Speeding up the Universe & late-time singularities

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Introduction

Introduction-1-: A brief sketch of the universe

- The universe is homogeneous and isotropic on large scales (cosmological principle)
- The matter content of the universe:
  - Standard matter
  - Dark matter
  - Something that induce the late-time acceleration of the Universe
- The acceleration of the universe is backed by several measurments: $H(z)$, SneIa, GRB, BAO, CMB, LSS (matter power spectrum, growth function)...

Introduction-2-

- The effective equation of state of whatever is driving the current speed up of the universe is roughly -1 (Please see Saridakis talk)..
- Such an acceleration could be due
  - A new component of the energy budget of the universe: dark energy; i.e. it could be Λ, quintessence or of a phantom(-like/effective) nature
  - A change on the behaviour of gravity on the largest scale. No new component on the budget of the universe but rather simply GR modifies its behaviour, within a metric, Palatini (affine metric) ....

Late-time acceleration of the Universe within GR: dark energy with a constant EoS

Constant equation of state for DE: background-1-

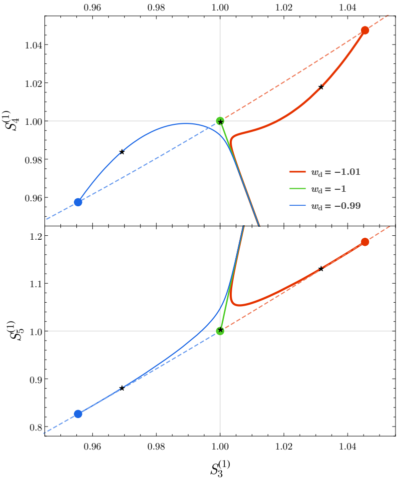
- Cosmic acceleration:

$$\frac{\ddot{a}}{a} = -\frac{4\pi \mathrm{G}}{3}(\rho_{\mathrm{m}} + \rho_{\mathrm{de}} + 3p_{\mathrm{de}})$$

- Observation indicates that for $w_{\mathrm{de}} \sim -1$ where $w_{\mathrm{de}} = p_{\mathrm{de}}/\rho_{\mathrm{de}}$.
- Therefore, as soon as DE starts dominating the Universe starts accelerating, i.e. $\ddot{a} > 0$.
- Simplest cases ΛCDM or *w*CDM.

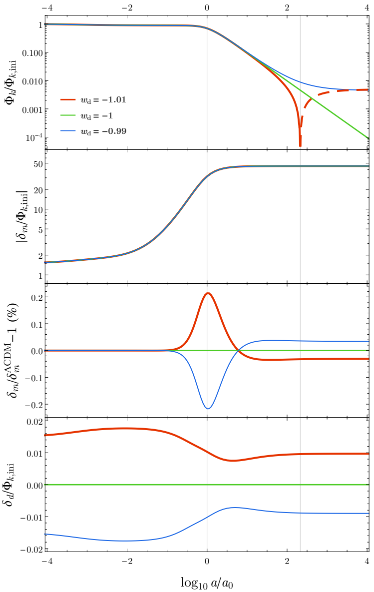
Constant equation of state for DE: background-2-

- State finders approach (Sahni, Saini and Starobinsky JETP Lett. [arXiv:astro-ph/0201498])
    - Scale factor: $\frac{a(t)}{a_0} = 1 + \sum_{n=1}^{\infty} \frac{A_n(t_0)}{n!} \left[H_0 \left(t - t_0\right)\right]^n$, where $A_n := a^{(n)}/(a\, H^n)$, $n \in \mathbb{N}$.
    - State finders parameters: $S_3^{(1)} = A_3$, $S_4^{(1)} = A_4 + 3\left(1 - A_2\right)$, $S_5^{(1)} = A_5 - 2\left(4 - 3A_2\right)\left(1 - A_2\right)$
    - $\Omega_{\rm m} = 0.309$, $\Omega_{\rm d} = 0.691$ and $H_0 = 67.74$ km s^{-1} Mpc^{-1} (according to Planck).



Albarran, B.L. and Morais, EPJC 2018 [arXiv:1706.01484]

Constant equation of state for DE: perturbations-1-



- Example of the evolution of the perturbations: $k = 10^{-3}$ Mpc^{-1}
- ΛCDM model: Φ_k vanishes asymptotically
- Phantom model: Φ_k also evolves towards a constant in the far future but a change of sign occurs roughly at $\log_{10} a/a_0 \simeq 2.33$, corresponding to 8.84×10^{10} years in the future. A dashed line indicates negative values of Φ_k
- Quintessence model: Φ_k evolves towards a constant in the far future without changing sign

Albarran, B.L. and Morais, EPJC 2018 [arXiv:1706.01484]

Constant equation of state for DE: perturbations-2-

- What about $f\sigma_8$ for the three different DE models?

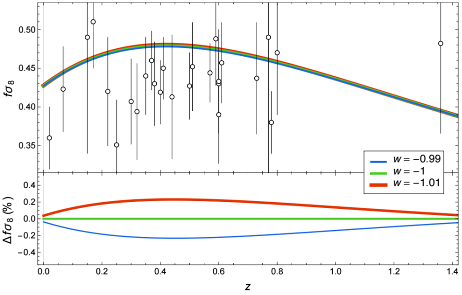


Figure 2: (Top panel) evolution of $f\sigma_8$ for low red-shift $z \in (0, 1.4)$ for three dark energy models: (blue) $w = -0.99$, (green) $w = -1$ and (red) $w = -1.01$. White circles and vertical bars indicate the available data points and corresponding error bars (cf. Table I of [13]). (Bottom panel) evolution of the relative differences of $f\sigma_8$ for each model with regard to ΛCDM ($w = -1$). $\Delta f\sigma_8$ is positive in the phantom case and negative in the quintessence case. For all the models, it was considered that σ_8 evolves linearly with δ_m and that $\sigma_8 = 0.816$ at the present time [7].

$$f \equiv \frac{d\left(\ln \delta_{\rm m}\right)}{d\left(\ln a\right)}, \qquad \sigma_8\left(z,\ k_{\sigma_8}\right) = \sigma_8\left(0,\ k_{\sigma_8}\right) \frac{\delta_{\rm m}\left(z,\ k_{\sigma_8}\right)}{\delta_{\rm m}\left(0,\ k_{\sigma_8}\right)}$$

$k_{\sigma_8} = 0.125$ hMpc^{-1}, $\sigma_8(0,\ k_{\sigma_8}) = 0.820$ (Planck)

Albarran, B.L. and Morais [arXiv:1706.01484]

Late-time acceleration of the Universe within GR and with a phantom fluid

Late-time acceleration of the Universe within GR and with a phantom fluid

The models

The models

- We are going to focus on the genuinely phantom matter. i.e. when the Equation of State satisfies $w < -1$. (not only not excluded but even favoured observationally)
- The phantom matter violates the Null energy condition. In consequence, the rest of the energy conditions are violated.
    - Null energy condition $\Rightarrow$ $p + \rho \geqslant 0$.
    - Weak energy condition $\Rightarrow p + \rho \geqslant 0$, $\rho \geqslant 0$.
    - Dominant energy condition $\Rightarrow$ $\rho \geqslant |p|$.
    - Strong energy condition $\Rightarrow$ $p + \rho \geqslant 0$, $3p + \rho \geqslant 0$.
- For example, a suitable way to write the Equation of State of a phantom fluid is

$$p = -\rho - C\rho^{\alpha},$$

where C is a positive constant and α is a real number. We are going to focus on the cases $\alpha = 1, 1/2, 0$.

Genuine phantom models: BR, LR and LSBR

- The DE content can be described for example with a perfect fluid or a scalar field

Event	EoS for a perfect fluid	Potential for a scalar field
BR	$p_d = w_{\rm d}\rho_d$	$V(\phi) = C_{br} e^{\lambda\phi}$
LR	$p_{\rm d} = -\rho - B\sqrt{\rho_{\rm d}}$	$V(\phi) = C_{lr}\phi^4 + D_{lr}\phi^2$
LSBR	$p_{\rm d} = -\rho_{\rm d} - A/3$	$V(\phi) = C_{ls}\phi^2 + D_{ls}$

Where $w_d < -1$, the parameters A and B are positive and C_{br}, C_{lr}, D_{lr}, C_{ls} and D_{lr} are constants.

- The lower is the power on ϕ of $V(\phi)$, the smoother is the abrupt event.

(1) A.A. Starobinsky. astro-ph 9912054; R.R. Cadwell astro-ph 9908168; Cadwell *et al.* astro-ph/0301273

(2) H. Štefančić. astro-ph 0411630; S. Nojiri, S. Odintsov and S. Tsujikawa hep-th/0501025; M. Bouhmadi-López arXiv:astro-ph/0512124.

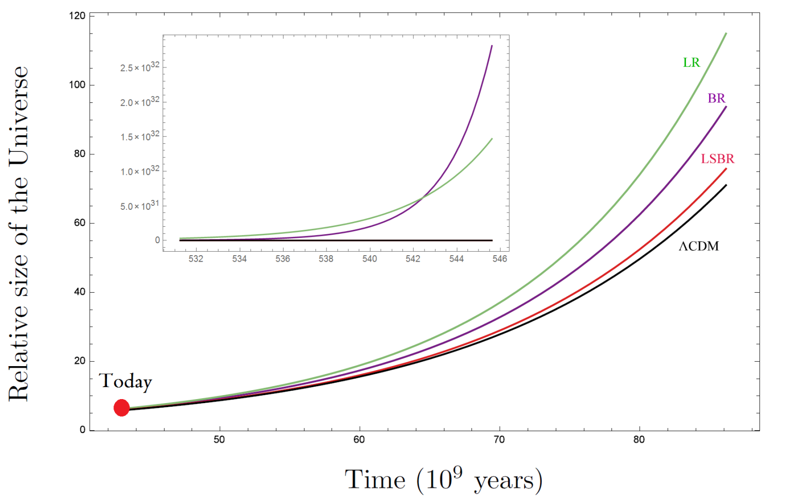
(3) M. P. Dąbrowski, C. Kiefer and B. Sandhöfer. hep-th/0605229

(4) M. Bouhmadi-López , A. Errahmani, P. Martín-Moruno, T. Ouali and Y. Tavakoli. arXiv:1407.2446

Recent Review ⟶ de Haro, Nojiri, Odintsov, Oikonomou and Pan arXiv: 2309.07465 (Phys. Rept)

Phantom energy: Should we be afraid?

- Evolution of the scale factor for different models vs cosmic time.



Late-time singularities

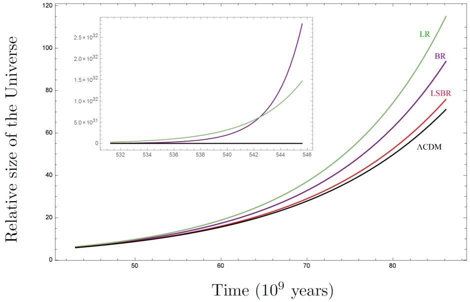
DE might induce a future cosmic singularity

Some of the cosmological parameters:

- t → Cosmic time
- a → Scale factor (relative size)
- H → Hubble parameter (growth rate)
- $\dot{H}$ → Time derivative of H

Singularity	t	a	H	$\dot{H}$	$\ddot{H}, \dddot{H}$ ...
Big Bang	0	0	∞	∞	∞
De Sitter $_{(\Lambda CDM)}$	∞	∞	H_{ds}	0	0
Big Rip	t_s	∞	∞	∞	∞
LR	∞	∞	∞	∞	∞
LSBR	∞	∞	∞	$\dot{H}_s$	0
Big Freeze	t_s	a_s	∞	∞	∞
Sudden. S.	t_s	a_s	H_s	∞	∞
Type IV	t_s	a_s	H_s	$\dot{H}_s$	∞

Asymptotic evolution of the scale factor



Bouhmadi-López, Kiefer, Martín-Moruno, arXiv:1904.01836 [gr-qc] (review published in GRG)
Borislavov Vasilev, Bouhmadi-López, Martín-Moruno, arxiv: arXiv:2106.12050 (review published in Universe)

Late-time acceleration of the Universe within GR and with a phantom fluid

Observational data and constraints

Observational data

- The Pantheon compilation: 1048 SNeIa dataset $0.01 < z < 2.26$
- The power spectrum of CMB affects crucially the physics, from the decoupling epoch till today. Effects are mainly quantified by the acoustic scale l_a and the shift parameter R Komatsu et 2008
- The BAO peaks present in the matter power spectrum can be used to determine the Hubble parameter $H(z)$ and the angular diameter distance $D_A(z)$
- $H(z)$ data

Model fitted

- BR model: $p_{\mathrm{d}} = w_{\mathrm{d}}\rho_{\mathrm{d}}$

$$E(a)^2 = \Omega_{\mathrm{r}} a^{-4} + \Omega_{\mathrm{m}} a^{-3} + \Omega_{\mathrm{d}} a^{-3(1+w_{\mathrm{d}})}.$$

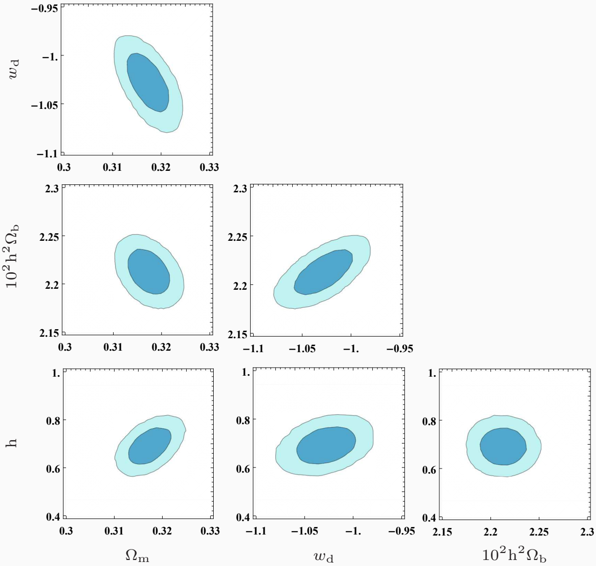
- LR model: $p_{\mathrm{d}} = -\left(\rho_{\mathrm{d}} + \beta\rho_{\mathrm{d}}^{\frac{1}{2}}\right)$

$$E^2(a) = \Omega_{\mathrm{r}} a^{-4} + \Omega_{\mathrm{m}} a^{-3} + \Omega_{\mathrm{d}} \left(1 + \frac{3}{2}\sqrt{\frac{\Omega_{\mathrm{lr}}}{\Omega_{\mathrm{d}}}} \ln(a)\right)^2.$$

- LSBR model: $p_{\mathrm{d}} = -\left(\rho_{\mathrm{d}} + \frac{\alpha}{3}\right)$

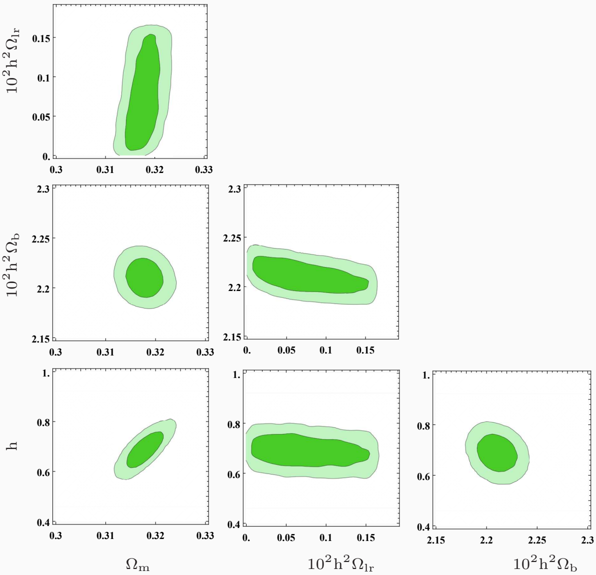
$$E^2(a) = \Omega_{\mathrm{r}} a^{-4} + \Omega_{\mathrm{m}} a^{-3} + \Omega_{\mathrm{d}} \left(1 - \frac{\Omega_{\mathrm{lsbr}}}{\Omega_{\mathrm{d}}} \ln(a)\right).$$

BR Model



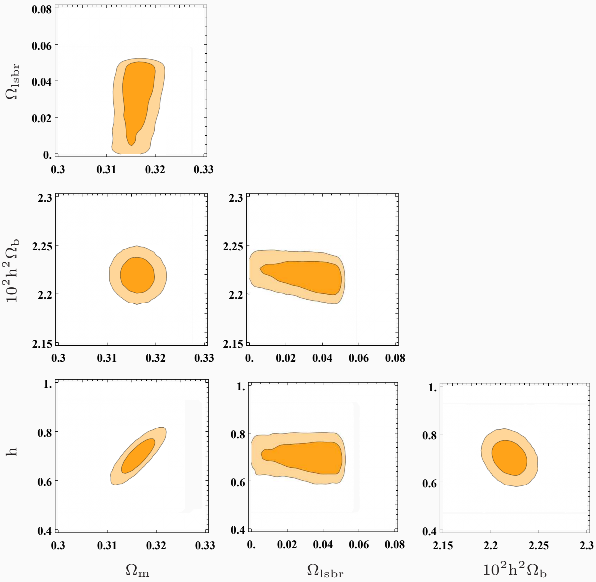
Bouali, Albarran, Bouhmadi-López and Ouali, arXiv:1905.07304 [astro-ph.CO]. Published in Physics of the Dark Universe.

LR Model



Bouali, Albarran, Bouhmadi-López and Ouali, arXiv:1905.07304 [astro-ph.CO]. Published in Physics of the Dark Universe.

LSBR Model



Bouali, Albarran, Bouhmadi-López and Ouali, arXiv:1905.07304 [astro-ph.CO]. Published in Physics of the Dark Universe.

Comparison with LCDM

Model	Par	Best fit	Mean	χ^2_{tot}	$\chi^2_{\mathrm{tot}}{}^{\mathrm{red}}$	AIC_c	ΔAIC_c
ΛCDM	Ω_{m}	$0.318349^{+0.00248001}_{-0.00248001}$	$0.31834^{+0.00248987}_{-0.00248987}$	1047.42	0.957422	1053.441953	0
	h	$0.69814^{+0.0480814}_{-0.0480814}$	$0.698602^{+0.0481787}_{-0.0481787}$				
	$\Omega_{\mathrm{b}}h^2$	$0.022218^{+0.000120872}_{-0.000120872}$	$0.0222202^{+0.000122619}_{-0.000122619}$				
BR	Ω_{m}	$0.317173^{+0.00318473}_{-0.00318473}$	$0.317327^{+0.0031808}_{-0.0031808}$	1047.51	0.958380	1055.54663	2.104677
	w_{br}	$-1.02758^{+0.0240102}_{-0.0240102}$	$-1.02874^{+0.0239306}_{-0.0239306}$				
	h	$0.691013^{+0.0507771}_{-0.0507771}$	$0.691523^{+0.0507536}_{-0.0507536}$				
	$\Omega_{\mathrm{b}}h^2$	$0.0221218^{+0.000170789}_{-0.000170789}$	$0.022123^{+0.000170538}_{-0.000170538}$				
LR	Ω_{m}	$0.317198^{+0.00276851}_{-0.00276851}$	$0.317705^{+0.00280131}_{-0.00280131}$	1047.53	0.958398	1055.56663	2.124677
	Ω_{lr}	$0.000445721^{+0.000416159}_{-0.000416159}$	$0.000763824^{+0.000416359}_{-0.000416359}$				
	h	$0.694604^{+0.0494111}_{-0.0494111}$	$0.688584^{+0.0493315}_{-0.0493315}$				
	$\Omega_{\mathrm{b}}h^2$	$0.0221295^{+0.000130585}_{-0.000130585}$	$0.0221028^{+0.000132755}_{-0.000132755}$				
LSBR	Ω_{m}	$0.317115^{+0.00253975}_{-0.00253975}$	$0.316144^{+0.00253899}_{-0.00253899}$	1047.56	0.958426	1055.59663	2.154677
	Ω_{lsbr}	$0.0500261^{+0.0130141}_{-0.0130141}$	$0.0299424^{+0.0133398}_{-0.0133398}$				
	h	$0.695705^{+0.0481201}_{-0.0481201}$	$0.701962^{+0.0481465}_{-0.0481465}$				
	$\Omega_{\mathrm{b}}h^2$	$0.022138^{+0.000121724}_{-0.000121724}$	$0.0221928^{+0.000121811}_{-0.000121811}$				

Table III. Summary of the best fit and the mean values of the cosmological parameters.

Akaike information criterion (AIC)

Late-time acceleration of the Universe within GR and with a phantom fluid

A perturbative approach: GR and phantom fluids

Our approach

- We start considering that the late-time acceleration of the universe is described by a dark energy component effectively encapsulated within a perfect fluid with energy density ρ_d and pressure p_d. On this setup, we consider two simple scenario:
  - A constant equation of state for DE
  - A DE in an effective and genuinely phantom DE universe. The reason of this second choice will become clear after considering the first case.
- Of course, on top of this we invoke a dark matter component.
- Given that to get the matter power spectrum, we start our numerical integration since the radiation dominated epoch, we will consider as radiation as well on our model.

Cosmological perturbations: GR and for the late Universe-1

- We worked on the Newtonian gauge and carried the first order perturbations considering DM, DE and radiation on GR. Radiation was included because our numerical integrations start from well inside the radiation dominated epoch (to get the matter power spectrum)
- We assumed initial adiabatic conditions for the different fractional energy density perturbations
- The total fractional energy density is fixed by Planck measurments; i.e. through A_s and n_s
- The speed of sound for DE:
  - The pressure perturbation of DE reads: $\delta p_d = c_{sd}^2 \delta\rho_d - 3\mathcal{H}\left(1+w_d\right)\left(c_{sd}^2 - c_{ad}^2\right)\rho_d v_d$, where $c_{sd}^2 = \left.\frac{\delta p_d}{\delta \rho_d}\right|_{r.f.}$ and $c_{aA}^2 = \frac{p_d'}{\rho_d'}$
  - Given that c_{sd}^2 is negative, we can end up with a problem (this is not intrisic to phantom matter as it can happen for example with fluids with a negative constant equation of state larger than -1)
  - We choose $c_{sd}^2 = 1$ as a phenomenological parameter

Cosmological perturbations: GR and for the late Universe-2

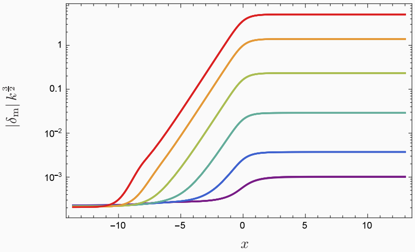
- Adiabatic conditions:

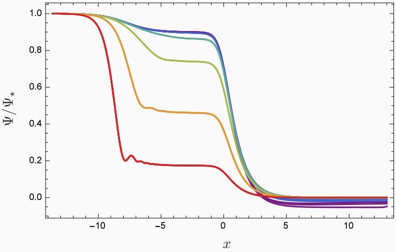
$$\frac{3}{4}\delta_{\mathrm{r,ini}} = \delta_{\mathrm{m,ini}} = \frac{\delta_{\mathrm{d,ini}}}{1 + w_{\mathrm{d,ini}}} \approx \frac{3}{4}\delta_{\mathrm{ini}}$$

$$v_{\mathrm{r,ini}} = v_{\mathrm{m,ini}} = v_{\mathrm{d,ini}} \approx \frac{\delta_{\mathrm{ini}}}{4\mathcal{H}_{\mathrm{ini}}}$$

- Initial conditions for δ are fixed through the amplitude and spectral index of the primordial inflationary power spectrum: $A_s = 2.143 \times 10^{-9}$, $n_s = 0.9681$ and $k_* = 0.05$ Mpc^{-1} (Planck values): $\Phi_{\mathrm{ini}} = \frac{2\pi}{3}\sqrt{2A_s}\left(\frac{k}{k_*}\right)^{n_s-1} k^{-3/2}$
- Well inside the radiation era: $\Phi_{\mathrm{ini}} \approx -\frac{1}{2}\delta_{\mathrm{tot,ini}}$ and $\Phi_{\mathrm{ini}} \approx -2\mathcal{H}_{\mathrm{ini}} v_{\mathrm{tot,ini}}$
- We choose $c_{sd}^2 = 1$ as a phenomenological parameter
- The parameters of the models will be fixed through the fitting we did previously.

Results: DM perturbations and the gravitational potential



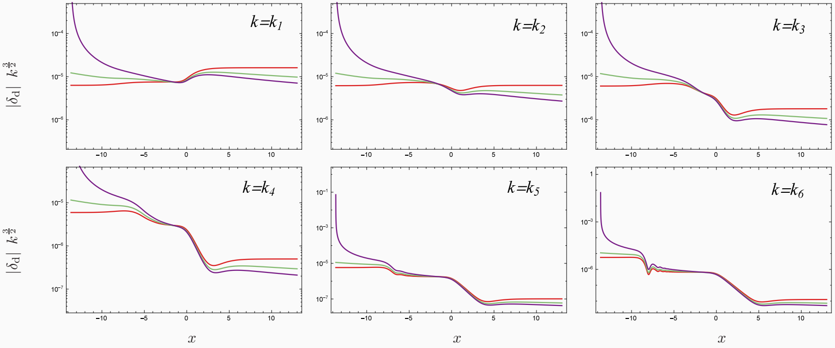


$k_1 = 3.33 \times 10^{-4}\mathrm{h\ Mpc}^{-1}$, $k_4 = 1.02 \times 10^{-2}\mathrm{h\ Mpc}^{-1}$,

$k_2 = 1.04 \times 10^{-4}\mathrm{h\ Mpc}^{-1}$, $k_5 = 3.19 \times 10^{-2}\mathrm{h\ Mpc}^{-1}$,

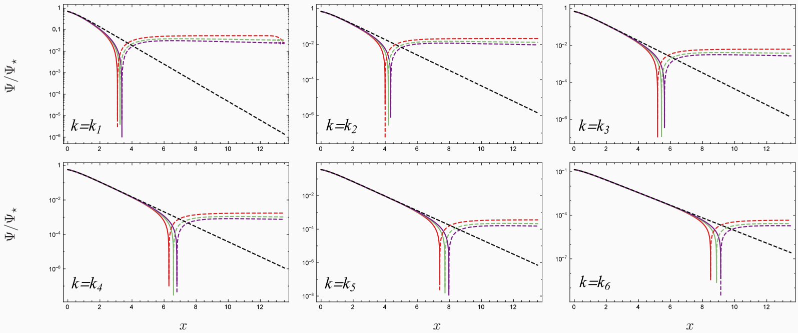
$k_3 = 3.26 \times 10^{-3}\mathrm{h\ Mpc}^{-1}$, $k_6 = 1.00 \times 10^{-1}\mathrm{h\ Mpc}^{-1}$.

Results: DE perturbations



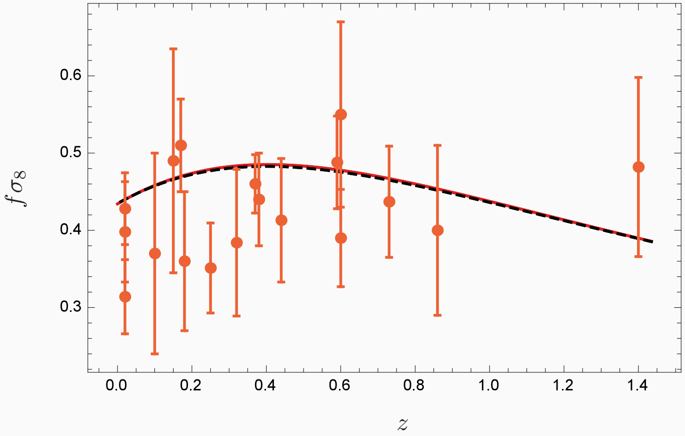
$$k_1 = 3.33 \times 10^{-4} \mathrm{h\ Mpc}^{-1}, \qquad k_4 = 1.02 \times 10^{-2} \mathrm{h\ Mpc}^{-1},$$
$$k_2 = 1.04 \times 10^{-4} \mathrm{h\ Mpc}^{-1}, \qquad k_5 = 3.19 \times 10^{-2} \mathrm{h\ Mpc}^{-1},$$
$$k_3 = 3.26 \times 10^{-3} \mathrm{h\ Mpc}^{-1}, \qquad k_6 = 1.00 \times 10^{-1} \mathrm{h\ Mpc}^{-1}.$$

Results: a closer look at the gravitational potential



$$k_1 = 3.33 \times 10^{-4} \mathrm{h\ Mpc}^{-1}, \qquad k_4 = 1.02 \times 10^{-2} \mathrm{h\ Mpc}^{-1},$$
$$k_2 = 1.04 \times 10^{-4} \mathrm{h\ Mpc}^{-1}, \qquad k_5 = 3.19 \times 10^{-2} \mathrm{h\ Mpc}^{-1},$$
$$k_3 = 3.26 \times 10^{-3} \mathrm{h\ Mpc}^{-1}, \qquad k_6 = 1.00 \times 10^{-1} \mathrm{h\ Mpc}^{-1}.$$

Results: The evolution of $f\sigma_8$ (growth rate)-1-

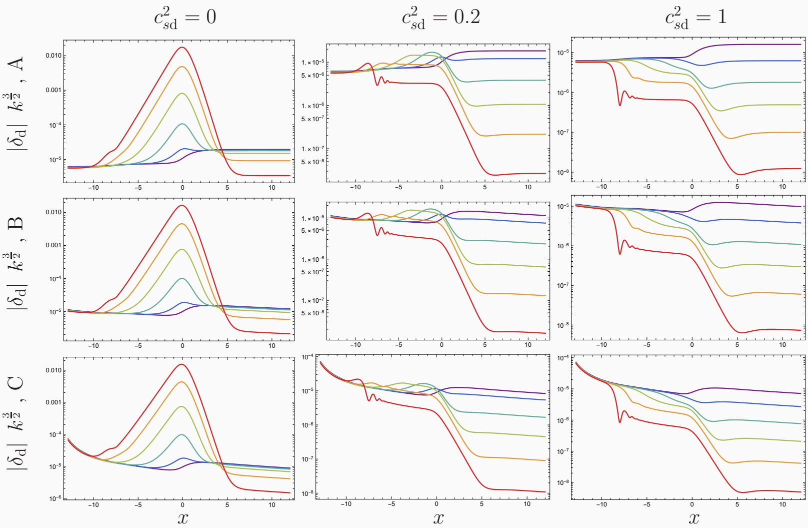


The evolution of $f\sigma_8$ for the 3 models.

$$f \equiv \frac{d\left(\ln \delta_{\mathrm{m}}\right)}{d\left(\ln a\right)}, \qquad \sigma_8\left(z,\ k_{\sigma_8}\right) = \sigma_8\left(0,\ k_{\sigma_8}\right) \frac{\delta_{\mathrm{m}}\left(z,\ k_{\sigma_8}\right)}{\delta_{\mathrm{m}}\left(0,\ k_{\sigma_8}\right)}$$

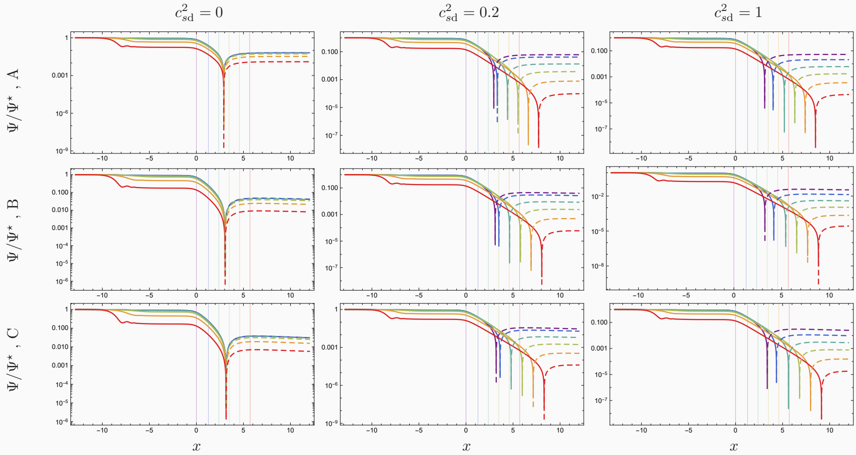
$k_{\sigma_8} = 0.125$ hMpc^{-1}, $\sigma_8(0,\ k_{\sigma_8}) = 0.820$ (Planck)

Effect of the speed of sound, C_{sd}^2, on DE perturbations

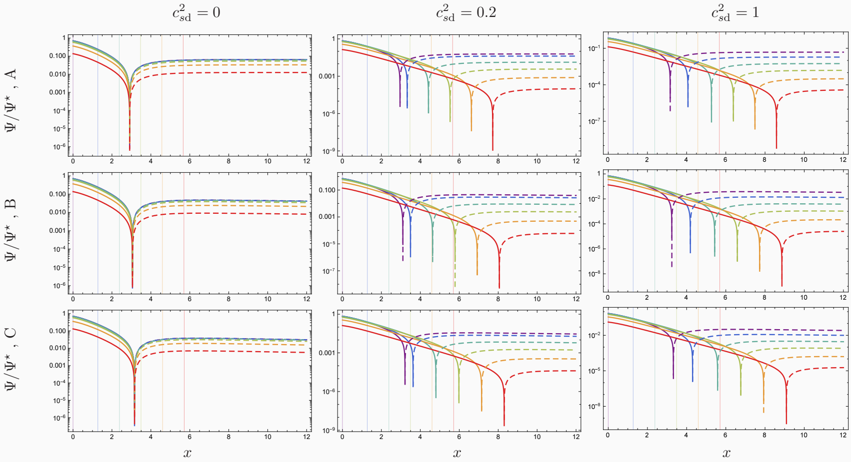


Albarran, Bouhmadi-López and Marto, arXiv:2011.08222 [gr-qc.CO]. Published in European Physical Journal C.

Effect of the speed of sound, C_{sd}^2, on the gravitational potential-1-



Effect of the speed of sound, C_{sd}^2, on the gravitational potential-2-



Late-time acceleration of the Universe within GR and with a phantom fluid

DE and DM models with interactions

DE and DM models with interaction

♯Motivations

- Solving the coincidence problem.
- Check if an interaction between DE and DM could statistically improve the previous models.
- Check if the interaction could attenuate or modify the nature of future cosmological events induced by the former models corresponding to BR, LR and LSBR.
- Check whether the previous models A, B and C might respond differently to the interaction, as they exhibited very similar behaviour both at the background level and at the perturbative level.

$$\begin{cases} \dot{\rho}_{\mathrm{m}} + 3H\rho_{\mathrm{m}} = -Q, \\ \dot{\rho}_{\mathrm{d}} + 3H(1+w_{\mathrm{d}})\rho_{\mathrm{d}} = Q. \end{cases}$$

where

$$Q = \lambda H \rho_{\mathrm{d}}$$

Comparing Models-1-

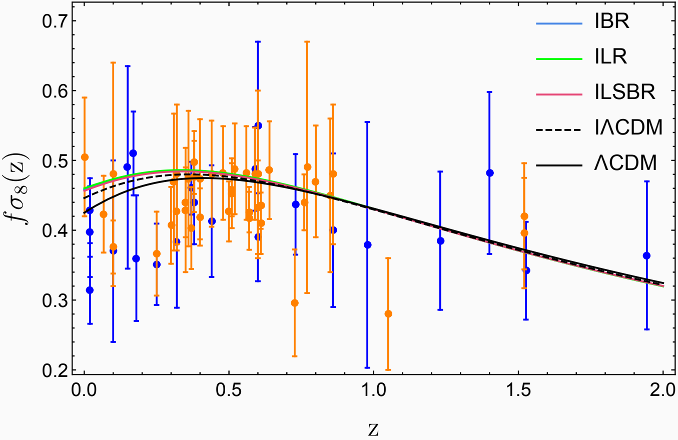
Model	Par	Best fit	Mean
ΛCDM	$\Omega_{\rm m}$	$0.312308^{+0.00607812}_{-0.00607812}$	$0.312583^{+0.00607362}_{-0.00607362}$
	h	$0.678603^{+0.00447778}_{-0.00447778}$	$0.678435^{+0.00447417}_{-0.00447417}$
	$\Omega_b h^2$	$0.0224102^{+0.000134672}_{-0.000134672}$	$0.0224002^{+0.00013449}_{-0.00013449}$
IΛCDM	$\Omega_{\rm m}$	$0.314738^{+0.00663245}_{-0.00663245}$	$0.315252^{+0.00663212}_{-0.00663212}$
	h	$0.676461^{+0.00504971}_{-0.00504971}$	$0.675976^{+0.005048282}_{-0.005048282}$
	λ	$0.00992076^{+0.0116578}_{-0.0116578}$	$0.011897^{+0.011643}_{-0.011643}$
	$\Omega_b h^2$	$0.0224698^{+0.000159949}_{-0.000159949}$	$0.0224845^{+0.000159746}_{-0.000159746}$
IBR	$\Omega_{\rm m}$	$0.310664^{+0.00830269}_{-0.00830269}$	$0.31219^{+0.00828119}_{-0.00828119}$
	$w_{\rm d}$	$-1.03248^{+0.0482072}_{-0.0482072}$	$-1.03728^{+0.0482912}_{-0.0482912}$
	λ	$0.0168888^{+0.0154053}_{-0.0154053}$	$0.0193522^{+0.0155017}_{-0.0155017}$
	h	$0.682033^{+0.0092436}_{-0.0092436}$	$0.681365^{+0.00922296}_{-0.00922296}$
	$\Omega_b h^2$	$0.0224867^{+0.000166489}_{-0.000166489}$	$0.0224764^{+0.000166426}_{-0.000166426}$
ILR	$\Omega_{\rm m}$	$0.310516^{+0.00726847}_{-0.00726847}$	$0.309924^{+0.00726091}_{-0.00726091}$
	$\Omega_{\rm lr}$	$0.00105782^{+0.00182933}_{-0.00182933}$	$0.00252318^{+0.0018261}_{-0.0018261}$
	λ	$0.017768^{+0.0129577}_{-0.0129577}$	$0.0231745^{+0.0129438}_{-0.0129438}$
	h	$0.682915^{+0.00661987}_{-0.00661987}$	$0.684813^{+0.00661268}_{-0.00661268}$
	$\Omega_b h^2$	$0.0224682^{+0.00016143}_{-0.00016143}$	$0.0224685^{+0.000161631}_{-0.000161631}$
ILSBR	$\Omega_{\rm m}$	$0.313552^{+0.00671554}_{-0.00671554}$	$0.314018^{+0.00670695}_{-0.00670695}$
	Ω_{lsbr}	$0.0458419^{+0.0144588}_{-0.0144588}$	$0.0295684^{+0.0145073}_{-0.0145073}$
	λ	$0.0154082^{+0.0120793}_{-0.0120793}$	$0.0148823^{+0.0120577}_{-0.0120577}$
	h	$0.679071^{+0.00524713}_{-0.00524713}$	$0.678149^{+0.00524195}_{-0.00524195}$
	$\Omega_b h^2$	$0.0224517^{+0.000162054}_{-0.000162054}$	$0.0224808^{+0.000161849}_{-0.000161849}$

Comparing Models-2-

Model	$\chi^2_{min}{}^{tot}$	AIC_c	ΔAIC_c
ΛCDM	1073.9795	1080.0014	0
IΛCDM	1073.1076	1081.1443	1.1429
IBR	1072.6870	1082.7420	2.7406
ILR	1072.6477	1082.7028	2.7014
ILSBR	1072.6200	1082.7600	2.6685

Notice that All the interacting models will induce a BR!!!

Results: The evolution of $f\sigma_8$ (growth rate)



Bouali, Albarran, Bouhmadi-López, Errahmani and Ouali, Phys. of The Dark Universe, arXiv:2103.13432[astro-ph.CO].

We will post the effect of a varying c_s^2 (hopefully) soon arXiv:24XX.XXXXX

- Note: you have as well phantom dark energy models that approaches a de Sitter Universe asymptotically and give good fitting

(work in collaboration with Fernández-Jambrina, Lazkoz and Salzano, arXiv:2311.10526, Phys.Dark Univ. 45 (2024) 101511)

Speeding up with fields

Speeding up with fields

Late-time acceleration through a 3-form field

Can we have something more fundamental to describe DE?

- Can we have something more fundamental to describe phantom DE models?
  - A possibility come in the form of 3-forms.
  - Inspired in string theory: Copeland, Lahiri,Wands (1995)
  - Massless 3-form as Cosmological Constant (solving CC problem): Turok, Hawking (1998)
  - Inflation or late time acceleration driven by self-interacting 3-forms: Koivisto, Nunes (2009) and (2010)
  - Non-Gaussianity: Kumar, Mulryne, Nunes, Marto, Moniz (2016)
  - Quantum cosmology with 3-forms: Bouhmadi-López, Brizuela, Garay (2018)
- The answer as we will see in a momment is yes:

Phantom DE models (LSBR): Morais, Bouhmadi-López, Kumar, Marto, Tavakoli (2017),Bouhmadi-López, Marto, Morais and Silva (2018), C.G. Boiza, M.B.-L, H.-W. Chiang and P. Chen work in progress (2024)

p-forms in Cosmology

A p-form is a totally anti-symmetric covariant tensor:

$$\omega_{\mu_1 \ldots \mu_p} = \omega_{[\mu_! \ldots \mu_p]} \, .$$

In D-dimensions, the number of degrees of freedom of a massive p-form is

$$\text{degrees of freedom} = \frac{(D-1)!}{(D-1-p)!p!} \, .$$

In a 4-dimensional space-time:

- $p = 0$ (scalar field) $\Rightarrow$ 1 degree of freedom
- $p = 1$ (vector field) $\Rightarrow$ 3 degrees of freedom
- $p = 2 \Rightarrow$ 3 degrees of freedom
- $p = 3 \Rightarrow$ 1 degree of freedom

$\Rightarrow$ The scalar field and the 3-form are the only ones compatible with a homogeneous and isotropic universe (in an easy way).

C. Germani and A. Kehagias, J. Cosmol. Astropart. Phys. 2009, 28 (2009)
T. S. Koivisto, D. F. Mota, and C. Pitrou, J. High Energy Phys. 2009, 92 (2009)

The 3-form action

- We will consider the following action for a massive 3-form, $A_{\mu\nu\rho}$, minimally coupled to gravity

$$S^A = \int \mathrm{d}^4\mathbf{x}\sqrt{|\det g_{\mu\nu}|}\left[-\frac{1}{48}F^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma} - V\left(A^{\mu\nu\rho}A_{\mu\nu\rho}\right)\right].$$

- The strength tensor, a 4-form, is defined through the exterior derivative: $F_{\mu\nu\rho\sigma} \equiv 4\nabla_{[\mu}A_{\nu\rho\sigma]}$
- The equation of motion, obtained from variation of S^A, is

$$\nabla_\sigma F^\sigma{}_{\mu\nu\rho} - 12\frac{\partial V}{\partial\left(A^2\right)}A_{\mu\nu\rho} = 0$$

- $\Rightarrow$ a massless 3-form is equivalent to a cosmological constanst

C. Germani and A. Kehagias, J. Cosmol. Astropart. Phys. 2009, 28 (2009)
T. S. Koivisto, D. F. Mota, and C. Pitrou, J. High Energy Phys. 2009, 92 (2009)
M. Duff and P. Van Nieuwenhuizen, Phys. Lett. B 94, 179 (1980)

3-form Cosmology

We consider a homogeneous and isotropic universe described by the Friedmann-Lemaître-Robertson-Walker line element

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j \,.$$

t - cosmic time, $\dot{\{\}} = d\{\}/dt$
a - scale factor
x^i - comoving spatial coordinates (roman indices run from 1 to 3).

Only the purely spatial components of the 3-form are dynamical:

$$A_{0ij} = 0 \,, \qquad A_{ijk} = a^3(t)\chi(t)\epsilon_{ijk} \,.$$

T. S. Koivisto, D. F. Mota, and C. Pitrou, J. High Energy Phys. 2009, 92 (2009)
Koivisto and Nunes PLB [arXiv:0907.3883], idem PRD [arXiv:0908.0920]

3-form Cosmology: background equations

⇒ Friedmann Equation

$$3H^2 = \kappa^2 \rho_\chi = \kappa^2 \left[\frac{1}{2} \left(\dot{\chi} + 3H\chi \right)^2 + V(\chi^2) \right] .$$

⇒ Raychaudhuri equation

$$\dot{H} = -\frac{\kappa^2}{2} \left(\rho_\chi + P_\chi \right) = -\frac{\kappa^2}{2} \chi \frac{\partial V}{\partial \chi} .$$

A 3-form can show phantom-like behavior if $\partial V / \partial \chi^2 < 0$.

⇒ Equation of motion

$$\ddot{\chi} + 3H\dot{\chi} + 3\dot{H}\chi + \frac{\partial V}{\partial \chi} = 0 .$$

3-form Cosmology: evolution of χ-1-

Combining the Raychaudhuri equation and the equation of motion for χ:

$$\ddot{\chi} + 3H\dot{\chi} + \left(1 - \frac{\chi^2}{\chi_c^2}\right)\frac{\partial V}{\partial \chi} = 0\,.$$

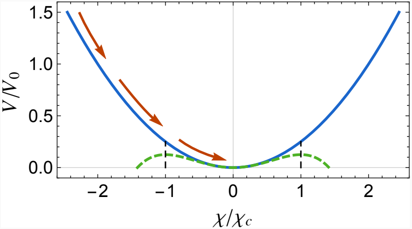
The static solutions are:

- the critical points of the potential: $\dfrac{\partial V}{\partial \chi} = 0$,
- the limiting points: $\chi = \pm\chi_c$.

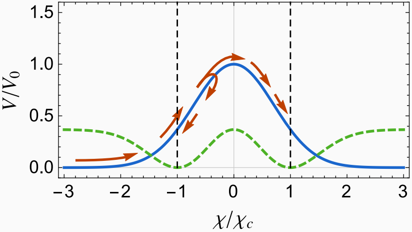
Once inside the interval $[-\chi_c, \chi_c]$, the field χ evolves towards a local minimum of V. However...

3-form Cosmology: evolution of χ-2-

- Independently of the shape of a regular potential, in absence of DM interaction, the 3-form decays rapidly towards the interval $[-\chi_c, \chi_c]$ Koivisto and Nunes PLB [arXiv:0907.3883], idem PRD [arXiv:0908.0920]



- In an expanding Universe, once inside the interval $[-\chi_c, \chi_c]$, the 3-form will end up in one of the minima of the potential (notice $V_{\rm eff} \neq V$).

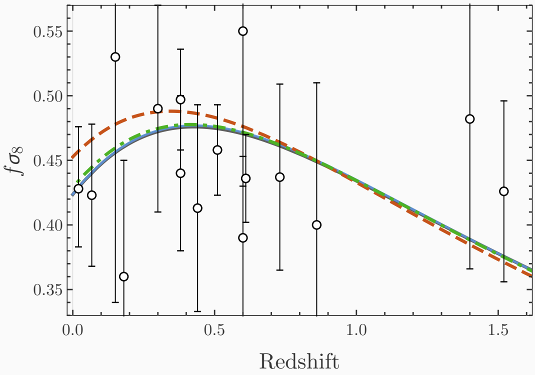


- If the 3-form stops at the limits of this interval:

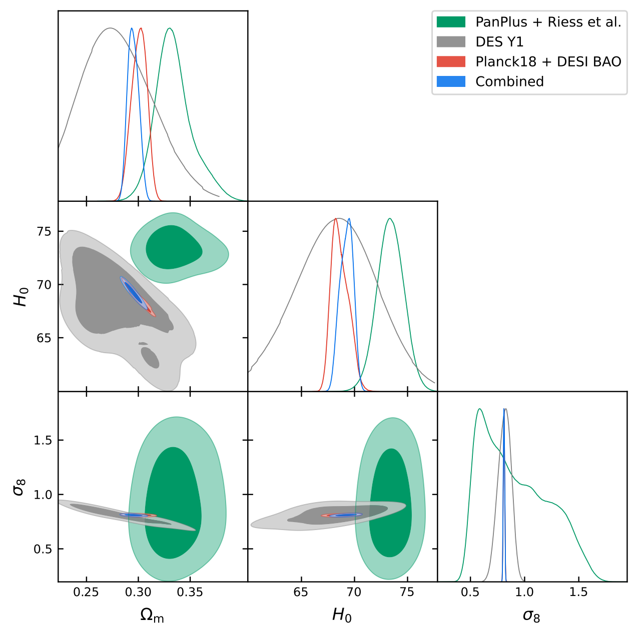
$$\chi = \pm\chi_c \quad \text{and} \quad \dot{\chi} = 0$$

- $\longrightarrow$ Universe heads towards a LSBR event $\left(\chi_c = \sqrt{2/3\kappa^2}\right)$

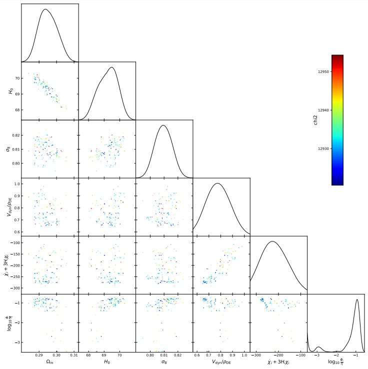
Behaviour of fσ_8

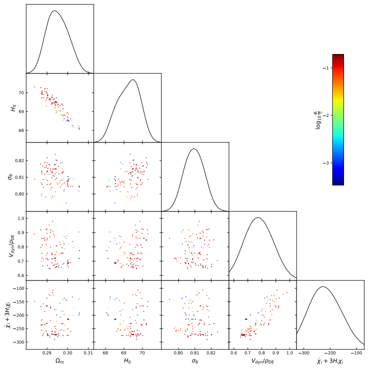


Fitting the model-1-



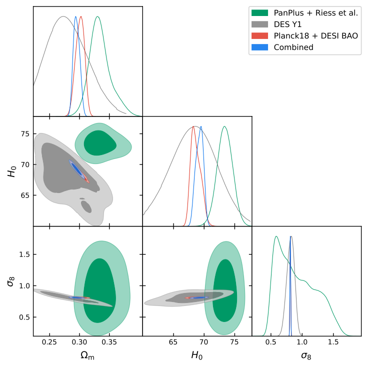
Fitting the model-2-

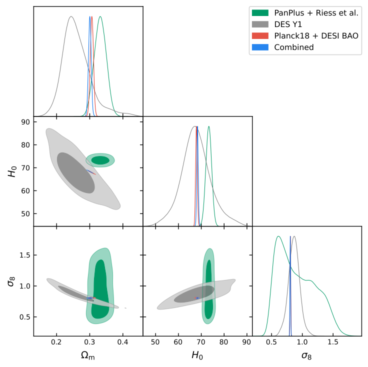




C.G. Boiza, M.B.-L, H.-W. Chiang and P. Chen work in progress (2024)

Comparing the model to LCDM





C.G. Boiza, M.B.-L, H.-W. Chiang and P. Chen work in progress (2024)

Further consideration with 3-forms from a gravitational point of view

Let me add that 3-forms can be quite interesting for further reasons as:

- They allow naturally for regular BHs (Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2005.13260 [gr-qc]. Published in EPJC 2021)
- They naturally support wormholes without changing the sign of the kinetic energy (Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2108.07302 [gr-qc]. Published in JCAP 2021).

Speeding up with fields

Late-time acceleration of the universe within Kinetic Gravity Braiding theories

Kinetic gravity Braiding theories

- The gravitational action (Deffayet, Pujolas, Sawiki and Vikman, arXiv: 1008.0048 [hep-th]. Published in JCAP 2010) :

$$S = \int d^4\mathbf{x}\sqrt{-g}\left[\frac{1}{2}R + K(\phi, X) - G(\phi, X)\Box\phi\right]$$

- Passes constraints from GR170817
- Essential mixing
- (Im)Perfect fluid
- Self-tuning de Sitter solution
- Phantom behaviour without ghost nor gradient instabilities (where the equation of state of the scalar field < -1)
- We will assume the shift-symmetric case: the action is invariant under $\phi \to \phi + c$ where c is a constant
- Our goal is to analyse the future evolution of the shift-symmetric Kinetic Gravity Braiding theories. (Borislavov, Bouhmadi-López, and Martín-Moruno arXiv:2210.07276, 2212.02547, PLB 2023, JCAP 2023)

Background dynamics

- Conserved shift current: $J = \dot{\phi} K_X + 6 X G_X H$
- Background gravitational equations:
  - Friedmann eq: $3H^2 = \rho_m + \rho_r - K + \dot{\phi} J$
  - Raychaudhuri eq: $\dot{H} = -\frac{1}{2}\left(\rho_m + \frac{4}{3}\rho_r\right) + X G_X \ddot{\phi} - \frac{1}{2}\dot{\phi} J$
- Conservation equations:
  - $\dot{\rho}_m = -3H\rho_m$
  - $\dot{\rho}_r = -4H\rho_r$
  - $\dot{J} = -3HJ$, therefore, $J = Q_0 \left(\frac{a}{a_0}\right)^{-3}$.

A dynamical system approach: suitable definition of the variables

- The matter dimensionless variables reads

$$\Omega_r := \frac{\rho_r}{3H^2},$$
$$\Omega_m := \frac{\rho_m}{3H^2},$$
$$\Omega_\phi := \frac{\epsilon\sqrt{2X}J - K}{3H^2}$$

- We assume Ω_ϕ to be positive since we are mainly interested in the future attractors of expanding FLRW models. Hence, $\Omega_i \in [0,1]$ for $i \in \{r, m, \phi\}$.
- We carry out our analysis for expanding solutions; i.e. H positive

$$\frac{H}{H_0} = \frac{h}{1-h^2} \quad \longrightarrow h \in [0,1]$$

- Through the above definition we obtain new solutions that were overlooked previously.
- The Friedmann constraint reads: $\Omega_r + \Omega_m + \Omega_\phi = 1$

A dynamical system approach: evolution of the system

- Evolution:

$$h' = \frac{(1-h^2)h}{1+h^2} C_1,$$
$$\Omega_r' = -2\Omega_r (2 + C_1),$$
$$\Omega_\phi' = C_2 - 2\Omega_\phi C_1,$$

- Auxiliary functions:

$$C_1 := \frac{H'}{H}, \qquad C_2 := \frac{\epsilon\sqrt{2X}}{H^2} (H G_X X' - J)$$

- Equations of state:

$$w_{\text{eff}} := \frac{P_{\text{tot}}}{\rho_{\text{tot}}} = -1 - \frac{2}{3} C_1, \qquad w_\phi := \frac{P_\phi}{\rho_\phi} = -1 - \frac{1}{3\Omega_\phi} C_2.$$

- A prime stands for derivative respect to $\ln(a)$.

A dynamical system approach: fixed points

1. Vacuum solutions dominated by matter or the scalar field: ($h^{\mathrm{fp}} = \Omega_r^{\mathrm{fp}} = 0$, $C_1^{\mathrm{fp}} \neq -2$ and $C_2^{\mathrm{fp}} = 2C_1^{\mathrm{fp}}\Omega_\phi^{\mathrm{fp}}$)
2. Vacuum solutions where radiation like effects dominates the nearby evolution of the system, i.e. $w_{\mathrm{eff}}^{\mathrm{fp}} = 1/3$. Scaling solutions for the scalar field. ($h^{\mathrm{fp}} = 0$, $C_1^{\mathrm{fp}} = -2$ and $C_2^{\mathrm{fp}} = -4\Omega_\phi^{\mathrm{fp}}$):
3. Cosmological singularities (Ex. BR) ($h^{\mathrm{fp}} = 1$, $\Omega_r^{\mathrm{fp}} = 0$, $C_1^{\mathrm{fp}} \neq -2$ and $C_2^{\mathrm{fp}} = 2C_1^{\mathrm{fp}}\Omega_\phi^{\mathrm{fp}}$)
4. Initial cosmological singularities. It is a radiation dominated regime ($w_{\mathrm{eff}}^{\mathrm{fp}} = 1/3$). ($h^{\mathrm{fp}} = 1$, $C_1^{\mathrm{fp}} = -2$ and $C_2^{\mathrm{fp}} = -4\Omega_\phi^{\mathrm{fp}}$)
5. de Sitter solutions Ex. ($h^{\mathrm{fp}} \neq \{0, 1\}$, $\Omega_r^{\mathrm{fp}} = 0$ and $C_1^{\mathrm{fp}} = C_2^{\mathrm{fp}} = 0$):

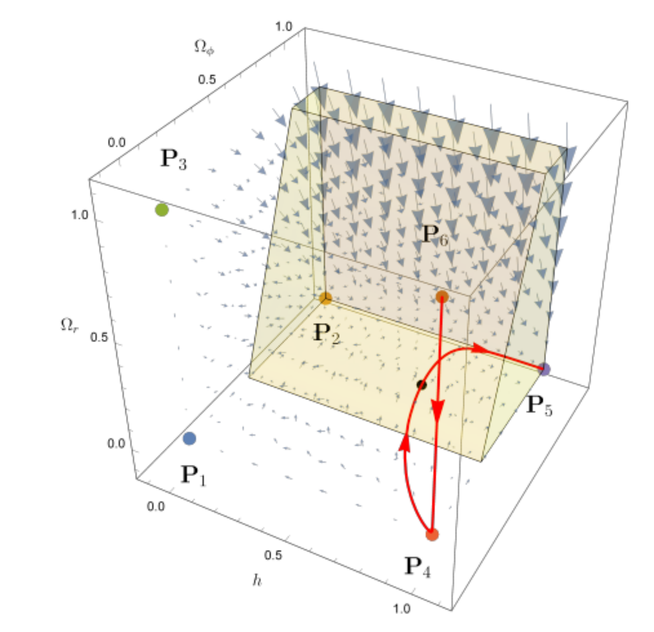
Teodor Borislavov, Mariam Bouhmadi-López, and Prado Martín-Moruno arXiv:2210.07276, 2212.02547, PLB 2023, JCAP 2023

Proxy model: $K(X) = 0$ and $G(X) = c_G X^\beta$

Fixed Point	$(h^{\rm fp}, \Omega_\phi^{\rm fp}, \Omega_r^{\rm fp})$	$w_\phi^{\rm fp}$	$w_{\rm eff}^{\rm fp}$	$\beta < -\frac{1}{2}$	$\beta = -\frac{1}{2}$	$-\frac{1}{2} < \beta < -\frac{1}{4}$	$\beta = -\frac{1}{4}$	$-\frac{1}{4} < \beta < 0$	$0 < \beta < \frac{1}{2}$	$\beta = \frac{1}{2}$	$\frac{1}{2} < \beta$
P_1 (vacuum)	$(0, 0, 0)$	$\frac{1}{4\beta}$	0	saddle	saddle	saddle	saddle	saddle	attractor	attractor	attractor
P_2 (vacuum)	$(0, 1, 0)$	$\frac{1}{4\beta+1}$	$\frac{1}{4\beta+1}$	attractor	—	saddle	—	saddle	saddle	—	saddle
P_3 (vacuum)	$(0, 0, 1)$	$\frac{1}{6\beta}$	$\frac{1}{3}$	saddle	saddle	saddle	saddle	saddle	saddle	—	saddle
P_4 (BB)	$(1, 0, 0)$	$\frac{1}{4\beta}$	0	saddle	saddle	saddle	saddle	saddle	saddle	saddle	saddle
P_5 (BB/BR)	$(1, 1, 0)$	$\frac{1}{4\beta+1}$	$\frac{1}{4\beta+1}$	saddle	—	attractor	—	repeller	repeller	—	saddle
P_6 (BB)	$(1, 0, 1)$	$\frac{1}{6\beta}$	$\frac{1}{3}$	repeller	repeller	repeller	repeller	repeller	saddle	—	repeller
P_7 (BF)	$(1, 1, 0)$	$-\infty$	$-\infty$	—	—	—	attractor*	—	—	—	—
L_1(dS)	$(h^{\rm fp}, 1, 0)$	-1	-1	—	attractor	—	—	—	—	—	—
L_2 (sudden)	$(h^{\rm fp}, -4\beta, \Omega_r^{\rm fp})$	$-\infty$	$-\infty$	—	—	—	—	attractor*	—	—	—
L_3 (vacuum)	$(0, \Omega_\phi^{\rm fp}, \Omega_r^{\rm fp})$	$\frac{1}{3}$	$\frac{1}{3}$	—	—	—	—	—	—	saddle	—
L_4 (BB)	$(1, \Omega_\phi^{\rm fp}, \Omega_r^{\rm fp})$	$\frac{1}{3}$	$\frac{1}{3}$	—	—	—	—	—	—	repeller	—

TABLE I. Classification and linear stability of the fixed points of our model. A superscript "fp" indicates evaluation at the fixed point. A horizontal bar denotes that the corresponding fixed point does not exist. The physical interpretation of each point is shown in brackets where BB stands for Big Bang and BF for Big Freeze. The labels L_1, L_2, L_3 and L_4 represent sets of non-isolated fixed points where $h^{\rm fp}$ can take any values. In addition, $\Omega_r^{\rm fp} \in [0, 1 + 4\beta]$ holds for L_2, and $\Omega_\phi^{\rm fp} + \Omega_r^{\rm fp} = 1$ for L_3 and L_4. The starred quantities designate fixed points that have eluded our dynamical system analysis because of the choice of the dynamical variables but whose existence and stability follows directly from the Friedmann equations.

Proxy model: $K(X) = 0$ and $G(X) = c_G X^\beta$



Proxy model: $K(X) = 0$ and $G(X) = c_G X^\beta$

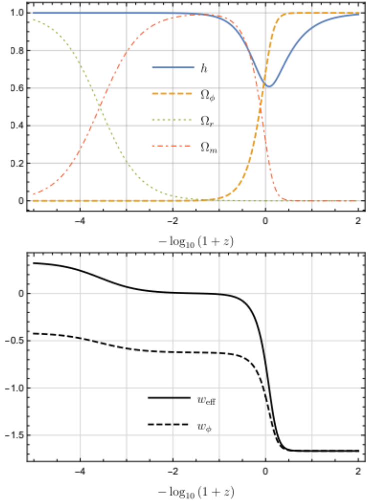


FIG. 2. Numerical evolution of the dynamical system (19)-(21) for $\beta = -2/5$ with the same initial conditions as in figure 1. Top panel: the variable h and the partial densities Ω_i for $i = \{m, r, \phi\}$. Bottom panel: the effective equation of state parameter $w_{\rm eff}$ and the equation of state parameter w_ϕ for the scalar field.

Stability of the system we have analysed at the pertubative level

1. At zero order (background): the system is stable, i.e. there are attractor solutions.
2. At first order: (at least for the simplest model we have analysed) a tachyonic or a ghost issue can arise and they are complementary; i.e. if we avoid one, the other one shows up. We think the tachyonic one is more problematic as it can affect the large scale structure. The other one can be shown to be (potentially) avoided when quantising gravity and the matter fields.
3. Dark-energy fluctuations features ghost and/or gradient instabilities for gravitational-wave amplitudes that are produced by typical binary systems.

P. Creminelli, G. Tambalo, F. Vernizzi and V. Yingcharoenrat, arXiv:1910.14035, JCAP 2020

Teodor Borislavov, Mariam Bouhmadi-López, and Prado Martín-Moruno arXiv:2406.12576

DE singularities: a quantum approach

On the quantum fate of singularities in a dark-energy dominated universe

- So far, there is no fully successful quantum gravity theory that would lead to THE theory of quantum cosmology
- There are, however, several approaches in this direction as we have heard on this conference. Here we will follow the most conservative one which corresponds to the Wheeler deWitt approach.
- The Wheeler DeWitt equation is the equivalent to Schrödinger like equation

On the quantum fate of singularities in a dark-energy dominated universe within GR

Within the framework of quantum geometrodynamics and mainly within a Born Oppenheimer approximation

- It was shown that the big rip can be removed Dabrowski, Kiefer and Sandhöfer, PRD, [arXiv:hep-th/0605229], Alonso, B.L. and Martín-Moruno, PRD, [arXiv:1802.03290 [gr-qc]].
- It was shown the avoidance of a big brake singularity Kamenshchik, Kiefer and Sandhöfer 07', PRD, [arXiv:0705.1688].
- It was shown also the avoidance of a big démarrage singularity and a big freeze BL, Kiefer, Sandhöfer and Moniz, PRD, [arXiv:0905.2421]
- Type IV singularity is removed BL, Krämer and Kiefer, PRD, [arXiv:1312.5976].
- It has been shown as well that LR can be removed Albarran, BL, Kiefer, Marto, Moniz, PRD, [arXiv:1604.08365].

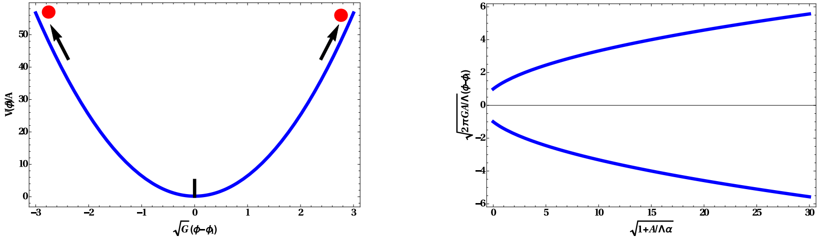
Reviews on the topic by B.L., Kiefer and Martín-Moruno arXiv:1904.01836 (GRG), T. Borislavov, B.L. and Martín-Moruno arXiv: 2106.12050 (Universe), On DE singularities in general: de Haro, Nojiri, Odintsov, Oikonomou and Pan arXiv: 2309.07465 (Phys. Rept)

LSBR driven by a scalar field

- Phantom scalar field ϕ: $\rho_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi)\ , \quad p_\phi = -\frac{1}{2}\dot{\phi}^2 - V(\phi)$

$$\ddot{\phi} + 3H\dot{\phi} - V'(\phi) = 0, \quad V(\phi) = \frac{A}{6} + 2\pi AG\left(\phi - \phi_1\right)^2$$

$\alpha = \ln(a/a_0)$



LSBR: Quantisation with a scalar field

- The Wheeler-DeWitt equation:

$$\frac{\hbar^2}{2}\left[\frac{\kappa^2}{6}\frac{\partial^2}{\partial\alpha^2}+\frac{\partial^2}{\partial\phi^2}\right]\psi(\alpha,\phi)+a_0^6e^{6\alpha}V(\phi)\psi(\alpha,\phi)=0$$

- Can be solved within the BO:
  - The gravitational part are oscillatory or exponential functions.
  - The matter part can be written as parabolic cylinder functions that decay to zero at large value of the scale factor.
- It can be shown that there are solutions (wave functions) that vanishe close to the classically abrupt event. Therefore, the DeWitt condition is fullfilled. This result can be interpreted as an "abrupt event" avoidance.

Albarran, BL, Cabral, Martín-Moruno, JCAP, [arXiv:1509.07398] (minimally coupled scalar field)

LSBR: Quantisation with a 3-form-1-

- The classical action for a FLRW universe (spatially flat) reads:

$$S = S_A + S_{EH} = \int dt\, \mathcal{V} N \left(\frac{-a\dot{a}^2}{2\kappa N^2} + \frac{\dot{\phi}^2}{2a^3 N^2} - a^3 V \right),$$

where $\phi = a^3 \chi$.

- The classical Hamiltonian for a FLRW universe (spatially flat) reads:

$$\mathcal{H} = N \left(-\frac{\kappa}{2a} p_a^2 + \frac{a^3}{2} p_\phi^2 + a^3 V \right),$$

where

$$p_a = \frac{\delta L}{\delta \dot{a}} = -\frac{a\dot{a}}{\kappa N}, \quad p_\phi = \frac{\delta L}{\delta \dot{\phi}} = \frac{\dot{\phi}}{a^3 N}.$$

B.L., Brizuela and Garay, JCAP, [arXiv:1802.05164 [gr-qc]] (3-forms)

LSBR: Quantisation with a 3-form-2-

- The classical Hamiltonian for a FLRW universe (spatially flat) can be rewritten as:

$$\mathcal{H} = N\left(\frac{1}{2}G^{AB}p_A p_B + a^3 V\left(6(a^{-3}\phi)^2\right)\right),$$

with A and B indices referring to a or ϕ and the mini-superspace metric given by

$$G^{AB} = \begin{pmatrix} -\frac{\kappa}{a} & 0 \\ 0 & a^3 \end{pmatrix}. \qquad G_{AB} = \begin{pmatrix} -\frac{a}{\kappa} & 0 \\ 0 & \frac{1}{a^3} \end{pmatrix}.$$

LSBR: Quantisation with a 3-form-3-

- With the usual prescription, we transform the classical dynamical variables into quantum operators by means of the Laplace-Beltrami operator:

$$G^{AB}p_A p_B \to -\frac{\hbar^2}{\sqrt{-G}}\partial_A(\sqrt{-G}\,G^{AB}\partial_B)\,,$$

  where G is the determinant of G_{AB}.

- The Wheeler-DeWitt equation then reads:

$$\left(\hbar^2\kappa\partial_\beta^2 - \hbar^2\partial_\phi^2 + 2V\right)\psi(\beta,\phi) = 0,$$

  and $\beta = a^3/3$.

LSBR: Quantisation with a 3-form-4-

- The potential:

$$V = V_0 e^{-\lambda^2 \chi^2} = V_0 e^{-9\lambda^2 \phi^2/\beta^2}\,,$$

  with $\lambda^2 = \kappa/2\sigma^2$, and σ the dimensionless width of the Gaussian potential.
- The WDW equation is given now by:

$$\left(\hbar^2 \kappa \partial_\beta^2 - \hbar^2 \partial_\phi^2 + 2V_0 e^{-9\lambda^2 \phi^2/\beta^2}\right) \psi(\beta, \phi) = 0.$$

LSBR: Quantisation with a 3-form-5-

- The WdW eq. can be solved as follows:
  - The matter part can be written in different ways depending on the approximations used
    - Constant potential (exponential decreasing functions)
    - Linear approximation for the potential + a B.O. approximation (Airy functions)
    - quadratic approximation for the potential + a B.O. approximation (Bessel functions)
  - The matter part decays to zero at large value of the scale factor.
  - The gravitational part are oscillatory or exponential (decaying) functions.
- it can be shown that there are solutions (wave functions) that vanishe close to the classically abrupt event. Therefore, the DeWitt condition is fullfilled. This result can be interpreted as an "abrupt event" avoidance.
- A similar approach can be implemnted in modified theories of gravity within a metric and a Palatini approach.

Conclusions

Conclusions

- We started reviewing the late-time acceleration of the universe through a *wcdm* phenomenological approach
- We have also shown that the late-time acceleration of the Universe can be described through a phantom DE component
- We have looked at the observational fit and the perturbations
- We have described phantom DE through a more fundamental field encoded in a 3-form or a DE KGB model.
- We have discussed if some of these DE models can help to release the H_0 and σ_8 tensions
- Then finally, we have shown using the WDW equation that the DE singularities or abrupt event can be unharmful in a quantum context.

Xie Xie!!!

Thank you for your attention !!!

A quantum approach within metric $f(R)$ theory

- The modified WdW equation (first obtained by Vilenkin back in the 80')

$$\left[\partial_q^2 - \frac{1}{q^2}\partial_x^2 - V(q,x)\right]\Psi(q,x) = 0,$$

where the potential is given by

$$V(q,x) = \frac{q^2}{\lambda^2}\left[k + \frac{f_{R0}}{6R_0}\left(f - Rf_R\right)\frac{q^2}{f_R^2}\right],$$

and

$$q = \sqrt{R_0}\, a\left(f_R/f_{R0}\right)^{1/2} \text{ and } x = \ln\left(f_R/f_{R0}\right)^{1/2},$$

- It is a PDE because in metric $f(R)$ theories there is an extra degree of freedom, the scaleron.

$f(R)$ quantum cosmology and the BR

- A suitable choice would be

$$f(R) = \alpha_+ R^\gamma, \qquad \text{with} \qquad \gamma = 2 + \sqrt{3/2},$$

- Then

$$V(q,x) = -\frac{A}{\lambda^2} e^{-Bx} q^4,$$

$$q = \sqrt{R_0}\, a\, (R/R_0)^{\frac{\gamma-1}{2}} \;, \quad x = \ln{(R/R_0)}^{\frac{\gamma-1}{2}} \;,$$

where $A = \frac{\gamma-1}{6\gamma} = \frac{1}{30}(1+\sqrt{6})$, and $B = 2\frac{\gamma-2}{\gamma-1} = 6 - 2\sqrt{6}$.

- The WdW equation becomes

$$\left[q^2 \partial_q^2 - \partial_x^2 + \frac{A}{\lambda^2} e^{-Bx} q^6 \right] \Psi(q,x) = 0.$$

- It cannot be solved exactly but it can be shown there are approximate solutions (wave functions) that vanishe close to the classically singularity. Therefore, the DeWitt condition is fullfilled. This result can be interpreted as an singularity avoidance.

Alonso-Serrano, Bouhmadi-López and Martín-Moruno, PRD [arXiv:1802.03290 [gr-qc]].Borislavov Vasilev, B.L., Martín-Moruno, PRD,

[arXiv:1907.13081 [gr-qc], PRD, arXiv:2103.12786[gr-qc] ($f(R)$ metric theories)