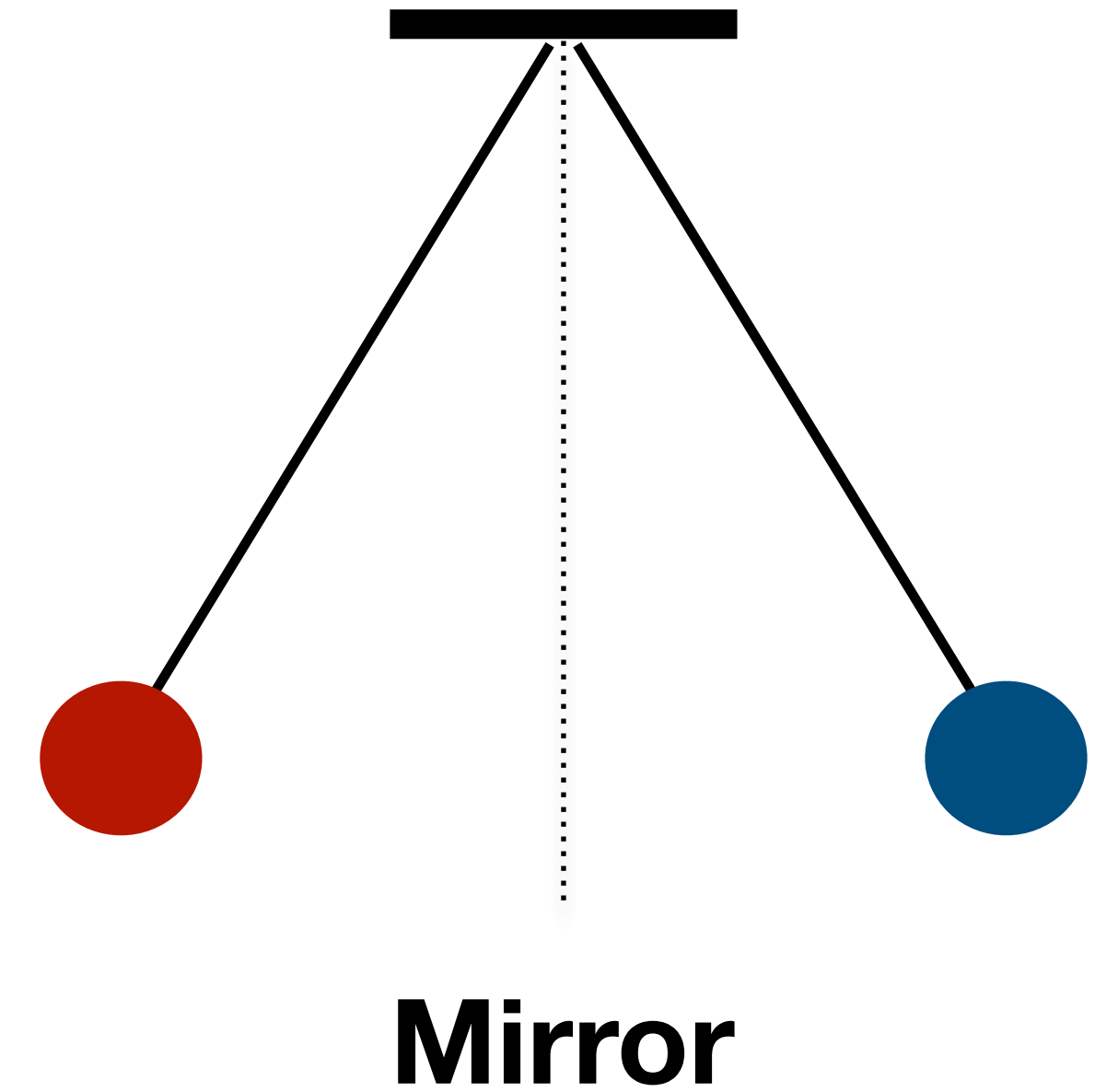
Direct-sum Schrodinger equation is a PT symmetric formulation of quantum mechanics

$$i \frac{\partial |\Psi\rangle}{\partial t_p} = \begin{pmatrix} \hat{H}_+ & 0 \\ 0 & -\hat{H}_- \end{pmatrix} |\Psi\rangle$$

According to this a single-quantum state is expressed as a direct-sum of a component evolving forward in time at position \mathbf{x} and another component evolving backward in time at position

-x

One can define positive energy without referencing to the arrow of time.



$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|\Psi_+\rangle \oplus |\Psi_-\rangle) = \frac{1}{\sqrt{2}} \begin{pmatrix} |\Psi_+\rangle \\ |\Psi_-\rangle \end{pmatrix}$$

Direct-Sum QM (PT symmetric QM)

K. Sravan Kumar, J. Marto, arXiv: 2305.06046 [hep-th] E. Gaztanaga, K. Sravan Kumar, JCAP 06 (2024) 001

$$i \frac{\partial |\Psi\rangle}{\partial t_p} = \begin{pmatrix} \hat{H}_+ & 0 \\ 0 & -\hat{H}_- \end{pmatrix} |\Psi\rangle$$

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|\Psi_+\rangle \oplus |\Psi_-\rangle) = \frac{1}{\sqrt{2}} \begin{pmatrix} |\Psi_+\rangle \\ |\Psi_-\rangle \end{pmatrix}$$

$$\int_{-\infty}^{\infty} \langle \Psi | \Psi \rangle dx = \frac{1}{2} \int_{-\infty}^{\infty} [\langle \Psi_+ | \Psi_+ \rangle + \langle \Psi_- | \Psi_- \rangle] dx = 1$$

Wave function is

$$\Psi(x, t_p) = \langle x | \Psi \rangle = \frac{1}{2} (\langle x_+ | \Psi_+ \rangle + \langle x_- | \Psi_- \rangle) = \frac{1}{2} (\Psi_+^0(x_+) e^{-i\hat{H}t_p} + \Psi_-^0(x_-) e^{i\hat{H}t_p}), \quad x_+ = x > 0, x_- = -x < 0$$

A quantum state is direct-sum of two positive energy components evolving forward $t : -\infty \rightarrow \infty$ and back ward $t : \infty \rightarrow -\infty$ in time at parity conjugate points.

$$\mathcal{PT} \Psi(x, t_p) = \Psi(x, t_p)$$

Wave function is PT symmetric

$$\hat{p}_+ = -i \frac{d}{dx_+}$$

$$\hat{p}_- = i \frac{d}{dx_-}$$

$$\hat{x} = \hat{x}_+ \oplus \hat{x}_-$$

$$\hat{p} = \hat{p}_+ \oplus \hat{p}_-$$

$$\hat{H} = \hat{H}_+(p_+, x_+) \oplus \hat{H}_-(p_-, x_-)$$

Direct-sum Hilbert spaces, Direct-sum rules

$|\Psi_+\rangle, |\Psi_-\rangle$ are the state vectors in direct-sum Hilbert space $\mathcal{H} = \mathcal{H}_+ \oplus \mathcal{H}_-$

These two Hilbert spaces are super-selection-sectors corresponding to

Parity conjugate regions of position space spanned by coordinates (x_+, x_-)

Position and momentum operators in these two Hilbert spaces commute

$$[\hat{x}_+, \hat{x}_-] = 0, \quad [\hat{p}_+, \hat{p}_-] = 0$$

Operators only act on the states of corresponding Hilbert space

$$\hat{H}|\Psi_{\pm}\rangle = \left(\hat{H}_+(p_+, x_+) \oplus \hat{H}_-(p_-, x_-) \right) |\Psi_{\pm}\rangle = \hat{H}_{\pm} |\Psi_{\pm}\rangle$$

Direct-sum quantum harmonic oscillator

$$\hat{H} = \left(\frac{\hat{p}^2}{2m} + \frac{1}{2}k^2\hat{x}^2 \right) = \left(\frac{\hat{p}_+^2}{2m} + \frac{1}{2}k^2\hat{x}_+^2 \right) \oplus \left(\frac{\hat{p}_-^2}{2m} + \frac{1}{2}k^2\hat{x}_-^2 \right) = \hat{H}_+ \oplus \hat{H}_-$$

$$\hat{x}_+ = \sqrt{\frac{1}{2}} (a + a^\dagger), \quad \hat{p}_+ = -i \frac{d}{dx_+} = \frac{i}{\sqrt{2}} (a^\dagger - a)$$

$$\hat{x}_- = \sqrt{\frac{1}{2}} (b + b^\dagger), \quad \hat{p}_- = i \frac{d}{dx_-} = -\frac{i}{\sqrt{2}} (b^\dagger - b)$$

$$\hat{x} = \hat{x}_+ \oplus \hat{x}_-$$

$$\hat{p} = \hat{p}_+ \oplus \hat{p}_-$$

$$[\hat{x}_+, \hat{p}_+] = i, \quad [\hat{x}_-, \hat{p}_-] = -i, \quad [\hat{x}, \hat{p}] = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

$$[a, a^\dagger] = [b, b^\dagger] = 1, \quad [a, b^\dagger] = [a, b] = 0$$

PT symmetric wavefunction of direct-sum harmonic oscillator

$$\begin{aligned}\Psi(x, t_p) &= \frac{1}{\sqrt{2}} \Psi_+(t_p, x_+) + \frac{1}{\sqrt{2}} \Psi_-(-t_p, x_-) \\ &= \frac{1}{\sqrt{2^{n+1}n!}} \left(\frac{1}{\pi}\right)^{1/4} e^{-x_+^2} H_n(x_+) e^{-iE_n t_p} + \frac{1}{\sqrt{2^{n+1}n!}} \left(\frac{1}{\pi}\right)^{1/4} e^{-x_-^2} H_n(x_-) e^{iE_n t_p}\end{aligned}$$

H_n are Hermitian polynomials

Direct-sum QM is a PT symmetric QM with Hermitian operators

Direct-sum quantum field theory (DQFT)

We take forward the construction to Minkowski spacetime

$$ds^2 = - dt_m^2 + d\mathbf{x}^2$$

We write the single quantum state as direct-sum of two components which describe the same field at parity conjugate points in physical space

We showed that this construction does not change any results of standard quantum theory because spacetime is PT symmetric:

$$\begin{aligned} \hat{\phi}(x) &= \frac{1}{\sqrt{2}} \hat{\phi}_+(t_m, \mathbf{x}) \oplus \frac{1}{\sqrt{2}} \hat{\phi}_-(-t_m, -\mathbf{x}) \\ &= \frac{1}{\sqrt{2}} \begin{pmatrix} \hat{\phi}_+ & 0 \\ 0 & \hat{\phi}_- \end{pmatrix} \end{aligned}$$

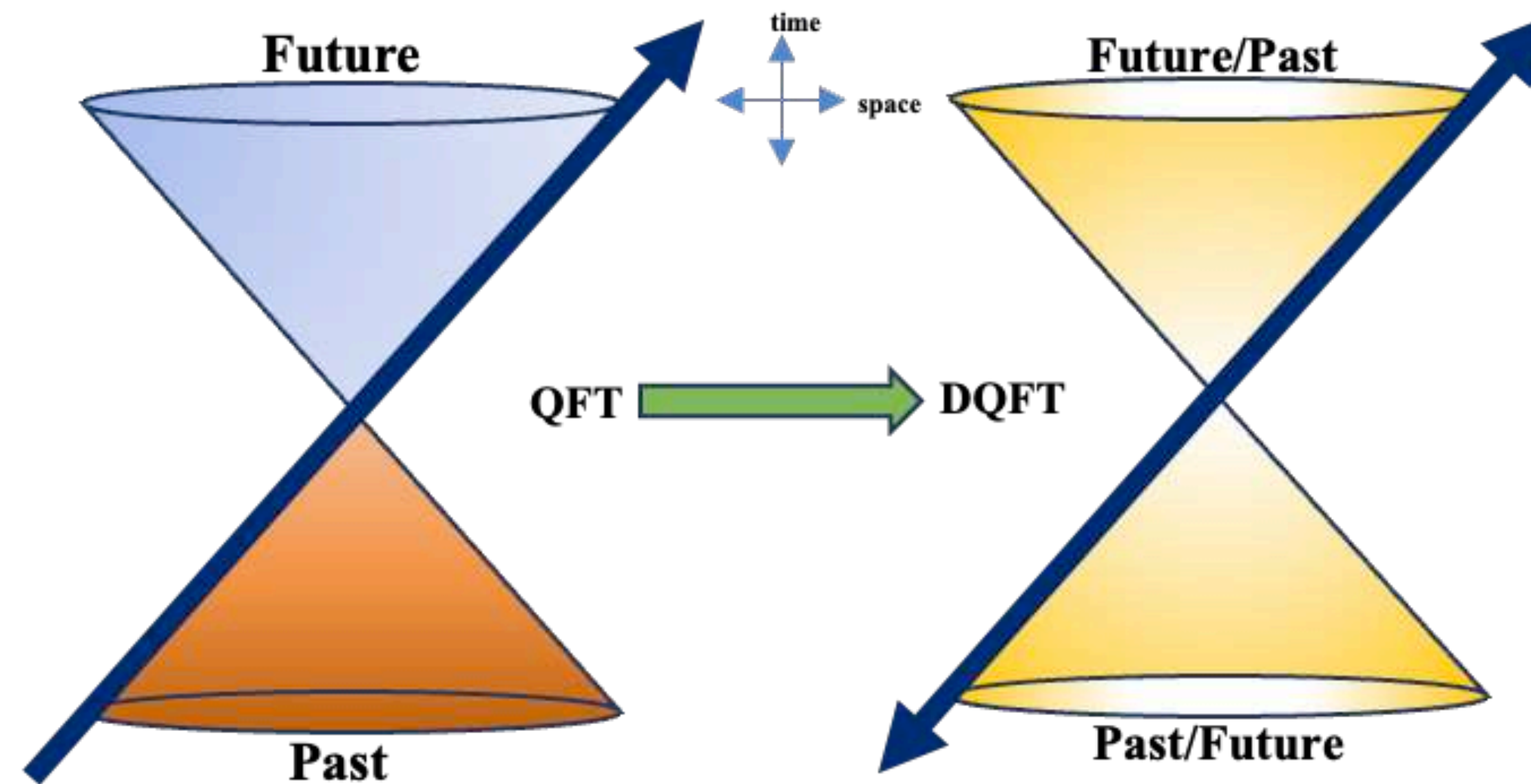
Causality $[\hat{\phi}_+, \hat{\phi}_-] = 0$
 $[\hat{\phi}(x), \hat{\phi}(y)] = 0, \quad (x - y)^2 > 0$

Vacuum $|0\rangle = |0_+\rangle \oplus |0_-\rangle = \begin{pmatrix} |0_-\rangle \\ |0_+\rangle \end{pmatrix}$

$$\hat{\phi}_{\pm}(x) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{\sqrt{2|k_0|}} \left[\hat{a}_{(\pm)\mathbf{k}} e^{\pm ik \cdot x} + \hat{a}_{(\pm)\mathbf{k}}^\dagger e^{\mp ik \cdot x} \right]$$

$$k \cdot x = -k_0 t + \mathbf{k} \cdot \mathbf{x}$$

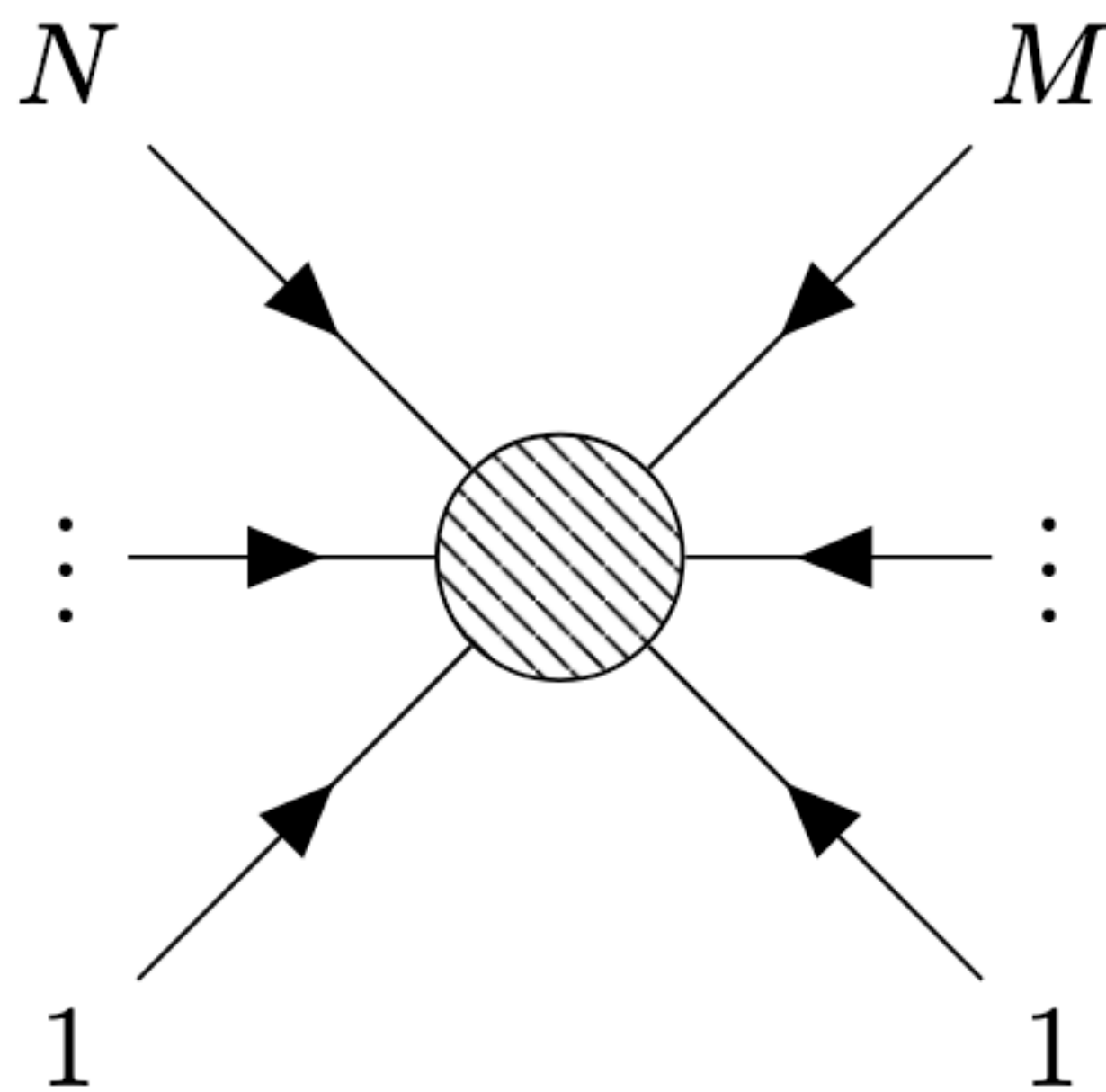
In a nutshell:



Special Relativity+ Quantum Mechanics: Field operators commute for spacelike distances $[\hat{\phi}(x), \hat{\phi}(y)] = 0, (x - y)^2 > 0$

A quantum field in DQFT has two direct-sum components with opposite time evolutions at parity conjugate regions satisfying an additional new causality condition $[\hat{\phi}_+(x), \hat{\phi}_-(-x)] = 0$

No observational implications for QFT in Minkowski spacetime



$$\mathcal{L}_{int} = -\frac{\lambda}{3} \hat{\phi}^3 = -\frac{\lambda}{3} \begin{pmatrix} \hat{\phi}_+^3 & 0 \\ 0 & \hat{\phi}_-^3 \end{pmatrix}$$

$$\langle 0_+ | \hat{\phi}_+(x) \hat{\phi}_+(x') | 0_+ \rangle = \langle 0_- | \hat{\phi}_-(x) \hat{\phi}_-(x') | 0_- \rangle$$

$$\mathcal{A} = \frac{\mathcal{A}_+^{N \rightarrow M}(p_a, -p_b) + \mathcal{A}_-^{N \rightarrow M}(-p_a, p_b)}{2}$$

$$\mathcal{A}_+^{N \rightarrow M} = \mathcal{A}_-^{N \rightarrow M} = \mathcal{A} \quad \text{Due to the (C)PT symmetry}$$

DSI calculations

The second order action for curvature perturbation

$$\delta^{(2)}S_s = \frac{1}{2} \int d\tau d^3x a^2 \frac{\dot{\phi}^2}{H^2} \left[\zeta'^2 - (\partial\zeta)^2 \right]$$

The Mukhanov-Sasaki variable (a classical field redefinition)

$$v = \frac{a\dot{\phi}}{H} \zeta$$

The quantum MS variable

$$\frac{1}{\sqrt{2}} \begin{pmatrix} \hat{v}_{(+)}(\tau, \mathbf{x}) & 0 \\ 0 & \hat{v}_{-}(-\tau, -\mathbf{x}) \end{pmatrix}, \quad |0\rangle_{qdS} = \begin{pmatrix} |0\rangle_{qdS_I} \\ |0\rangle_{qdS_{II}} \end{pmatrix}$$

$${}_{qdS} \langle 0 | \hat{v}(\tau, \mathbf{x}) \hat{v}(\tau, \mathbf{y}) | 0 \rangle_{qdS} =$$

$$\begin{aligned} & \frac{1}{2} {}_{qdS_I} \langle 0 | \hat{v}_{+}(\tau, \mathbf{x}) \hat{v}_{(+)}(\tau, \mathbf{y}) | 0 \rangle_{qdS_I} + \frac{1}{2} {}_{qdS_{II}} \langle 0 | \hat{v}_{-}(-\tau, -\mathbf{x}) \hat{v}_{-}(-\tau, -\mathbf{y}) | 0 \rangle_{qdS_{II}} \\ & = \frac{1}{2} \int \frac{dk}{k} \frac{k^3}{2\pi^2} \left(|v_{+k}|^2 + |v_{-k}|^2 \right) \frac{\sin kL}{kL} \end{aligned}$$

PT symmetry breaking (quantum mechanically): $\tau \rightarrow -\tau \implies (t, H, \epsilon, \eta) \rightarrow (-t, -H, -\epsilon - \eta)$

Power Spectrum

$$\mathcal{P}_{\tilde{\phi}}(k, \tau_0) = \int \frac{d^3x}{(2\pi)^{3/2}} e^{-i\mathbf{k}\cdot\mathbf{x}} G(\mathbf{x}, \tau_0), \quad \mathbf{x} = |\mathbf{x}_1 - \mathbf{x}_2|$$

$$G(\mathbf{x}, \tau_0) = \langle 0 | \tilde{\phi}(\mathbf{x}_1, \tau_0) \tilde{\phi}(\mathbf{x}_2, \tau_0) | 0 \rangle, \quad |0\rangle = \begin{pmatrix} |0_+\rangle \\ |0_-\rangle \end{pmatrix}, \quad \begin{pmatrix} \hat{\tilde{\phi}}_+ & 0 \\ 0 & \hat{\tilde{\phi}}_- \end{pmatrix}$$

$$G(\mathbf{x}, \tau_0) = \Theta(\tau_0)\theta(\mathbf{x})G_+(\mathbf{x}, \tau_0) + \Theta(-\tau_0)\theta(-\mathbf{x})G_-(\mathbf{x}, \tau_0)$$

$$\mathcal{P}_{\tilde{\phi}}(k, \tau_0) = \Theta(\tau_0)\theta(\mathbf{x})\mathcal{P}_{\tilde{\phi}_+} + \Theta(-\tau_0)\theta(-\mathbf{x})\mathcal{P}_{\tilde{\phi}_-}$$

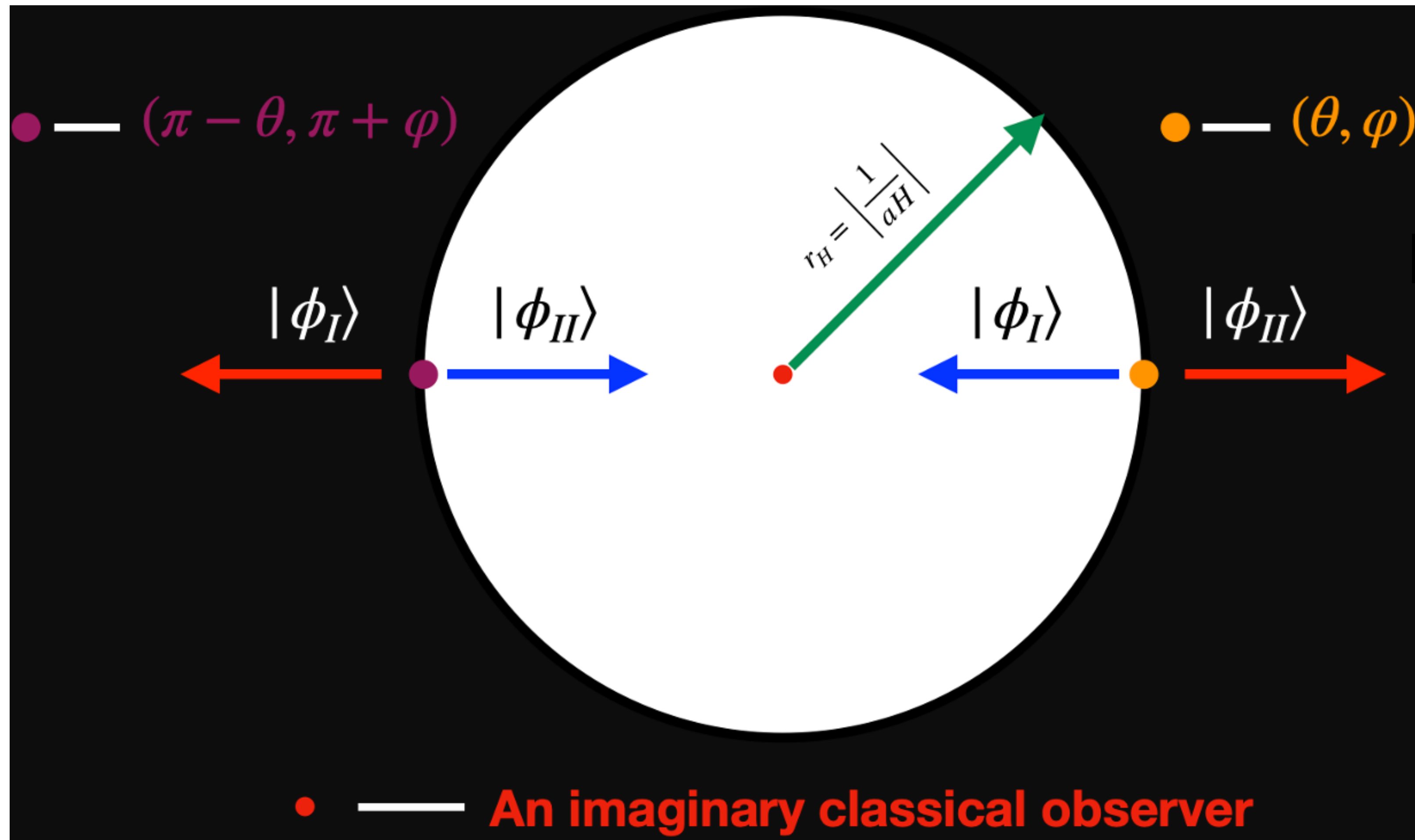
$$\mathcal{P}_\zeta = \frac{k^3}{2\pi^2} \frac{1}{2a^2\epsilon} \Bigg|_{\text{classical}} \mathcal{P}_\nu \Bigg|_{\tau=\mp\frac{1}{aH_*}}$$

$$\approx \frac{H_*^2}{8\pi\epsilon_*} \left(\frac{k}{k_*}\right)^{n_s-1} \frac{1}{2} \left[2 + \Theta(\tau)\Theta(\mathbf{x})\Delta\mathcal{P}_\nu\left(\frac{k}{k_*}\right) - \Theta(-\tau)\Theta(-\mathbf{x})\Delta\mathcal{P}_\nu\left(\frac{k}{k_*}\right) \right]$$

Horizon is a Mirror

DQFT brings back the unitarity that is lost

DQFT is a solution to information paradox



$$|\phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} |\phi_I\rangle \\ |\phi_{II}\rangle \end{pmatrix}$$

Pure states evolve into Pure states: Unitarity

$$|\psi_{12}\rangle = \sum_{mn} c_{mn} |\phi_1\rangle \otimes |\phi_2\rangle \quad c_{mn} \neq c_m c_n$$

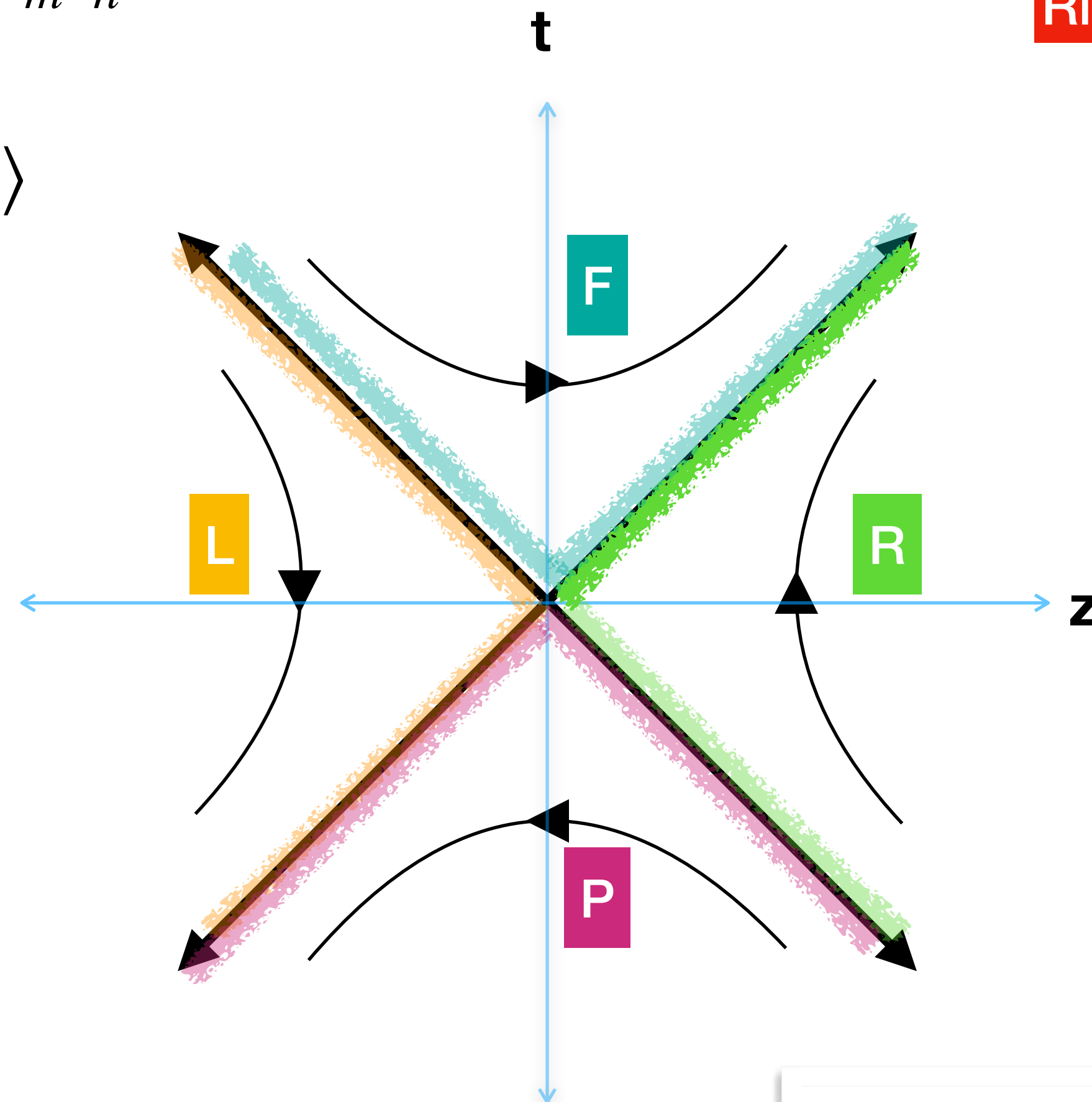
$$|\phi_1\rangle = \sum_m c_m |\phi_{m1}\rangle, \quad |\phi_2\rangle = \sum_n c_n |\phi_{n2}\rangle$$

$$\mathcal{H}_{\mathcal{A}} = \mathcal{H}_L \oplus \mathcal{H}_R$$

$$|\psi_{LR}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} |\psi_R\rangle \\ |\psi_L\rangle \end{pmatrix}$$

$$\rho = \frac{1}{2} \rho_L \oplus \frac{1}{2} \rho_R = \frac{1}{2} \rho_L^2 \oplus \frac{1}{2} \rho_R^2 = \rho^2$$

Rindler spacetime



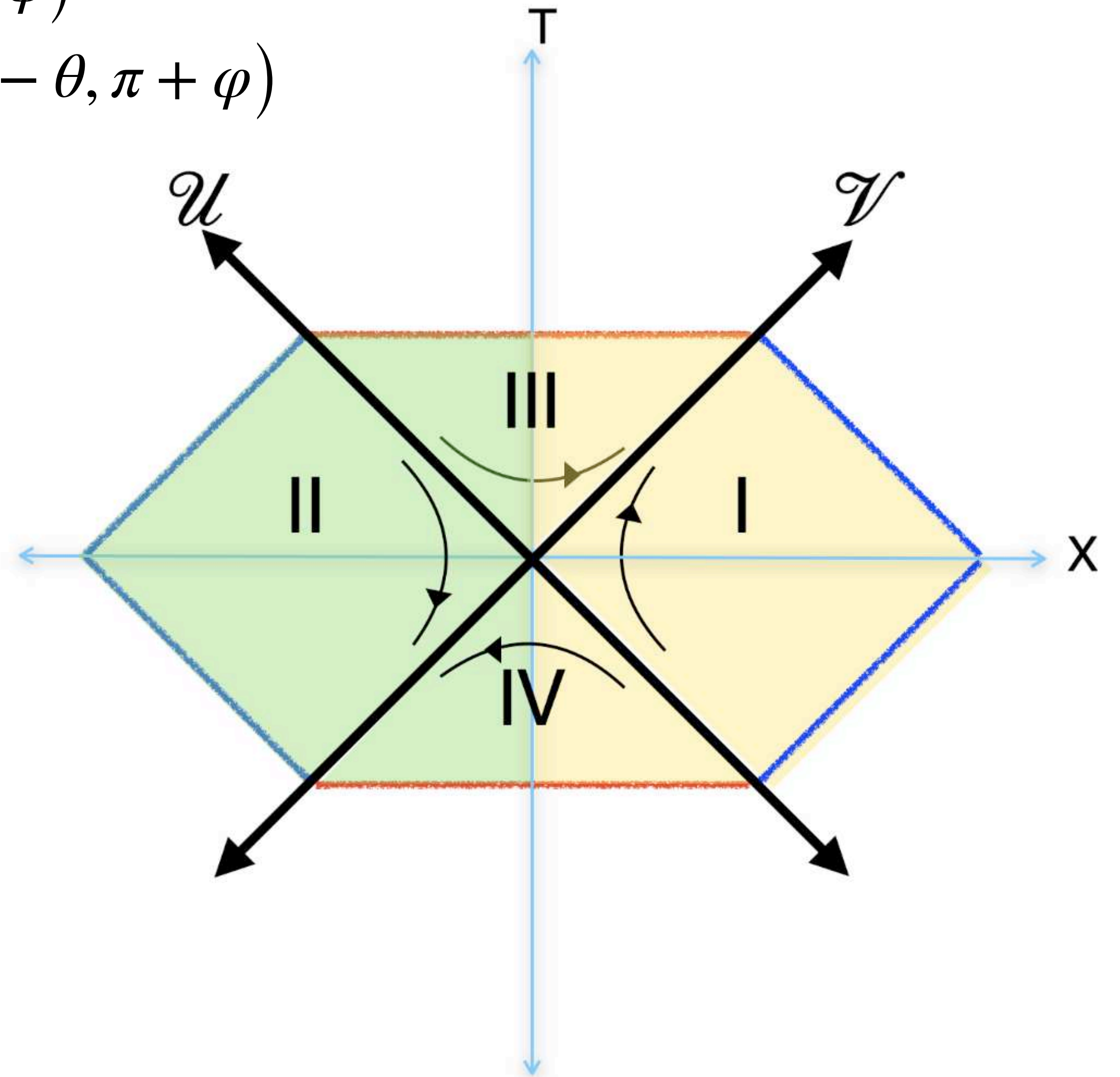
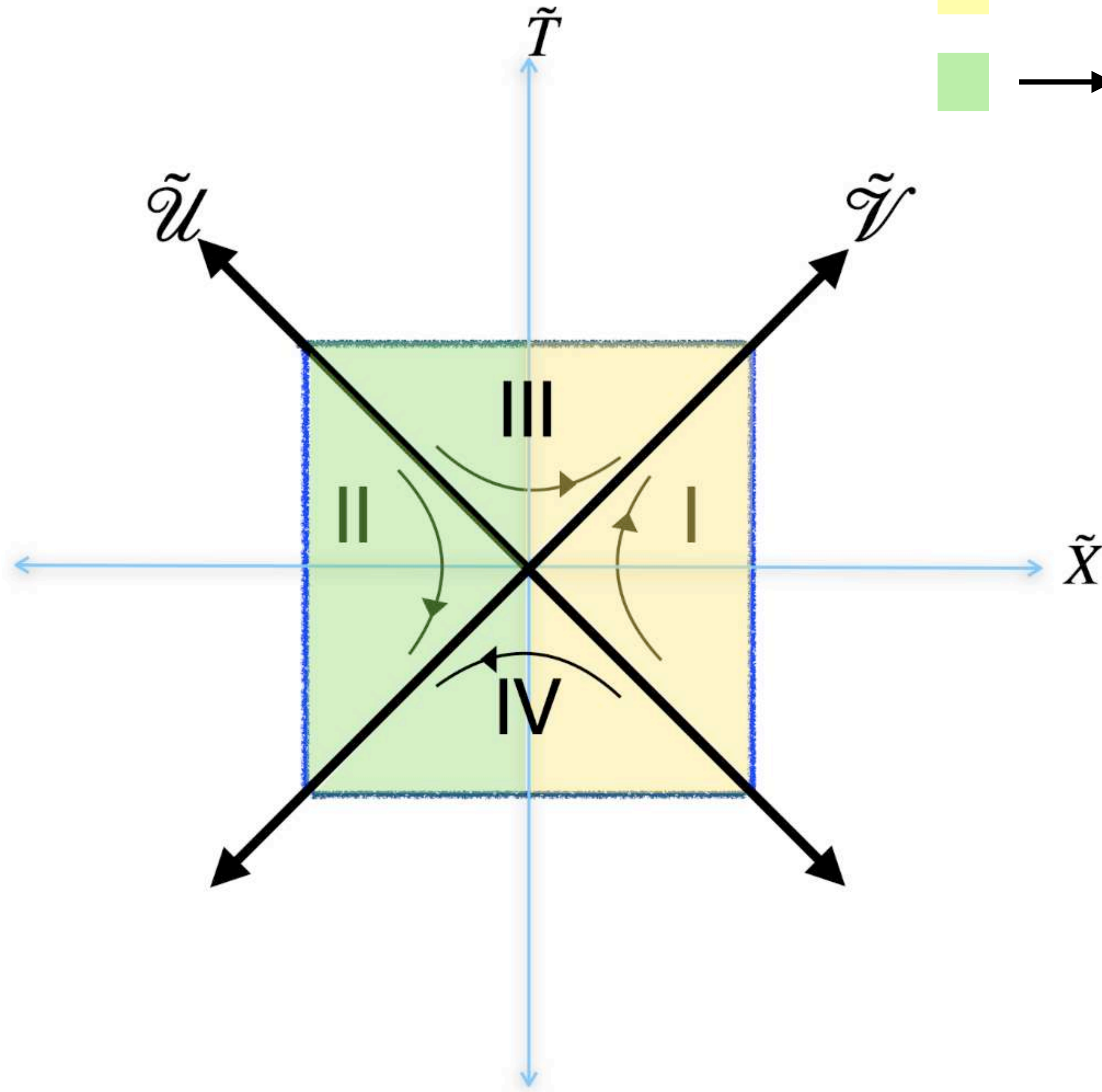
Horizon is a Mirror

Both left and right Rindler observer's QFT is unitary

Since left region is PT conjugate of Right, observers can reconstruct physics beyond the horizon.

Pure states evolve into Pure states: Unitarity in curved spacetime (Horizon is a Mirror)

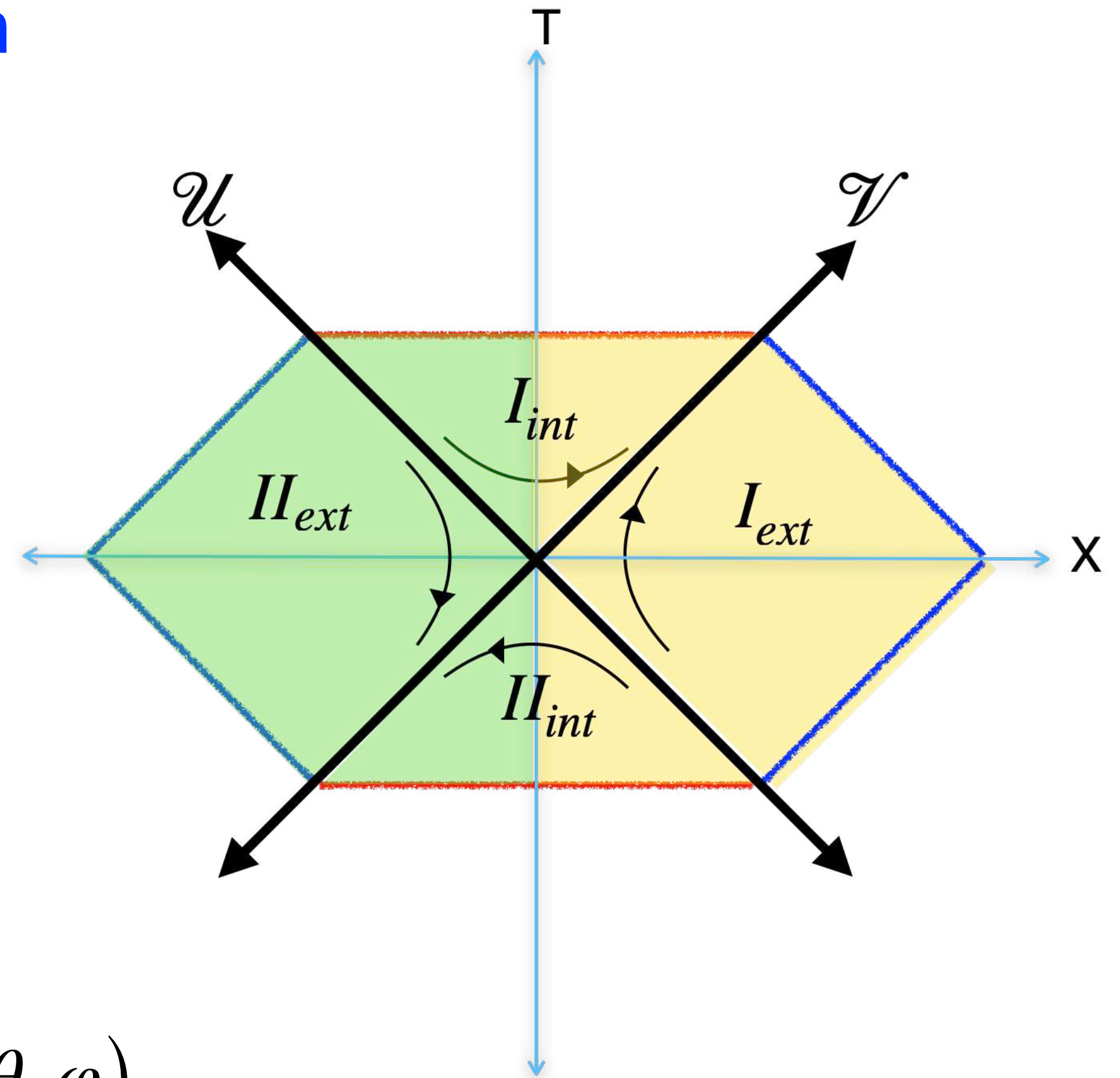
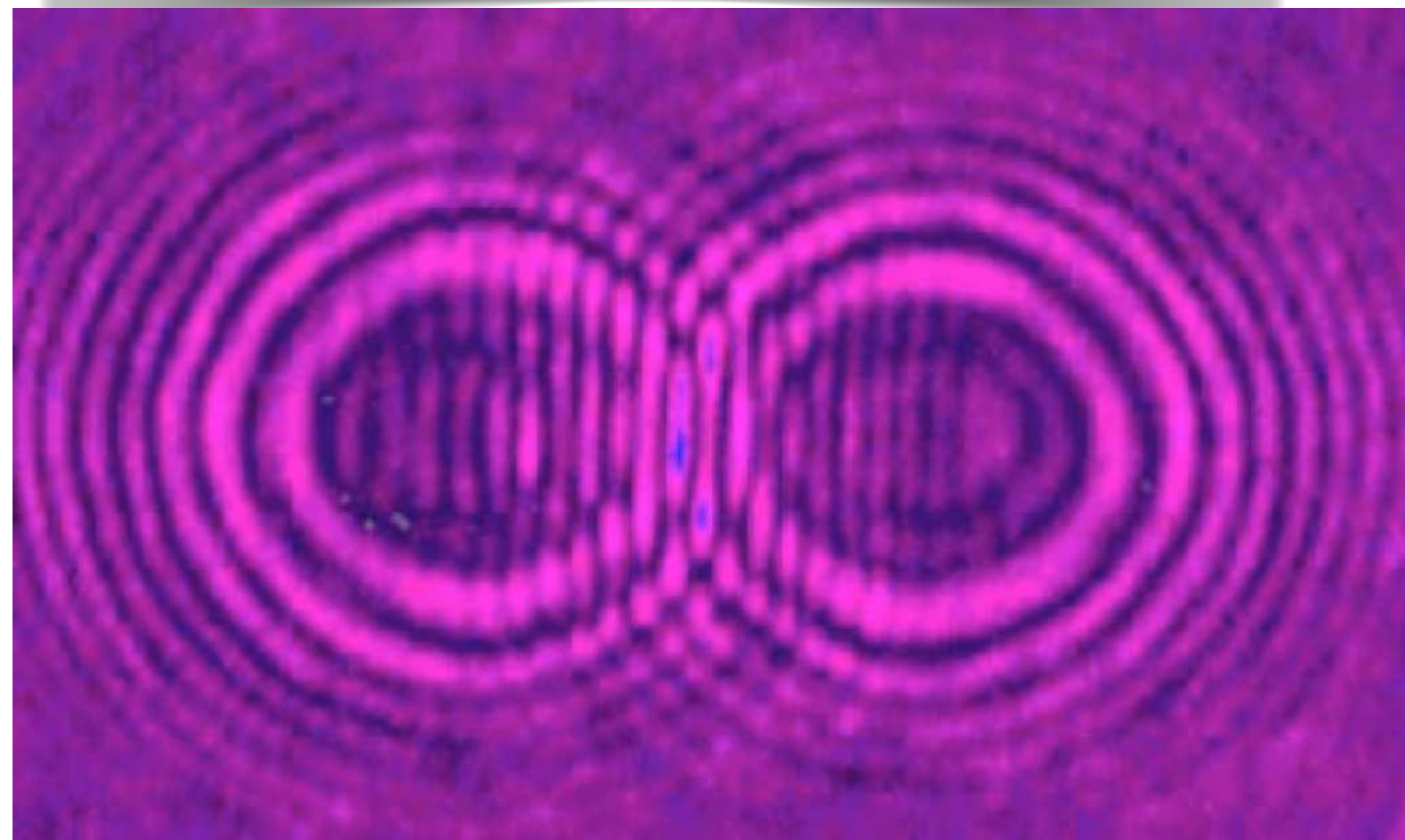
- $\longrightarrow (\theta, \varphi)$
- $\longrightarrow (\pi - \theta, \pi + \varphi)$



Quantum gravity at the black hole horizon

$$\left[\hat{\Phi}_{I_{ext}}, \hat{\Phi}_{I_{int}} \right] = i\hbar \frac{8\pi G}{r_S^2 (\ell^2 + \ell + 1)}, \quad \left[\hat{\Phi}_{II_{ext}}, \hat{\Phi}_{II_{int}} \right] = i\hbar \frac{8\pi G}{r_S^2 (\ell^2 + \ell + 1)}, \quad (c = 1)$$

Derived from GR+QM



$\longrightarrow (\theta, \varphi)$
 $\longrightarrow (\pi - \theta, \pi + \varphi)$

The interior and exterior quantum field components correspond to direct-sum Fock space $\mathcal{F} = \mathcal{F}_I \oplus \mathcal{F}_{II}$

Conclusions (Take away message-2)

- Quantum Field Theory in Curved Spacetime is need of the hour for both theory and observations.
- Without a consistent QFT in curved spacetime, one cannot achieve full quantum gravity.
- Gravitational Horizons are most important in our understanding of Universe. (i) In the context of dark energy: Black Hole Universe proposal [E. Gaztanaga *Symmetry* 14 \(2022\) 9, 1849, *Mon.Not.Roy.Astron.Soc.* 521 \(2023\) 1, L59-L63](#) (ii) In the context of understanding dark matter: Matter horizons proposed by [G. W. R Ellis and S. W. Stoeger *Mon.Not.Roy.Astron.Soc.* 398 \(2009\) 1527-1536](#)

An important message: Observational people should know the theoretical principles and theory people should understand observational analysis and principles for a coherent progress in physics.

QGRAV 2021



Thank you very much