Non-perturbative gravity: When singular black holes are forbidden?

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Based on:

A. Koshelev, A. Tokareva, ‘Non-perturbative quantum gravity denounces singular Black Holes, arXiv:2404.07925 (submitted to JHEP)

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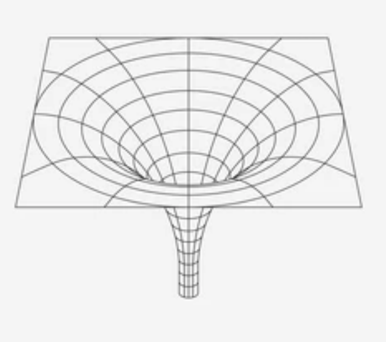
This talk is about...

Life of the action of 4D non-perturbative gravity

$$S = \int d^4x \, \sqrt{-g} \left(\frac{M_P^2}{2} R + \frac{\lambda}{2} \left[R \mathcal{F}_1(\Box) R + L_{\mu\nu} \mathcal{F}_2(\Box) L^{\mu\nu} + W_{\mu\nu\lambda\sigma} \mathcal{F}_4(\Box) W^{\mu\nu\lambda\sigma} \right] + \dots \right)$$

$$L_{\mu\nu} = R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R$$

in a black hole background



No strings attached!

T. Draper, B. Knorr, C. Ripken, F. Saueressig, ‘Finite Quantum Gravity Amplitudes: No Strings Attached’, Phys.Rev.Lett. 125 (2020) 18, 181301

Approaches to non-perturbative gravity

$$S=\int d^4x\,\sqrt{-g}\left(\frac{M_P^2}{2}R+\frac{\lambda}{2}\left[R\mathcal{F}_1(\Box)R+L_{\mu\nu}\mathcal{F}_2(\Box)L^{\mu\nu}+W_{\mu\nu\lambda\sigma}\mathcal{F}_4(\Box)W^{\mu\nu\lambda\sigma}\right]+\dots\right)$$

$$L_{\mu\nu}=R_{\mu\nu}-\frac{1}{4}g_{\mu\nu}R$$

- Quantum effective action determining the graviton propagator – no other contributions!
- Functions of d’Alambertian are unknown parameters

Consistency requirements:

- no ghosts, GR in the low energy limit
- unitary scattering amplitudes
- no causality violation
- …

Approaches to non-perturbative gravity

$$S=\int d^4x\,\sqrt{-g}\left(\frac{M_P^2}{2}R+\frac{\lambda}{2}\left[R\mathcal{F}_1(\Box)R+L_{\mu\nu}\mathcal{F}_2(\Box)L^{\mu\nu}+W_{\mu\nu\lambda\sigma}\mathcal{F}_4(\Box)W^{\mu\nu\lambda\sigma}\right]+\dots\right)$$

How to compute form-factors of gravity?

- Perturbative quantum gravity, EFT of gravity (can be brought to this form after field redefinitions)
  Z. Bern, D. Kosmopoulos, A. Zhiboedov, Phys.A 54 (2021) 34, C. de Rham, S. Jaitly, A. Tolley, Phys.Rev.D 108 (2023) 4, 046011, ...

- Asymptotic safety program: functional renormalization group
  M. Reuter, Phys. Rev. D57 (1998) 971, E. Manrique, S. Rechenberger and F. Saueressig, **PRL** 106 (2011) 251302, T.Draper, B. Knorr, C. Ripken, F. Saueressig, **Phys.Rev.Lett.** 125 (2020) 18, J. Fehre, D. Litim, J.Pawlowski, M.Reichert, **Phys.Rev.Lett.** 130 (2023) 8, 081501 A. Platania, JHEP 09 (2022) 167, ...
  Talks by Benjamin Knorr and Alessia Platania

- Recontstruction of graviton scattering amplitudes from unitarity and analyticity (bootstrap)
  A. Zhiboedov, P. Tourkine, JHEP 11 (2023) 005, L. Alberte, C. de Rham, A. Tolley, **Phys.Rev.Lett.** 128 (2022) 5, …

- Analytic infinite derivative gravity (renormalizable by power-counting) and loop corrections to it,
  Y.V. Kuzmin, Sov. J. Nucl. Phys. 50 (1989) 1011, E.T. Tomboulis, hep-th/9702146, A. Koshelev, L. Rachwal, L. Modesto, A. Starobinsky, JHEP 11 (2016) 067, ...
- …

Talks by Gianluca Calcagni and Leonardo Modesto

Schwartzshield black hole background

Analytic infinite derivative gravity: action and equations of motion

$$S=\int d^4x\,\sqrt{-g}\left(\frac{M_P^2}{2}R+\frac{\lambda}{2}\left[R\mathcal{F}_1(\Box)R+L_{\mu\nu}\mathcal{F}_2(\Box)L^{\mu\nu}+W_{\mu\nu\lambda\sigma}\mathcal{F}_4(\Box)W^{\mu\nu\lambda\sigma}\right]+\dots\right)$$

$$\mathcal{F}_i=\sum_{n=0}^{\infty}f_{i_n}(\Box/\mathcal{M}^2)^n$$

Trace of Einstein equations

$$M_P^2R-6\lambda\Box\mathcal{F}_1(\Box)R-\lambda(\mathcal{L}_1{}^{\mu}_{\mu}+2\bar{\mathcal{L}}_1)-$$
$$-2\lambda\nabla_\rho\nabla_\mu\mathcal{F}_2(\Box)L^{\mu\rho}-\lambda(\mathcal{L}_2{}^{\mu}_{\mu}+2\bar{\mathcal{L}}_2)+2\lambda\Delta_2{}^{\mu}_{\mu}-\lambda(\mathcal{L}_4+2\bar{\mathcal{L}}_4)+4\lambda\Delta_4{}^{\mu}_{\mu}=-T$$

$$\mathcal{L}_1{}^{\mu}_{\nu}=\sum_{n=1}^{\infty}f_{1_n}\sum_{l=0}^{n-1}\partial^\mu R^{(l)}\partial_\nu R^{(n-l-1)},\quad \bar{\mathcal{L}}_1=\sum_{n=1}^{\infty}f_{1_n}\sum_{l=0}^{n-1}R^{(l)}R^{(n-l)},\qquad \dots$$

$$\mathcal{F}_4(\Box)=-3\mathcal{F}_1(\Box)$$

No scalar degree of freedom

$$F_W(\Box)=M_P^2\frac{e^{\sigma(\Box)}-1}{2\Box}$$

No-ghost condition

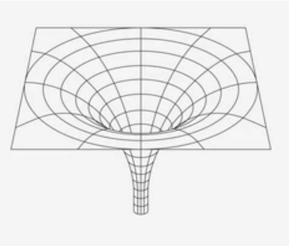
Biswas, Conroy, Koshelev, Mazumdar, Class.Quant.Grav. 31 (2014) 015022

Static black hole solution? How to substitute there Schwarzschild metric and not to die?

We take a spacetime integral and omit total derivatives

Huge simplification!

$$\frac{1}{\Delta t}\int d^4x\sqrt{-g}T^{\mu}_{\mu}$$



$$M-4\pi\lambda\int_0^\infty\left(R\Box\mathcal{F}_1'(\Box)R+L_{\alpha\beta}\Box\mathcal{F}_2'(\Box)L^{\alpha\beta}+W_{\alpha\beta\gamma\delta}\Box\mathcal{F}_4'(\Box)W^{\alpha\beta\gamma\delta}\right)r^2dr=-E.$$

$R\propto T\propto M\delta^3(r)$ - another challenge: square of delta-function is not well-defined object...

A. Koshelev, A. Tokareva, ‘Non-perturbative quantum gravity denounces singular Black Holes, arXiv:2404.07925

Schwarzschild metric as a limit of the regular one

$$M - 4\pi\lambda \int_0^\infty \left(R\Box\mathcal{F}_1'(\Box)R + L_{\alpha\beta}\Box\mathcal{F}_2'(\Box)L^{\alpha\beta} + W_{\alpha\beta\gamma\delta}\Box\mathcal{F}_4'(\Box)W^{\alpha\beta\gamma\delta} \right) r^2 dr = -E.$$

Regularization suitable for infinite derivative theory – all invariants are regular!

$$ds^2 = -A(r)dt^2 + A(r)^{-1}dr^2 + r^2 d\Omega^2$$

*Power-law regularization is not enough

Our approach:
- static metric with regular invariants (including those with infinite number of derivatives…)
- A(r) can be differentiated infinitely many times
- There is a smooth Schwarzschild limit $\alpha \to 0$

$$\boxed{A(r) = 1 - \frac{2GM}{r} e^{-\alpha r^{-p}}, \quad p > 0.}$$

Still complicated but we have xAct, Mathematica, and r-integrals can be computed analytically

Non-local contribution can be expanded in powers of α after integration over the whole spacetime

Existence of the Schwarzschild metric limit

$$M-4\pi\lambda\int_0^\infty\left(R\Box\mathcal{F}_1'(\Box)R+L_{\alpha\beta}\Box\mathcal{F}_2'(\Box)L^{\alpha\beta}+W_{\alpha\beta\gamma\delta}\Box\mathcal{F}_4'(\Box)W^{\alpha\beta\gamma\delta}\right)r^2dr=-E.$$

$$\Box\mathcal{F}_i'(\Box)=\sum_{n>0}\hat{f}_{in}\Box^n$$

Trace of the Einstein equations integrated over the whole space – should be a total BH mass

$$\begin{aligned}M-4\pi\lambda M^2(2\alpha)^{-\frac{3}{p}}\Bigg[\left(\sum_{n=0}^{\infty}(-1)^n\beta_{1n}(p)\hat{f}_{1n}(2\alpha)^{-\frac{2n}{p}}\right)+\left(\sum_{n=0}^{\infty}(-1)^n\beta_{2n}(p)\hat{f}_{2n}(2\alpha)^{-\frac{2n}{p}}\right)+\\
+\left(\sum_{n=0}^{\infty}(-1)^n\beta_{4n}(p)\hat{f}_{4n}(2\alpha)^{-\frac{2n}{p}}\right)\Bigg]-\\
-4\pi\lambda M^3(2\alpha)^{-\frac{6}{p}}\Bigg[\left(\sum_{n=1}^{\infty}(-1)^{n-1}\gamma_{1n}(p)\hat{f}_{1n}(2\alpha)^{-\frac{2(n-1)}{p}}\right)+\left(\sum_{n=1}^{\infty}(-1)^{n-1}\gamma_{2n}(p)\hat{f}_{2n}(2\alpha)^{-\frac{2(n-1)}{p}}\right)+\\
+\left(\sum_{n=1}^{\infty}(-1)^{n-1}\gamma_{4n}(p)\hat{f}_{4n}(2\alpha)^{-\frac{2(n-1)}{p}}\right)\Bigg]+O(M^4)=-E\end{aligned}$$

Computed up to n=11

Negative powers of α – in there a limit? ⇒ $$\sum_{k=0}^{\infty}\frac{(-1)^k}{k!\,\alpha^k}=e^{-1/\alpha}\xrightarrow{\alpha\to 0}0,\quad\alpha>0.$$

Resummation is required!

Existence of the Schwarzschild metric limit

$$M - 4\pi\lambda \int_0^\infty \left(R\Box\mathcal{F}_1'(\Box)R + L_{\alpha\beta}\Box\mathcal{F}_2'(\Box)L^{\alpha\beta} + W_{\alpha\beta\gamma\delta}\Box\mathcal{F}_4'(\Box)W^{\alpha\beta\gamma\delta} \right) r^2 dr = -E.$$

$$\Box\mathcal{F}_i'(\Box) = \sum_{n>0} \hat{f}_{in}\Box^n$$

- Series for β_{1n}.

$$\beta_{10} = \frac{1}{4}p\left(2p^2 + 3p + 25\right)\Gamma\left(2 + \frac{3}{p}\right)$$

$$\beta_{11} = \frac{1}{4}p\left(16p^4 + 70p^3 + 175p^2 + 250p + 49\right)\Gamma\left(2 + \frac{5}{p}\right)$$

$$\beta_{12} = \frac{1}{16}p\left(272p^6 + 2212p^5 + 7336p^4 + 14063p^3 + 15015p^2 + 5733p + 729\right)\Gamma\left(2 + \frac{7}{p}\right)$$

$$\beta_{13} = \frac{1}{64}p\left(7936p^8 + 99216p^7 + 498300p^6 + 1362204p^5 + 2277129p^4 + 2272104p^3 + 1155170p^2 + \right.$$
$$\left. + 284076p + 27225\right)\Gamma\left(2 + \frac{9}{p}\right)$$

$$\beta_{14} = \frac{1}{256}p\left(353792p^{10} + 6132896p^9 + 43636648p^8 + 169875772p^7 + 407283954p^6 + 632696295p^5 + \right.$$
$$\left. + 629270917p^4 + 376198438p^3 + 130175144p^2 + 24132119p + 1863225\right)\Gamma\left(2 + \frac{11}{p}\right)$$

When can we do resummation?

$$M - 4\pi\lambda \int_0^{\infty} \left(R\Box\mathcal{F}_1'(\Box)R + L_{\alpha\beta}\Box\mathcal{F}_2'(\Box)L^{\alpha\beta} + W_{\alpha\beta\gamma\delta}\Box\mathcal{F}_4'(\Box)W^{\alpha\beta\gamma\delta} \right) r^2 dr = -E.$$

$$\Box\mathcal{F}_i'(\Box) = \sum_{n>0} \hat{f}_{in}\Box^n$$

We found numerically the asymptotics of the series coefficients

ξ(p) varies from −1/2 for p = 1/2 to 1/3 for p → 0

$$\beta_{in}(p) \propto O(1)B(p,n), \quad B(p,n) = p^{2n+1}\,\Gamma\left(2 + \frac{2n+1}{p}\right)\Gamma\left(\frac{3n}{2}\right).$$ - large p

$$\beta_{in}(p) \propto O(1)C(p,n), \quad C(p,n) = p\,\Gamma\left(2 + \frac{2n+1}{p}\right)\Gamma(2n)\,e^{-(1+\xi(p))}$$ - small p

We need convergence of the series in non-zero size circle!

$$M - 4\pi\lambda M^2(2\alpha)^{-\frac{3}{p}} \left[\left(\sum_{n=0}^{\infty} (-1)^n \beta_{1n}(p) \hat{f}_{1n}(2\alpha)^{-\frac{2n}{p}} \right) + \left(\sum_{n=0}^{\infty} (-1)^n \beta_{2n}(p) \hat{f}_{2n}(2\alpha)^{-\frac{2n}{p}} \right) + \right.$$
$$\left. + \left(\sum_{n=0}^{\infty} (-1)^n \beta_{4n}(p) \hat{f}_{4n}(2\alpha)^{-\frac{2n}{p}} \right) \right]$$

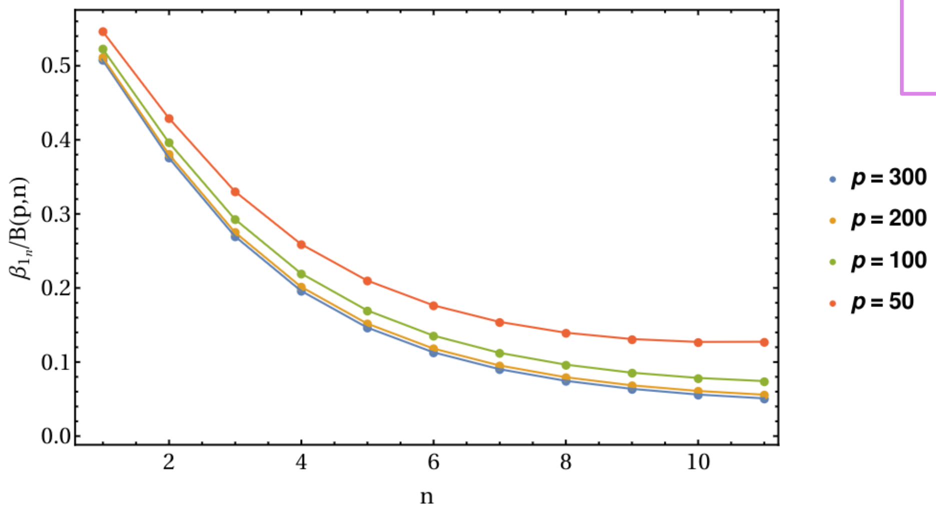
Zero radius of convergence means
infinite trace of energy-momentum tensor integrated over the space
– not a BH-like compact object… → singular Shwarzshield BH is disfavoured

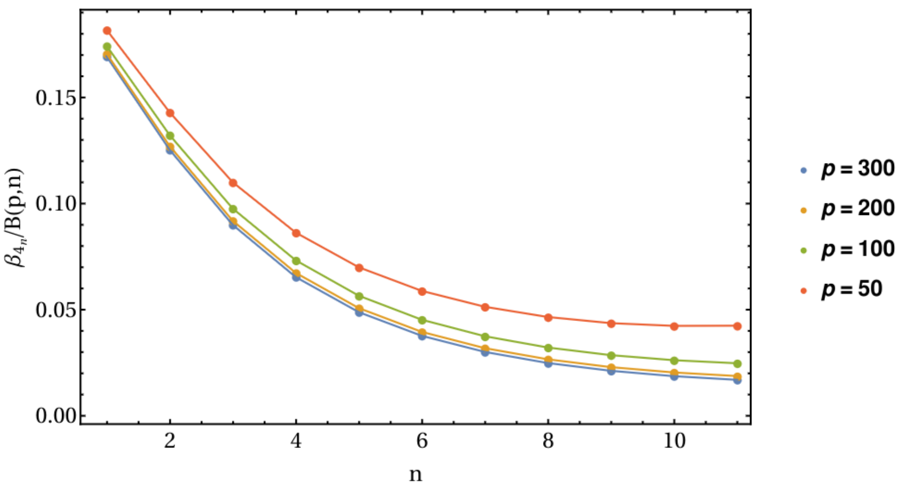
Asymptotics of the series coefficients

$$\beta_{in}(p) \propto O(1)B(p,n), \quad B(p,n) = p^{2n+1}\,\Gamma\left(2+\frac{2n+1}{p}\right)\Gamma\left(\frac{3n}{2}\right)$$ - large p

$$\beta_{in}(p) \propto O(1)C(p,n), \quad C(p,n) = p\,\Gamma\left(2+\frac{2n+1}{p}\right)\Gamma(2n)\,e^{-(1+\xi(p))}$$ - small p

$$M - 4\pi\lambda M^2(2\alpha)^{-\frac{3}{p}}\left[\left(\sum_{n=0}^{\infty}(-1)^n\beta_{1n}(p)\hat{f}_{1n}(2\alpha)^{-\frac{2n}{p}}\right) + \left(\sum_{n=0}^{\infty}(-1)^n\beta_{2n}(p)\hat{f}_{2n}(2\alpha)^{-\frac{2n}{p}}\right) + \right.$$
$$\left. + \left(\sum_{n=0}^{\infty}(-1)^n\beta_{4n}(p)\hat{f}_{4n}(2\alpha)^{-\frac{2n}{p}}\right)\right]$$





Asymptotics of the series coefficients

$$\beta_{in}(p) \propto O(1)B(p,n), \quad B(p,n) = p^{2n+1}\,\Gamma\left(2+\frac{2n+1}{p}\right)\Gamma\left(\frac{3n}{2}\right)$$ - large p

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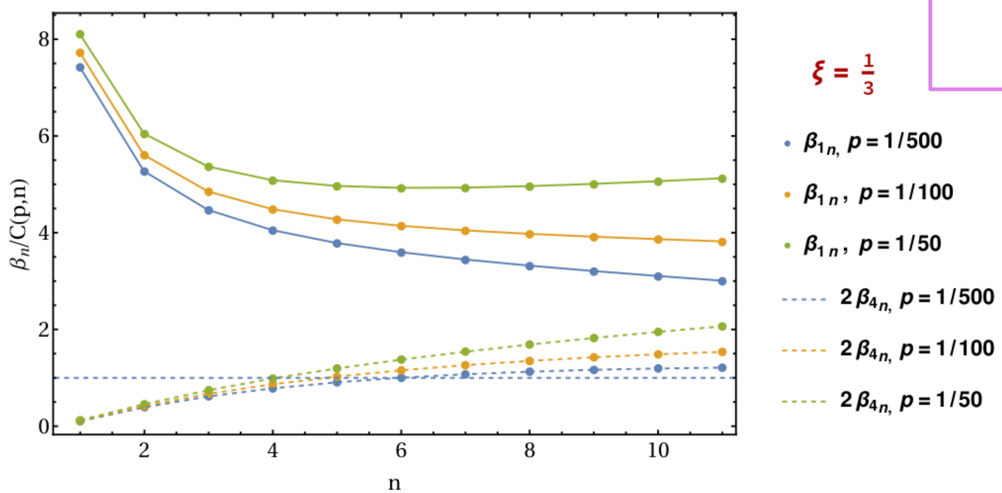
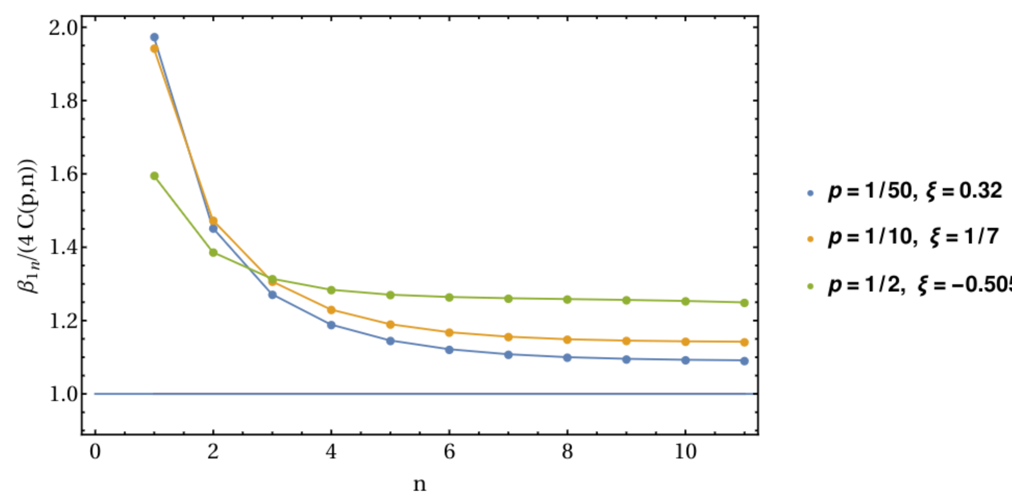
$$M - 4\pi\lambda M^2 (2\alpha)^{-\frac{3}{p}}\left[\left(\sum_{n=0}^{\infty}(-1)^n \beta_{1n}(p)\hat{f}_{1n}(2\alpha)^{-\frac{2n}{p}}\right) + \left(\sum_{n=0}^{\infty}(-1)^n \beta_{2n}(p)\hat{f}_{2n}(2\alpha)^{-\frac{2n}{p}}\right) + \right.$$
$$\left. + \left(\sum_{n=0}^{\infty}(-1)^n \beta_{4n}(p)\hat{f}_{4n}(2\alpha)^{-\frac{2n}{p}}\right)\right]$$





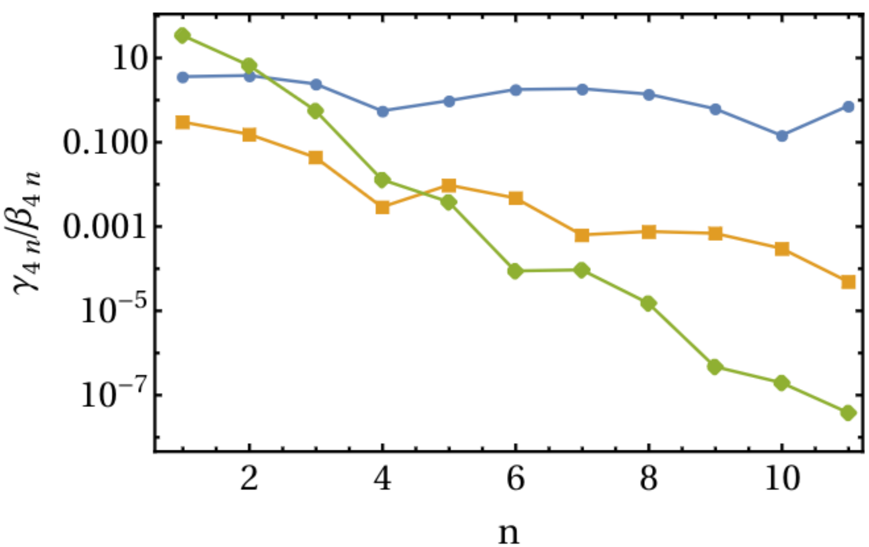
M^3 terms suppression

$$\beta_{in}(p) \propto O(1) B(p,n), \quad B(p,n) = p^{2n+1}\,\Gamma\left(2+\frac{2n+1}{p}\right)\Gamma\left(\frac{3n}{2}\right)$$ - large p

$$\beta_{in}(p) \propto O(1) C(p,n), \quad C(p,n) = p\,\Gamma\left(2+\frac{2n+1}{p}\right)\Gamma(2n)\, e^{-(1+\xi(p))}$$ - small p

$$M - 4\pi\lambda M^2 (2\alpha)^{-\frac{3}{p}} \left[\left(\sum_{n=0}^{\infty}(-1)^n \beta_{1n}(p)\hat{f}_{1n}(2\alpha)^{-\frac{2n}{p}}\right) + \left(\sum_{n=0}^{\infty}(-1)^n \beta_{2n}(p)\hat{f}_{2n}(2\alpha)^{-\frac{2n}{p}}\right) + \right.$$
$$\left. + \left(\sum_{n=0}^{\infty}(-1)^n \beta_{4n}(p)\hat{f}_{4n}(2\alpha)^{-\frac{2n}{p}}\right)\right] -$$
$$\left[\left(\sum_{n=1}^{\infty}(-1)^{n-1} \gamma_{1n}(p)\hat{f}_{1n}(2\alpha)^{-\frac{2(n-1)}{p}}\right) + \left(\sum_{n=1}^{\infty}(-1)^{n-1} \gamma_{2n}(p)\hat{f}_{2n}(2\alpha)^{-\frac{2(n-1)}{p}}\right) + \right.$$
$$\left. - \left(\sum_{n=1}^{\infty}(-1)^{n-1} \gamma_{4n}(p)\hat{f}_{4n}(2\alpha)^{-\frac{2(n-1)}{p}}\right)\right] + O(M^4) = -E$$

Series in front of M^3 have the same or better convergence than M^2



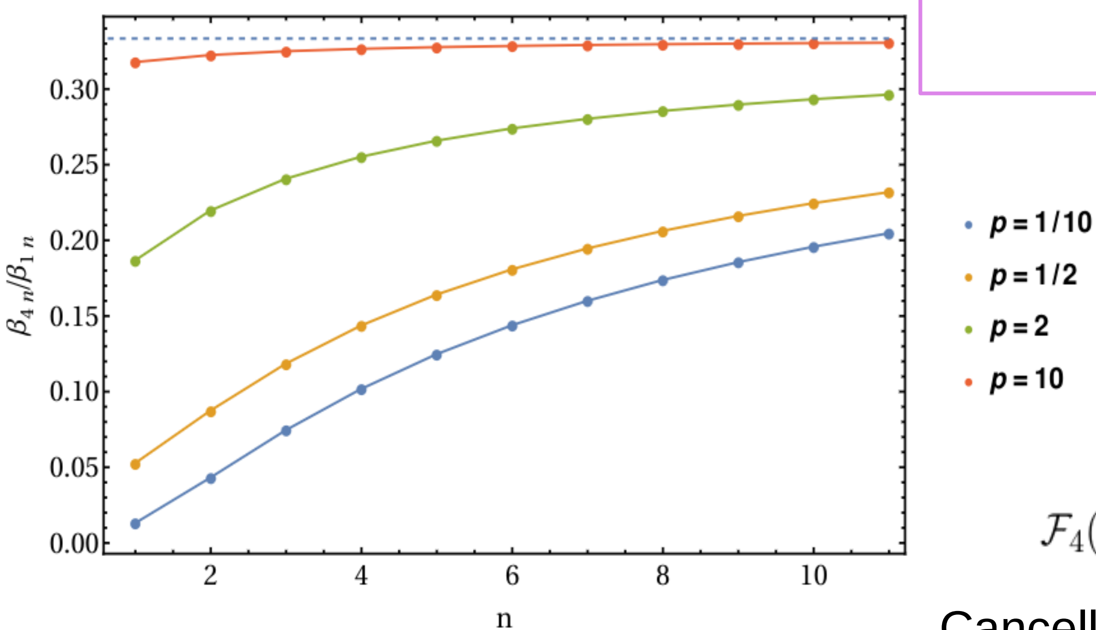
We expect that higher orders do not affect the conclusion. Though it may be not true if there are Riem^3 terms which are dominating over 3-point interaction from quadratic action

Possible cancellations?

$$\beta_{in}(p) \propto O(1)B(p,n), \quad B(p,n) = p^{2n+1}\,\Gamma\left(2+\frac{2n+1}{p}\right)\Gamma\left(\frac{3n}{2}\right). \qquad \text{- large p}$$

$$\beta_{in}(p) \propto O(1)C(p,n), \quad C(p,n) = p\,\Gamma\left(2+\frac{2n+1}{p}\right)\Gamma(2n)\,e^{-(1+\xi(p))} \qquad \text{- small p}$$

$$M - 4\pi\lambda M^2(2\alpha)^{-\frac{3}{p}}\left[\left(\sum_{n=0}^{\infty}(-1)^n\beta_{1n}(p)\hat{f}_{1n}(2\alpha)^{-\frac{2n}{p}}\right)+\left(\sum_{n=0}^{\infty}(-1)^n\beta_{2n}(p)\hat{f}_{2n}(2\alpha)^{-\frac{2n}{p}}\right)+\right.$$
$$\left.+\left(\sum_{n=0}^{\infty}(-1)^n\beta_{4n}(p)\hat{f}_{4n}(2\alpha)^{-\frac{2n}{p}}\right)\right]-$$



For large p we found a sudden cancellation if

$$\hat{f}_{4n} = -3\hat{f}_{1n}$$

$\mathcal{F}_4(\Box) = -3\mathcal{F}_1(\Box)$ ⇨ No scalar degree of freedom

Cancellations are not complete, leading factorial asymptotics is the same

Classification of entire functions by their maximal growth

Relevant quantity:

Maximal growth rate of the function on complex plane (order and type)

$$\rho = \max \left(\lim_{|z|\to\infty} \frac{\log\left(\log|\phi(z)|\right)}{\log|z|} \right) = \lim_{n\to\infty} \frac{n\log n}{-\log a_n}.$$

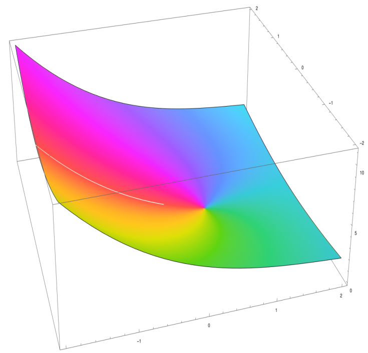
$$s = \max \left(\lim_{|z|\to\infty} \frac{\log|\phi(z)|}{|z|^\rho} \right) = \lim_{|z|\to\infty} \left(\frac{n}{e\,\rho} \right) |a_n|^{\frac{\rho}{n}}$$

Convergence radius (Cauchy–Hadamard theorem)

$$\phi(z) = \sum_{n=0}^{\infty} a_n z^n. \qquad z_r = \lim_{n\to\infty} |a_n|^{-\frac{1}{n}}.$$

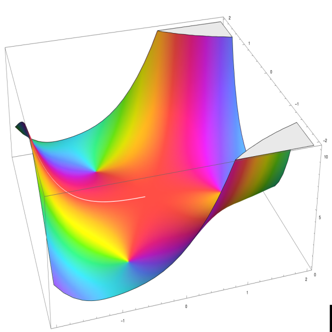
Zero-order entire function (subexponential)

$$\phi(z) = \sum_{n\geq 0} e^{-n^2} z^n. \qquad \sim e^{(\log z)^2}$$



Propagator in a local theory

Exponential type functions



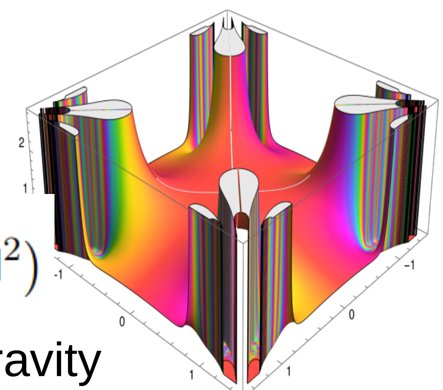
$$\varphi(z) \propto e^{s z^\rho}$$

Exponential non-locality

Exponential functions of infinite order exp(exp(z))

$$\Pi(k^2) = \frac{e^{2\sigma(k^2)}}{k^2},$$

$$\sigma(\Box) = \frac{1}{4}\left(\Gamma(0,\Box^2) + \gamma_E + \log\Box^2\right)$$



Propagator in infinite derivative gravity (limit of Stelle gravity for physical directions)

Main result: singularities in non-perturbative gravity

In classical GR singularity is unavoidable at the final stage of matter collapse

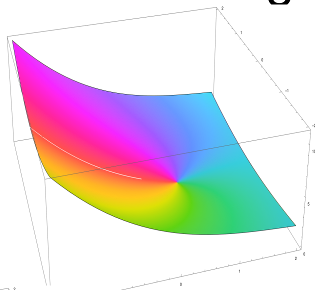
$$M - 4\pi\lambda \int_0^\infty \left(R\Box\mathcal{F}_1'(\Box)R + L_{\alpha\beta}\Box\mathcal{F}_2'(\Box)L^{\alpha\beta} + W_{\alpha\beta\gamma\delta}\Box\mathcal{F}_4'(\Box)W^{\alpha\beta\gamma\delta} \right) r^2 dr = -E.$$

In non-perturbative gravity:

Connection between the possibility to have singularity and graviton propagator!

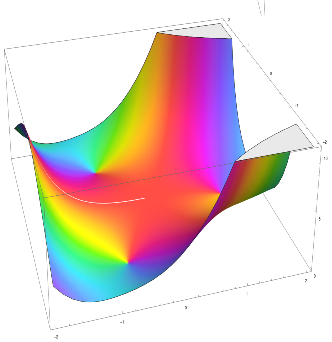
Relevant quantity:

Maximal growth rate of the function on complex plane (order and type)



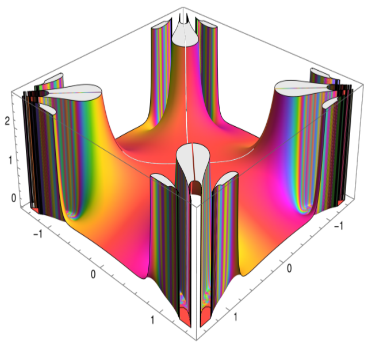
Zero-order entire function (subexponential)

Singular solutions are possible



Exponential functions growing slower than $\exp(z^{3/2})$

Convergence depend on regularization → uncertain



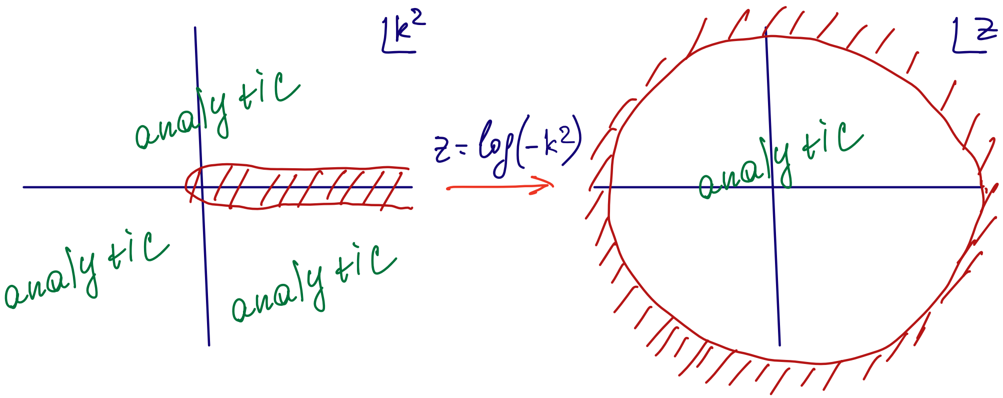
Exponential functions growing faster than $\exp(z^{3/2})$

Singularity is not allowed!

(only infinite mass of black hole leads to singular solution)

A. Koshelev, A. Tokareva, 'Non-perturbative quantum gravity denounces singular Black Holes, arXiv:2404.07925

From tree-level to non-perturbative action, from entire functions to logs



$P(k^2) = \gamma(\log(-k^2))$, γ - entire!

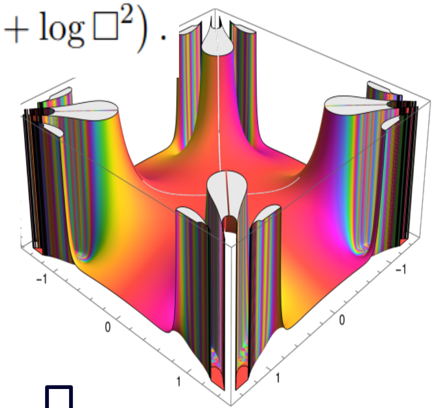
no poles, no zeros $\rightarrow$ $\gamma(z) = e^{\beta(z)}$

$P(k^2) = \exp\left(e^{\beta(\log(-k^2))}\right)$

At complex infinity it can be approximated by entire function

$$\sigma(\Box) = \frac{1}{4}\left(\Gamma(0,\Box^2) + \gamma_E + \log \Box^2\right).$$

$$\Pi(k^2) = \frac{e^{2\sigma(k^2)}}{k^2},$$



$e^{\sigma(k^2)}$

$\downarrow$

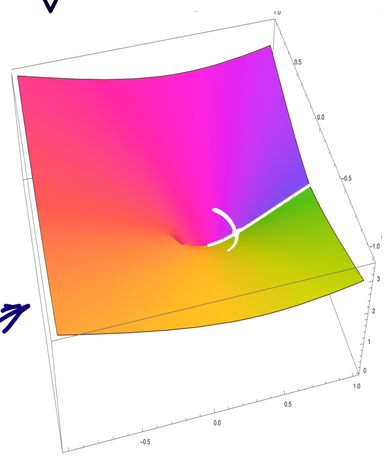
$\exp\left(e^{\beta(\log k^2)}\right)$

σ, β - entire

(ongoing work)

?

Non-perturbative resummation – open question



J. Fehre, D. Litim, J.Pawlowski, M.Reichert, **Phys.Rev.Lett.** 130 (2023) 8

Conclusions

- There are many different proposal formulating non-perturbative gravity. However, for the graviton two-point function all of them lead to the same form of effective action
- Substitution of Schwarzshild metric to infinite derivative equations of motion is a non-trivial computation which we suggest to approach by taking exponentially regularized metric first and getting a limit afterwards
- The Schwarzshild metric limit may not exist. It doesn’t exist for infinite derivative gravity with polynomial propagator along the physical and Euclidian momentum
- Not all effective actions for gravity are compatible with singular black holes which might be a resolution to a number of problems related to their description
- We formulate criteria for non-perturbative propagator which relate its maximal growth rate in the complex plane to the existence of singular spherically-symmetric solutions

Thank you !