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Interaction between several types of cosmic strings

Based on **JHEP12(2023)115**

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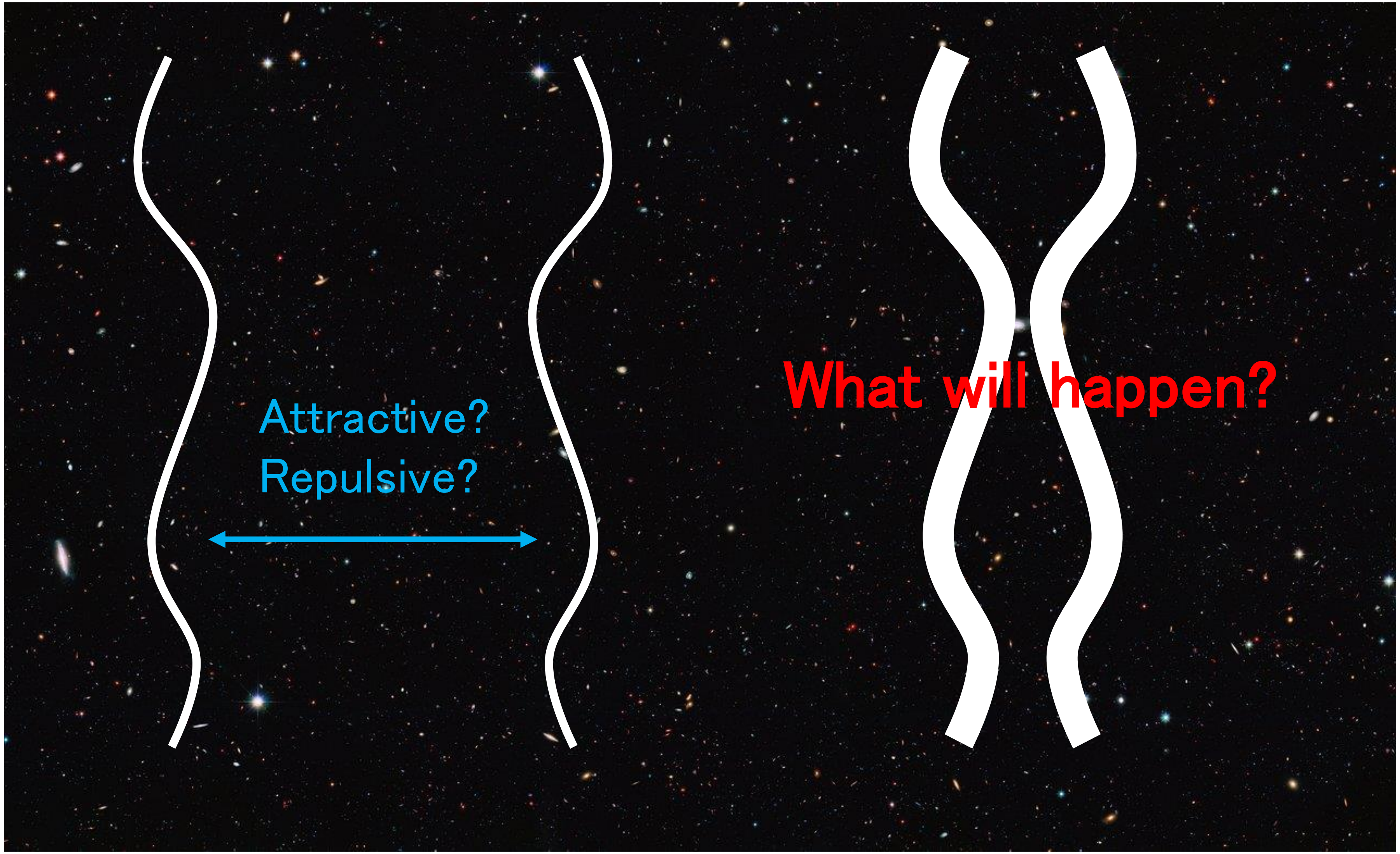
Introduction



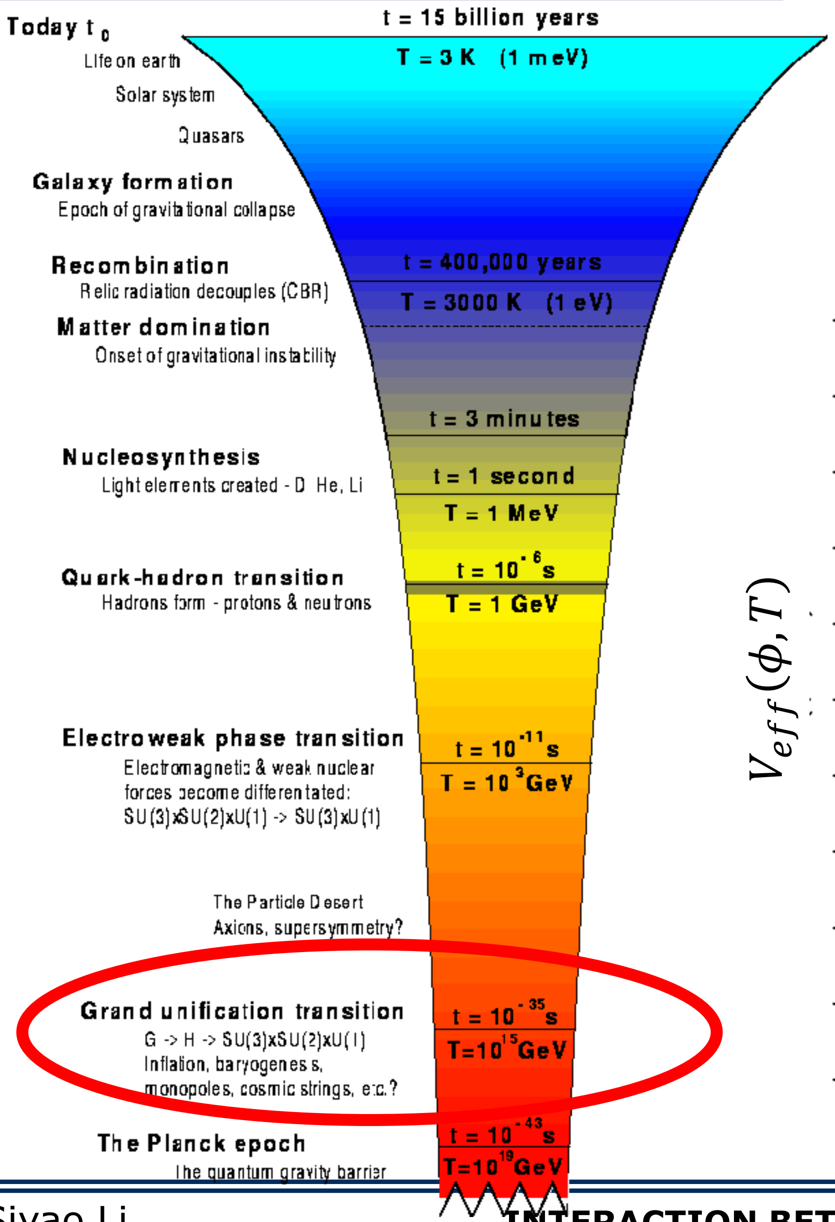
Purpose of this work:
to investigate interaction between two cosmic strings

Contents

- Introduction
- Apply **source approximation method** to derive interaction energy of two cosmic strings for:
  - Local Cosmic Strings
  - Bosonic Superconducting Cosmic Strings
- **Numerical** calculation with gradient flow method for interaction energy of two-string systems
- Summary

Phase Transition in the early universe

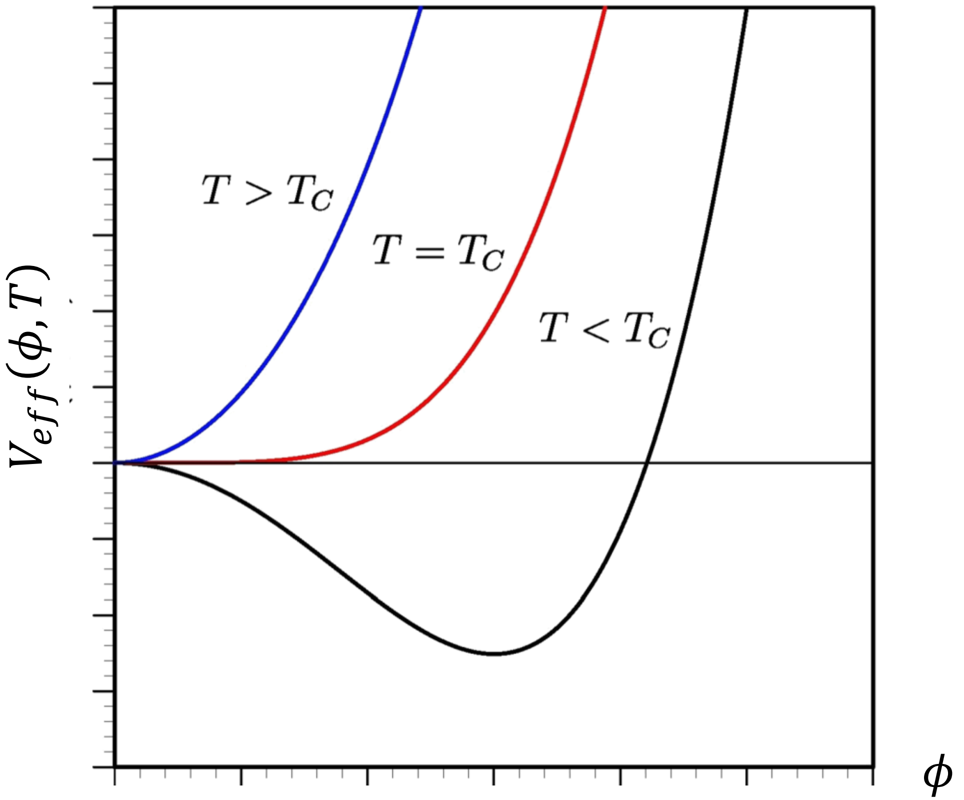
[A. Vilenkin & E. Shellard, 1994]



[Brandenberger ,1994]

Finite temperature effective potential

$$V_{eff}(\phi,T) = \frac{1}{4}\lambda\phi^4 - \frac{1}{2}\left(\lambda\eta^2 - \hat{\lambda}T^2\right)\phi^2 + \frac{1}{4}\lambda\eta^4$$



temperature decrease

topology of vacuum manifold changed

⇩

spontaneous symmetry breaking

⇩

Cosmic strings (topological defects) can form

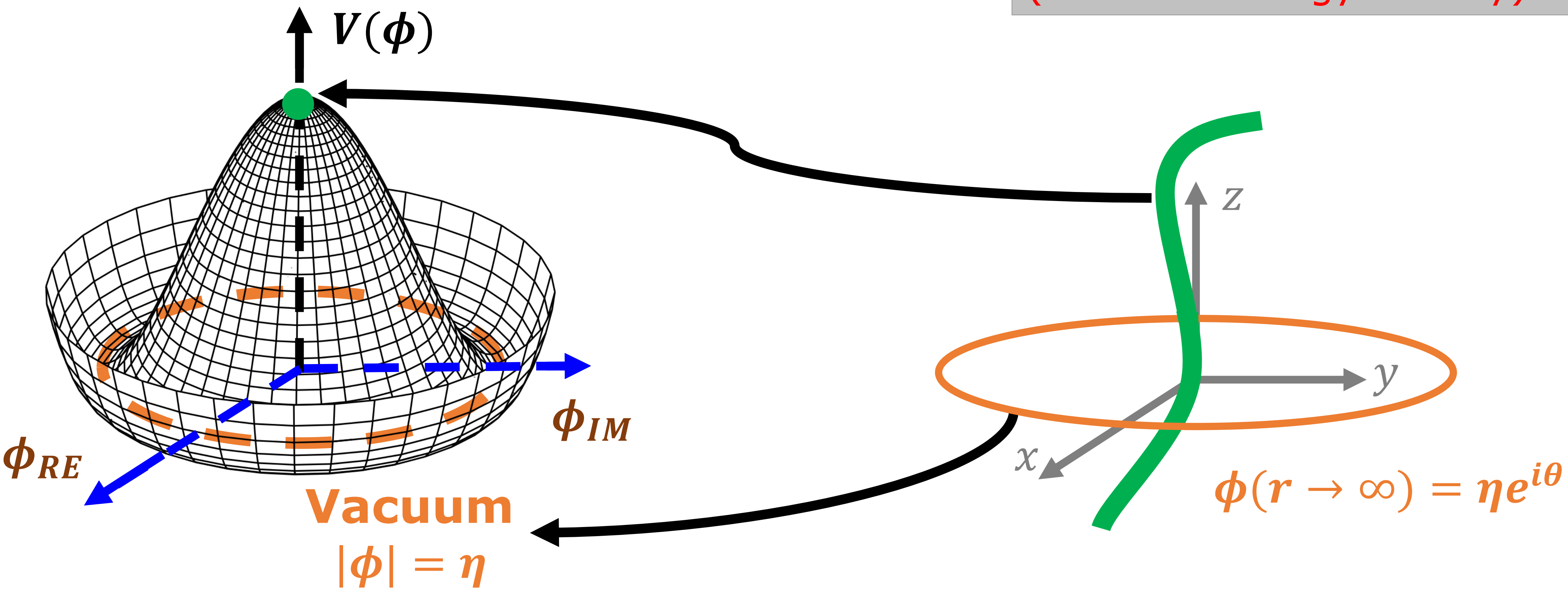
Cosmic strings

U(1) Goldstone model

Spontaneous symmetry breaking

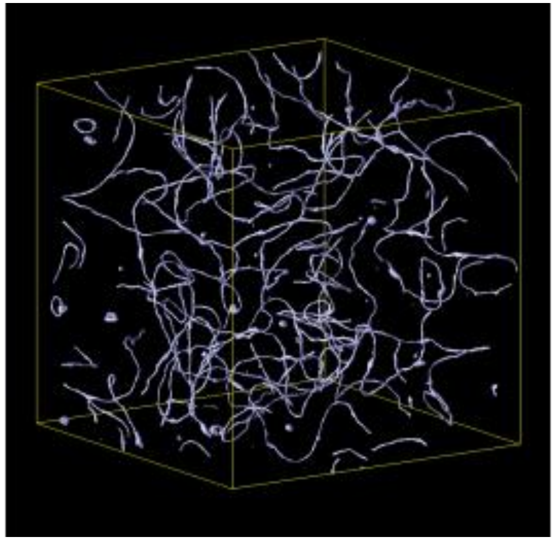
$$\mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - V(\phi), \ \ V(\phi) = \frac{1}{4}\lambda(\phi^2 - \eta^2)^2$$

Cosmic string =
linear topological defect
(localized energy density).



Cosmic string network and Observations

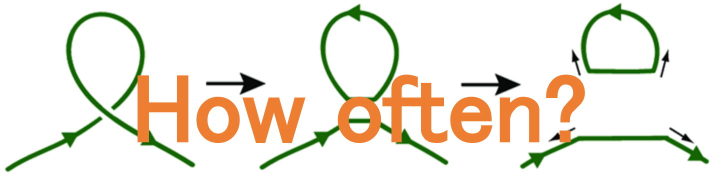
Cosmic strings network



[Hiramatsu et al. 2013]

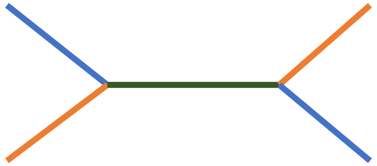
our purpose

interactions, dynamics… ⇒ reconnect, collapse… ⇒ kinks, cusps, Y-junctions



How often?

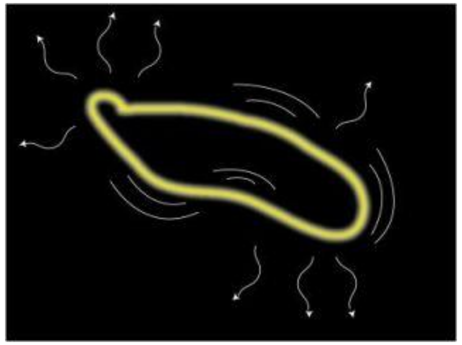
How many?



Y-junction

⇓

Lensing, GW burst…



→ local strings: gravitational waves

→ global strings: axion production

→ superconducting: electromagnetic waves

Derive interaction energy of two cosmic strings for:

- Local Cosmic Strings
- Bosonic Superconducting Cosmic Strings

with **source approximation method**

(applicable when distance between two cosmic strings are far)

Local cosmic strings

Abelian-Higgs model

$$\mathcal{L}_{AH} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \left|D_\mu\phi\right|^2 - V(\phi),$$

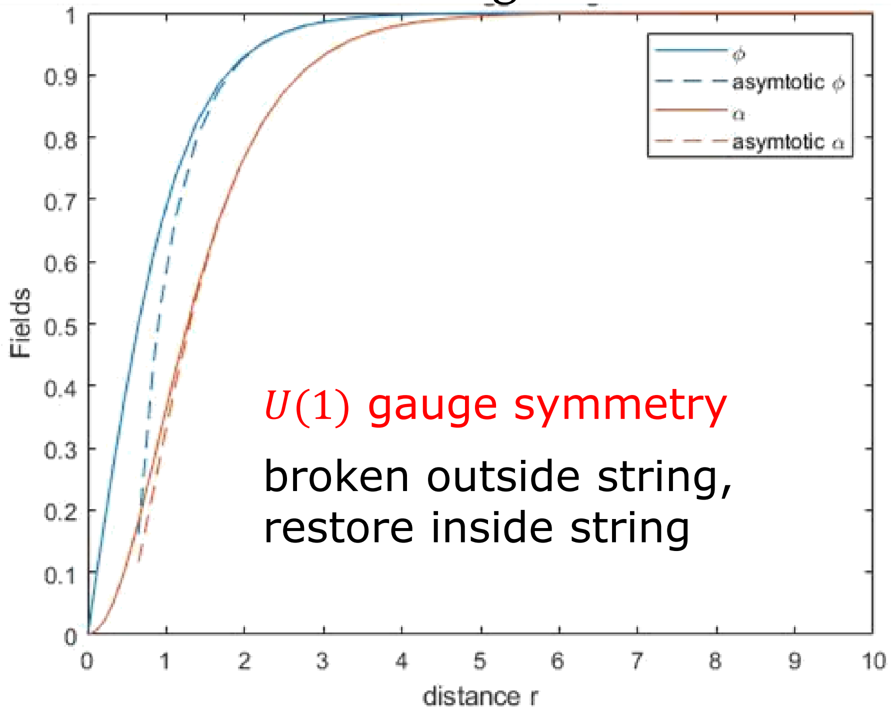
$$V(\phi) = \frac{1}{4}\lambda(|\phi|^2 - \eta^2)^2$$

A cosmic string solution can be
static, straight, circular symmetric

Boundary conditions:

- Regularity at the origin
- Finite total energy

Local string solutions



$U(1)$ gauge symmetry

broken outside string,
restore inside string

Asymptotic solutions at $r \to \infty$:

$$\phi(r) = [\eta + k_\phi K_0(m_\phi r)]e^{in\theta},$$

$$A_\theta(r) = k_e r K_1(m_e r) + \frac{n}{e}$$

modified Bessel function $K_i(mx) \propto e^{-mx}$ at $x \to \infty$

$$m_\phi \equiv \sqrt{\lambda}\eta, \qquad m_e \equiv \sqrt{2}e\eta$$

n: winding number

$k_\phi \propto -|n|$

$k_e \propto -n$

Point source approximation

$$\phi(r) = \left(\eta_\phi + \frac{\sigma(r)}{\sqrt{2}}\right)e^{in\theta},\ A_\theta(r) = U_\theta(r) + \frac{n}{e}$$

$\mathcal{L}$ =free field Lagrangian $-J_\sigma \sigma - j_\mu U^\mu$

external sources

substitute the asymptotic solutions

$$J_\sigma = -k_\phi \frac{\delta(r)}{r}$$

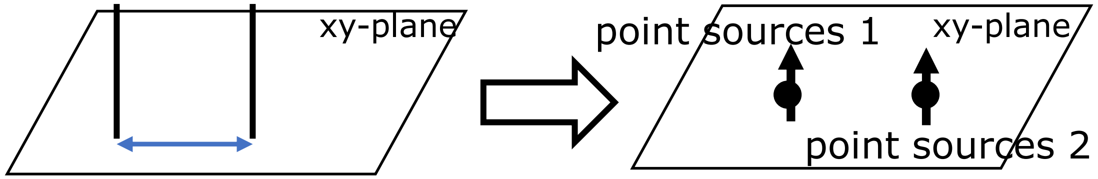
scalar monopole

$$j_\theta = -\frac{k_e}{m_e}\partial_r\left(\frac{1}{r}\delta(r)\right)$$

magnetic dipole moment

point-like sources in 2-dimensions

parallel, **well-separated** strings



$d >$ string widths

Then, interaction energy can be computed analogously to Yukawa potential.

Point source approximation

$\phi(r) = \left(\eta_\phi + \frac{\sigma(r)}{\sqrt{2}}\right) e^{in\theta}$, $A_\theta(r) = \mathrm{U}_\theta(r) + \frac{n}{e}$

$\mathcal{L}$ =free field Lagrangian$-J_\sigma \sigma - j_\mu U^\mu$

substitute the asymptotic solutions ⇒

$$J_\sigma = -k_\phi \frac{\delta(r)}{r}$$

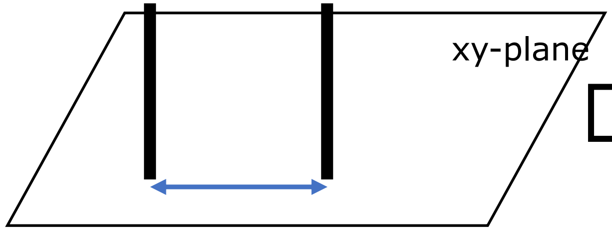
scalar monopole

$$j_\theta = -\frac{k_e}{m_e} \partial_r\left(\frac{1}{r}\delta(r)\right)$$

magnetic dipole moment

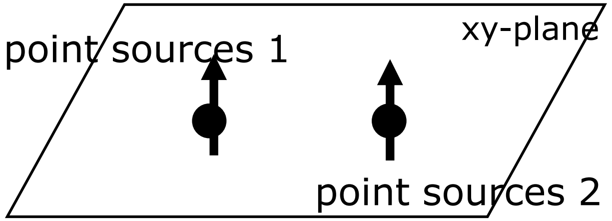
point-like sources in 2-dimensions

parallel, **well-separated** two strings

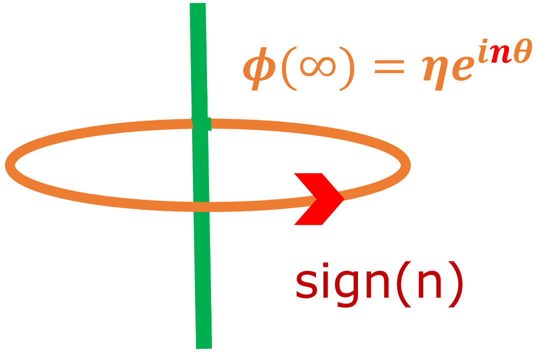


$d >$ string widths

⇒



$$E_{int} = 2\pi \int dz \left[-k_{\phi 1} k_{\phi 2} K_0(m_\phi d) + k_{A1} k_{A2} K_0(m_e d)\right]$$



Scalar field contribution

always attractive

Gauge field contribution

determined by **winding direction** :

- Same direction: repulsive
- Opposite: attractive

Bosonic superconducting strings

Bosonic superconducting model

$$\mathcal{L}_{BC} = \mathcal{L}_{AH} - \frac{1}{4}\widetilde{F^{\mu\nu}}\widetilde{F_{\mu\nu}} + \left|\widetilde{D_\mu}\tilde{\phi}\right|^2 - V(\phi, \tilde{\phi}),$$

$$V(\phi, \tilde{\phi}) = V_{AH} + \frac{1}{4}\lambda_{\tilde{\phi}}\left(\left|\tilde{\phi}\right|^2 - \eta_{\tilde{\phi}}{}^2\right)^2 + \beta|\phi|^2\left|\tilde{\phi}\right|^2$$

[Witten, 1985]

$U(1) \times \tilde{U}(1)$ gauge symmetry

- $U(1)$ symmetry to be spontaneously broken
  ⇩
  giving rise to underlying local string
- $\tilde{U}(1)$ symmetry to be unbroken outside the string
  ⇩
  making the string carry persistent current

Parameter space:

- $\tilde{U}(1)$ symmetry unbroken outside string

$$m_{\tilde{\phi}}^2(\mathrm{r} \to \infty) > 0$$

- $|\phi| = \eta_\phi,\ |\tilde{\phi}| = 0$ should be global minimum
- To make $|\tilde{\phi}| \neq 0$ energy favorable rather than trivial solution $|\tilde{\phi}| = 0$

(existence of negative energy state)

Asymptotic solutions

Boundary conditions:

- Regularity at the origin
- Finite energy

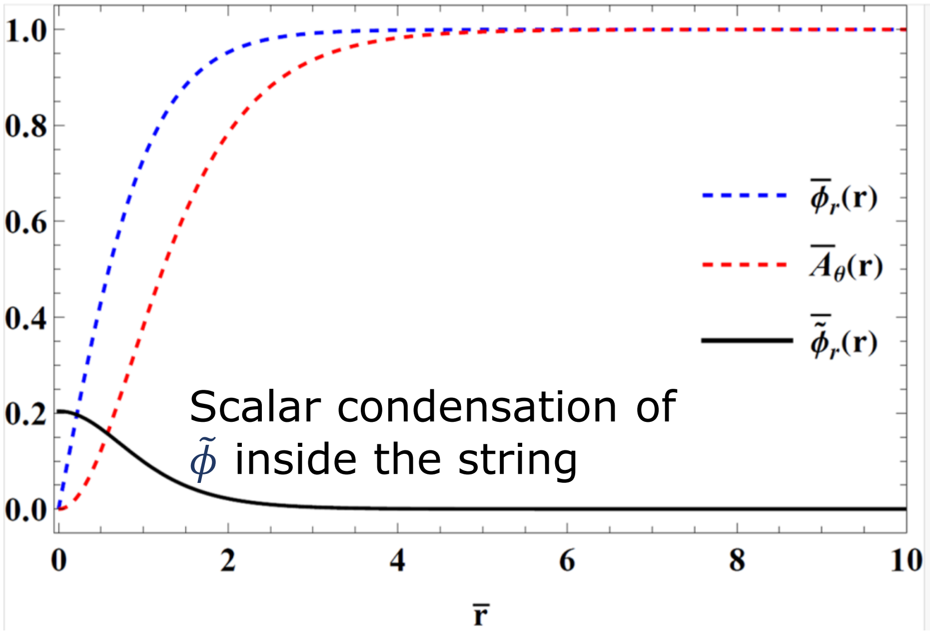
⇨

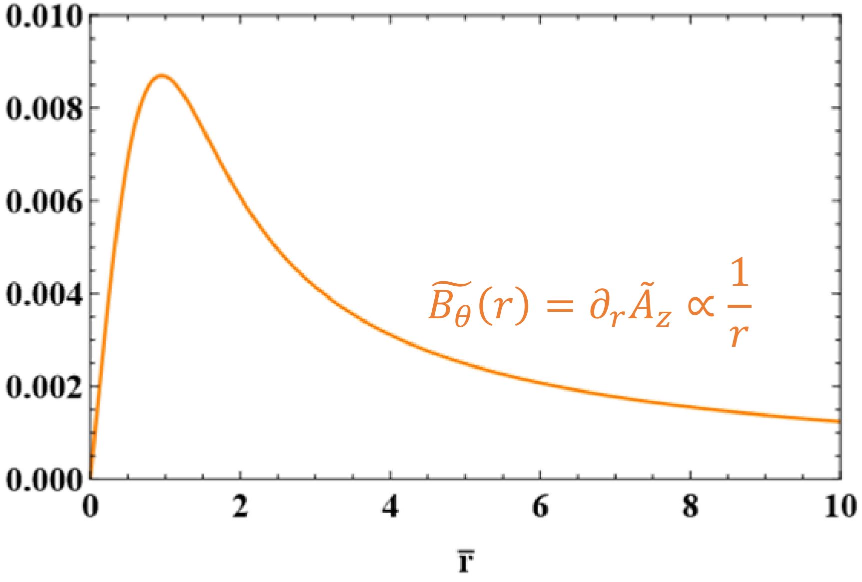
Asymptotic solutions at large distance:

$$\tilde{\phi}_r(r) = k_{\tilde{\phi}} K_0(m_{\tilde{\phi}} r)$$

$$\tilde{A}_z \propto \tilde{s}(r) = k_s \ln r \quad \text{massless}$$

modified Bessel function $K_i(mx) \propto e^{-mx}$ at $x \to \infty$





Interaction with source method

$$\mathcal{L} = \mathcal{L}_{AH} + \text{ free field Lagrangian } - J_{\tilde{\phi}}\tilde{\phi}_r - \tilde{j}_\mu \widetilde{A^\mu}$$

external sources

$$J_\sigma = -\frac{k_\phi}{r}\delta(r)$$

$$j_\theta = -\frac{k_e}{m_e}\partial_r(\frac{1}{r}\delta(r))$$

$$J_{\tilde{\phi}} = -k_{\tilde{\phi}}\frac{\delta(r)}{r}$$

$$\tilde{j}_\mu = -\frac{k_s}{g}\frac{\delta(r)}{r}$$

point-like sources in 2-dimensions

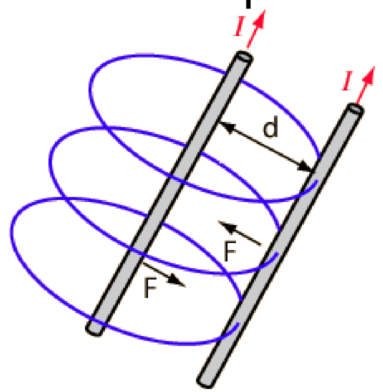
Point source approximation:

attractive scalar contribution

$$E_{int} = 2\pi \int dz\, [E_{AH} - k_{\tilde{\phi}1}k_{\tilde{\phi}2}K_0\left(m_{\tilde{\phi}}d\right) + k_{\tilde{A}1}k_{\tilde{A}2}\ln d]$$

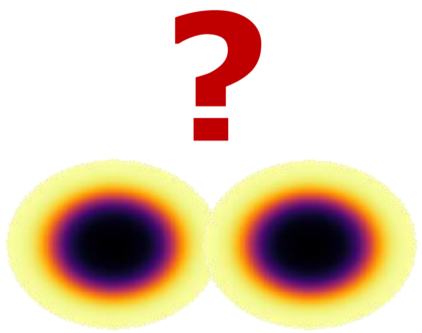
determined by direction of current

Similar as Ampere's Force

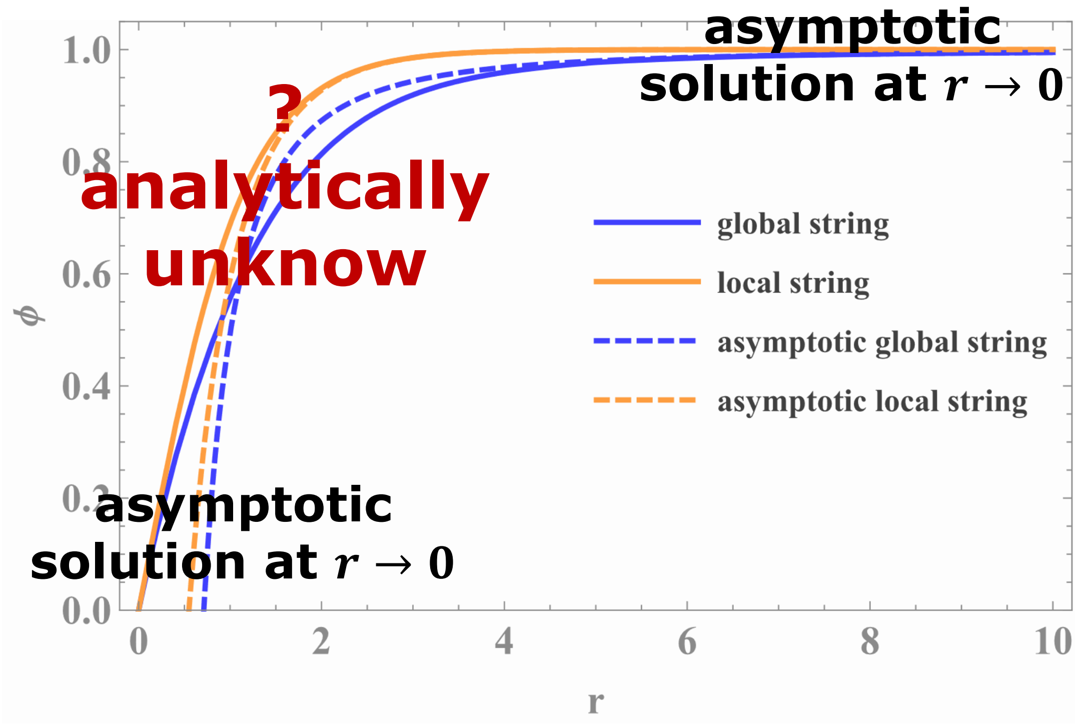


When two cosmic strings get closer…

Far



Close



Numerical simulation needed.

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Numerical computation for interaction energy of two-string systems for

- Local Cosmic Strings
- Bosonic Superconducting Cosmic Strings

with gradient flow method

(applicable for arbitrary distance between two cosmic strings)

Numerical calculation

- Aim: looking for static, lowest energy states of the system
- Method: **Gradient Flow**
  - initial guess satisfying boundary conditions
  - evolve the fields with time

field $X_i(r,\theta) \rightarrow X_i(t,r,\theta)$

$$EOM\ of\ X_i = 0 \rightarrow EOM\ of\ X_i = \partial_t X_i$$

Diffusion equation

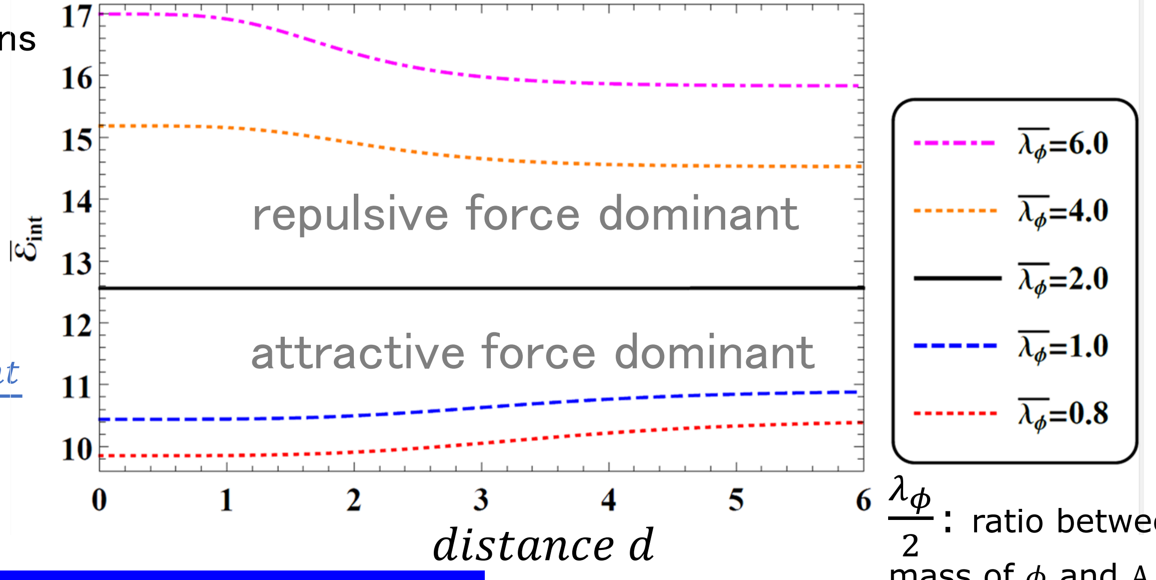
  - fixing the center of strings by hand
  - converge symbol: $\partial_t X_i = 0$

Interaction energy of two **local cosmic strings**

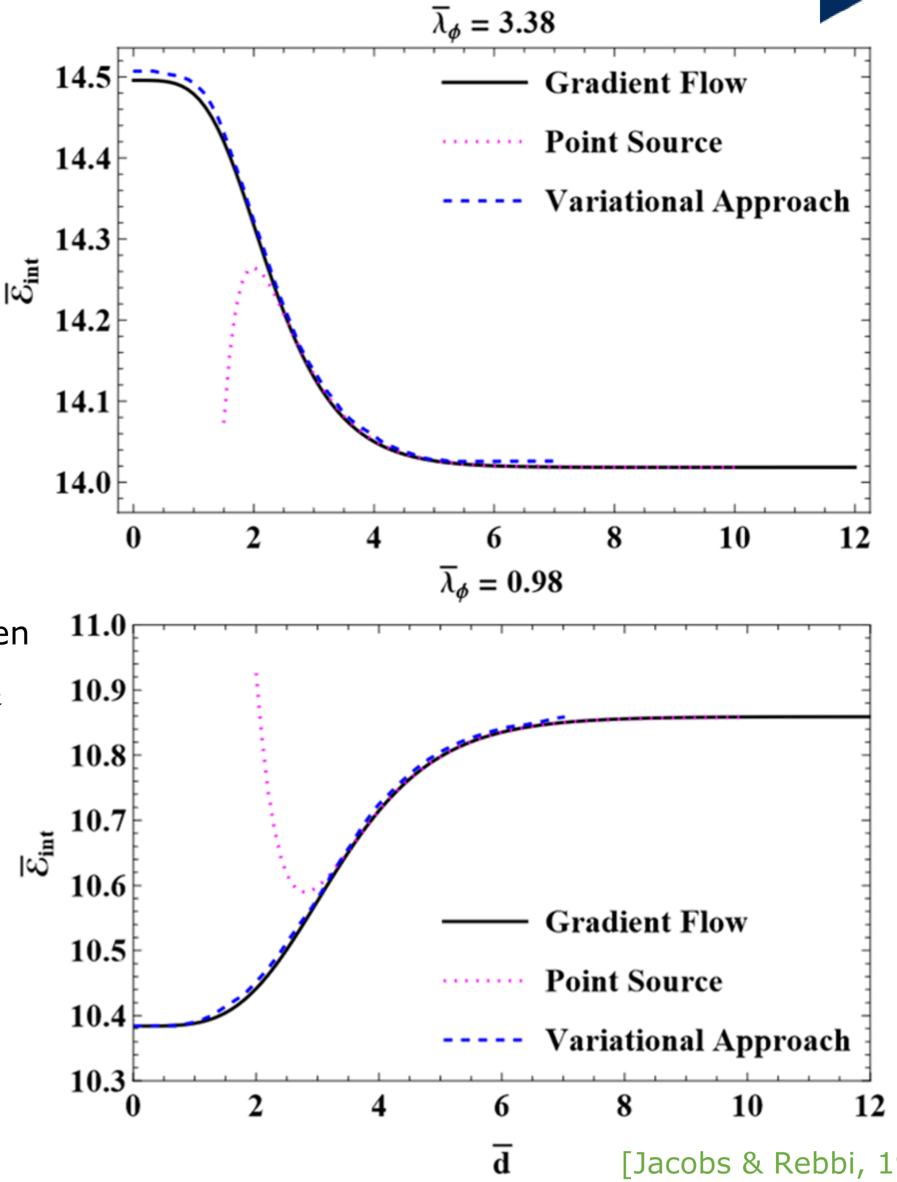
two local strings with same winding direction

At $d = 0$, solutions for $\mathrm{n} = 2$

Force $= \frac{\partial E_{int}}{\partial d}$



$\frac{\lambda_\phi}{2}$: ratio between mass of ϕ and A_μ



[Jacobs & Rebbi, 1979]

point source approximation

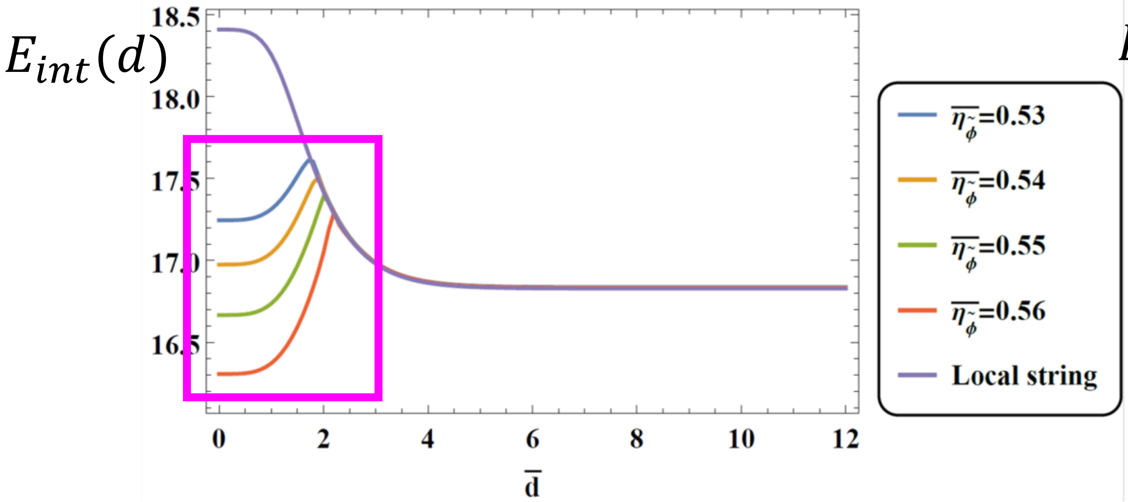
attractive / repulsive

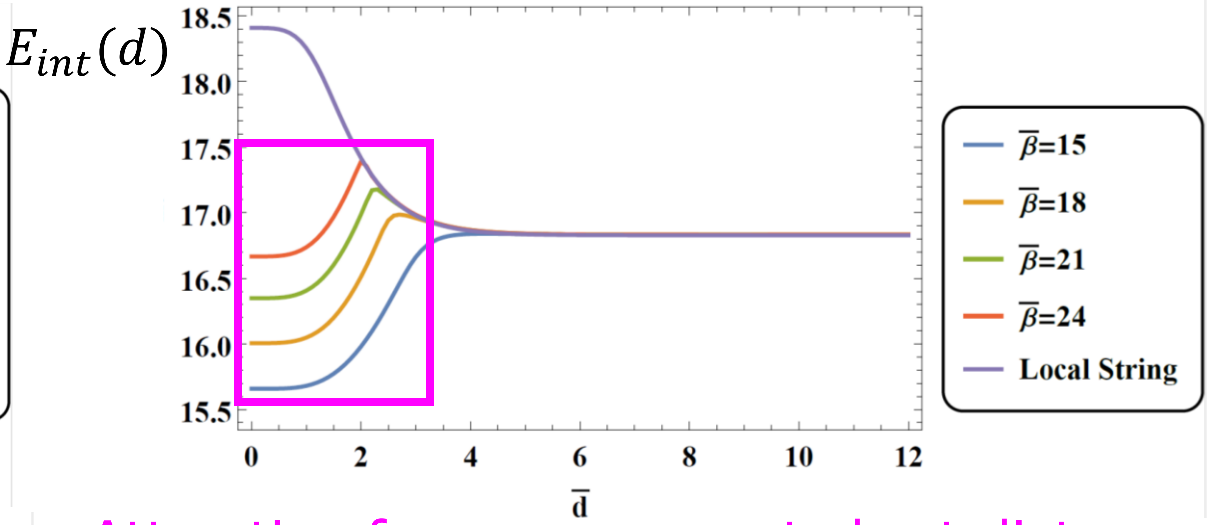
$$E_{int} = 2\pi \int dz \, [\underbrace{-k_{\phi 1} k_{\phi 2} K_0(m_\phi d)}_{\text{attractive}} + \underbrace{k_{A1} k_{A2} K_0(m_e d)}_{\text{repulsive}}]$$

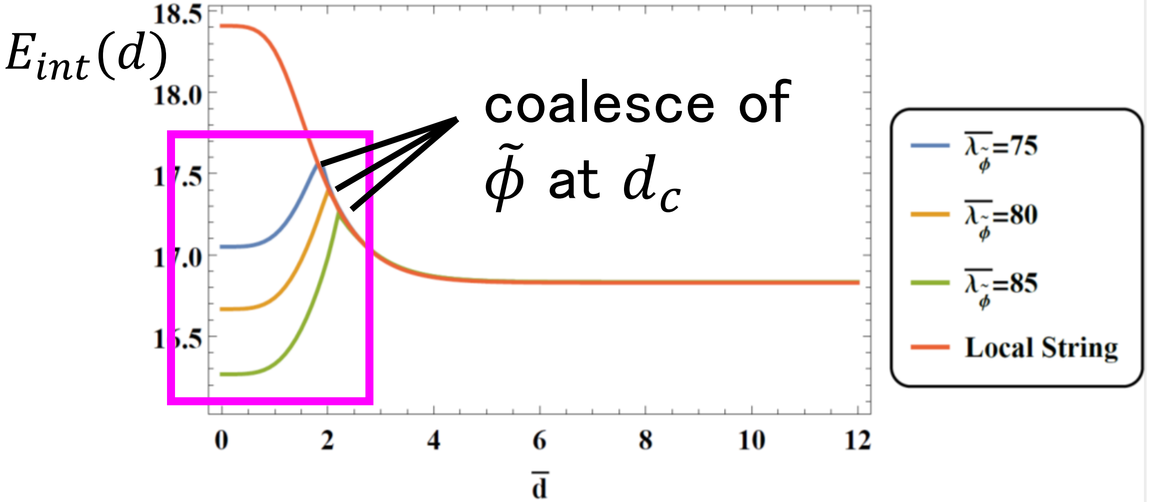
cancel at $\frac{m_\phi^2}{m_e^2} = 1$(i.e. $\lambda_\phi = 2$)

Interaction of **bosonic superconducting strings**

Two strings with **zero current** (only $\tilde{\phi}$ condensation, $\tilde{A}_\mu = 0$)):

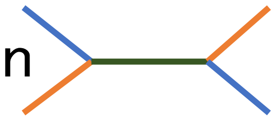






Attractive force appears at short distance **due to the gradient energy of $\tilde{\phi}$ condensation**

⇒ implying higher rate of Y–junction



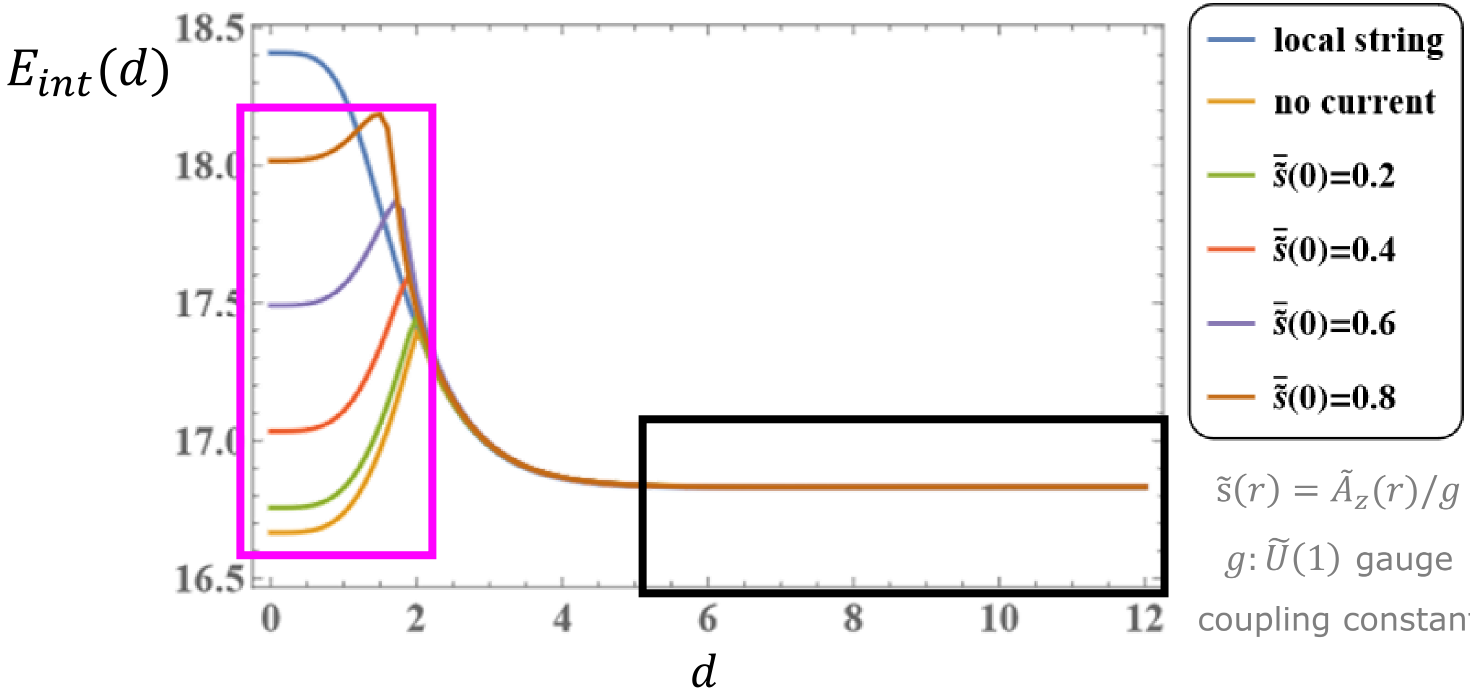
$$d_c \propto m_{\tilde{\phi}}^{-1}$$

effective mass of $\tilde{\phi}$:

$$m_{\tilde{\phi}} = \beta|\phi|^2 - \frac{1}{2}\lambda_{\tilde{\phi}}\eta_{\tilde{\phi}}^2 + \lambda_{\tilde{\phi}}|\tilde{\phi}|^2$$

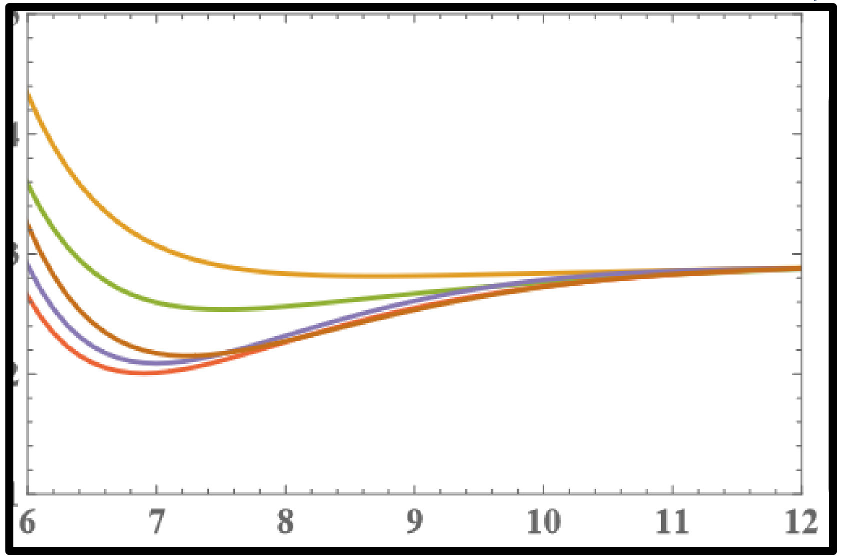
Interaction of **bosonic superconducting strings**

With non-zero current($\tilde{A}_\mu \neq 0$):



$\tilde{s}(r) = \tilde{A}_z(r)/g$

g: $\tilde{U}(1)$ gauge coupling constant



point source approximation:

$$E_{int}[\tilde{A}_\mu] = 2\pi \int dz \ \ [k_{\tilde{A}1} k_{\tilde{A}2} \ln d]$$

effective mass of $\tilde{\phi}$:

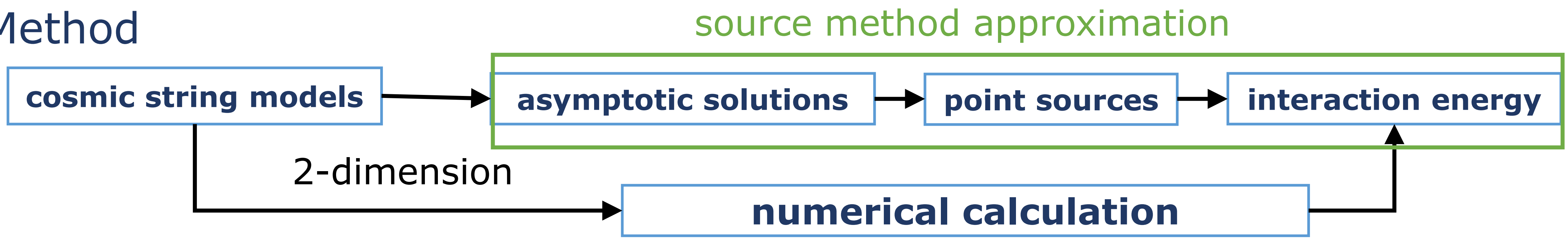
$$m_{\tilde{\phi}} = \beta|\phi|^2 - \frac{1}{2}\lambda_{\tilde{\phi}}\eta_{\tilde{\phi}}^2 + \lambda_{\tilde{\phi}}|\tilde{\phi}|^2 + \frac{1}{g^2}\tilde{A}_z^2$$

$\tilde{A}_z$ suppresses the condensation of $\tilde{\phi}$, then weaken the attractive force at short distance caused by $\tilde{\phi}$

Summary

- We investigated interaction between two straight, static, cylindrical symmetric cosmic strings for local strings, bosonic superconducting strings, global strings. (JHEP12(2023)115).

- Method



- Important conclusions
  - **well-separated**: interaction dominant by the field with smallest mass at large distance
  - **getting close**: strength of scalar condensate of bosonic superconducting string determines the short-distance attraction ⇨ implying higher rate of Y-junction formation

- Future work
  - Simulation of 3-d cosmic string network; Formation and distribution of substructures; Prospective observations…

Thank you very much for attention.

Siyao Li 李思遥

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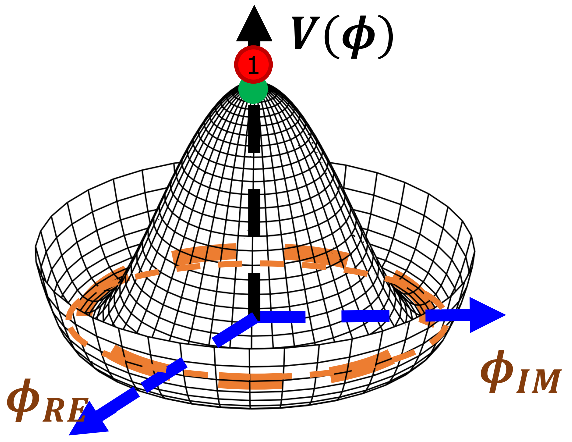
Cosmic string formation

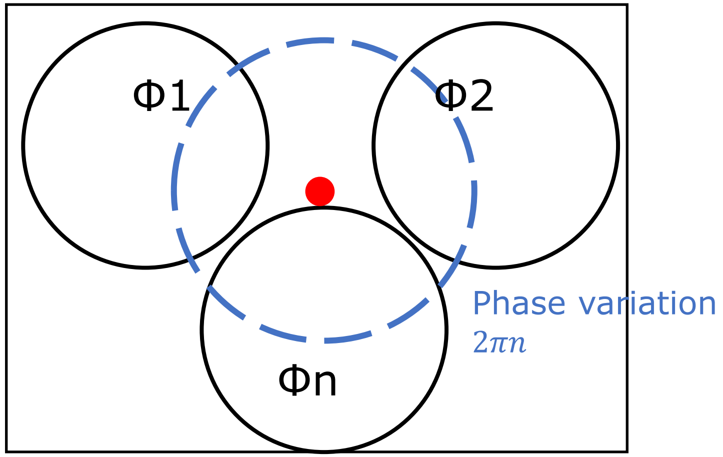


Spontaneous symmetry breaking happens at phase transition

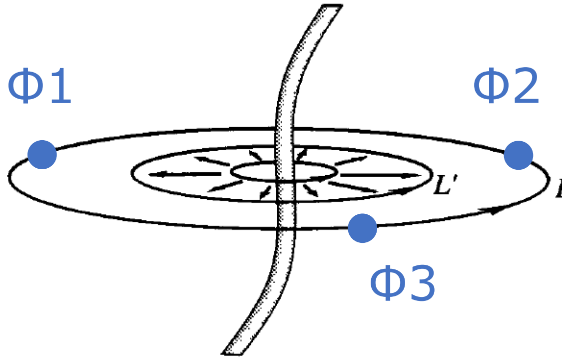
By causality, the values of ϕ in $\mathcal{M}$ cannot be correlated on scales larger than Hubble radius ~ t.

[Kibble, 1980]

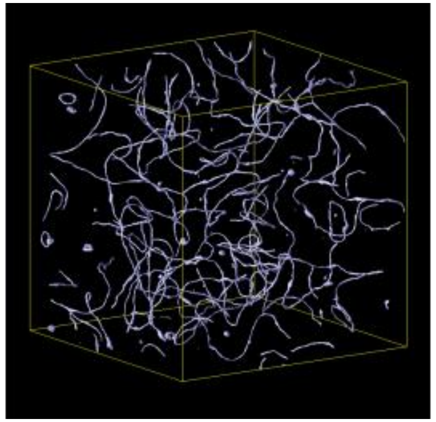
Independent regions



$\xi(\mathrm{t}) \lesssim t$: Correlation length



⇨ Cosmic strings inevitably form in the early universe.



Cosmic string network

[Hiramatsu et al. 2013]

Implications

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- Cosmic strings are predicted in many beyond Standard Model theories.
- Cosmic strings are inevitably formed in the early universe and can persist to the present time.
- Cosmic strings are considered to give contributions to CMB anisotropy and seeds of Large Scale Structure…
- Cosmic strings are constrained from cosmology: e.g. angular spectrum of CMB anisotropies gives a limit of only ~ 10% of the total power can come from strings [e.g. Wyman, Pogosian and Wasserman, 2005]. ⇨ Upper limit of mass per unit length $G\mu \sim G\eta^2 < 10^7$

Bosonic superconducting strings

static, straight, circular symmetric

Then a general ansatz is

$$\tilde{\phi} = \tilde{\phi}_r(r)e^{-is(r)\alpha(z)}, \widetilde{A_\mu} = -\frac{1}{g}\alpha(z)\,\partial_\mu s(r)$$

[Alford M et al. Nuclear Physics B, 1991]

$$\tilde{\phi} = \tilde{\phi}_r(r), \widetilde{A_\mu} = \frac{1}{g}\boxed{s(r)\partial_\mu\alpha(z)} \equiv \tilde{s}(r)$$

$$r\partial_r s\partial_z^2\alpha = 0$$

$$\frac{1}{r}\partial_r(r\partial_r)s(r) - 2g^2 s(r)\tilde{\phi}_r^2 = 0$$

⇒

$$\alpha(z) = \omega z$$

London Equation
with penetration depth
$\delta_A(r) = 1/g\tilde{\phi}_r(r)$

Superconducting current along the string

$$J_z = \int d^2x[-2g\omega s(r)\tilde{\phi}_r^2]$$

Numerical result for bosonic superconducting

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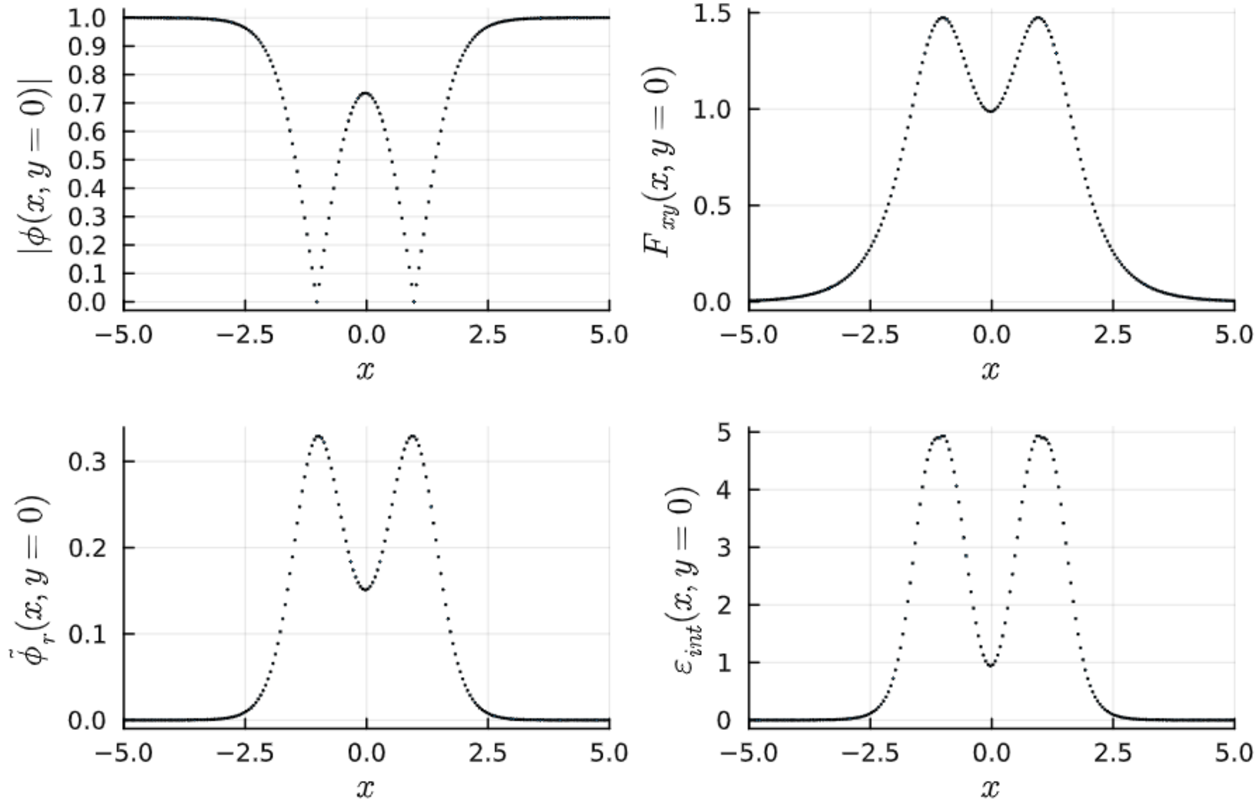
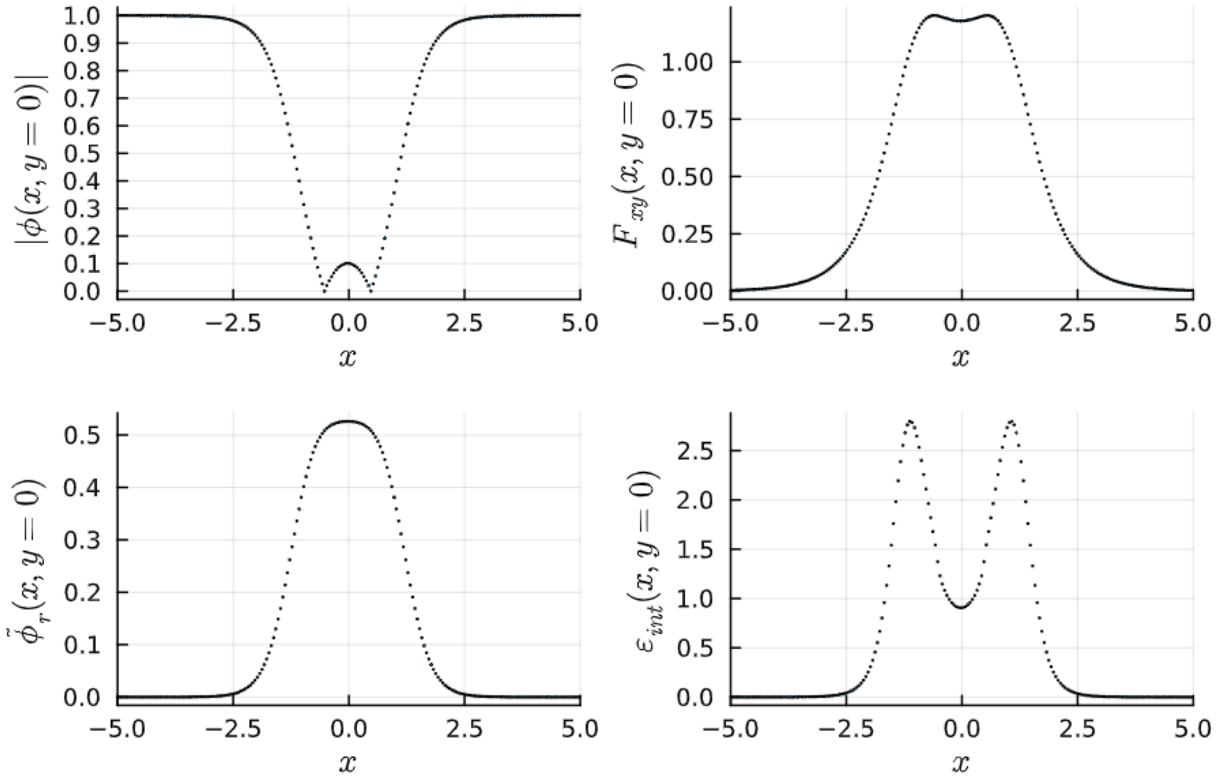
Zero current case: (without current: $\tilde{A}_\mu = 0(\tilde{s} = 0)$)

with parameters $\lambda_\phi = 8,\ \beta = 24,$
$\lambda_{\tilde{\phi}} = 80,\ \eta_{\tilde{\phi}} = 0.55, e\eta_\phi = 1.$
$(m_e^{-1} > m_\phi^{-1} > m_{\tilde{\phi}}^{-1})$

d=2



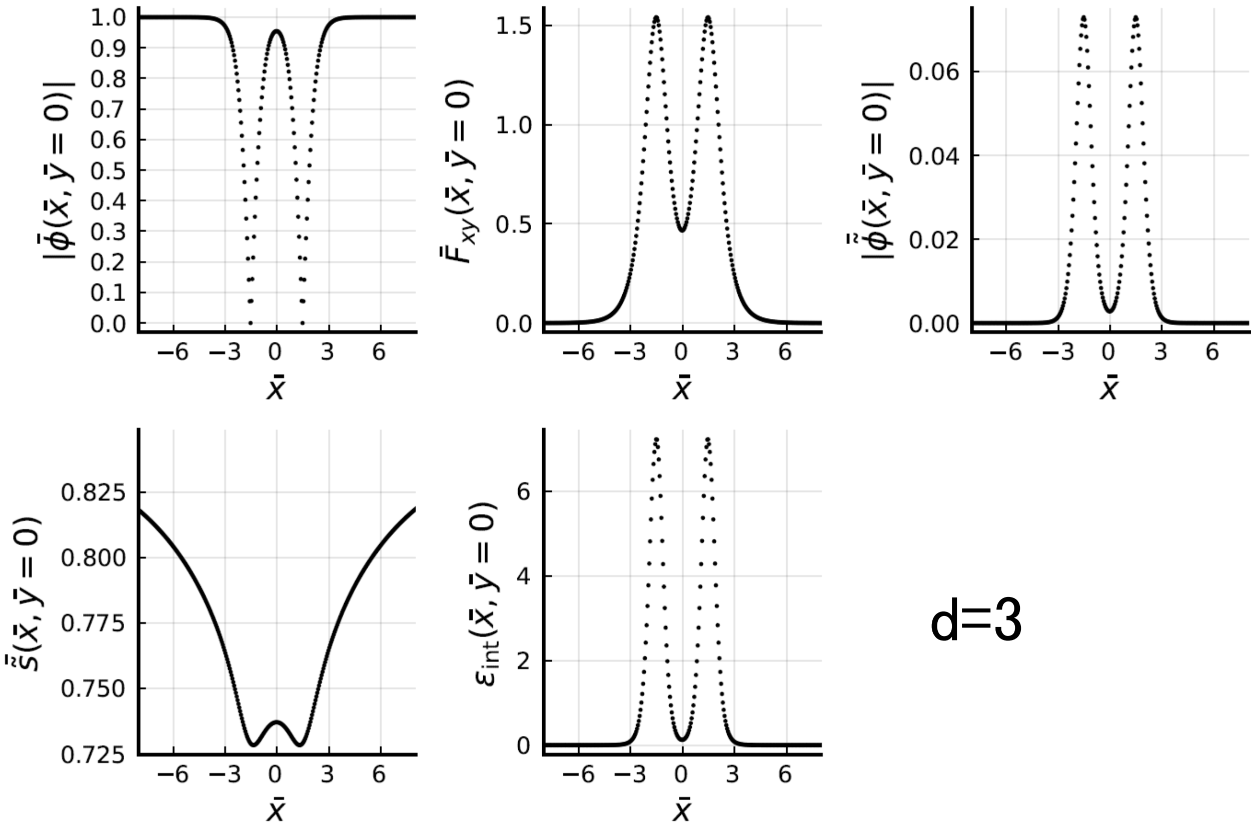
d=1



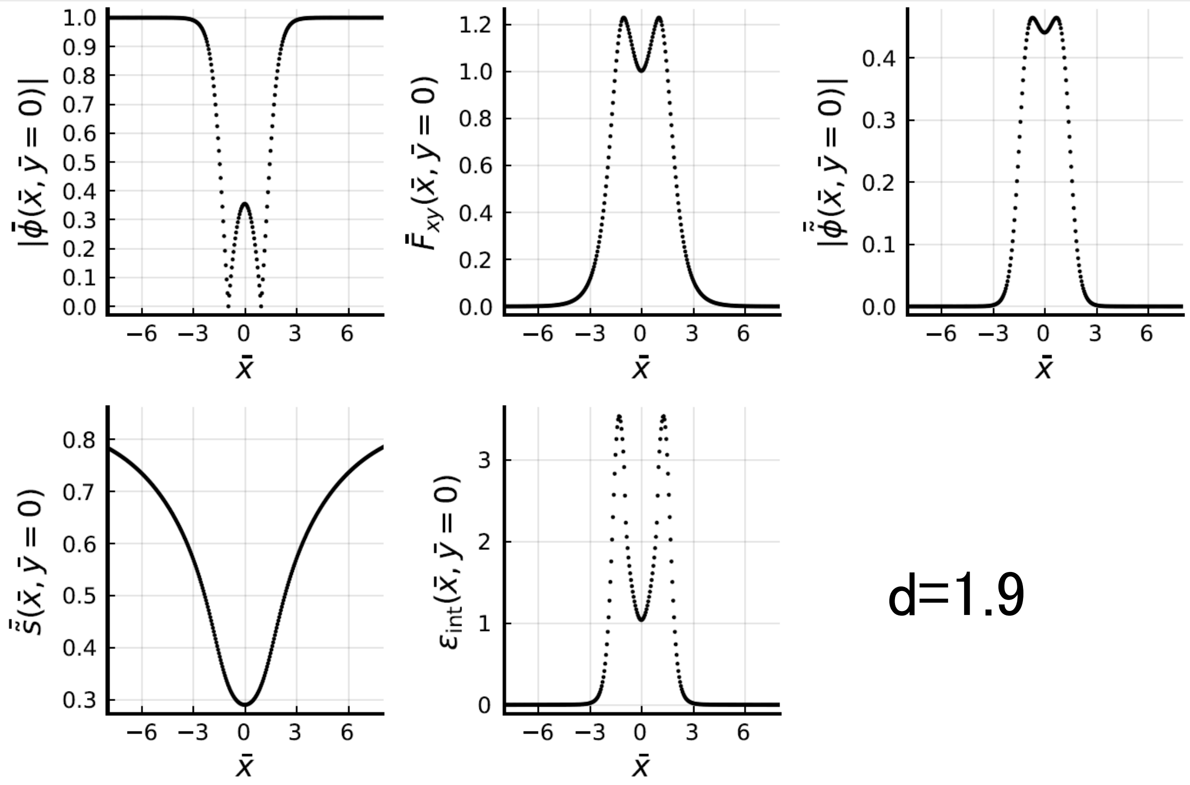
Numerical result for bosonic superconducting

Non-zero current case: ($\tilde{A}_z \neq 0(\tilde{s} \neq 0)$)

with parameters $\lambda_\phi = 8,\ \beta = 24,\ \lambda_{\tilde{\phi}} = 80,\ \eta_{\tilde{\phi}} = 0.55,$
$e\eta_\phi = 1, \tilde{s}0 = 0.4.$

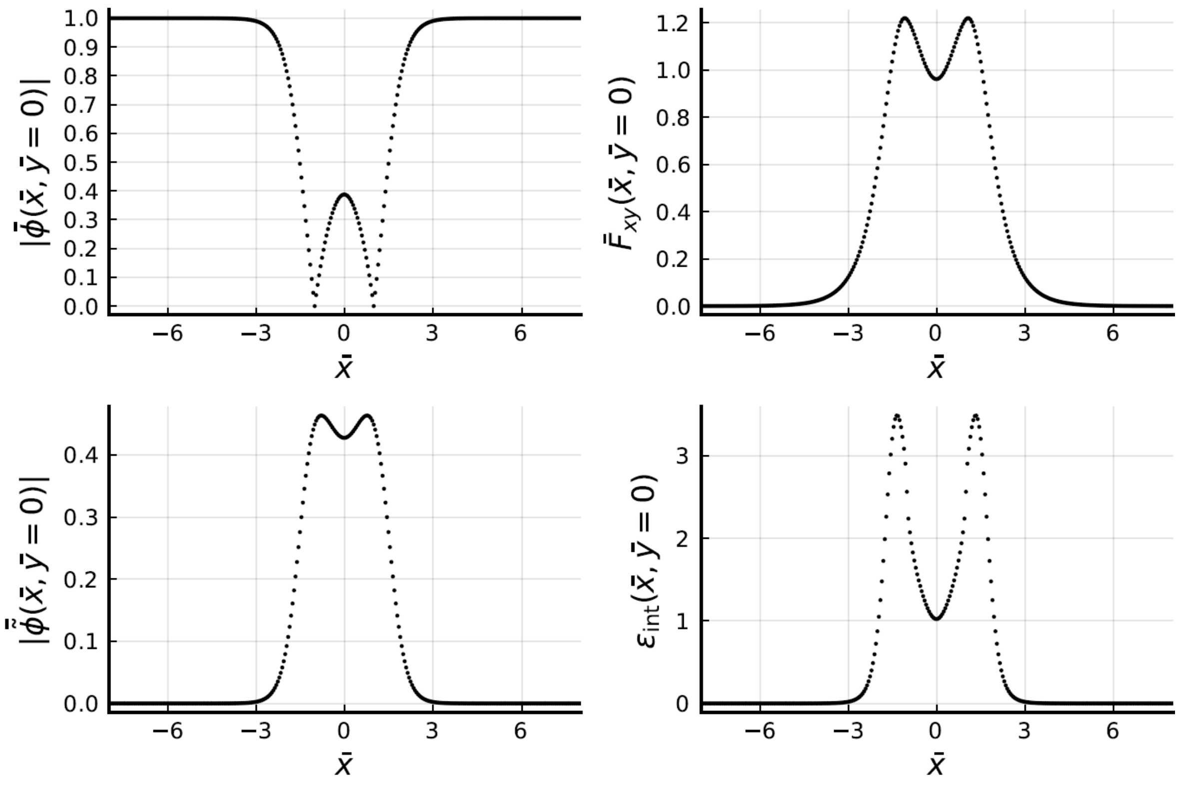


d=3

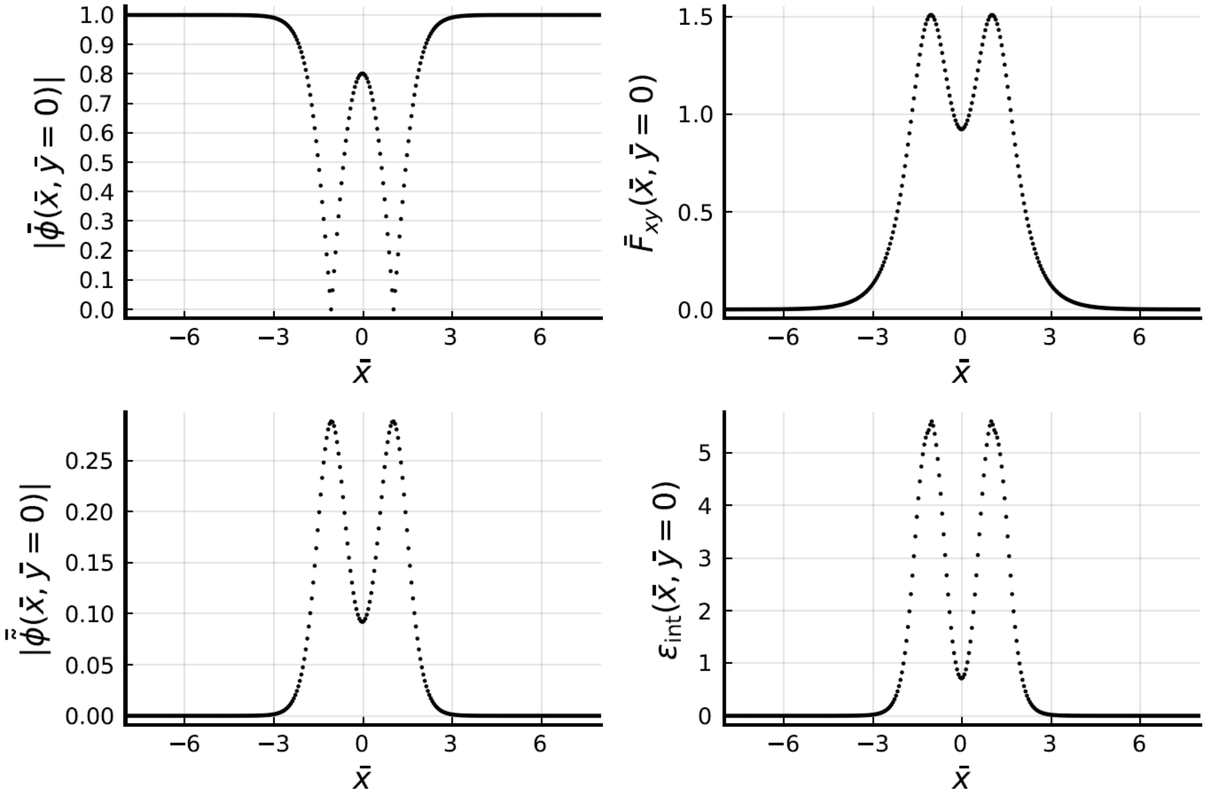


d=1.9

Coalesence of $\tilde{\phi}$



d=2



d=2.1

Current quenching

Effective mass of $\tilde{\phi}$:

$$m_{\tilde{\phi}}^2 = \beta|\phi|^2 - \frac{1}{2}\lambda_{\tilde{\phi}}\eta_{\tilde{\phi}}^2 + \lambda_{\tilde{\phi}}|\tilde{\phi}|^2 + \tilde{s}^2(r=0)$$

At $\mathrm{r} = 0, \ |\phi| = 0,$

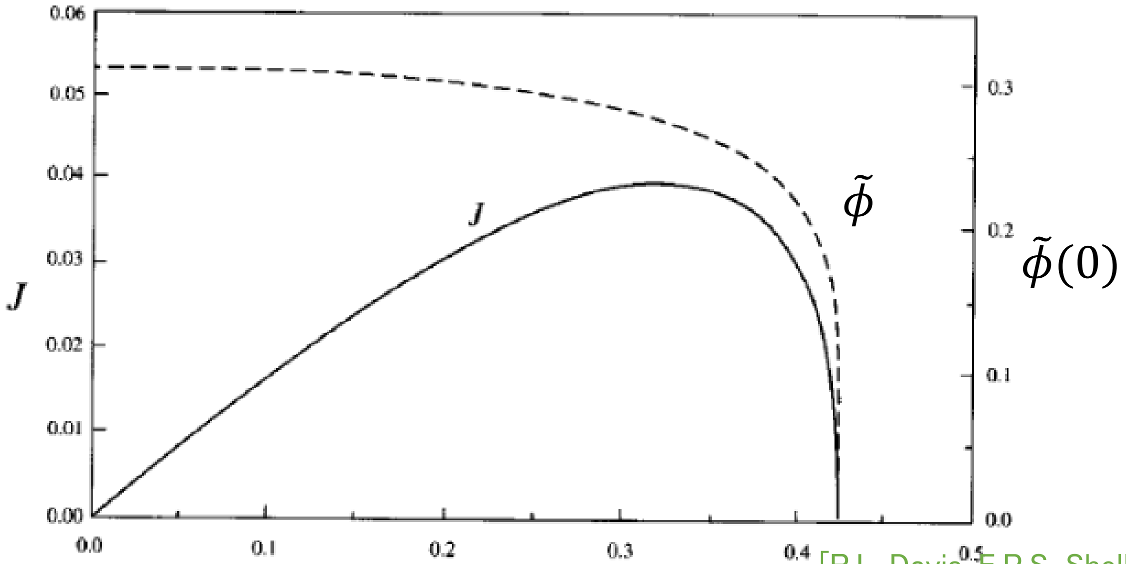
$$m_{\tilde{\phi}}^2(r=0) = -\frac{1}{2}\lambda_{\tilde{\phi}}\eta_{\tilde{\phi}}^2 + \lambda_{\tilde{\phi}}|\tilde{\phi}|^2 + \tilde{s}^2(r=0)$$

As $|\tilde{s}(r=0)|$ increases, $|\tilde{\phi}|$ is suppressed to zero.

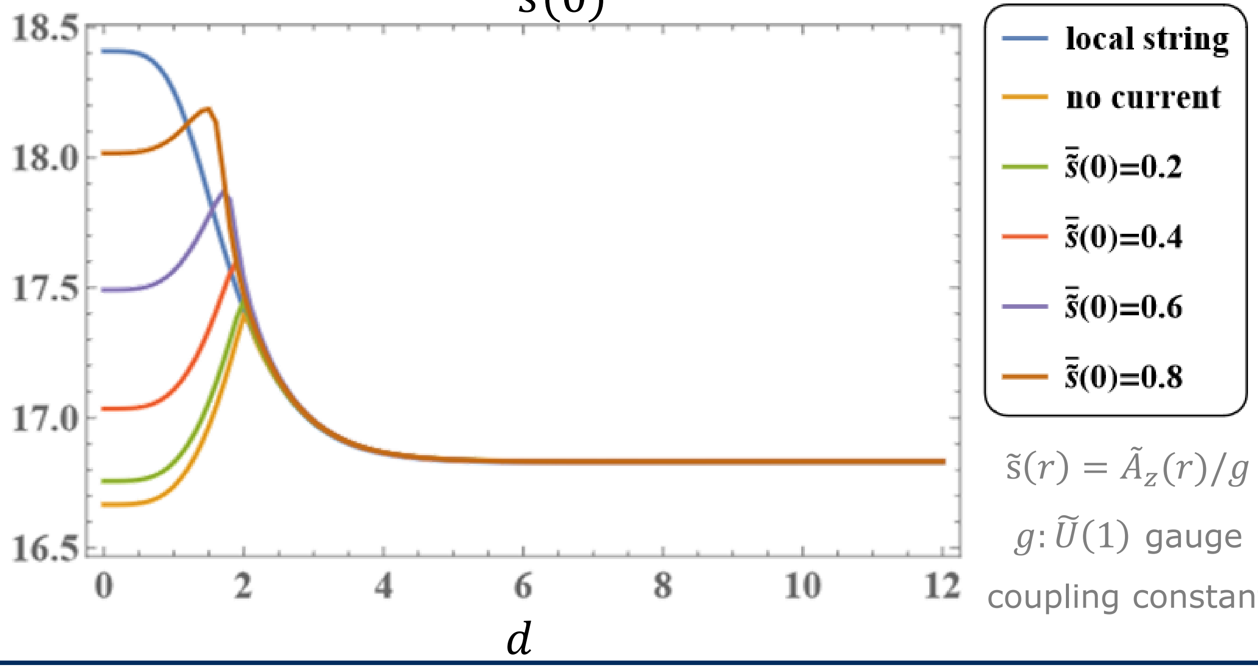
Current

$$J_z = \int d^2x[-2g\tilde{s}(r)\tilde{\phi}_r^2]$$

has a maximum value.



[R.L. Davis, E.P.S. Shellard, 1988]



$\tilde{s}(r) = \tilde{A}_z(r)/g$

g: $\tilde{U}(1)$ gauge coupling constant

Global strings

$$\mathcal{L} = |\partial_\mu \phi|^2 - V(\phi),$$
$$V(\phi) = \frac{1}{4}\lambda(|\phi|^2 - \eta^2)^2$$

global $U(1)$

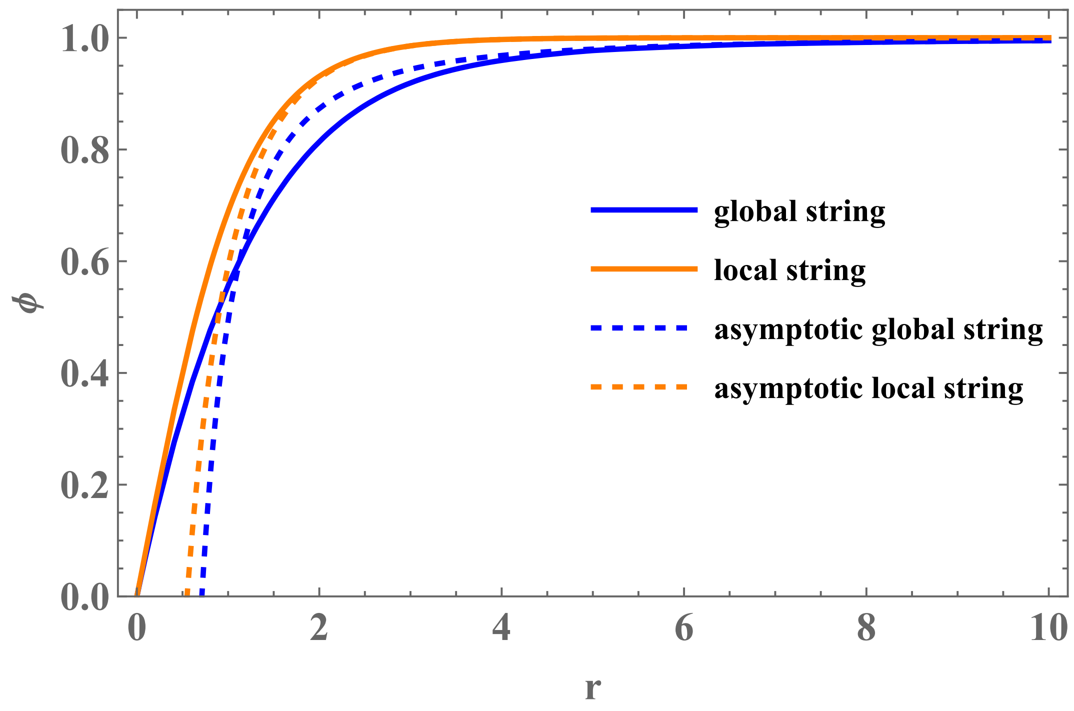
$$\phi(r) = \left(\eta + \frac{\sigma(r)}{\sqrt{2}}\right) e^{i\pi(\theta)}, \qquad \pi(\theta) = n\theta$$

massless Nambu-Goldston boson

$$\sigma(r) = -\frac{\sqrt{2}n^2\eta}{m^2 r^2} \qquad \mathrm{m} \equiv \sqrt{\lambda}\eta$$

$$J_\sigma = \frac{\sqrt{2}n^2\eta}{r^2}$$

$$J_\pi = \frac{n\eta}{r}\delta(r)$$



$$E_\pi = 2\pi\eta^2 \int dz\ [-n_1 n_2 \ln(\epsilon d)\ ]$$

cutoff at $\delta = \frac{1}{m}$

$$E_{int} = 2\pi\eta^2 \int dz\ [-n_1 n_2 \ln(md) - \frac{1}{\lambda} n_1^2 n_2^2 \frac{1}{d^2} \ln(m^2 d^2)]$$

suppressed at large distance

Numerical calculation for global strings

$$\mathcal{L} = |\partial_\mu \phi|^2 - V(\phi),$$
$$V(\phi) = \frac{1}{4}\lambda(|\phi|^2 - \eta^2)^2$$

global $U(1)$ symmetry

dominant at large distance

$$E_{int} = 2\pi\eta^2 \int dz\, [-n_1 n_2 \ln(md) - \frac{1}{\lambda} n_1^2 n_2^2 \frac{1}{d^2} \ln(m^2 d^2)]$$

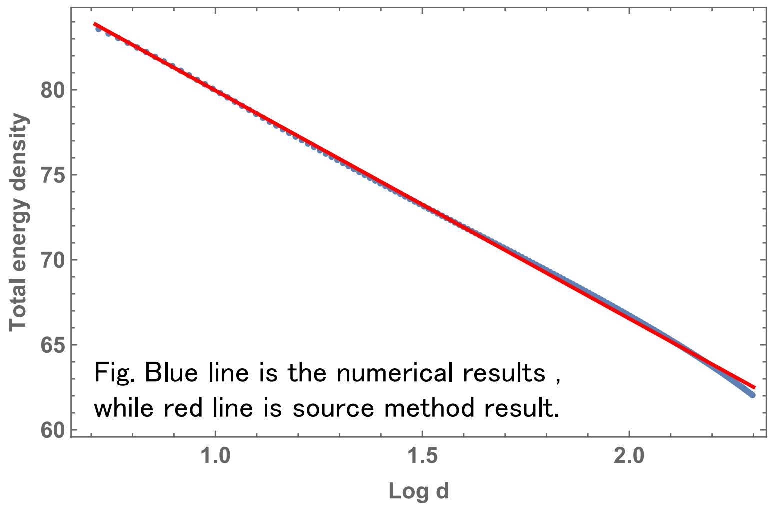
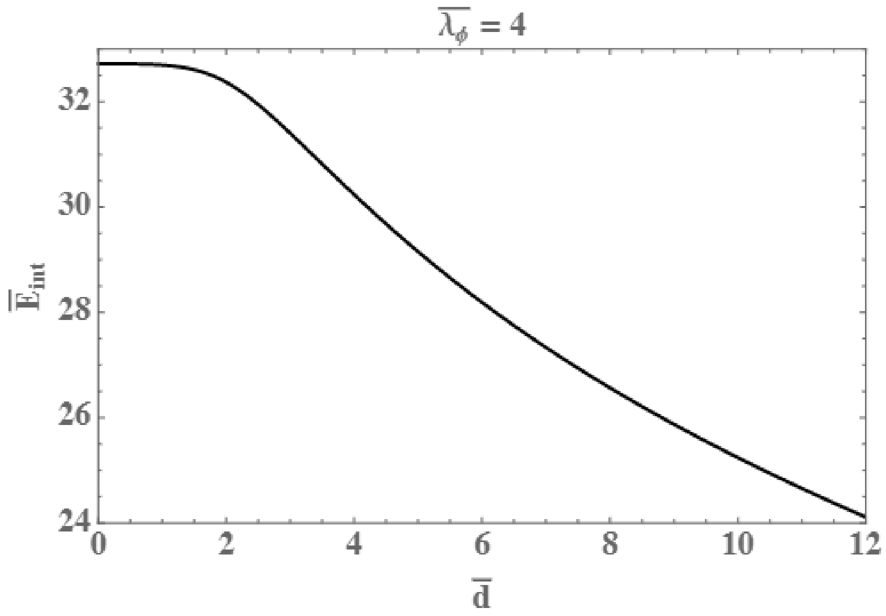
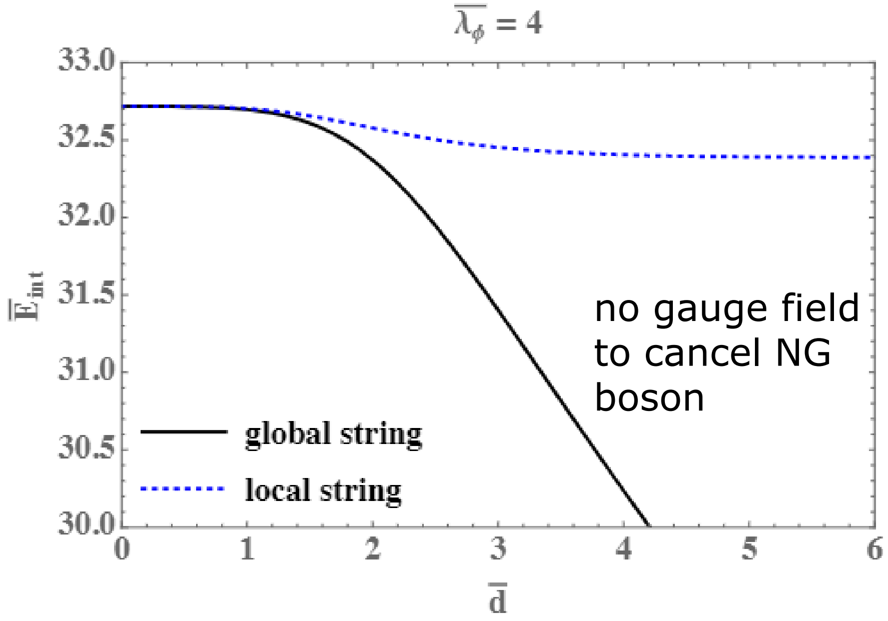


Fig. Blue line is the numerical results , while red line is source method result.

long-range interaction





no gauge field to cancel NG boson

Discussion

- Additional attractive sources are introduced in bosonic superconducting string model:

Scalar field $\tilde{\phi}$
- long-range attraction suppressed $\sim e^{-md}$
- significant attraction near the string core $d \sim \delta$

Current(gauge field $\tilde{A}_\mu$): dominant at long distance $d \gg \delta$.

⇨ leading to higher rate of Y-junction formation, for both large and small current strings and zero.

⇨ Corrections on predictions of gravitational lensing, gravitational wave bursts…