Resolving Singularities in General Relativity

A. Chamseddine, V. Mukhanov

Singularities

in the Universe
"inside" Black Holes

Energy dominance conditions:

$$\varepsilon > 0$$

$$\varepsilon + p > 0$$

$$\varepsilon + 3p > 0 \quad ?$$

$$p \approx -\varepsilon$$ *for de Sitter*

1980-1990

-Starobinsky

$$V = \frac{1}{2}m^2\varphi^2 \qquad R^2 \quad \longrightarrow \quad p \approx -\varepsilon$$

$$\varepsilon + 3p < 0$$

nonsingular Universe???

-Markov (1982)

$$3\left(\frac{\dot{a}}{a}\right)^2 = \varepsilon\left(1 - \frac{\varepsilon}{\varepsilon_m}\right) \qquad \varepsilon \propto \frac{1}{a^3},$$

???

limiting density

-Mukhanov, Brandenberger (1992)

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}R + \lambda(4R_{\mu\nu}R^{\mu\nu} - R^2\right) + V(\lambda) + \ldots)$$

nonsingular isotropic, homogeneous Universe

nonsingular 2d, black hole

BUT

what about Kasner universe and 4d black hole?

1990-... *Strings, LQG...*

Noncommutative geometry

$$g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi = 1$$

$$S = \int d^4x\sqrt{-g} \quad (-\frac{1}{2}R+\lambda\left((\partial\phi)^2 - 1\right) + f(\Box\phi))$$

- *Synchronous coordinates*

$$ds^2 = dt^2 - \gamma_{ik}dx^i dx^k \qquad \phi = \pm t + A,$$

$$\chi = \Box\phi = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial\phi}{\partial x^\nu}\right) = \frac{\dot{\gamma}}{2\gamma},$$

$$f\ -?$$

- $$f\left(\Box\phi\right) = 1 - \sqrt{1 - \frac{\left(\Box\phi\right)^2}{\varepsilon_m}} + \ldots,$$

$$f\left(\chi\right) = \chi_m^2 \left(1 + \frac{1}{3}\frac{\chi^2}{\chi_m^2} - \sqrt{\frac{2}{3}}\frac{\chi}{\chi_m} \arcsin\left(\sqrt{\frac{2}{3}}\frac{\chi}{\chi_m}\right) - \sqrt{1 - \frac{2}{3}\frac{\chi^2}{\chi_m^2}}\right)$$

Einstein equations

From i-k eqs. it follows

$$\varkappa^i_k = \frac{1}{3}\varkappa\delta^i_k + \frac{\lambda^i_k}{\sqrt{\gamma}},$$

constants of integration

where $\varkappa^i_k = \gamma^{im}\dot{\gamma}_{mk}$, $\varkappa = \varkappa^i_i = \dot{\gamma}/\gamma$.

0-0 equation gives

$$\frac{1}{12}\left(\frac{\dot{\gamma}}{\gamma}\right)^2 = \varepsilon\left(1 - \frac{\varepsilon}{\varepsilon_m}\right),$$

where $\varepsilon_m = 2\chi^2_m$.

mimetic

$$\varepsilon = \frac{\lambda^i_k\lambda^k_i}{8\gamma} + \frac{C}{\sqrt{\gamma}} + T^0_0.$$

Friedmann Universe

$$ds^2 = dt^2 - a^2(t)\,\delta_{ik}dx^i dx^k$$

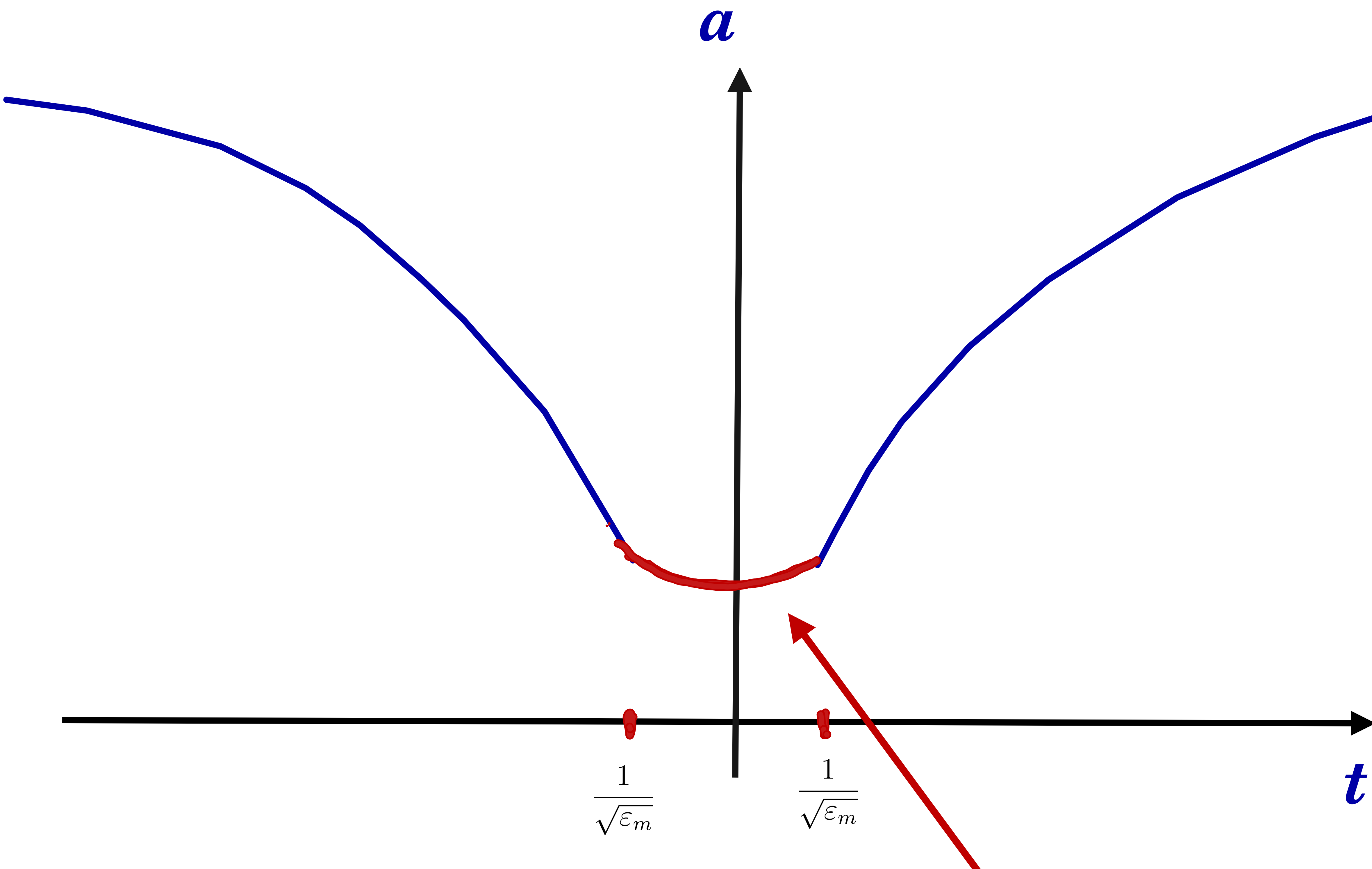
In this case $\gamma = a^6$, and $\lambda^i_k = 0$.

$$3\left(\frac{\dot{a}}{a}\right)^2 = \frac{\varepsilon_m}{a^{3(1+w)}}\left(1 - \frac{1}{a^{3(1+w)}}\right),$$

for $p = w\varepsilon$

Nonsingular solution

$$a = \left(1 + \frac{3}{4}(1+w)^2\,\varepsilon_m t^2\right)^{\frac{1}{3(1+w)}}$$



$$G^{\alpha}_{\beta} = T^{\alpha\,(eff)}_{\beta}$$

$$\varepsilon + p < 0 \qquad p < 0$$

Kasner Universe

$$ds^2 = dt^2 - t^{2p_1}dx^2 - t^{2p_2}dy^2 - t^{2p_3}dz^2,$$

$$p_1 + p_2 + p_3 = 1, \quad p_1^2 + p_2^2 + p_3^2 = 1.$$

- $$R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = -\frac{16}{t^4}p_1p_2p_3,$$

$$\gamma \propto t^2$$

$$\varepsilon = \frac{\lambda^i_k \lambda^k_i}{8\gamma},$$

$$\left(\frac{\dot{\gamma}}{\gamma}\right)^2 = \frac{3\bar{\lambda}^2}{2\gamma}\left(1 - \frac{\bar{\lambda}^2}{8\varepsilon_m \gamma}\right),$$

where we have denoted $\bar{\lambda}^2 \equiv \lambda^i_k \lambda^k_i$.

Solution $\quad \gamma = \frac{\bar{\lambda}^2}{8\varepsilon_m}\left(1 + 3\varepsilon_m t^2\right)$

At $\quad t = 0 \quad$ **we have** $\quad \gamma \neq 0$

$$\gamma_{ik} = \gamma_{(i)}(t)\,\delta_{ik},$$

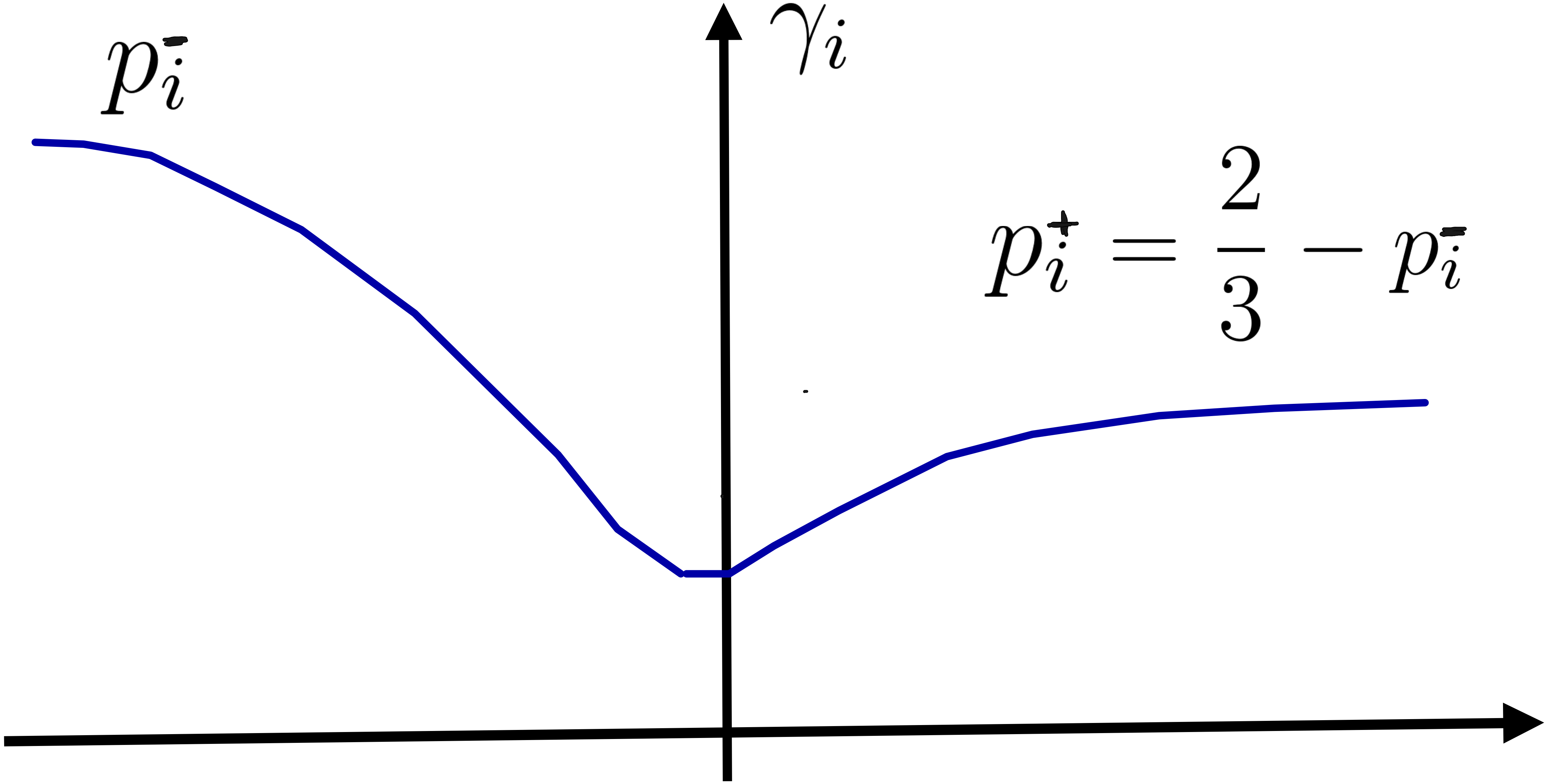
Exact solution

$$\gamma_{(i)} = \left(\frac{\bar{\lambda}^2}{8\varepsilon_m}\left(1+3\varepsilon_m t^2\right)\right)^{1/3} \exp\left(2\sqrt{\frac{2}{3}}\frac{\lambda_{(i)}}{\bar{\lambda}}\sinh^{-1}\left(\sqrt{3\varepsilon_m}t\right)\right),$$

For $|t| \gg \dfrac{1}{\sqrt{\varepsilon_m}}$

$$\gamma_{(i)} \simeq \left(\frac{\bar{\lambda}^2}{32\varepsilon_m}\right)^{1/3} \left(12\varepsilon_m t^2\right)^{p_i^{\pm}},$$

$$p_1^{\pm} + p_2^{\pm} + p_3^{\pm} = 1, \;\; \left(p_1^{\pm}\right)^2 + \left(p_2^{\pm}\right)^2 + \left(p_3^{\pm}\right)^2 = 1,$$



Kasner with $p_i^- = \left(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$ ***turns to***

Kasner with $p_i^+ = (1,0,0)$ $\equiv$ ***Minkowski***

Black Hole

$$ds^2 = \left(1 - \frac{r_g}{r}\right) dt_S^2 - \frac{dr^2}{\left(1 - \frac{r_g}{r}\right)} - r^2 d\Omega^2,$$

For $r < r_g$ ***it is a time coordinate***

$t_s \to R$ ***is a space coordinate***

"Inside" BH

$$ds^2 = dt^2 - a^2(t)\, dR^2 - b^2(t)\, d\Omega^2,$$

where for the Schwarzschild black hole

$$a^2(t) = \frac{1 - \tau^2(t)}{\tau^2(t)}, \;\; b^2(t) = \tau^4(t)\, r_g^2.$$

$$t = r_g \left(\arcsin \tau - \tau\sqrt{1 - \tau^2}\right)$$

Near horizon $(r_g - r \ll r_g)$

$$ds^2 = d\bar{t}^2 - \frac{1}{4}\left(\frac{\bar{t}}{r_g}\right)^2 dR^2 - r_g^2 d\Omega^2,$$

Similar to Kasner with $p = (1, 0, 0)$

Close to singularity $(r \ll r_g)$

$$ds^2 = dt^2 - \left(\frac{t}{t_0}\right)^{-2/3} dR^2 - \left(\frac{t}{t_0}\right)^{4/3} r_g^2 d\Omega^2,$$

Similar to Kasner with $p = \left(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$

Spatial curvature term from $d\Omega^2$ *is relevant only at* $r \sim r_g/2$.

In our theory singularity which would happen at $t = 0$ ***is avoided and "Kasner" with*** $p = \left(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$ ***goes to "Kasner" with*** $p = (1, 0, 0)$.

Metric for $t > \frac{1}{\sqrt{\varepsilon_m}}$.

$$ds^2 = dt^2 - Q_0^2 \left(\frac{t}{t_0}\right)^2 dR^2 - \frac{1}{Q_0} r_g^2 d\Omega^2,$$

where $Q_0 = \left(\frac{16}{3}\varepsilon_m r_g^2\right)^{2/3}$.

$$R_{g1} = r_g/Q_0^{1/2} \propto r_g^{1/3}$$

After bounce we find ourselves in near horizon inner region of the black hole of size $r_g^{1/3}$

$$p = (1,0,0) \longrightarrow p = \left(-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right) \longrightarrow p = (1,0,0)$$

within BH of size $r_g^{1/9}$

et. cet.