Consequences of phase transitions occurred during inflation

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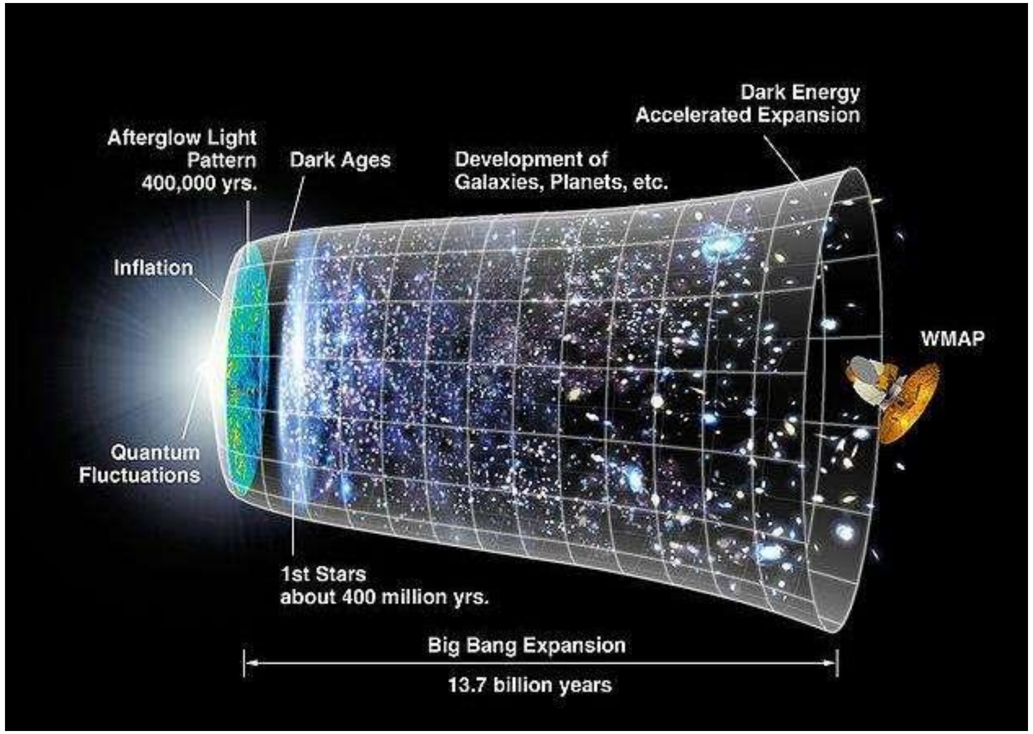
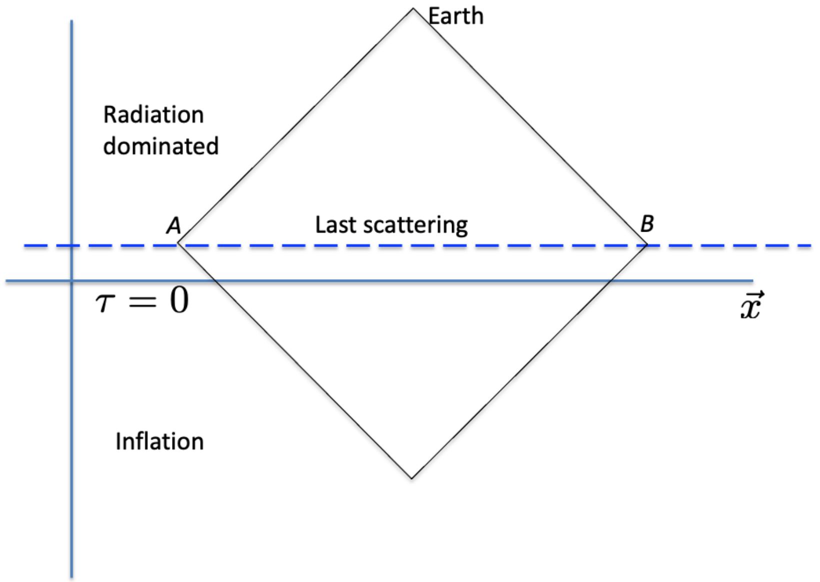
2009.12381, 2201.05171 w/ Kun-Feng Lyu, Lian-Tao Wang and Siyi Zhou
2208.14857 w/ Xi Tong and Siyi Zhou
2304.02361 w/ Chen Yang
2308.00070 w/ Boye Su, Hanwen Tai, Lian-Tao Wang, Chen Yang

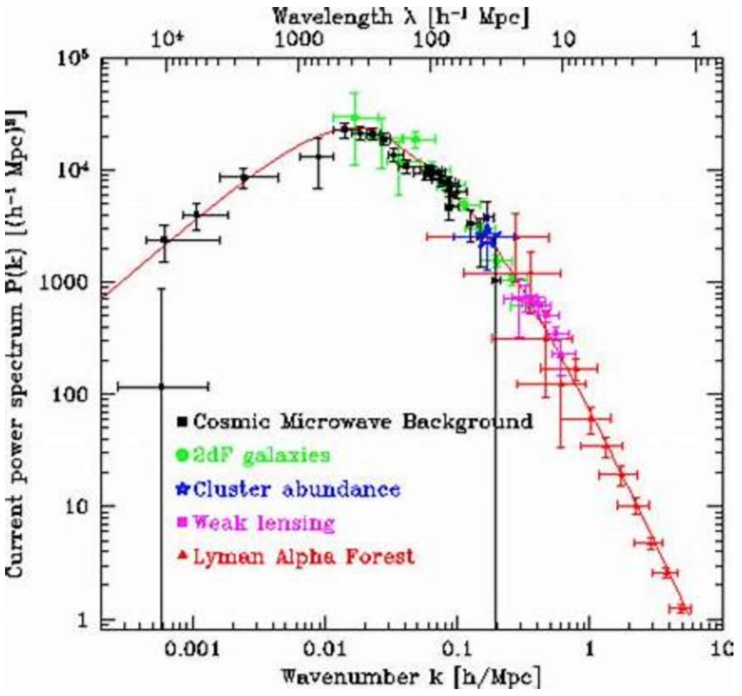
Very brief introduction of inflation

1. Solves the causality problem
2. Solves the flatness problem
3. Solves the magnetic monopole problem
4. Generates the seed of large scale structure



Very brief introduction of inflation

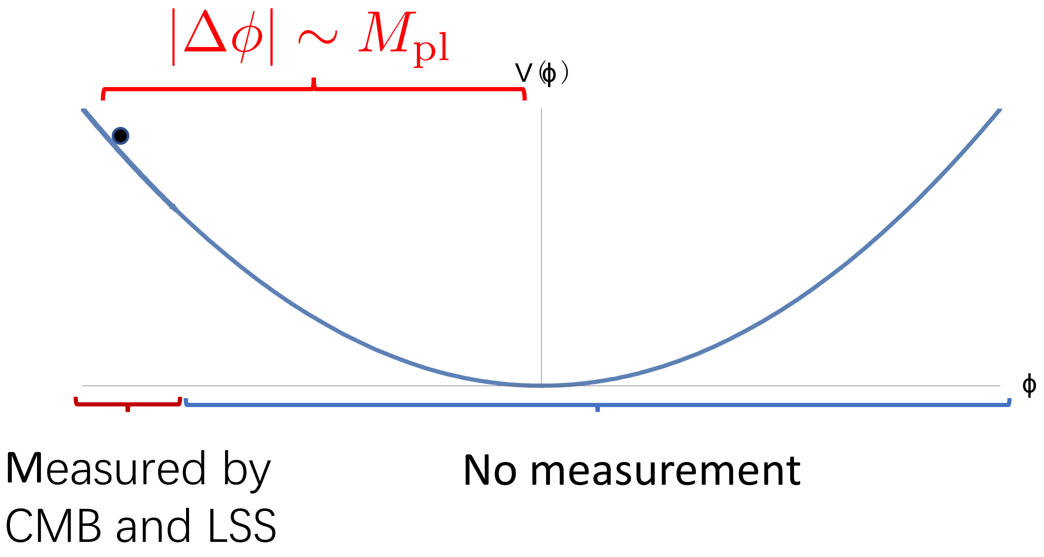




- To solve the problems, 40 to 60 e-folds is required, BUT we can only observe ten!

Slow roll models

- We usually assume a potential.
- Use it to calculate n_s, r ...

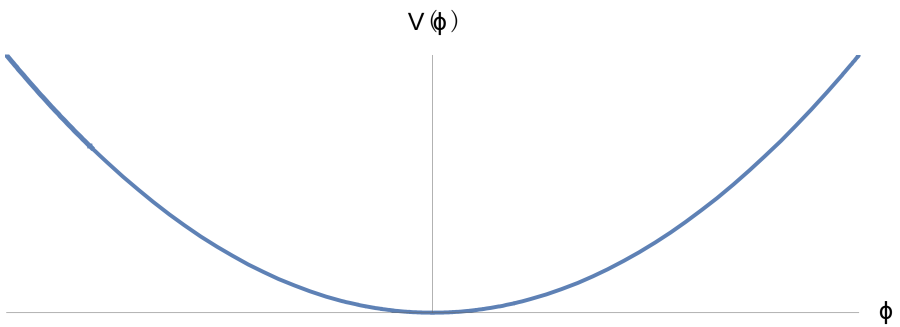


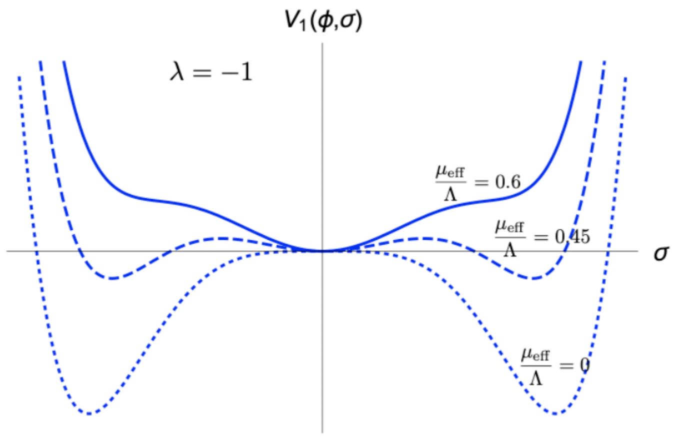
- The inflaton must couple to some spectator field.
- The masses or couplings in the spectator sector can be changed drastically due to the evolution of the inflaton field.

Induced phase transition in spectator sector

- ϕ: inflaton field σ: spectator field

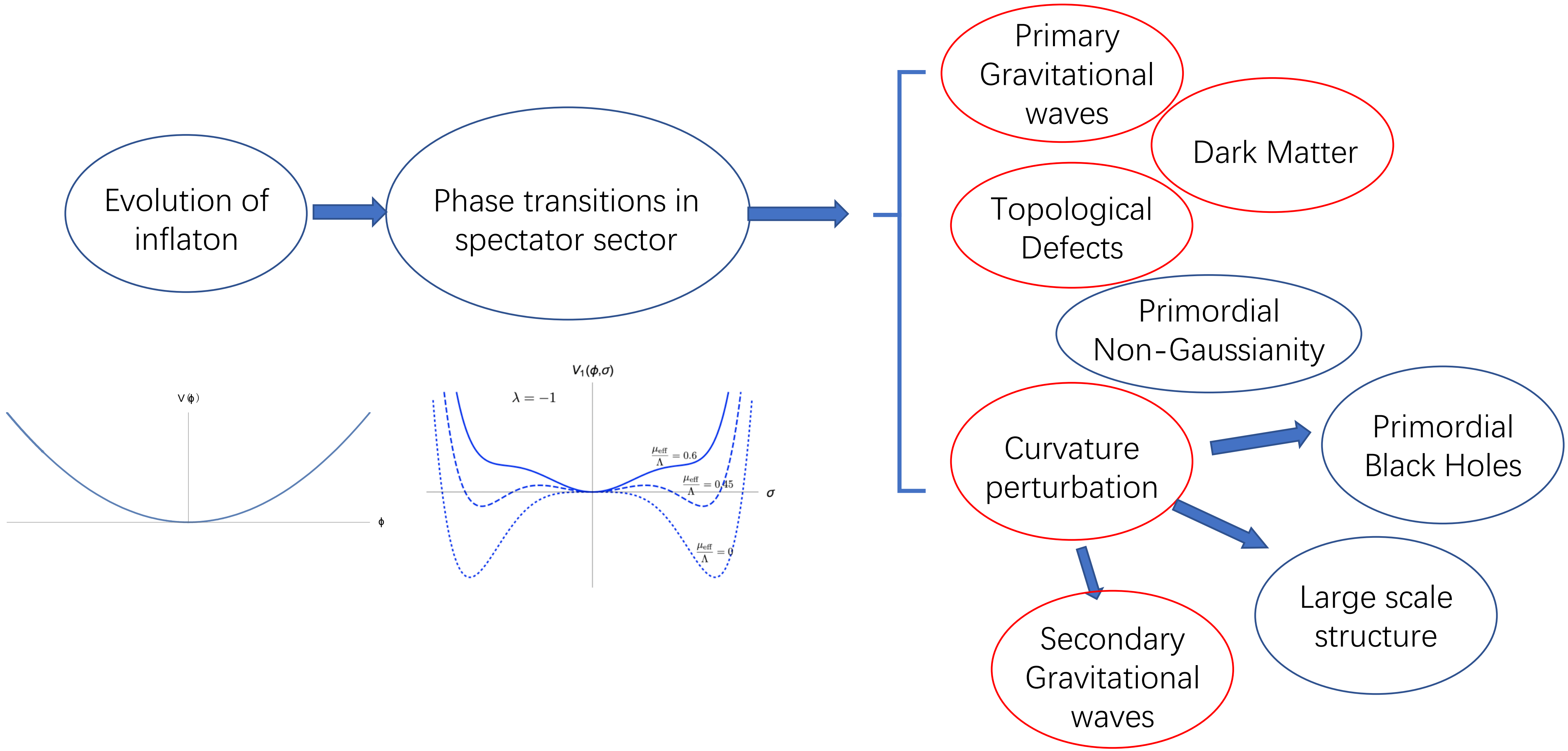
Example 1: $$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$





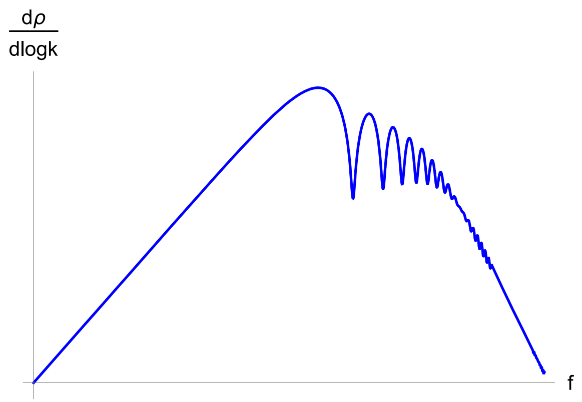
Example 2: $$\mathcal{L}_\sigma = -(1 - \frac{c^2\phi^2}{\Lambda^2})\frac{1}{4g^2}G^a_{\mu\nu}G^{a\mu\nu}$$

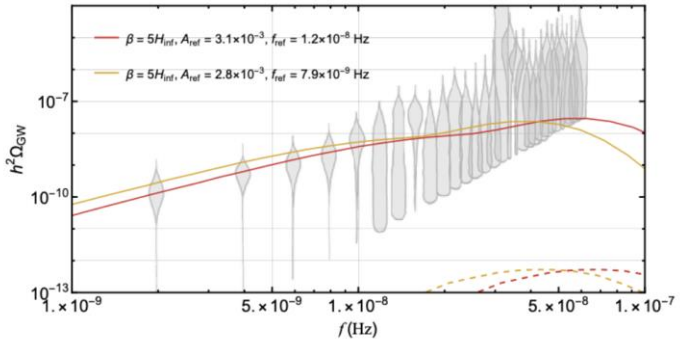
Induced phase transition in spectator sector

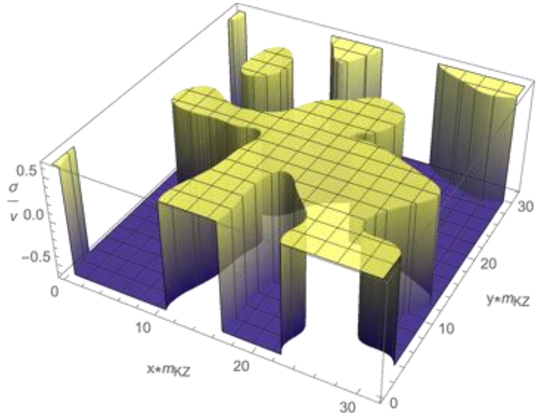


Outline

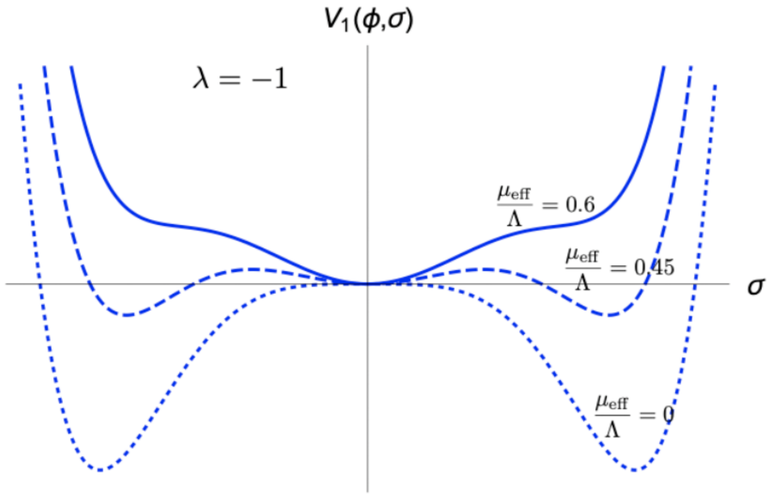
- First-order phase transitions during inflation.
    - Primary GWs
    - Dark Matter
    - Curvature perturbation
    - Secondary GWs
- GWs from second-order phase transitions (domain walls) during inflation.
- Summary and outlook



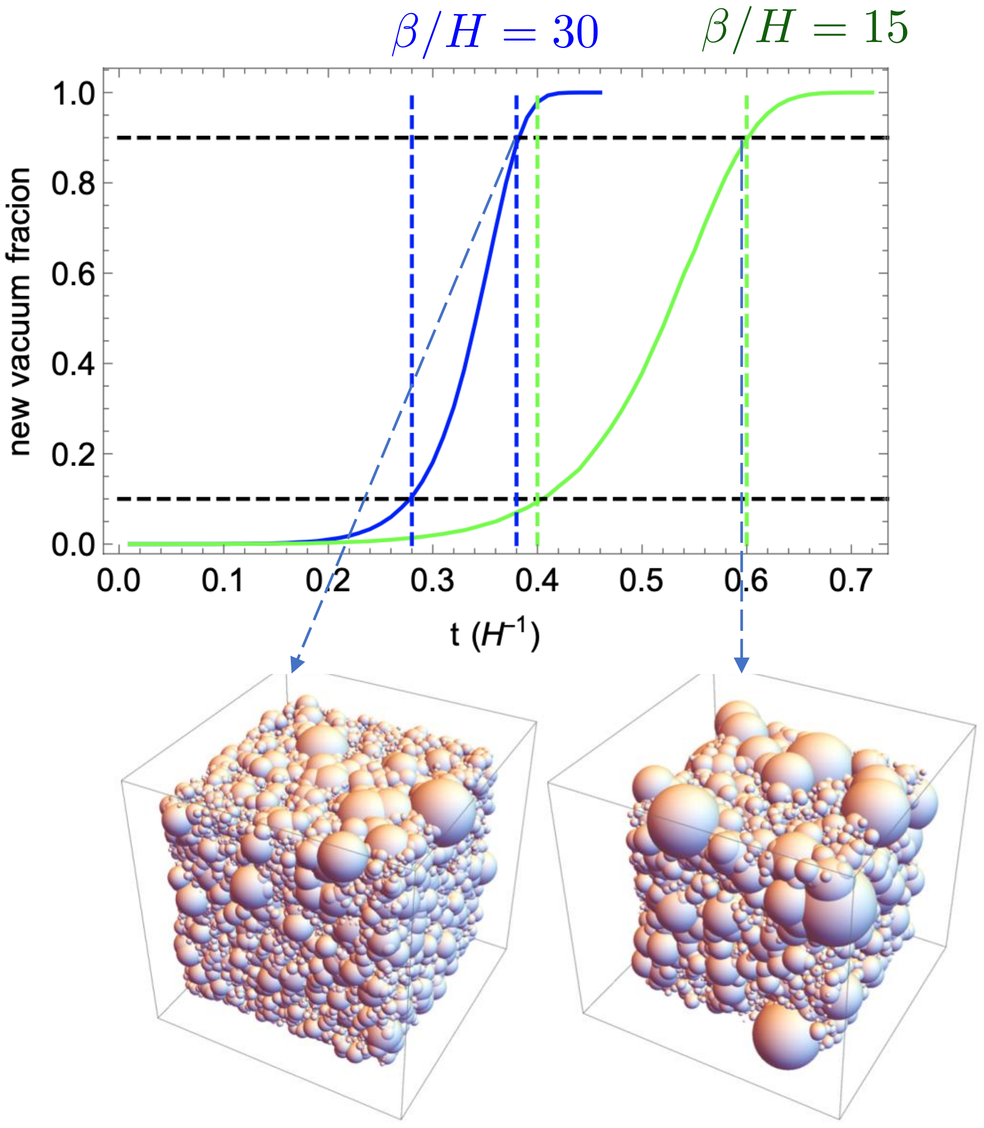




First-order phase transition during inflation

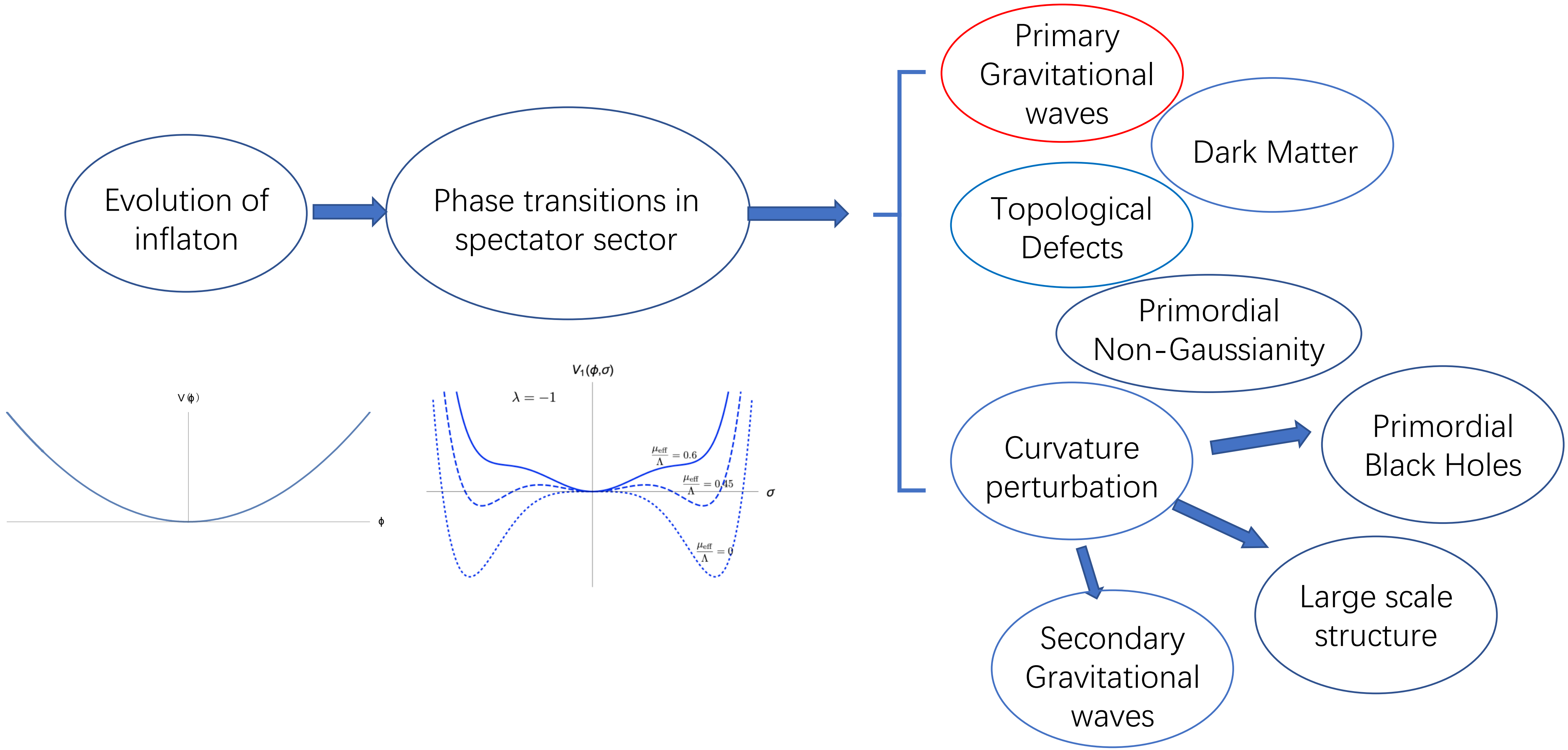


S_4 becomes smaller during



- $\beta = -\frac{dS_4}{dt}$, determines the rate of the phase transition.
- Phase transition completes if $\beta \gg H$.

Induced phase transition in spectator sector



GW from instantaneous and local sources (qualitative study)

- E.O.M. of GW

$$ds^2 = a^2(\tau)\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$$

Traceless and transverse

$$h''_{ij} + \frac{2a'}{a}h'_{ij} - \nabla^2 h_{ij} = 16\pi^2 G_N a^2 \sigma_{ij}$$

- For an instantaneous and local source,

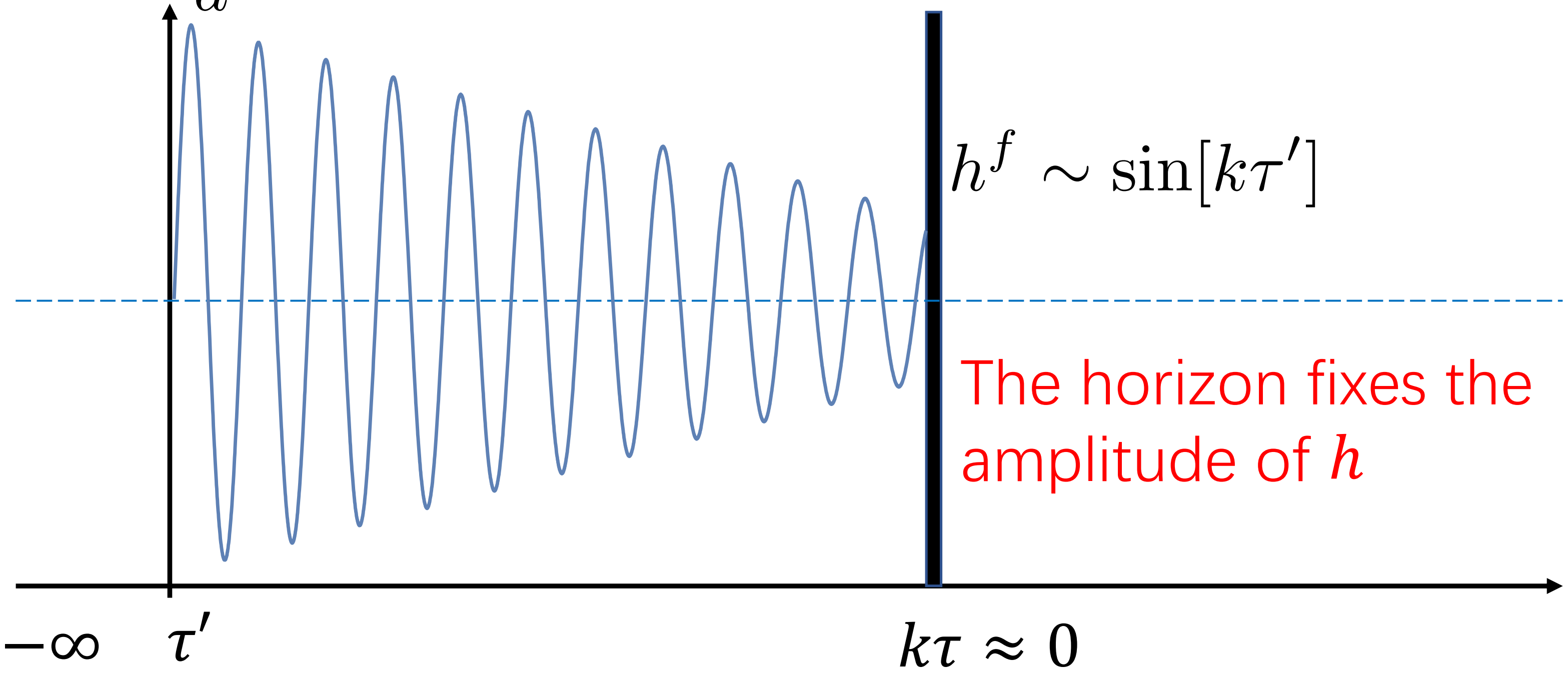
$$\sigma_{ij} \sim \delta(\mathbf{x})\delta(\tau - \tau')$$

- E.O.M. in Fourier space

$$h''(\tau,\mathbf{k}) + \frac{2a'}{a}h'(\tau,\mathbf{k}) + k^2 h(\tau,\mathbf{k}) = 16\pi G_N a^{-1} T\delta(\tau - \tau')$$

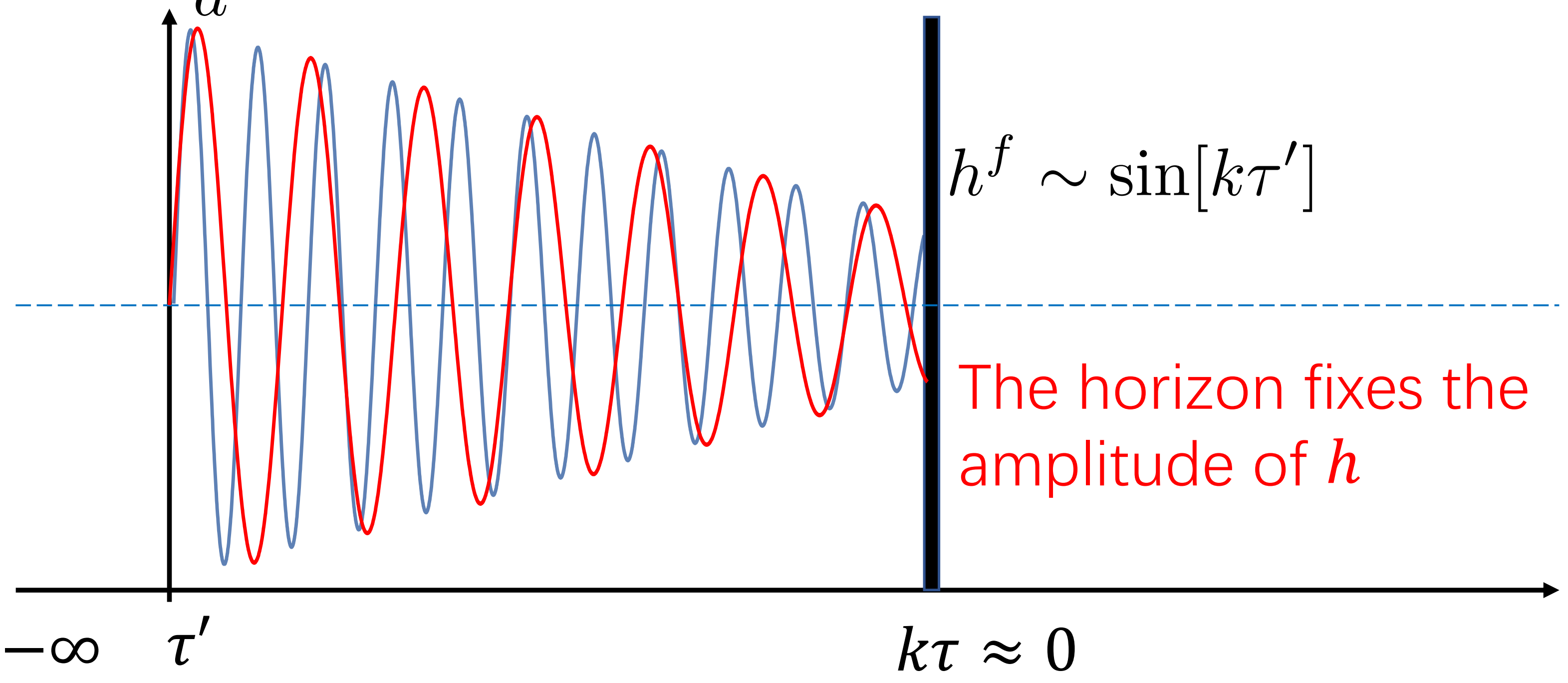
GW from instantaneous and local sources (qualitative study)

- $$h''(\tau,\mathbf{k})+\frac{2a'}{a}h'(\tau,\mathbf{k})+k^2h(\tau,\mathbf{k})=16\pi G_N a^{-1}T\delta(\tau-\tau')$$



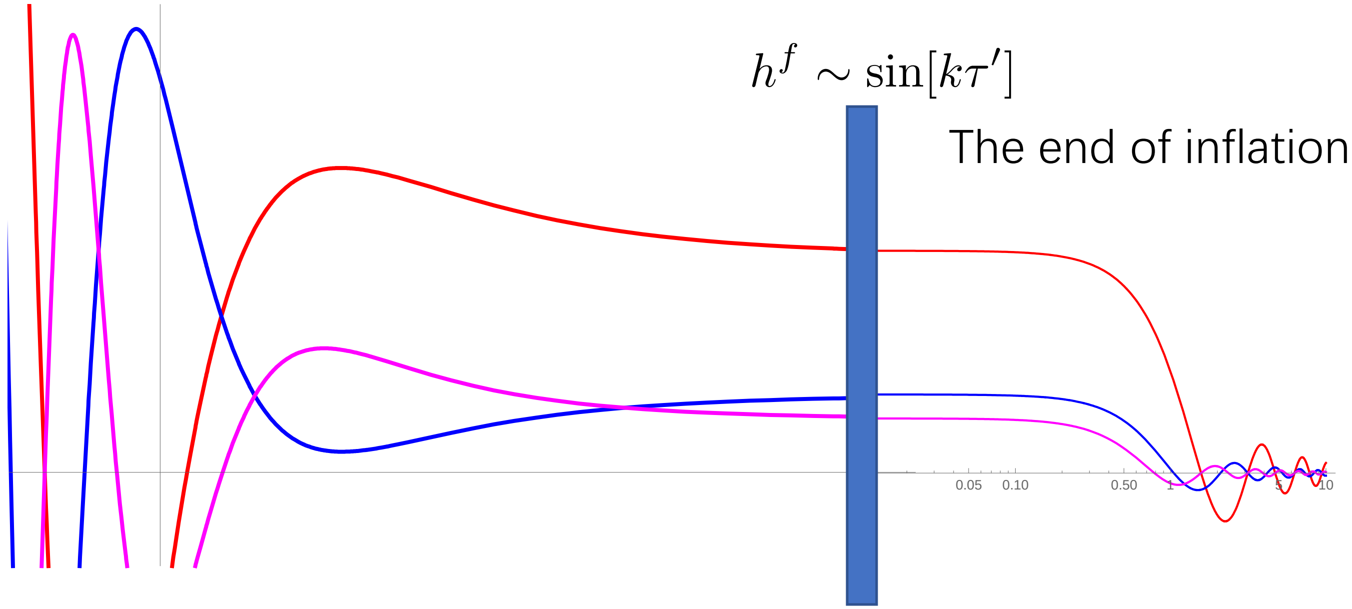
GW from instantaneous and local sources (qualitative study)

- $$h''(\tau,\mathbf{k}) + \frac{2a'}{a}h'(\tau,\mathbf{k}) + k^2 h(\tau,\mathbf{k}) = 16\pi G_N a^{-1} T\delta(\tau-\tau')$$

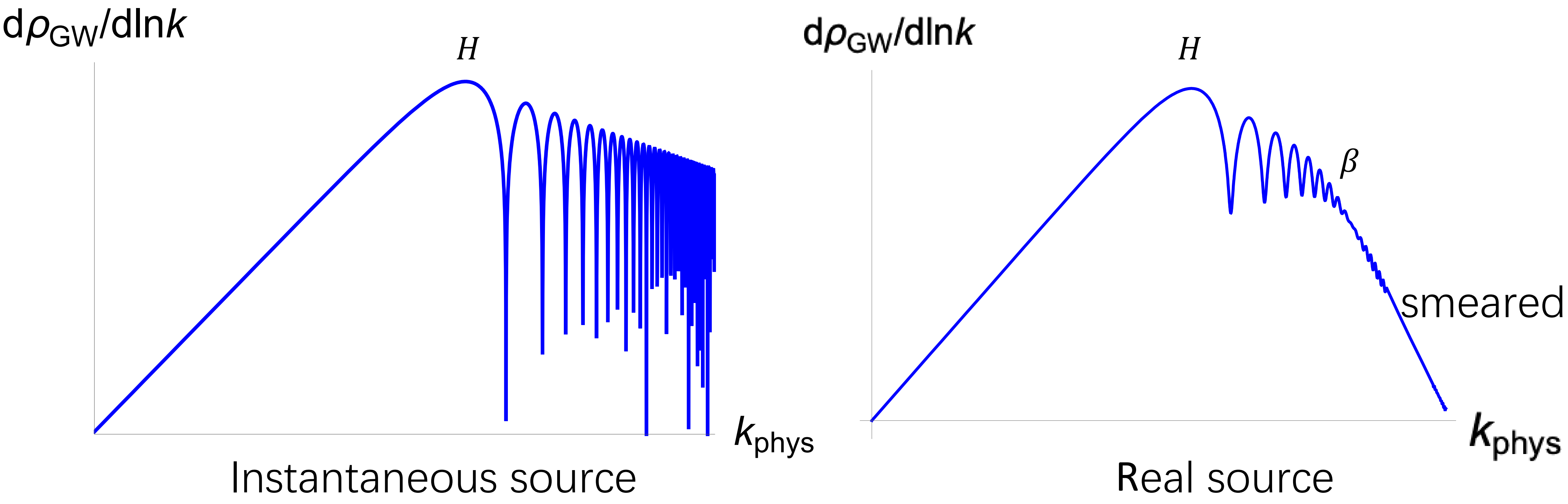


After inflation

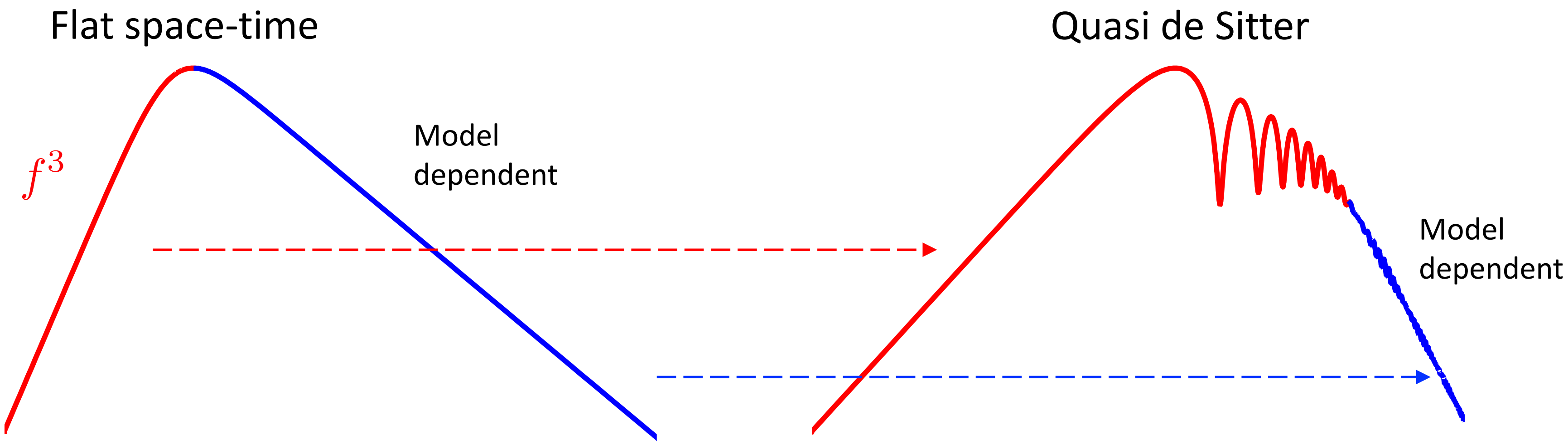
- $h^f(k)$ is the initial amplitude for the GW oscillation after inflation.
- All the modes start to oscillate with the same phase.
- Example, in RD, the oscillation is $\sin k\tau / k\tau$.



Spectrum of GW from a real source



Spectrum distortion by inflation



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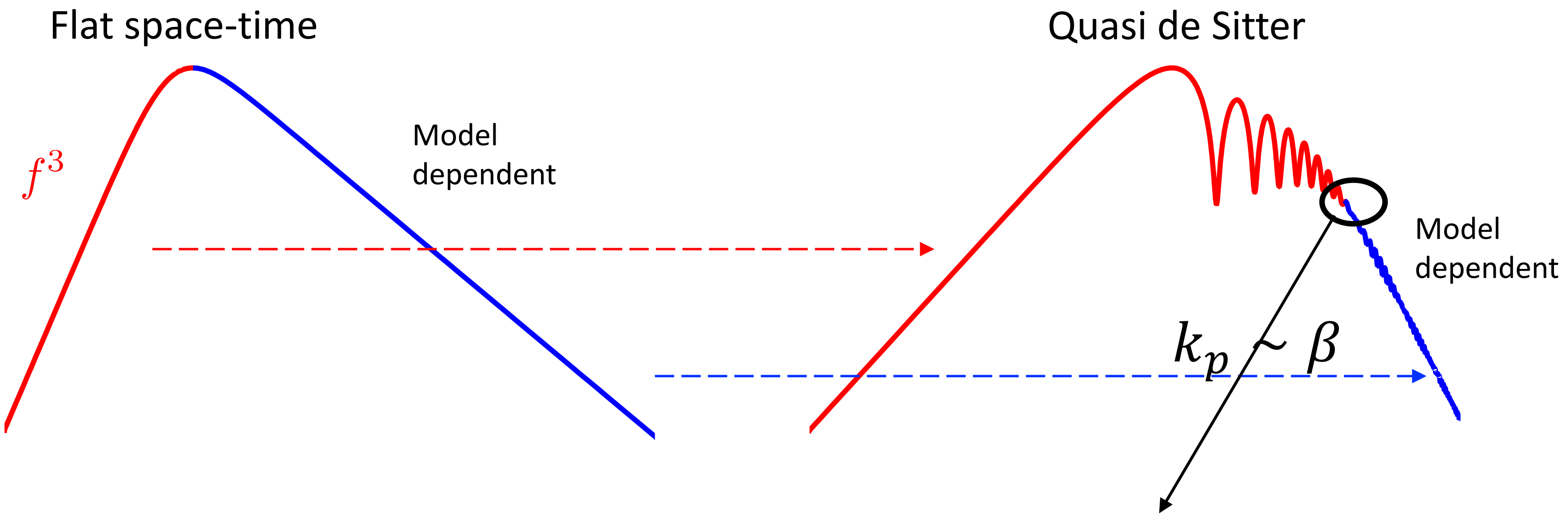
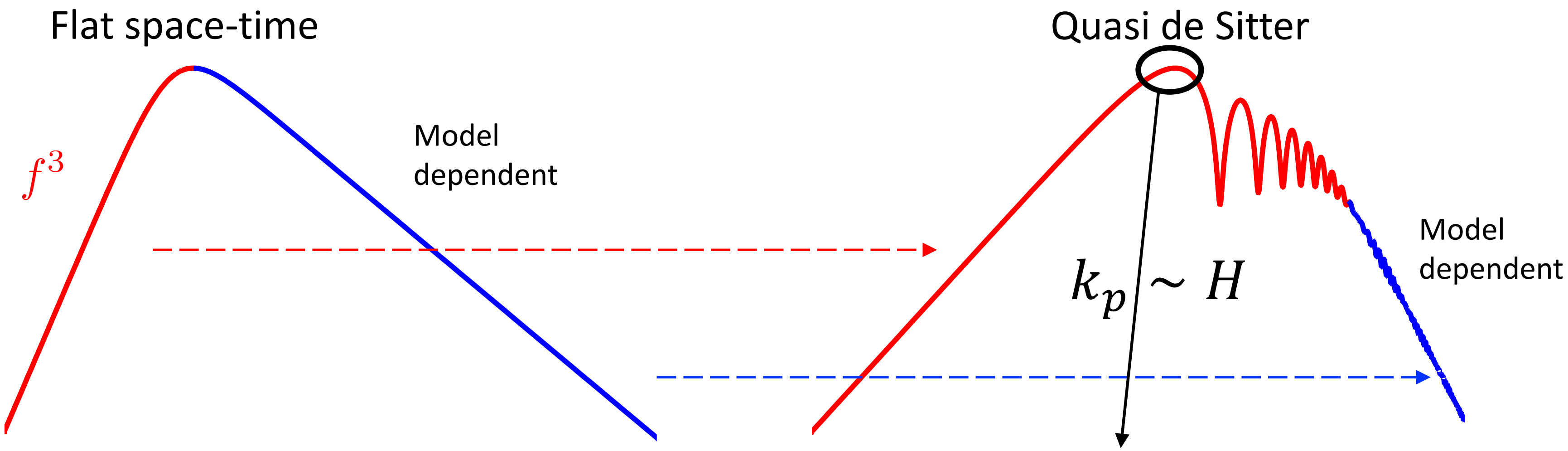


$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^6 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2$$

Spectrum distortion by inflation

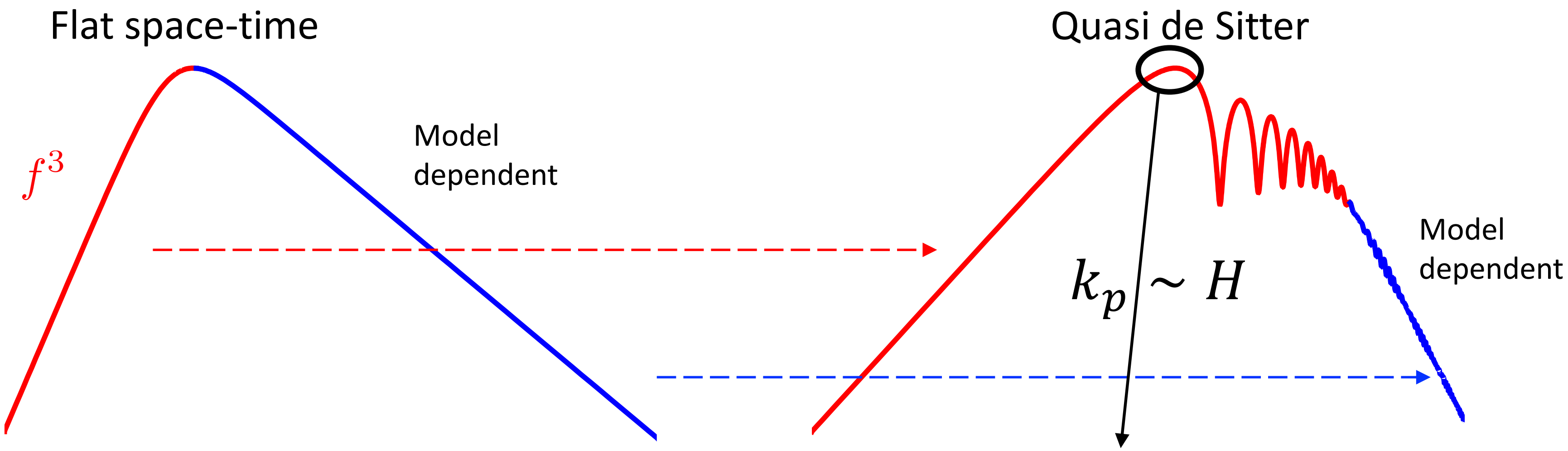


$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2$$

$$\approx 10^{-12} \times \left(\frac{H_{\rm inf}}{0.1\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{0.1\rho_{\rm inf}}\right)^2$$

$$\approx 10^{-17} \times \left(\frac{H_{\rm inf}}{0.01\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{0.1\rho_{\rm inf}}\right)^2$$

Spectrum distortion by inflation



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$$\approx 10^{-17} \times \left(\frac{H_{\mathrm{inf}}}{0.01\beta}\right)^5 \left(\frac{\Delta\rho_{\mathrm{vac}}}{0.1\rho_{\mathrm{inf}}}\right)^2$$

First-order phase transition during inflation

- Assume quasi-dS inflation, RD re-entering, and fast reheating

$$\Omega_{\rm GW}(k_{\rm today}) = \Omega_R \frac{H_{\rm inf}^4}{k_p^4}\left[\frac{1}{2} + \mathcal{S}(k_p\beta^{-1})\cos\left(\frac{2k_p}{H_{\rm inf}}\right)\right]\left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2 \frac{d\rho_{\rm GW}^{\rm flat}}{\Delta\rho_{\rm vac} d\log k_p}$$

Dilution factor

Smearing

Suppressed by the energy faction

Redshift

$$\frac{f_{\rm today}}{f_\star} = \underbrace{\frac{a(\tau_\star)}{a_1}}_{e^{-N_e}}\left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3} \frac{T_{\rm CMB}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_{\rm inf}^2}{8\pi G_N}\right)\right]^{1/4}}$$

N_e: e-folds before the end of inflation

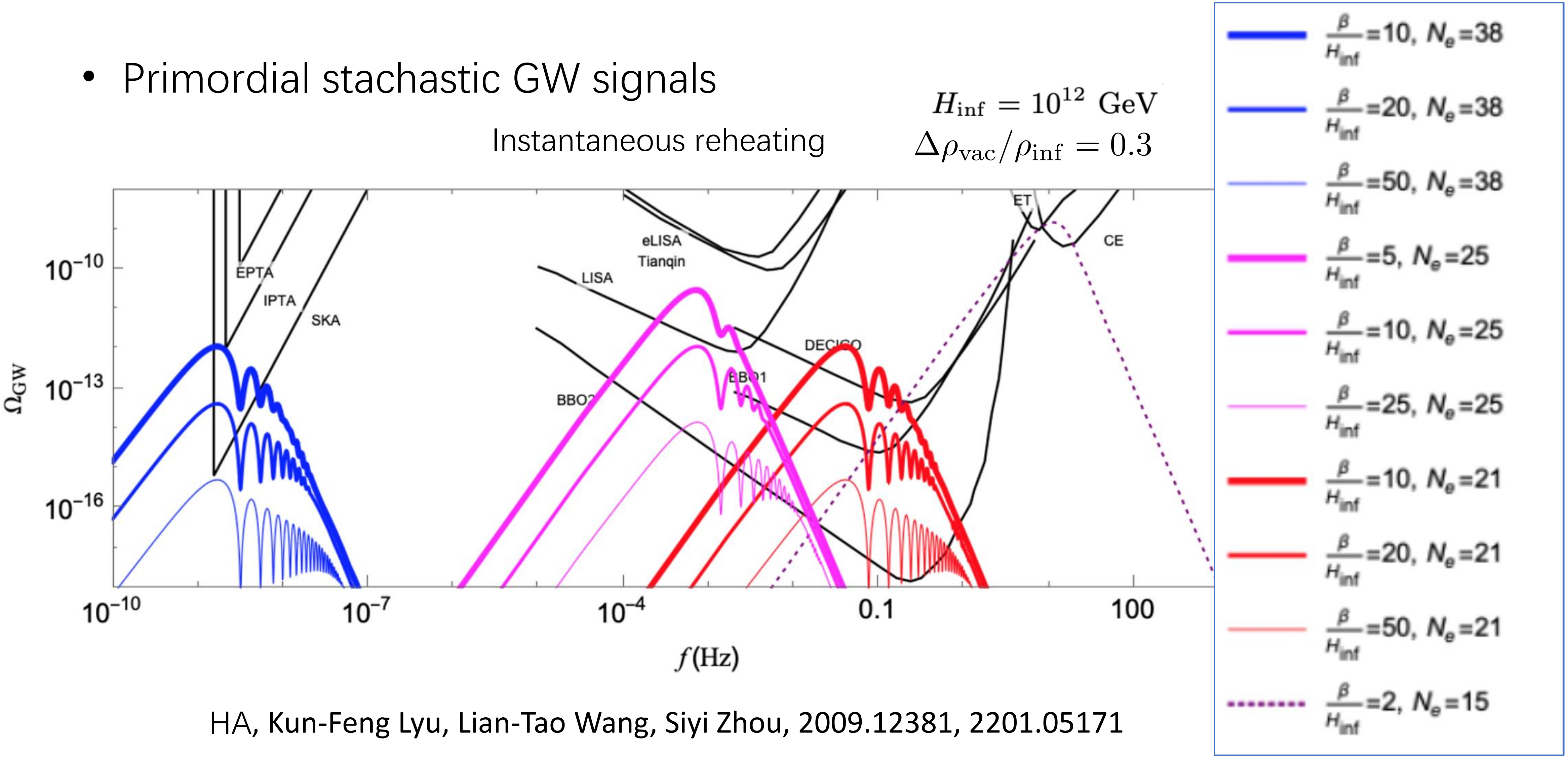
First-order phase transition during inflation

- Primordial stachastic GW signals

Instantaneous reheating

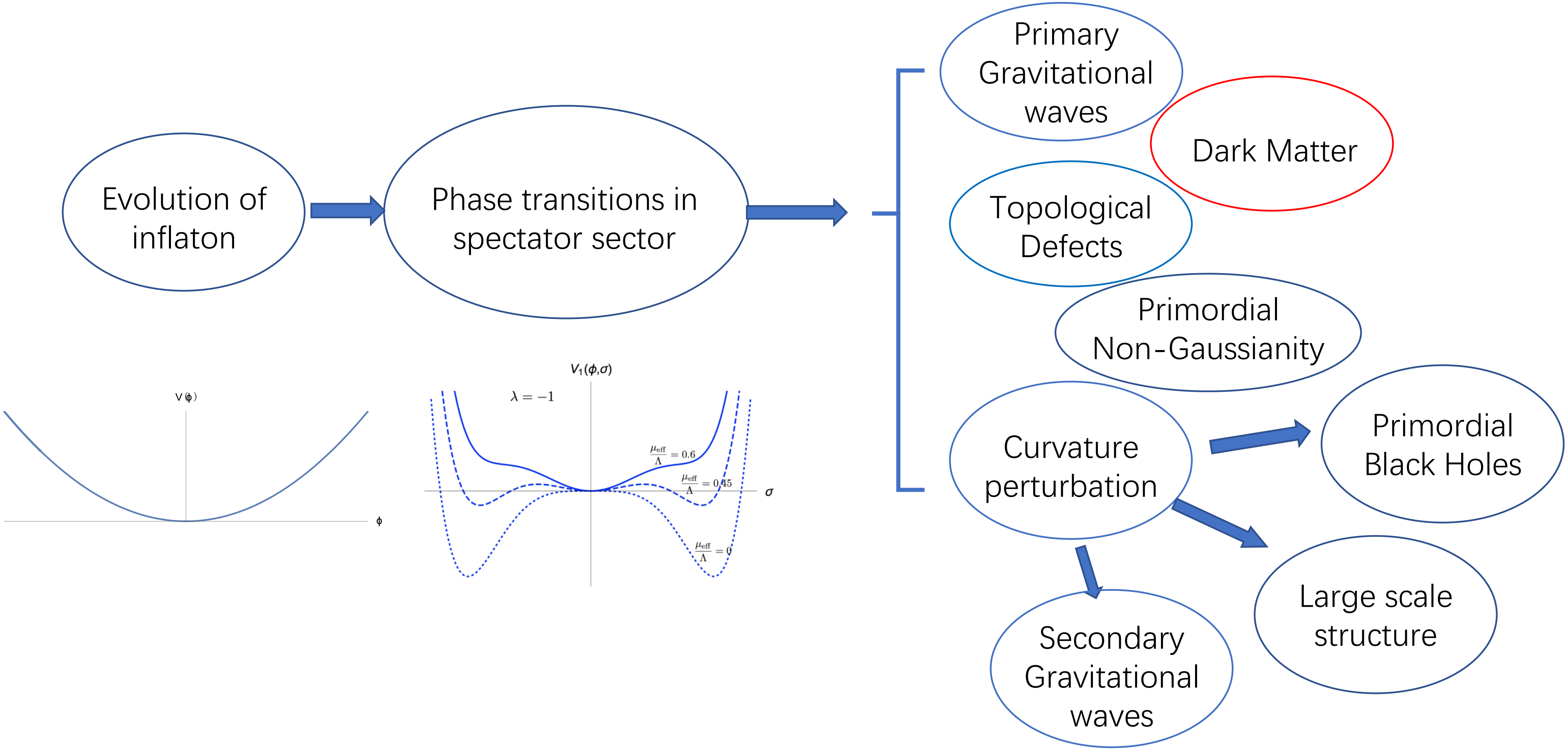
$H_{\rm inf} = 10^{12}$ GeV

$\Delta\rho_{\rm vac}/\rho_{\rm inf} = 0.3$



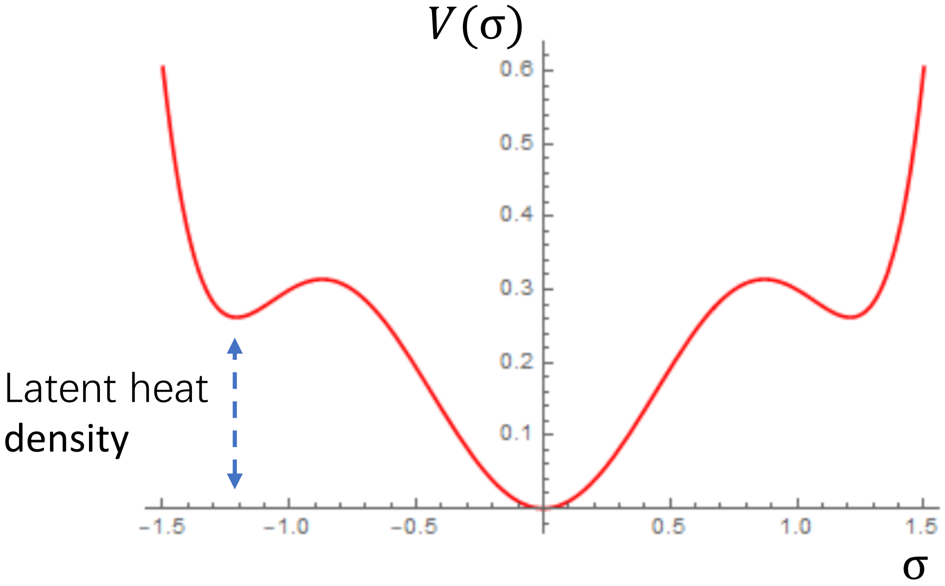
HA, Kun-Feng Lyu, Lian-Tao Wang, Siyi Zhou, 2009.12381, 2201.05171

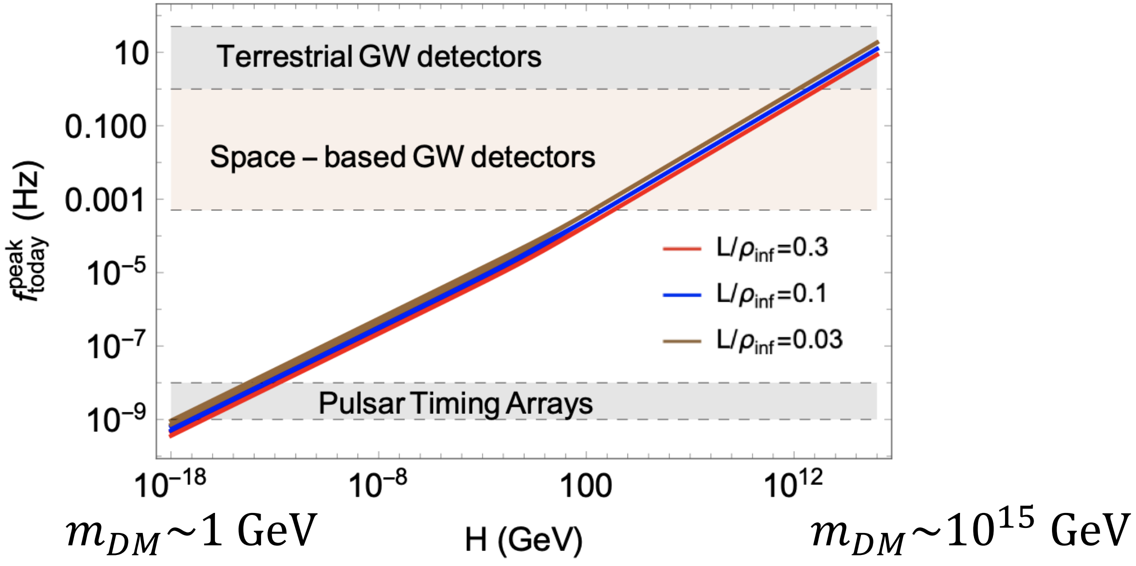
Induced phase transition in spectator sector



Producing super heavy DM

- Where does the latent heat go?
- σ particles produced during bubble collision and thermalization.
- If the phase transition is ***symmetry-restoration***, σ particles can be DM.





HA, Xi Tong, Siyi Zhou, 2208.14857

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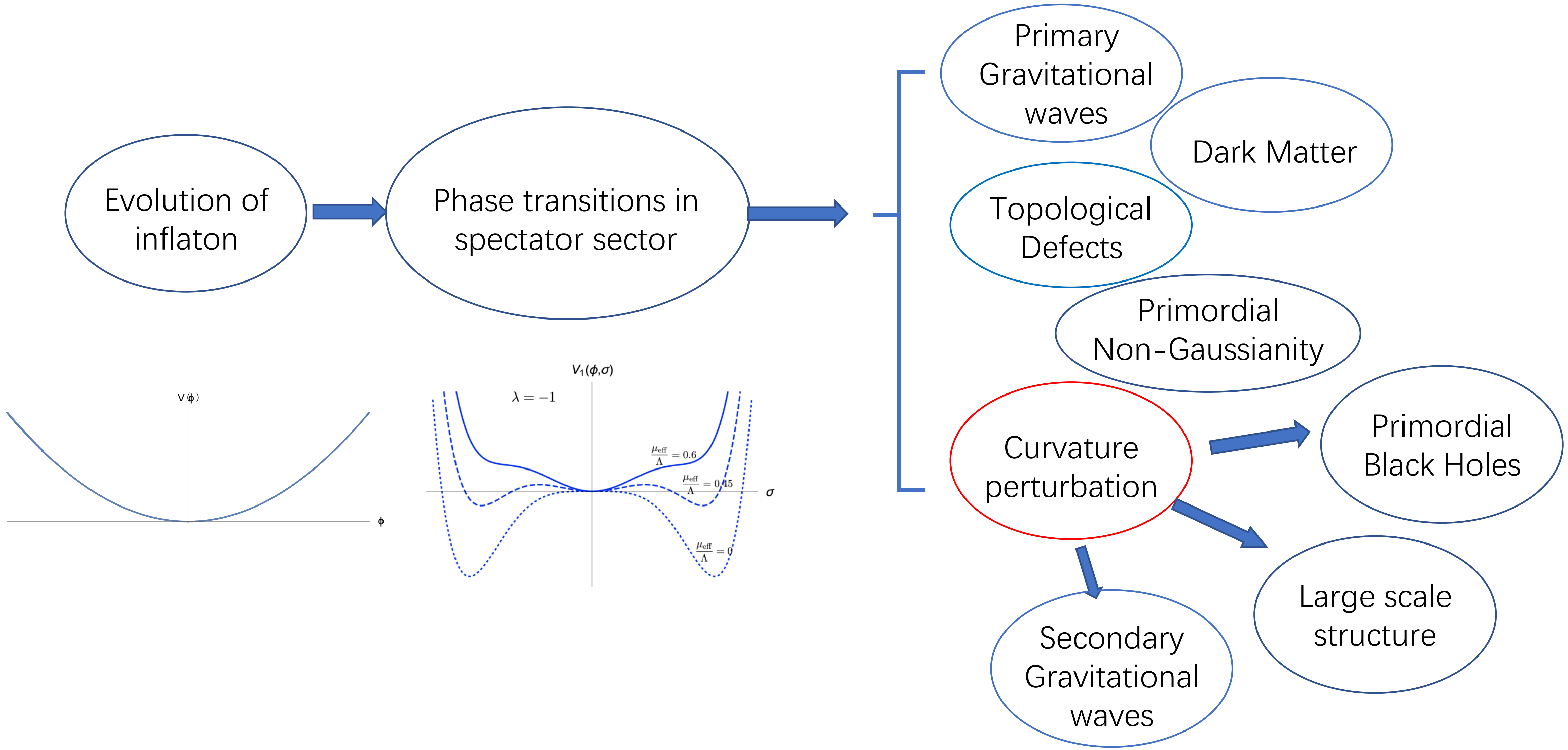


Induced scalar perturbation $\delta\phi$

- Interactions

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\phi,\sigma)$$

$$V(\phi,\sigma) = V_0(\phi) + V_1(\phi,\sigma) \xrightarrow{\phi = \phi_0 + \delta\phi} \frac{\partial V_1}{\partial\phi_0}\delta\phi$$

Source term for $\delta\phi$

$$\delta\tilde{\phi}''_{\mathbf{q}} - \frac{2}{\tau}\delta\tilde{\phi}'_{\mathbf{q}} + \left(q^2 + \frac{1}{H^2\tau^2}\frac{\partial^2 V_0}{\partial\phi_0^2}\right)\delta\tilde{\phi}_{\mathbf{q}} = \mathcal{S}_{\mathbf{q}}\ ,$$

$$\mathcal{S}_{\mathbf{q}} = -\frac{1}{H^2\tau^2}\left[\frac{\partial V_1}{\partial\phi}\right]_{\mathbf{q}} \underbrace{- \left\{\frac{2\Phi_{\mathbf{q}}}{H^2\tau^2}\left(\frac{\partial V_0}{\partial\phi_0} + \left[\frac{\partial V_1}{\partial\phi}\right]_0\right) + \frac{\dot{\phi}_0}{H\tau}\left(3\Psi'_{\mathbf{q}} + \Phi'_{\mathbf{q}}\right)\right\}}_{\text{Pure gravitational, subdominant}}$$

Induced curvature perturbation ζ

- We solve the following equations of motion numerically with a $1000 \times 1000 \times 1000$ lattice

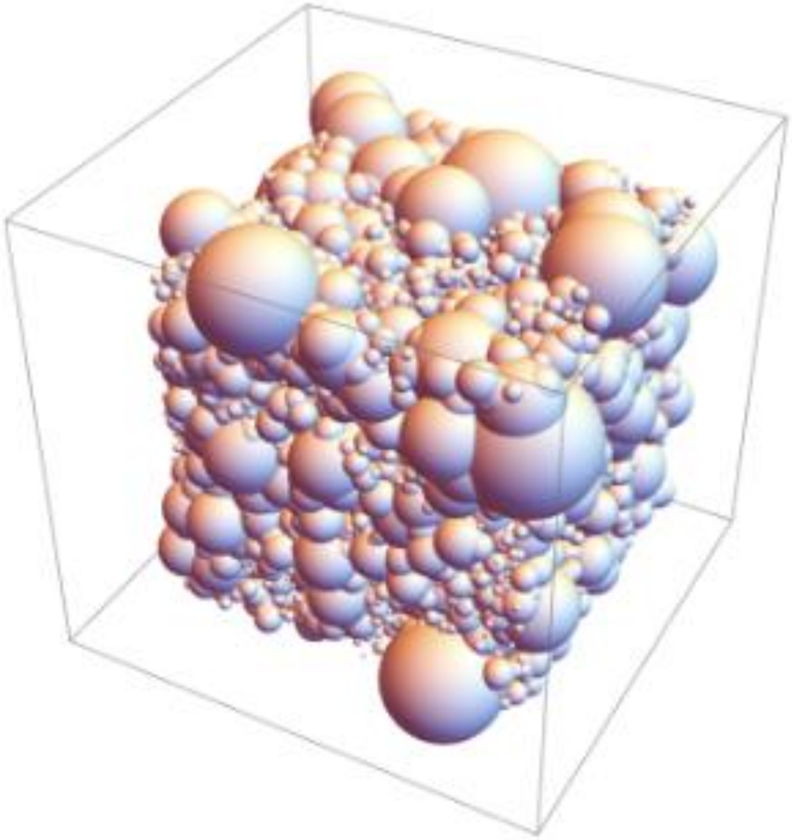
$$\delta\tilde{\phi}''_{\mathbf{q}} - \frac{2}{\tau}\delta\tilde{\phi}'_{\mathbf{q}} + \left(q^2 + \frac{1}{H^2\tau^2}\frac{\partial^2 V_0}{\partial\phi_0^2}\right)\delta\tilde{\phi}_{\mathbf{q}} = \mathcal{S}_{\mathbf{q}} \ ,$$

$$\tilde{\Psi}'_{\mathbf{q}} - \frac{\tilde{\Phi}_{\mathbf{q}}}{\tau} = -4\pi G_N \left(\frac{\dot{\phi}_0\delta\tilde{\phi}_{\mathbf{q}}}{H_{\inf}\tau} + \left[\frac{\partial_i}{\partial^2}(\sigma'\partial_i\sigma)\right]_{\mathbf{q}}\right)$$

$$\tilde{\pi}^S_{\mathbf{q}} = -\frac{3}{2}H_{\inf}^2\tau^2 q_i q_j q^{-4}\left[(\partial_i\sigma\partial_j\sigma)^{\mathrm{TL}}\right]_{\mathbf{q}}$$

- Conserved quantity after the phase transition

$$\zeta_{\mathbf{q}} = -\tilde{\Psi}_{\mathbf{q}} - \frac{H_{\inf}\delta\tilde{\phi}_{\mathbf{q}}}{\dot{\phi}_0}$$



Power spectrum of ζ

- After the collision of the bubbles, σ field oscillates and keeps producing ζ.
- The production of ζ lasts about H^{-1}, longer than β^{-1}.

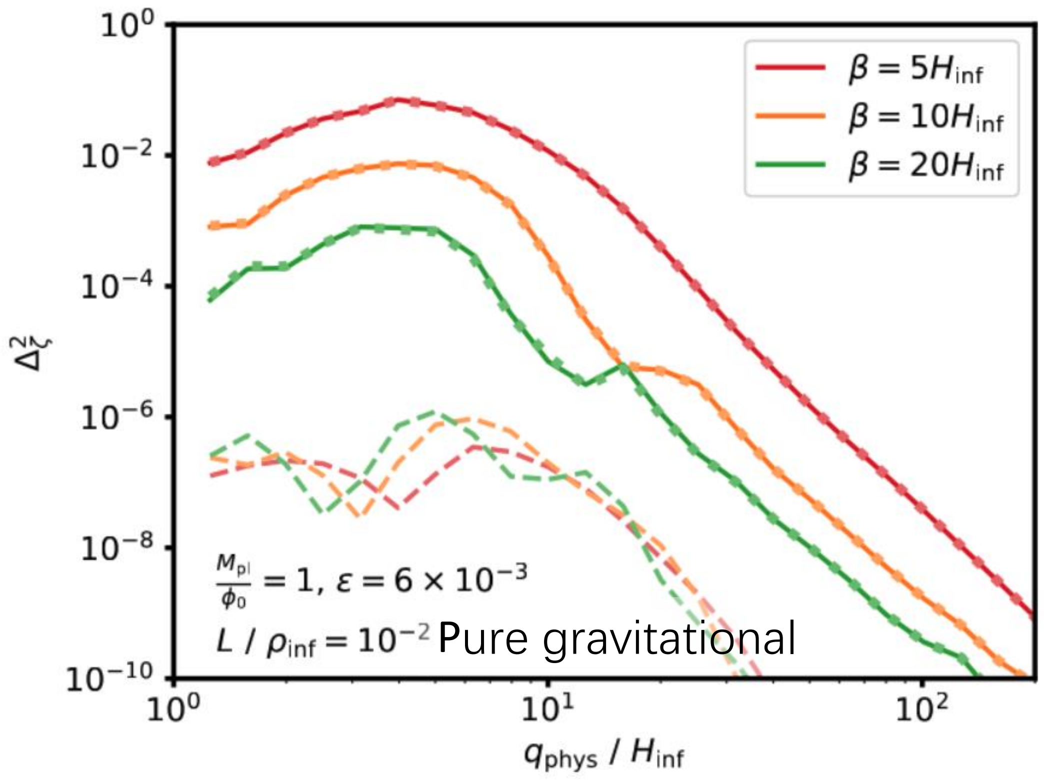
$$\zeta_{\mathbf{q}} \approx \frac{H_{\rm inf}}{\dot{\phi}_0 q^2} \int \frac{d\tau'}{\tau'} \left(\cos q\tau' - \frac{\sin q\tau'}{q\tau'}\right) \frac{c_m \phi_0 [\sigma^2(\tau')]_{\mathbf{q}}}{H_{\rm inf}^2 \tau'^2}$$

$$\Delta^{2\,({\rm emp})}_{\zeta}(q) = A_{\rm ref} \mathcal{F}\left(\frac{q_{\rm phys}}{H_{\rm inf}}\right)$$

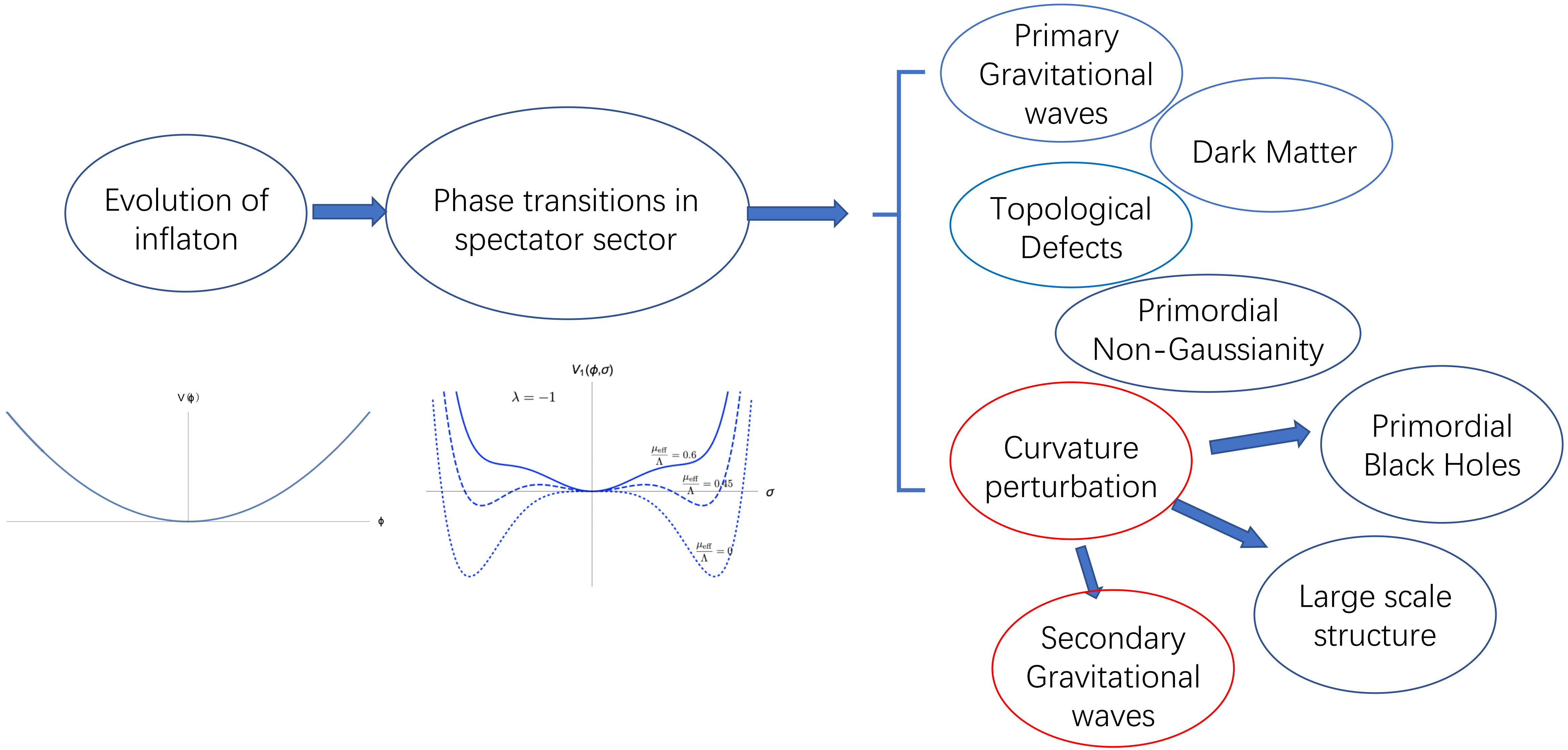
$$\mathcal{F}(x) = \frac{x^3}{1 + (\alpha_1 x)^4 + (\alpha_2 x)^9}$$

$$A_{\rm ref} = \frac{\mathcal{A}}{\epsilon} \left(\frac{M_{\rm pl}}{\phi_0}\right)^2 \left(\frac{H_{\rm inf}}{\beta}\right)^3 \left(\frac{L}{\rho_{\rm inf}}\right)^2$$

$\mathcal{A} \approx 24$ $\quad$ $L \equiv \Delta\rho$ $\quad$ $\alpha_1 \approx 0.31, \alpha_2 \approx 0.17$



Induced phase transition in spectator sector



Secondary GWs

- After inflation $\zeta \to \Phi, \Psi$
- Expand the Einstein equation to second order:

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\hat{\mathcal{T}}_{ij}{}^{lm}\mathcal{S}_{lm}\ ,$$

$$\begin{aligned}\mathcal{S}_{ij} \equiv\ & 2\Phi\partial^i\partial_j\Phi - 2\Psi\partial^i\partial_j\Phi + 4\Psi\partial^i\partial_j\Psi + \partial^i\Phi\partial_j\Phi - \partial^i\Phi\partial_j\Psi - \partial^i\Psi\partial_j\Phi + 3\partial^i\Psi\partial_j\Psi \\ & - \frac{4}{3(1+w)\mathcal{H}^2}\partial_i\left(\Psi' + \mathcal{H}\Phi\right)\partial_j\left(\Psi' + \mathcal{H}\Phi\right) - \frac{2c_s^2}{3w\mathcal{H}^2}\left[3\mathcal{H}(\mathcal{H}\Phi - \Psi') + \nabla^2\Psi\right]\partial_i\partial_j(\Phi - \Psi)\,.\end{aligned}$$

Scalar induced GWs

Matarrese, Mollerach, and Bruni, astro-hp/9707278
Mollerach, Harari, and Matarrese, astro-hp/0310711
Ananda, Clarkson, and Wands, gr-qc/0612013
Baumann, Steinhardt, Takahashi, Ichiki, hep-th/0703290
…

Secondary GWs

$$\Omega_{\rm GW}^{(2)}(f) = \Omega_R A_{\rm ref}^2 \mathcal{F}_2\left(\frac{q_{\rm phys}}{H_{\rm inf}}\right)$$

$$f = \frac{q}{2\pi a_0} = f_{\rm ref} \times \frac{q_{\rm phys}}{H_{\rm inf}}$$

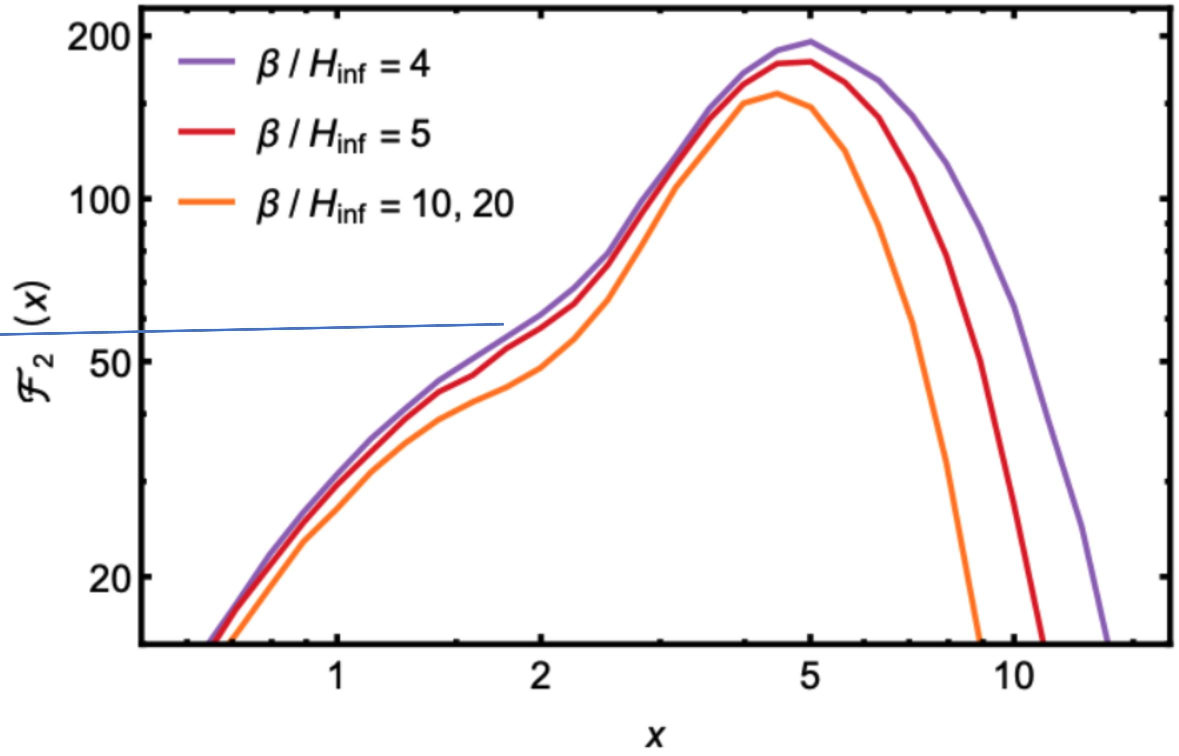
$$f_{\rm ref} = 10^{-9}\ {\rm Hz} \times e^{40-N_e}\left(\frac{H_{\rm inf}}{10^{14}\ {\rm GeV}}\right)^{1/2}$$

$$\mathcal{F}_2^{\rm IR}(x) \approx x^3\left(\frac{6}{5}\log^2 x + \frac{16}{25}\log x + \frac{28}{125}\right)$$

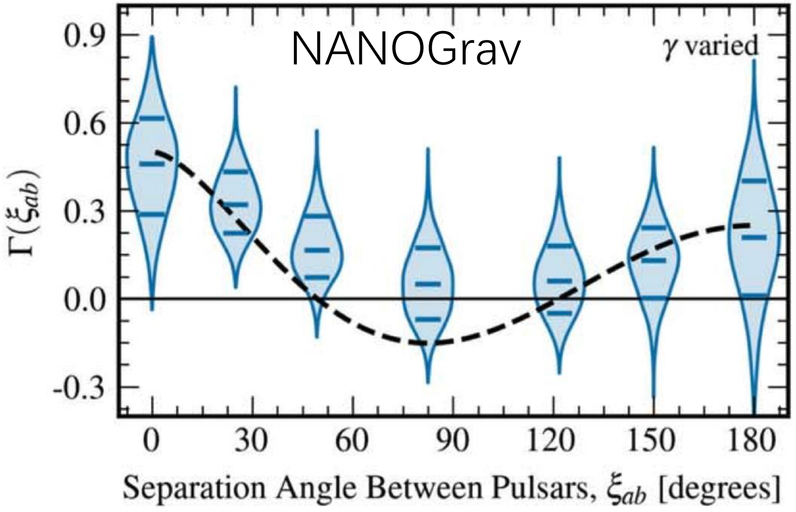
$\mathcal{F}_2$ collects information of the transfer functions.

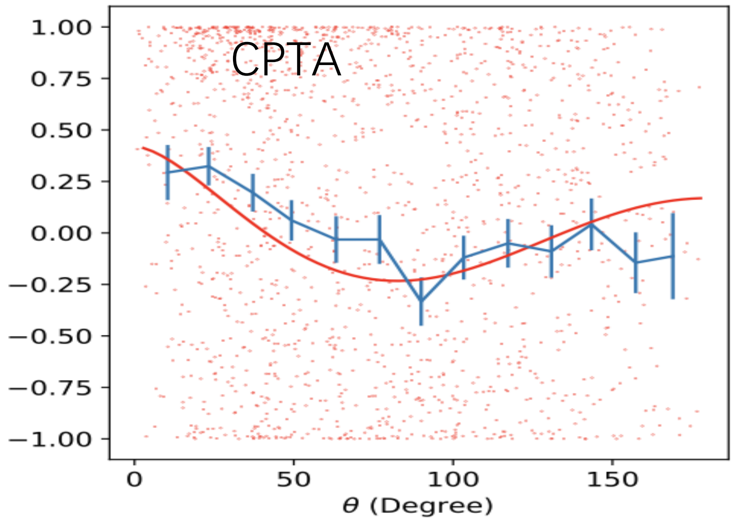
$$A_{\rm ref} = \frac{\mathcal{A}}{\epsilon}\left(\frac{M_{\rm pl}}{\phi_0}\right)^2\left(\frac{H_{\rm inf}}{\beta}\right)^3\left(\frac{\Delta\rho}{\rho_{\rm inf}}\right)^2$$

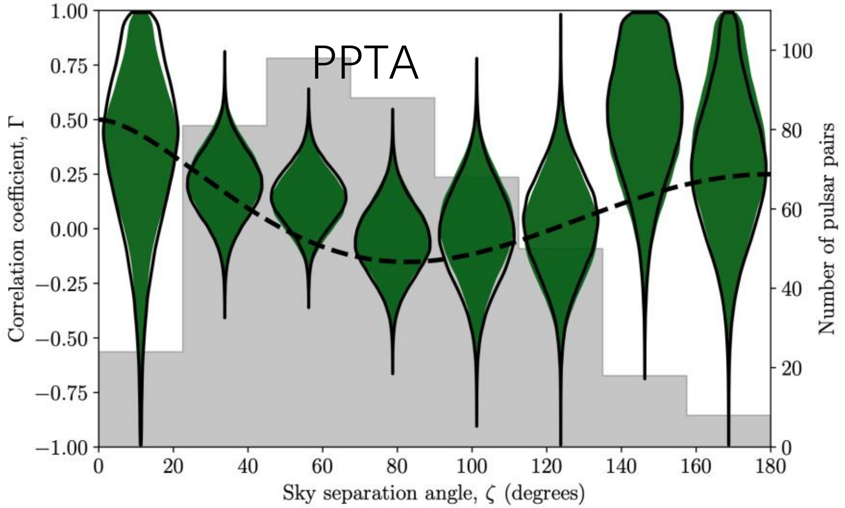
$$\mathcal{F}_2^{\rm max} \approx 200$$

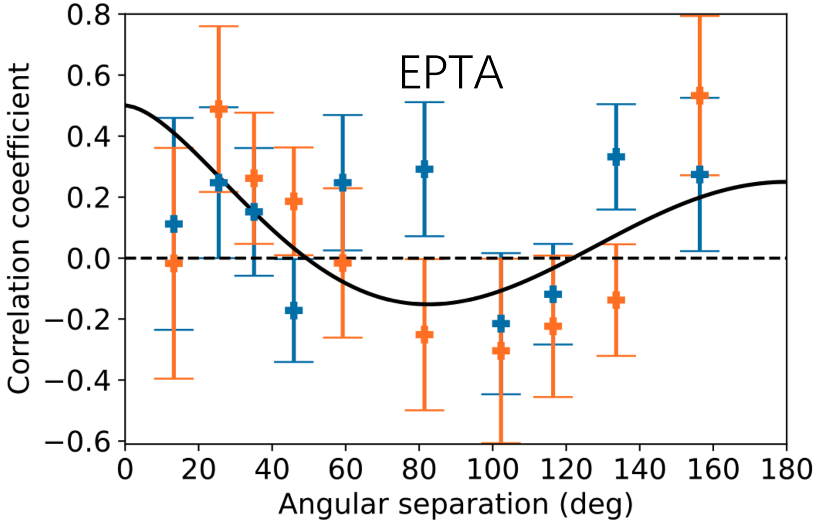


Observation from PTAs





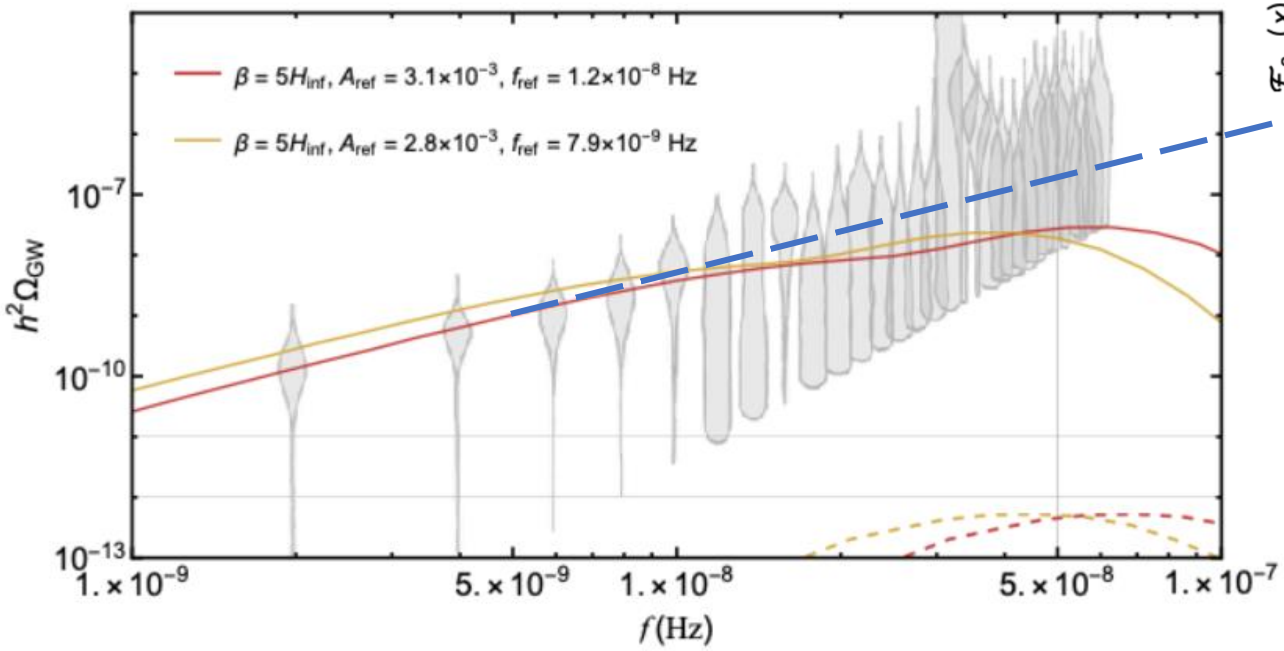


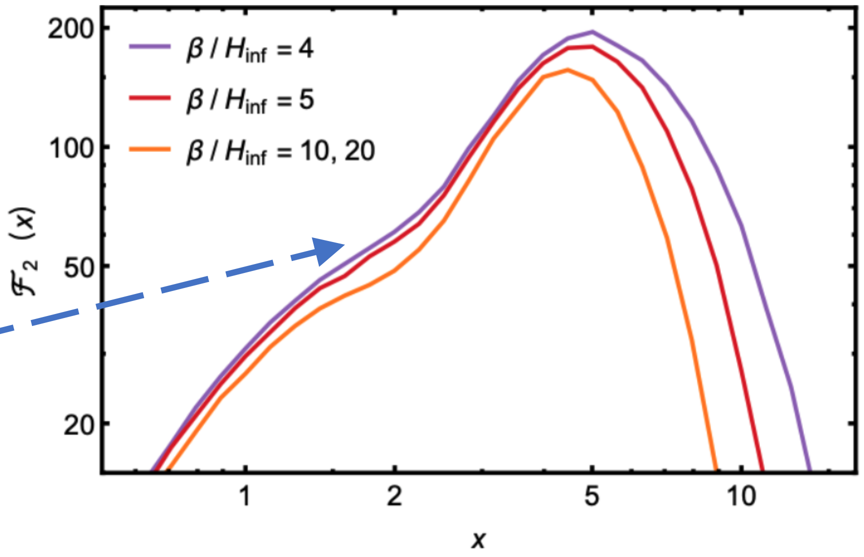


Observation from PTAs

$$\mathcal{F}_2^{\mathrm{IR}}(x) \approx x^3 \left(\frac{6}{5} \log^2 x + \frac{16}{25} \log x + \frac{28}{125} \right)$$

- The slope is around 2 in the IR region





$$\Omega_{\rm GW}^{(2)}(f) = \Omega_R A_{\rm ref}^2 \mathcal{F}_2 \left(\frac{q_{\rm phys}}{H_{\rm inf}} \right)$$

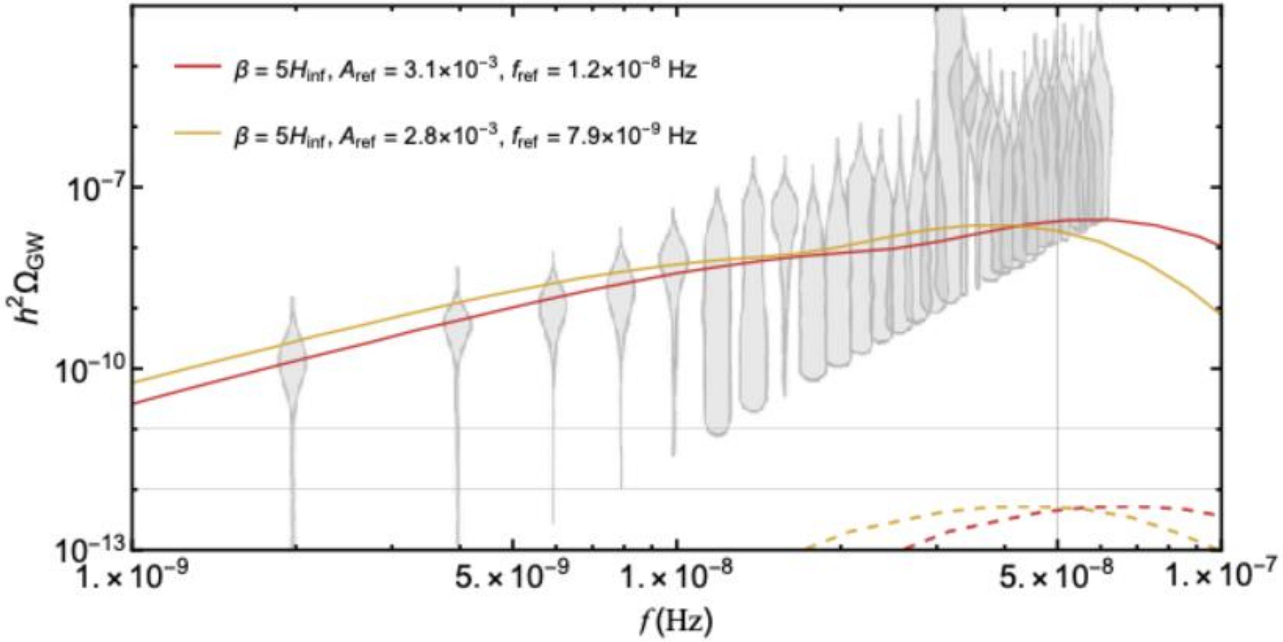
$$f = \frac{q}{2\pi a_0} = f_{\rm ref} \times \frac{q_{\rm phys}}{H_{\rm inf}}$$

$$f_{\rm ref} = 10^{-9}\ \mathrm{Hz} \times e^{40-N_e} \left(\frac{H_{\rm inf}}{10^{14}\ \mathrm{GeV}} \right)^{1/2}$$

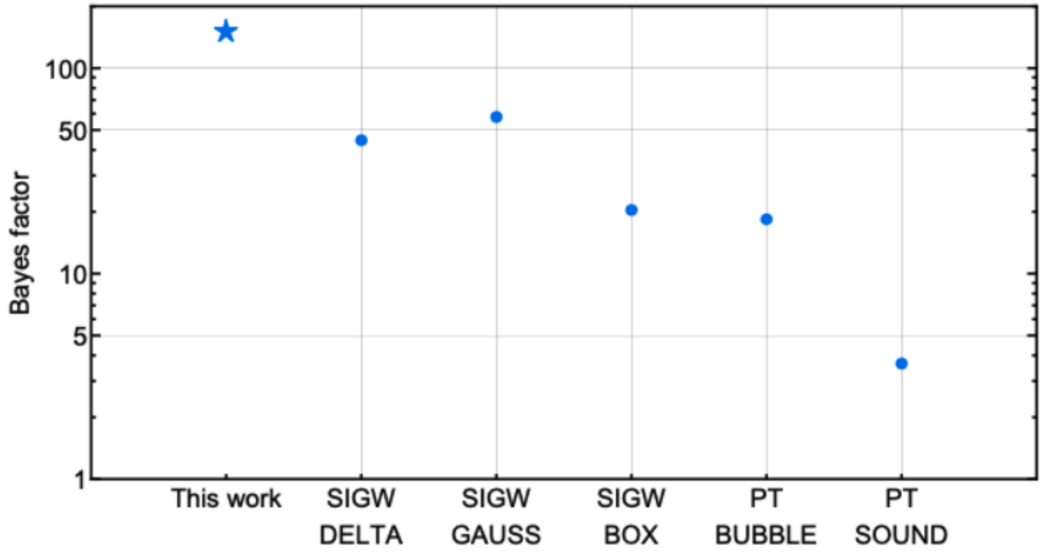
Observation from PTAs

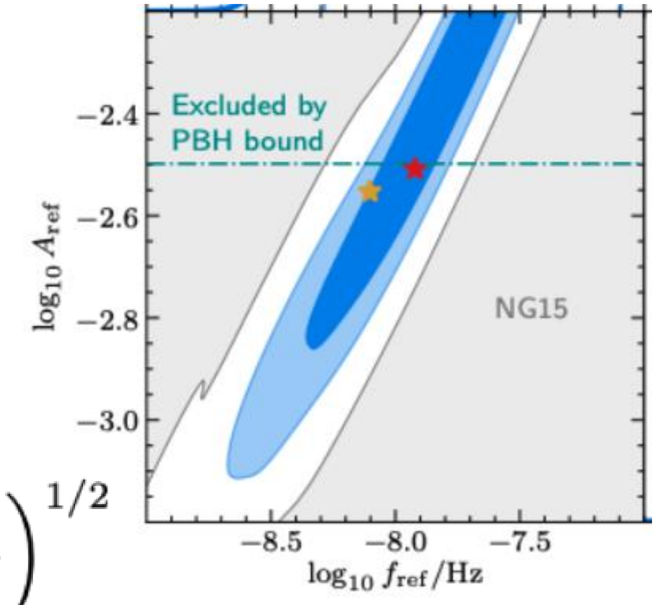
HA, Boye Su, Hanwen Tai, Lian-Tao Wang, Chen Yang, 2308.00070

- Bayes factor against SMBHB



Our model





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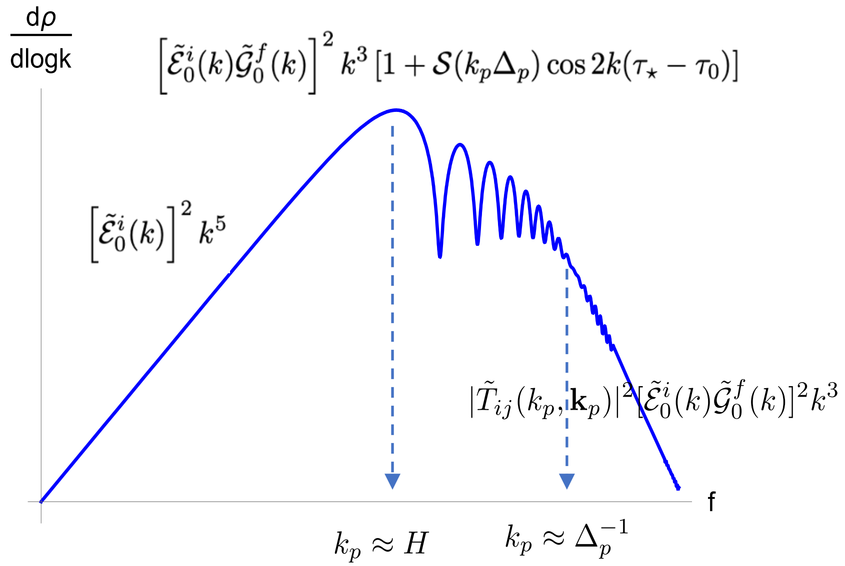
Comparison between primary GW and secondary GW

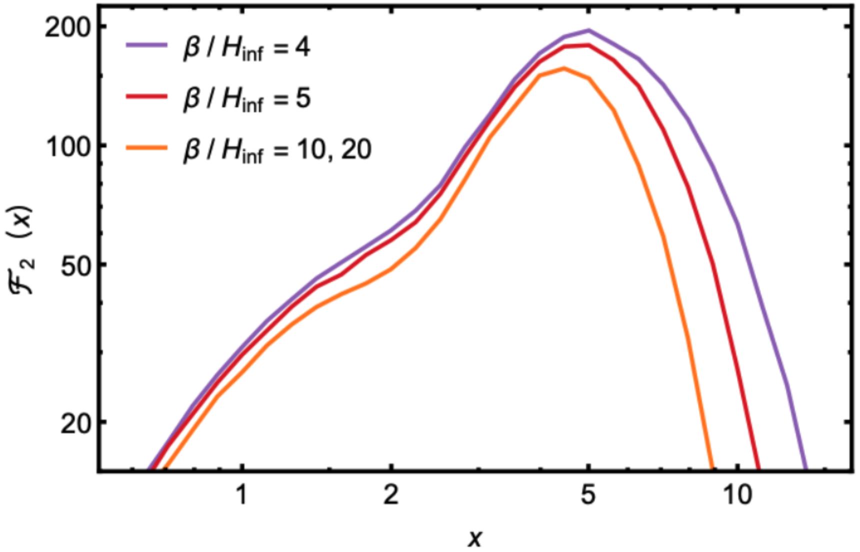
- Primary

$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2$$

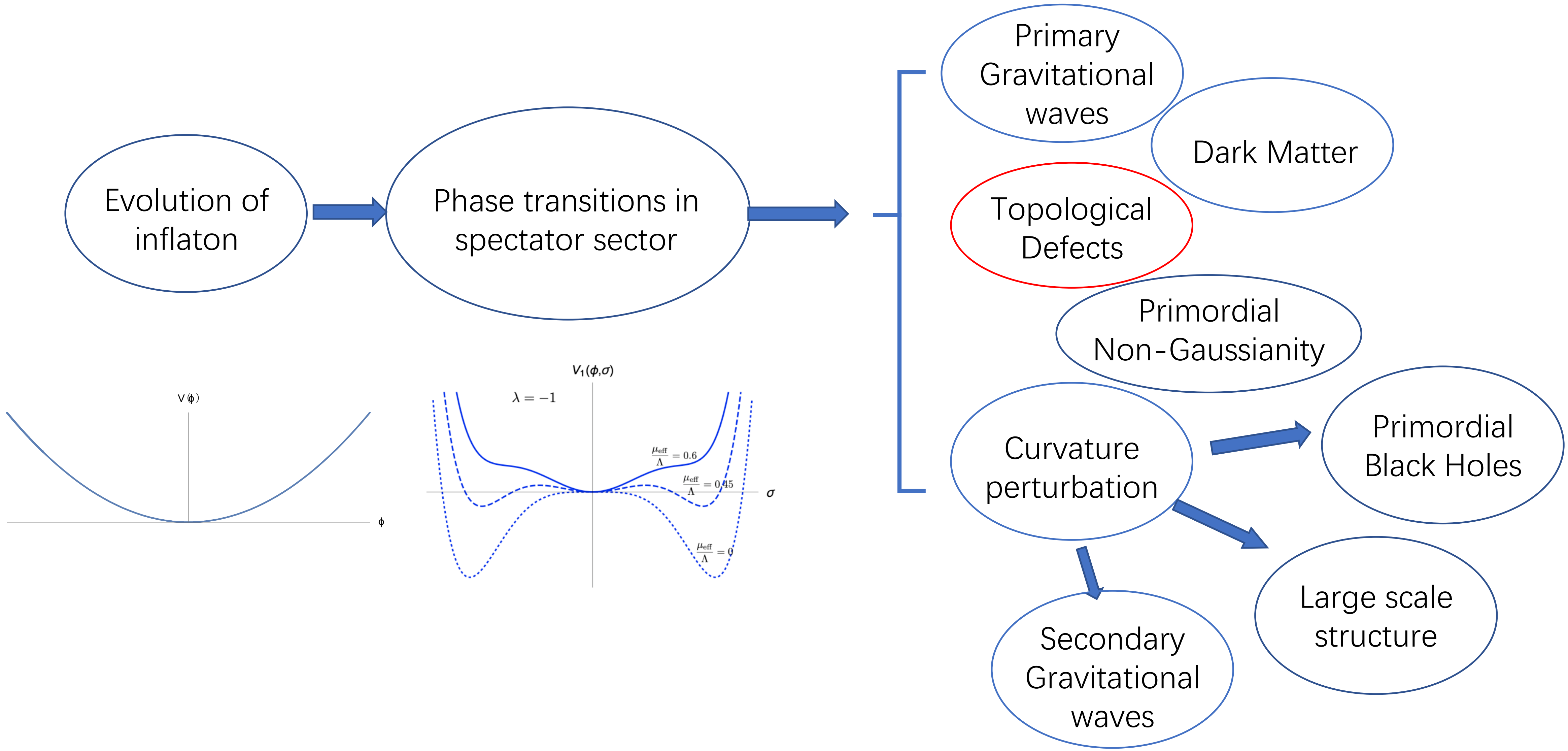


- Secondary

$$\Omega_{\rm GW} \sim \Omega_R \left(\frac{\mathcal{A}}{\epsilon}\right)^2 \left(\frac{M_{\rm pl}}{\phi_0}\right)^4 \left(\frac{H_{\rm inf}}{\beta}\right)^6 \left(\frac{\Delta\rho}{\rho_{\rm inf}}\right)^4$$

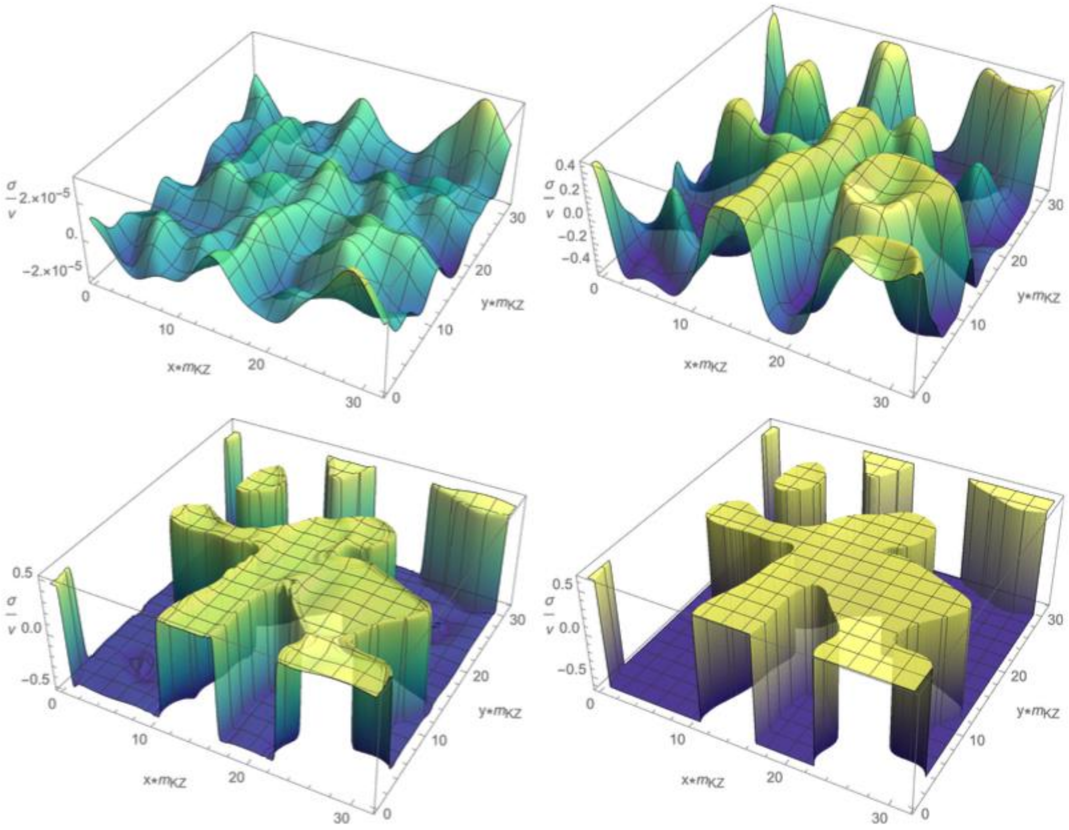


Induced phase transition in spectator sector



Second-order phase transition during inflation

- GWs directly from second-order phase transitions are small, usually cannot be detected.
- Phase transitions can produce topological defects:
  - Domain walls
  - Cosmic strings
  - Monopoles
- Domain walls soon become comovingly static after production.



Formation of domain walls

- Landau-Ginzburg type

$$V = -\frac{1}{2}m_{\text{eff}}^2\sigma^2 + \frac{\lambda}{4}\sigma^4$$

$$m_{\text{eff}}^2 = y\phi^2 - m^2$$

Inflaton field

- Kibble-Zurek mechanism

c for critial

$$V_{\text{KZ}} = -\frac{1}{2}m_{\text{KZ}}^3 a_c^{-1}(\tau - \tau_c)\sigma^2 + \frac{\lambda}{4}\sigma^4$$

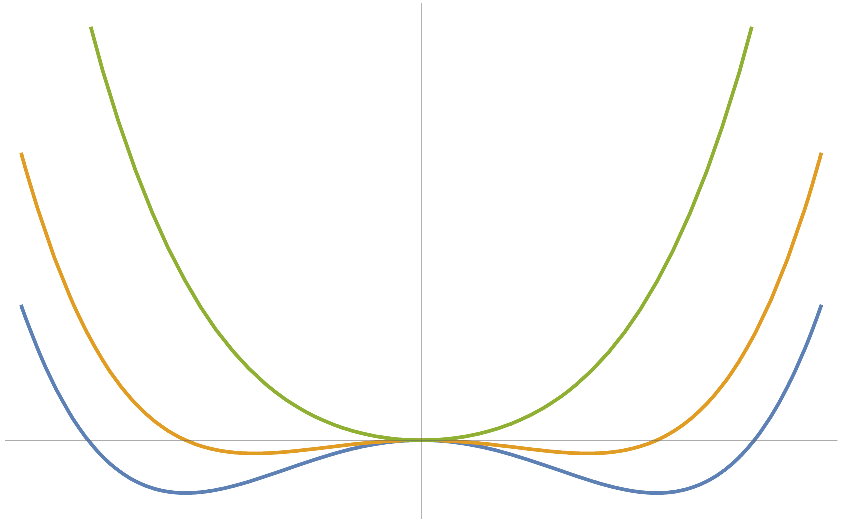
- m_{KZ} determines the average distances between the domain walls.

Kibble 1976, Zurek 1985

$$m_{\text{KZ}(B)}^3 = -ya_c\frac{d\phi_0^2}{d\tau} = \frac{2^{3/2}\varepsilon^{1/2}m^2 H M_{\text{pl}}}{\phi_0(\tau_c)}$$

Murayama & Shu, 0905.1720

$$H^2 \ll m_{\text{KZ}}^2 \ll m^2$$



Formation of domain walls

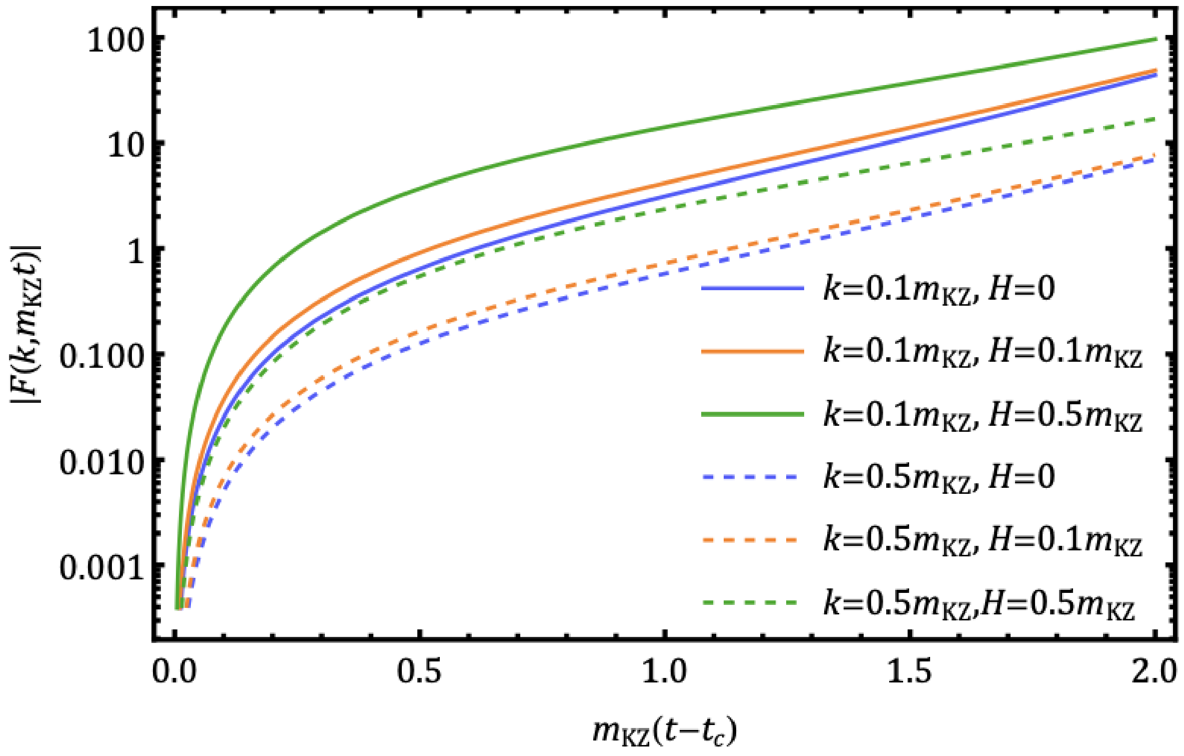
- Tachyonic growth

$$V_{\mathrm{KZ}} = -\frac{1}{2} m_{\mathrm{KZ}}^3 a_c^{-1} (\tau - \tau_c) \sigma^2 + \frac{\lambda}{4} \sigma^4$$

$$\sigma''_{\mathbf{k}} + \frac{2a'}{a} \sigma'_{\mathbf{k}} + \omega_{\mathbf{k}}^2(\tau) \sigma_{\mathbf{k}} = 0$$

$$k^2 - a_c^2 m_{\mathrm{KZ}}^3 (\tau - \tau_c) + \frac{\lambda}{2} \langle \sigma^2(\tau, \mathbf{x}) \rangle$$

$\omega_k^2 < 0$ for small k around τ_c.



$F(k, m_{KZ} t)$ can be seen as the occupation number in the k mode.

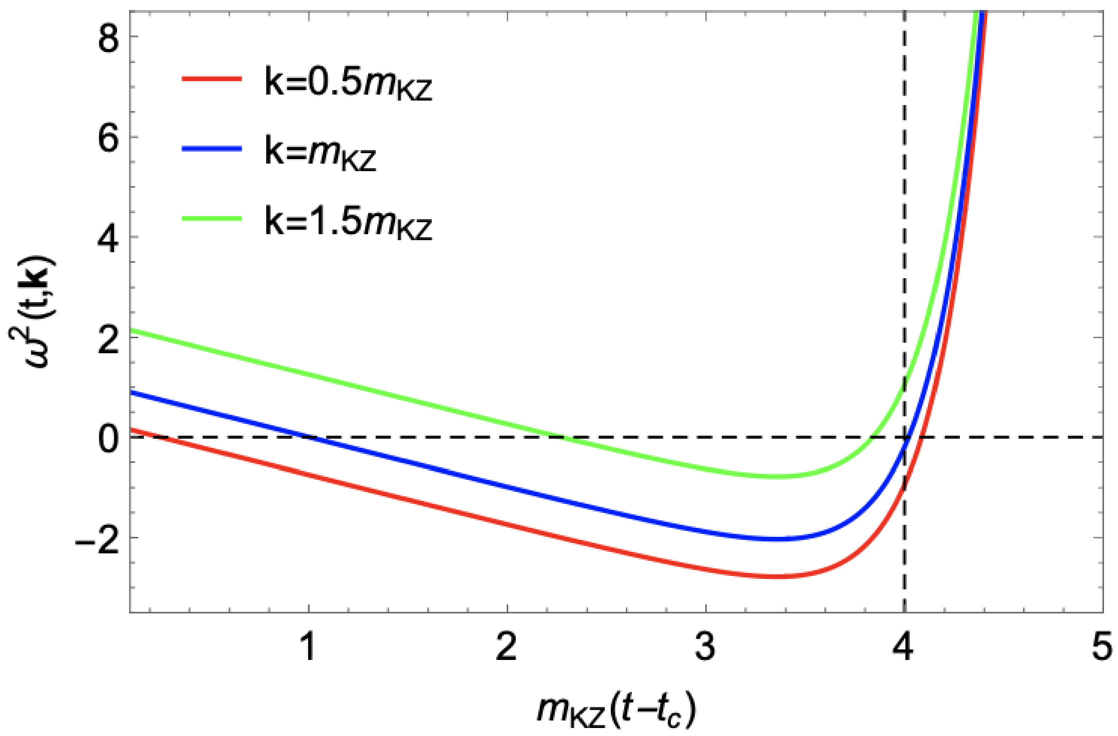
Formation of domain walls

- Stop of the tachyonic growth

$$k^2 - a_c^2 m_{\mathrm{KZ}}^3(\tau - \tau_c) + \frac{\lambda}{2}\langle \sigma^2(\tau, \mathbf{x}) \rangle$$

Growth exponentially

- Only modes with k smaller than about m_{KZ} can have a chance to grow exponentially.



Formation of domain walls

$$\tilde{\pi}(\mathbf{k},\tau) = a_{\mathbf{k}} a(\tau)^2 f'(k,\tau) + a^\dagger_{-\mathbf{k}} a(\tau)^2 f'^*(k,\tau),$$

$$\tilde{\sigma}(\mathbf{k},\tau) = a_{\mathbf{k}} f(k,\tau) + a^\dagger_{-\mathbf{k}} f^*(k,\tau).$$

$$F(k,\tau) = a(\tau)^2 \mathrm{Re}\left[f'(k,\tau) f^*(k,\tau)\right]$$

- Matching to classical nonlinear evolution

Quantum essemble $\Rightarrow$ Classical essemble

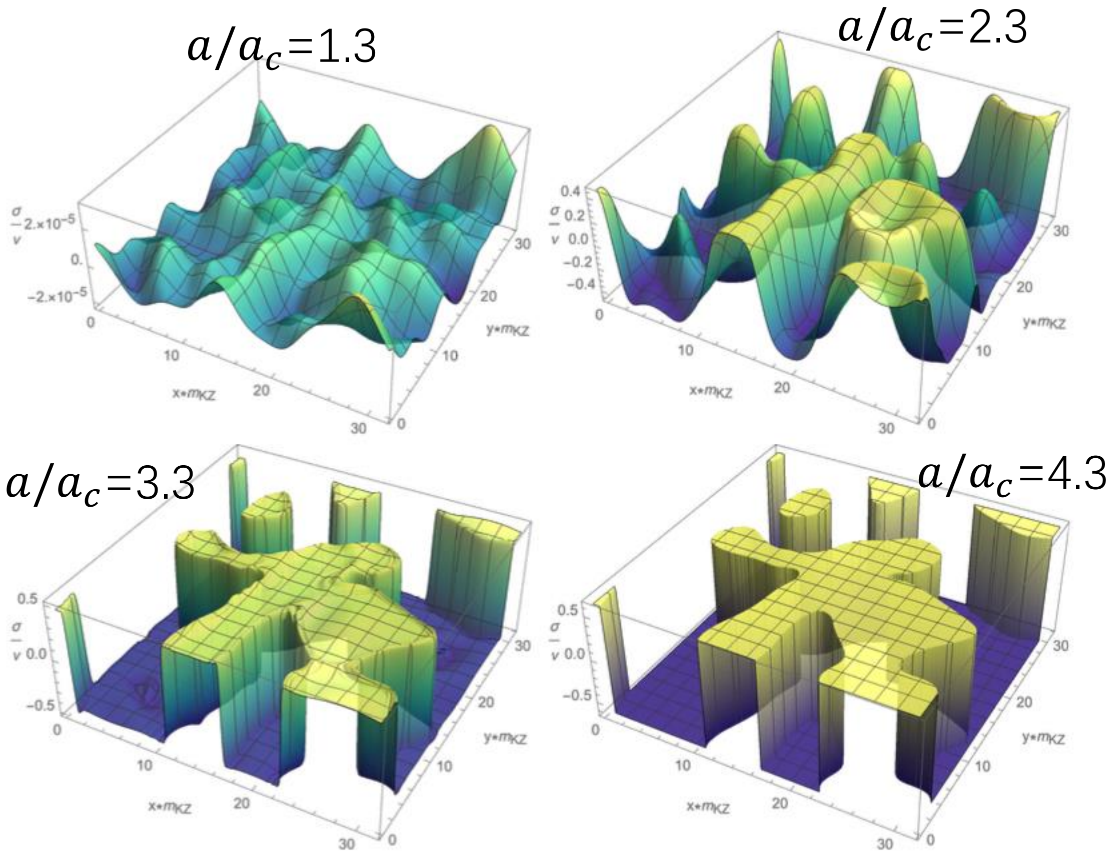
$$W(\sigma_{\mathbf{k}},\pi_{\mathbf{k}}) = \frac{1}{\pi^2}\exp\left[-\frac{|\sigma_{\mathbf{k}}|^2}{|f(\mathbf{k},\tau)|^2} - 4|f(\mathbf{k},\tau)|^2\left|\pi_{\mathbf{k}} - \frac{F(\mathbf{k},\tau)}{|f(\mathbf{k},\tau)|^2}\sigma_{\mathbf{k}}\right|^2\right]$$

We randomly generate the σ_k and π_k according to W as the initial condition for classical lattice simulation.

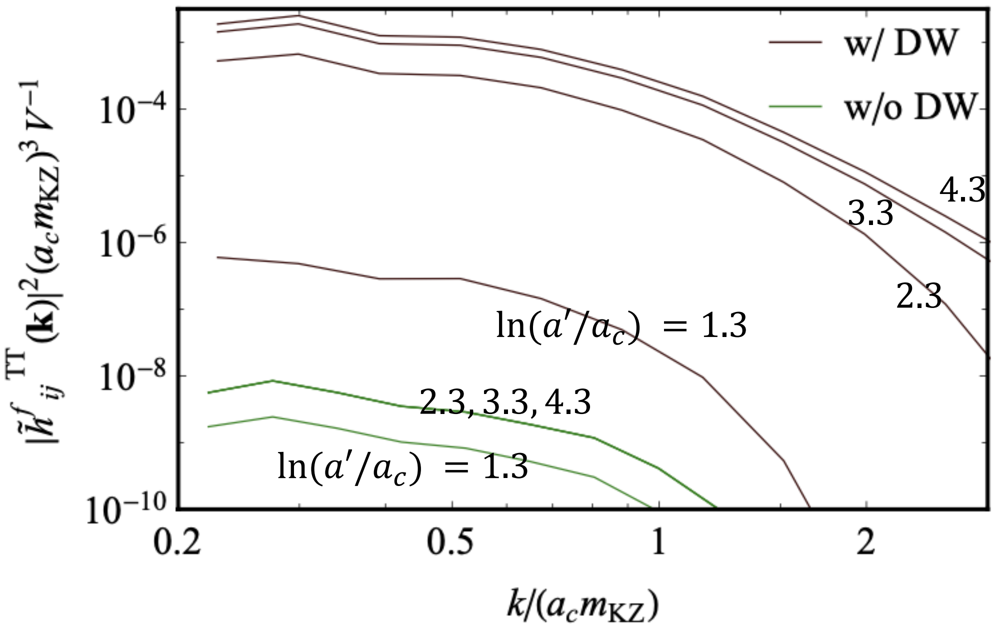
Polarski and Starobinsky 1996,
Lesgourgues, Polarski and Starobinsky, gr-qc/9611019
Kiefer, Polarski and Starobinsky, gr-qc/9802003
…

Formation of domain walls

- Symmetry breaking via a second order phase transition.
- We numerically solve the nonlinear evolution of σ field with $1000 \times 1000 \times 1000$ **lattice.**
- At the beginning there are fluctuations, dying out after a few e-folds.
- The configuration becomes comovingly static after a few e-folds.



Calculation of GWs



$$\tilde{h}^f_{ij}(\mathbf{k}) = \frac{16\pi G_N}{k} \int_{-\infty}^{0} d\tau' \mathcal{K}(k\tau') \tilde{T}^{TT}_{ij}(\tau', \mathbf{k})$$

With domains, the dominant contribution to $\tilde{h}^f$ happens around $\ln(a'/a_c) \sim 2$ to 3.

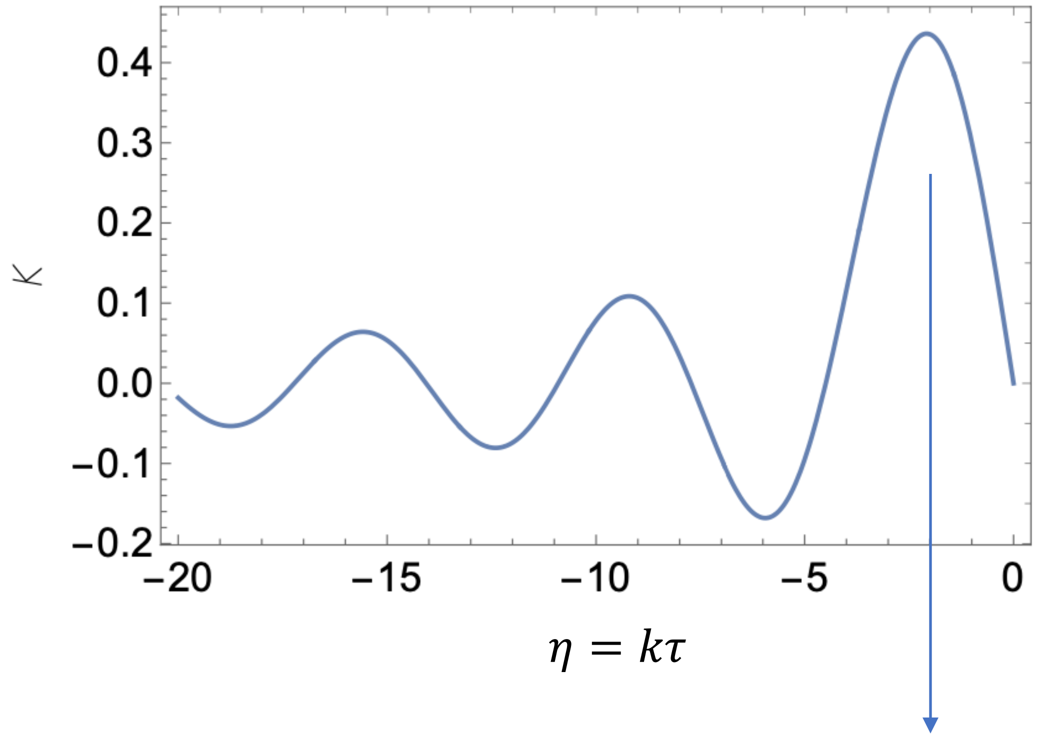
Without domains ($\delta\sigma \to |\delta\sigma|$), the dominant contribution to $\tilde{h}^f$ stops around $\ln(a'/a_c) \sim 2$, and the magnitude is much smaller.

The dominant contribution to GWs is from domain walls.

Calculation of GWs

- In Minkowski spacetime, static source cannot radiate due to energy-momentum conservation.
- During inflation, energy conservation is badly broken, so the even static source can produce GWs.

$$\tilde{h}^f_{ij}(\mathbf{k}) = \frac{16\pi G_N}{k}\int_{-\infty}^{0} d\tau' \mathcal{K}(k\tau')\tilde{T}^{TT}_{ij}(\tau', \mathbf{k})$$



The dominant contribution

Numerical results for GWs

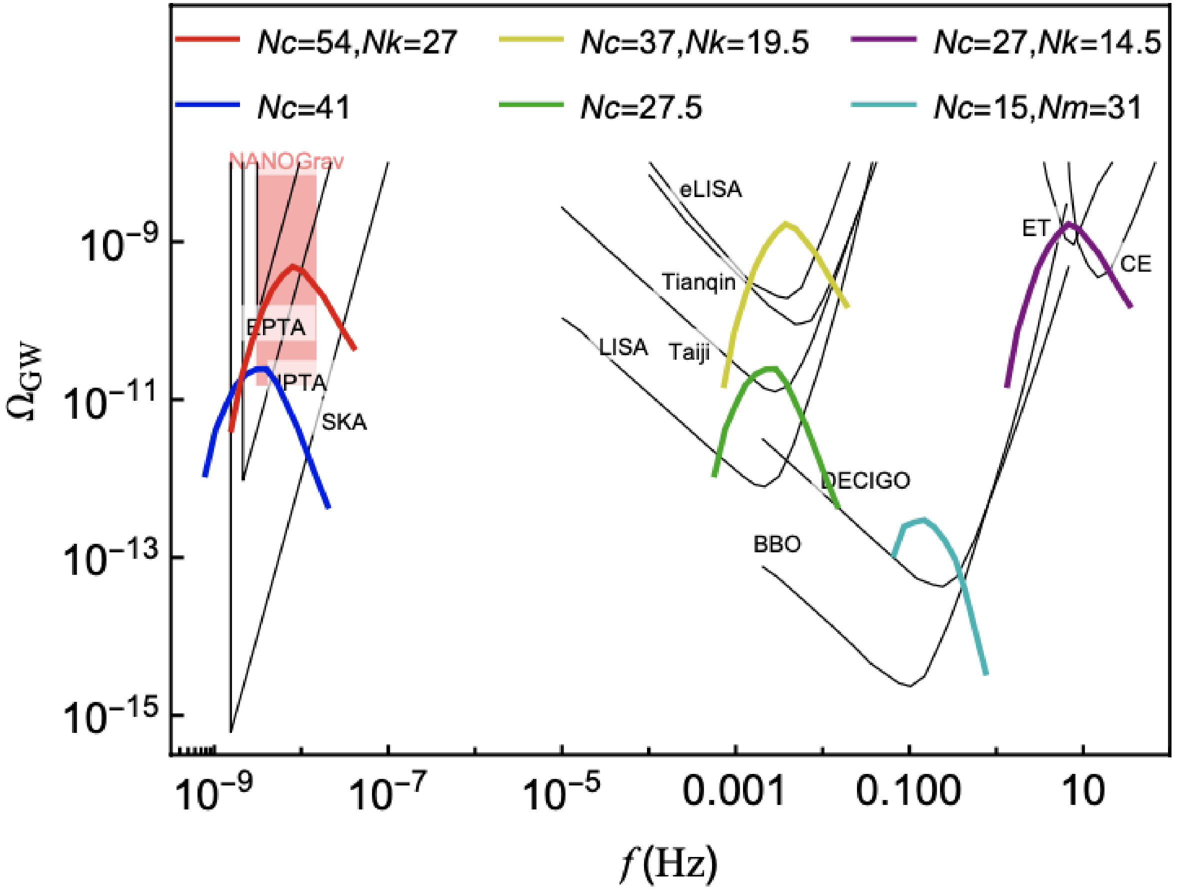
$$\Omega_{\rm GW}(f) = \Omega_R \times \rho_R^{-1} \left.\frac{d\rho_{\rm GW}}{d\ln f}\right|_{\rm today}$$

$$\frac{f_{\rm today}}{f_\star} = \frac{a(\tau_\star)}{a_1}\left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3} \frac{T_{\rm CMB}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_{\rm inf}^2}{8\pi G_N}\right)\right]^{1/4}}$$

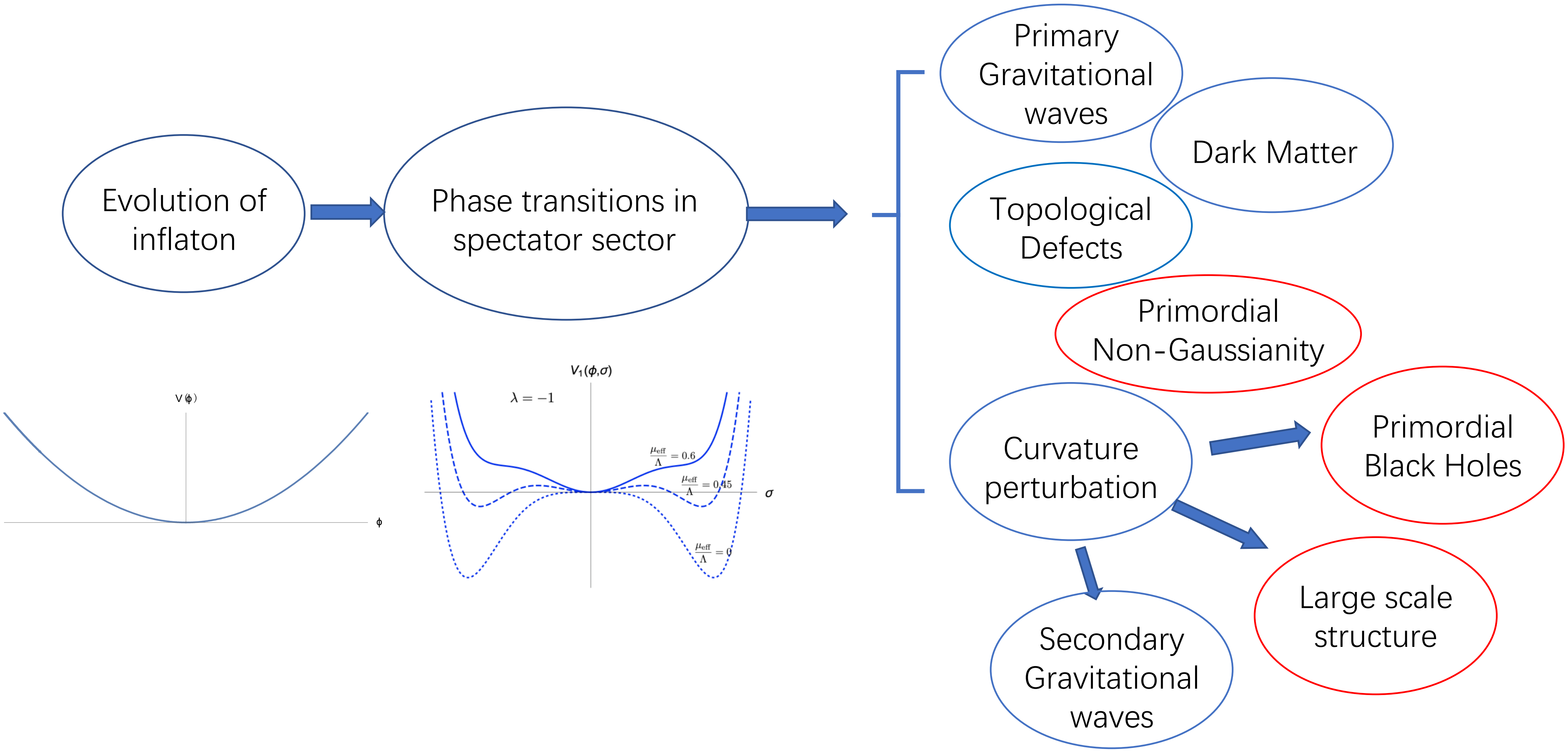
The detailed shape and strength also depends on the evolution of the universe.

- Instantaneous reheating,
- Matter dominated intermediate stage,
- Kination dominated intermeditate stage.

HA, Chen Yang, 2304.02361



Summary and outlook



Backups

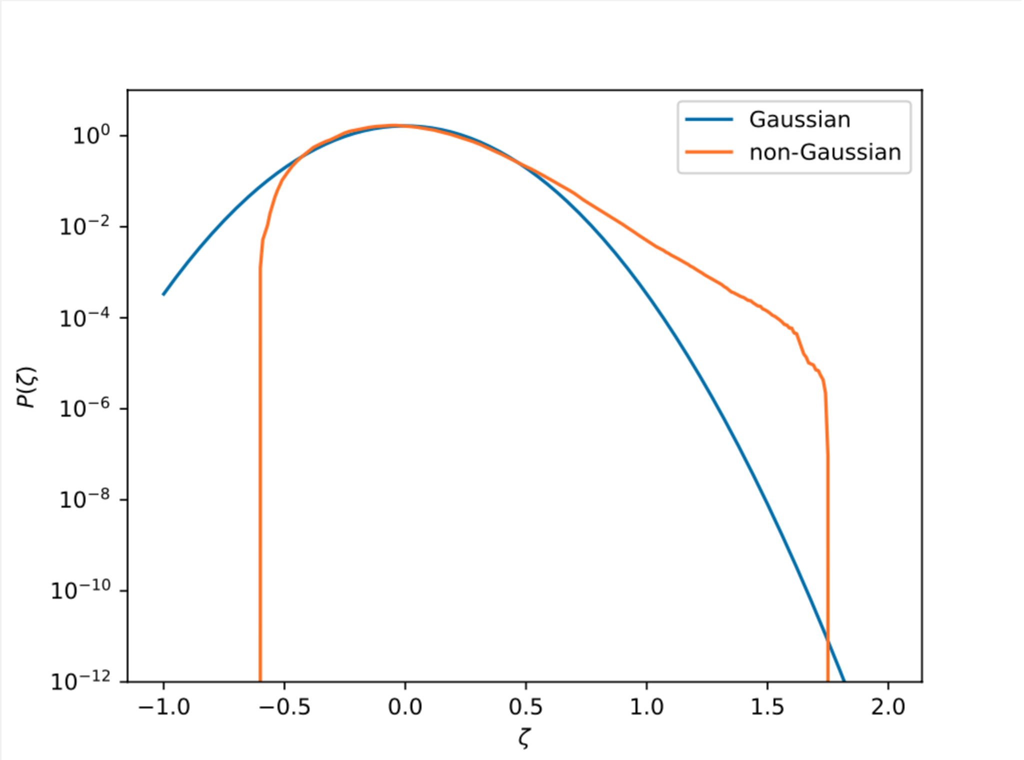
Outlook

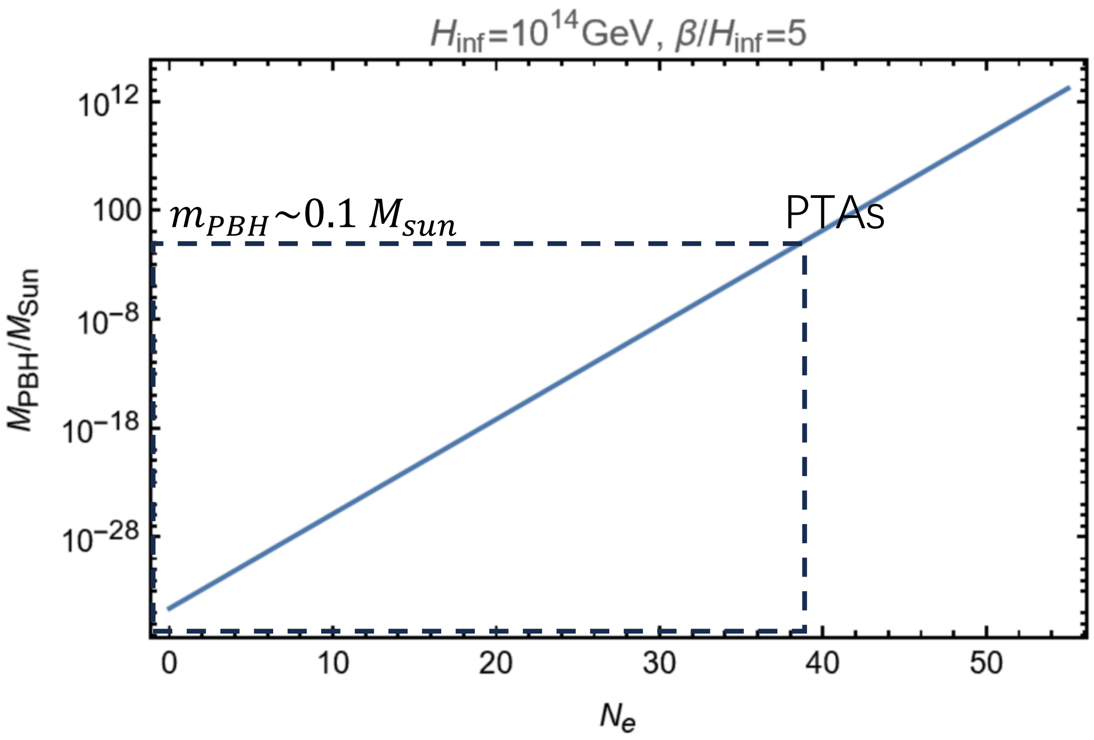
- The fate of the domain walls.
- Other topologcial defects.
- Application to high scale particle physics models.
- Baryogenesis (work in progress)

Primordial Black Holes

HA, Boye Su, Lian-Tao Wang, Chen Yang, work in progress

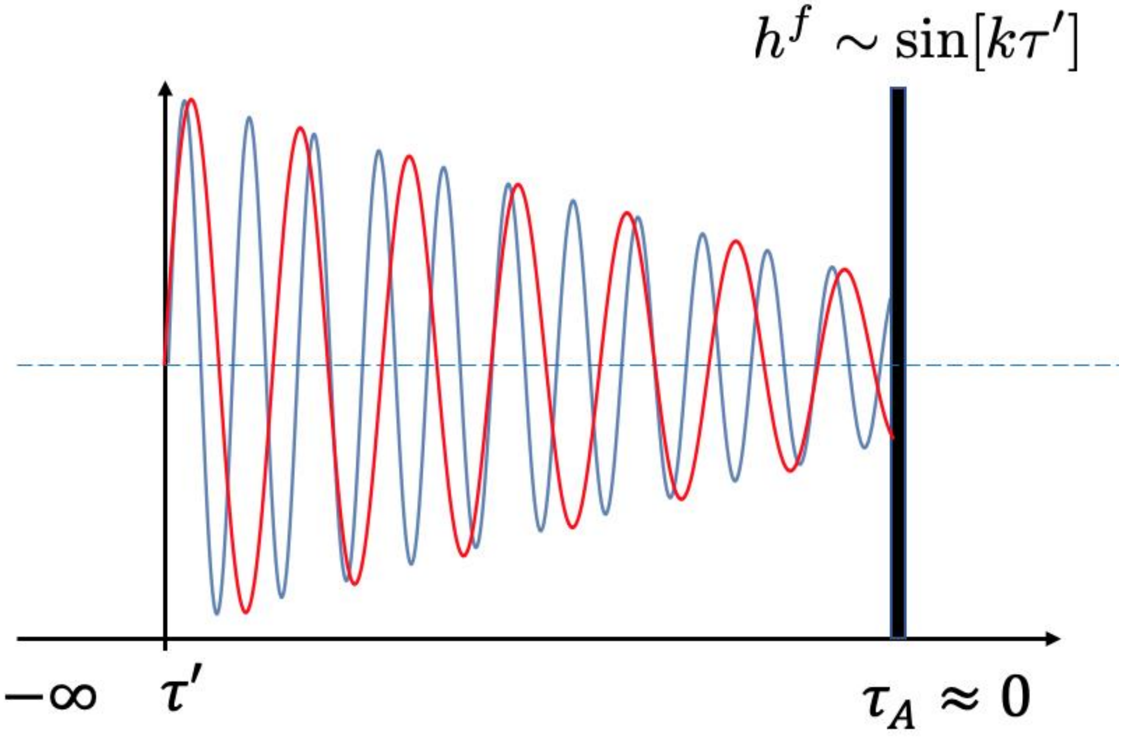
- PBHs will form if $\Delta_\zeta^2 \sim 0.01$
- The power spectrum is highly non-Gaussian





GW from instantaneous and local sources (qualitative study)

- The conformal time between the source and the horizon is fixed.
- The phase of h at the source is fixed.
- The value of h^f at the horizon oscillates with k.
- h^f is the initial condition for later evolution.



$k\tau_A \approx 0$

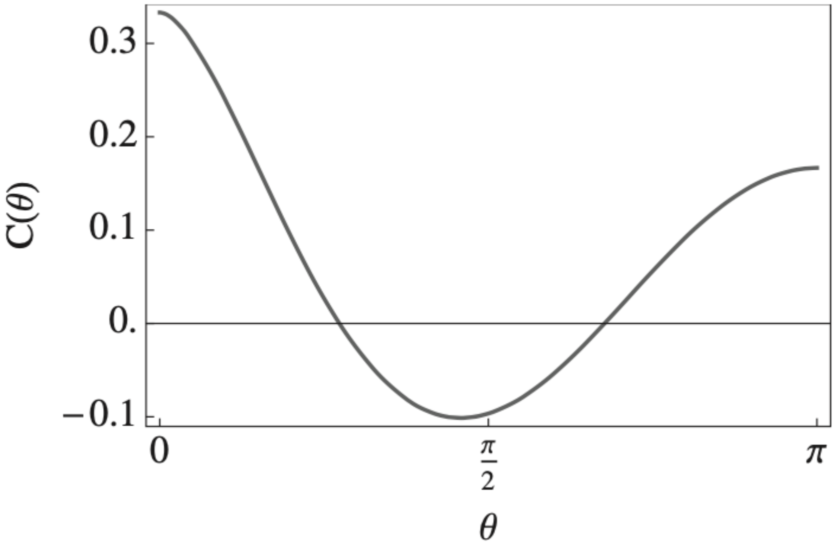
Observation from PTAs

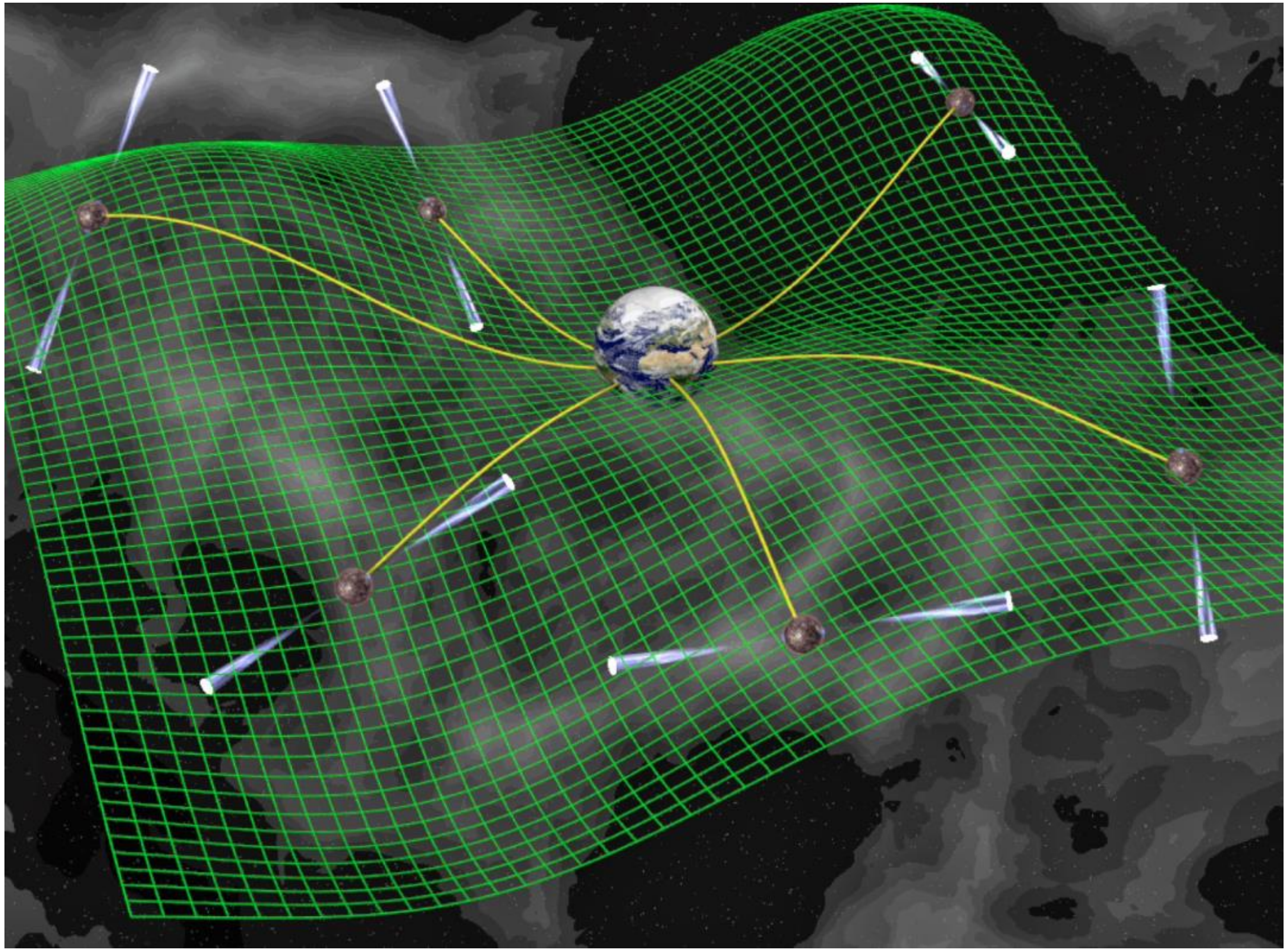
- Hellings-Downs curve

$$\langle z_a(t) z_b(t) \rangle = C(\theta_{ab}) \int_0^\infty df\, S_h(f)$$

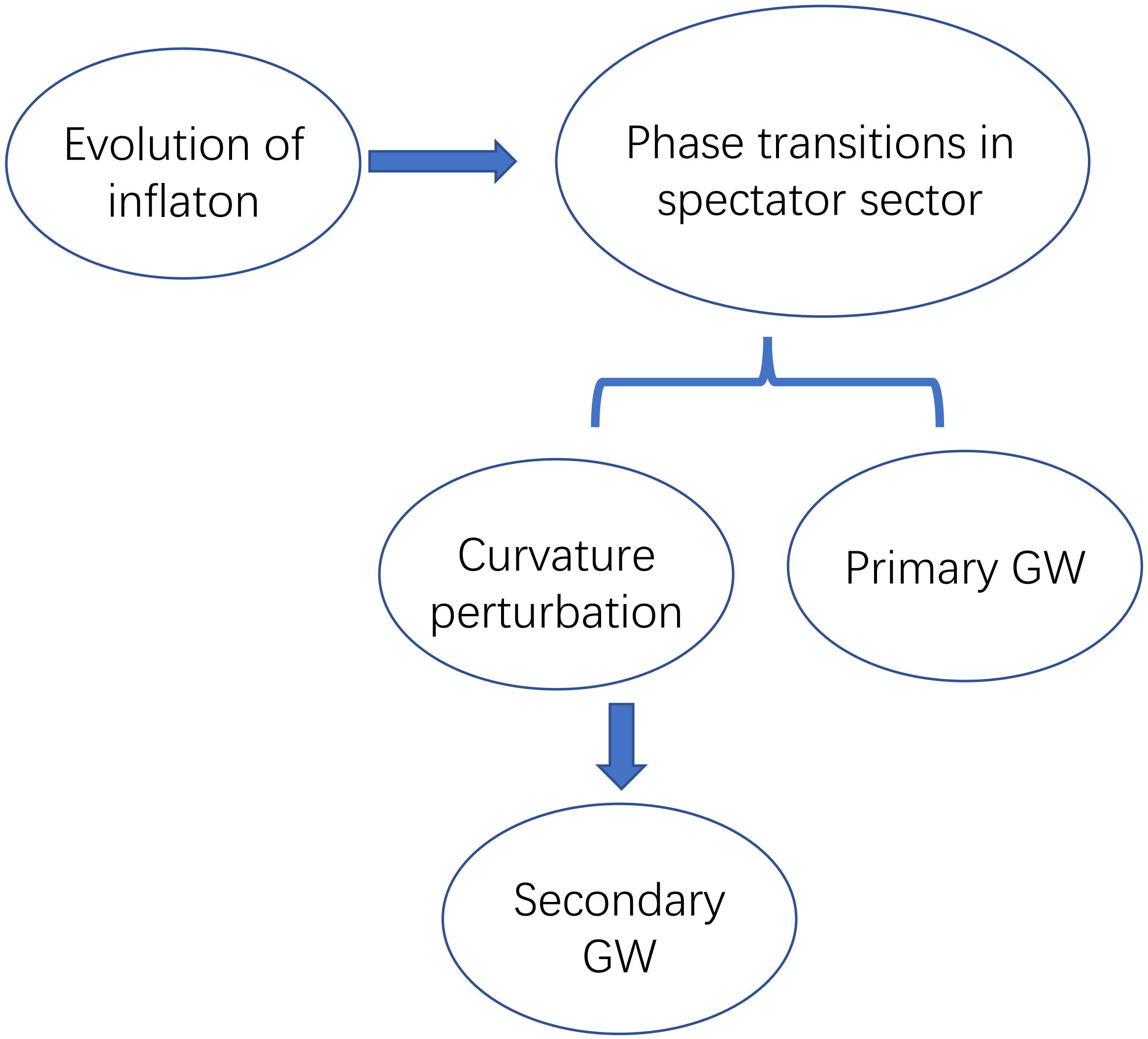
Angular correlation

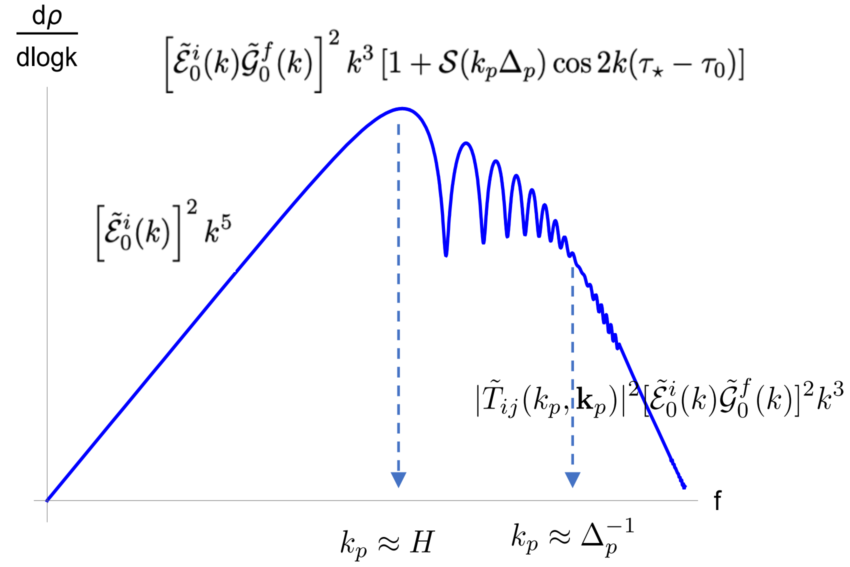
$$z_a(t) = -(\Delta\nu_a/\nu_a)(t) = \Delta T_a/T_a$$

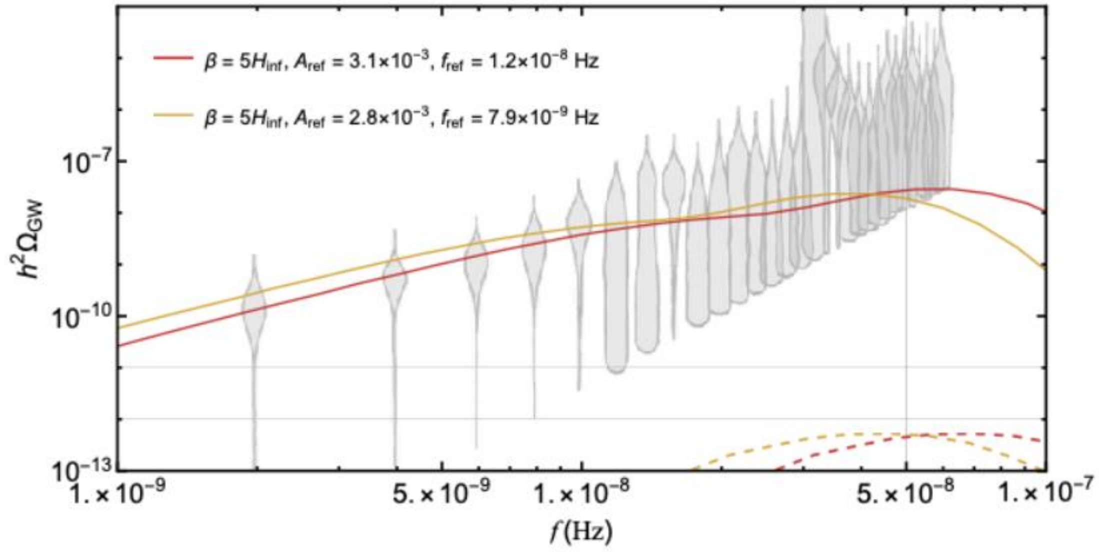




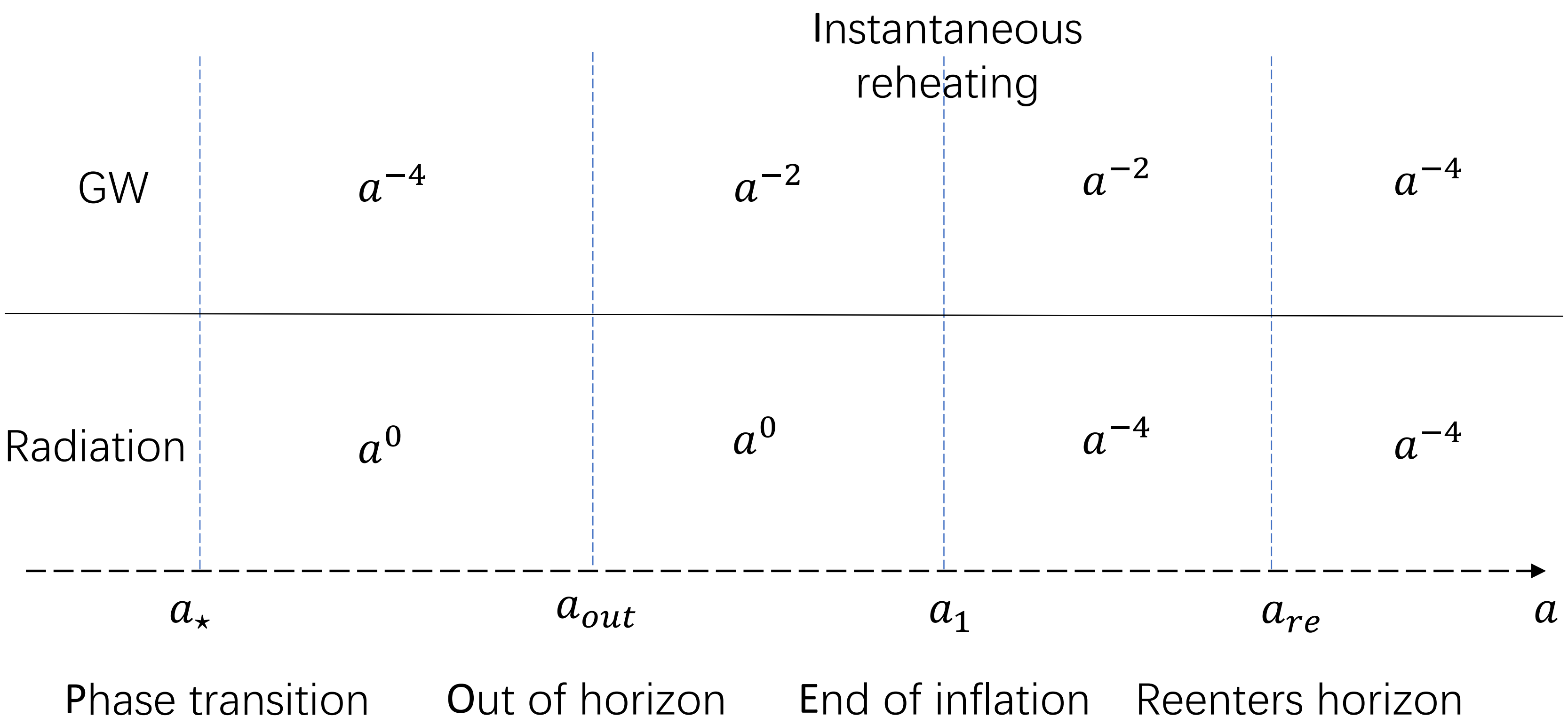
Summary for FOPT



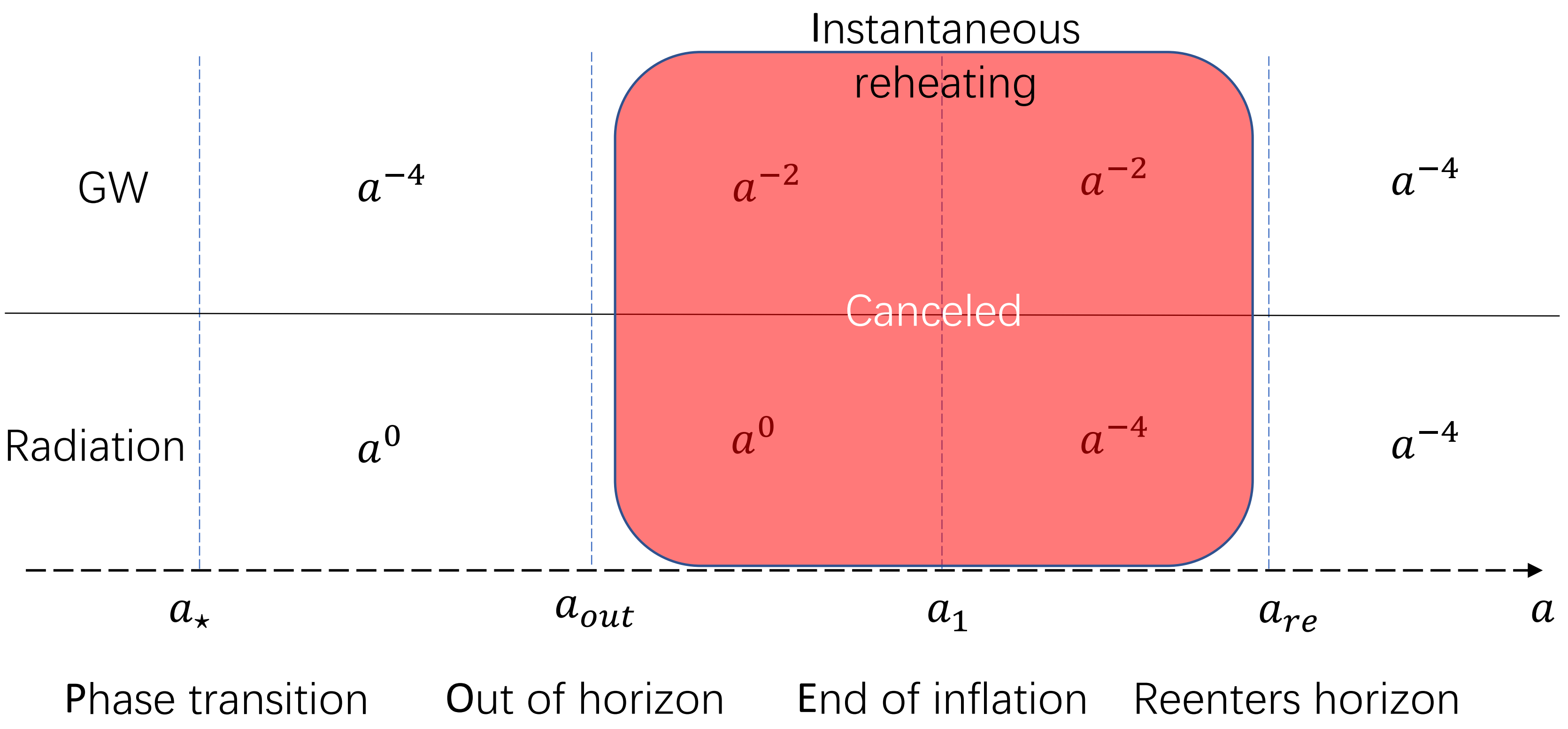




Redshifts of the GW signal

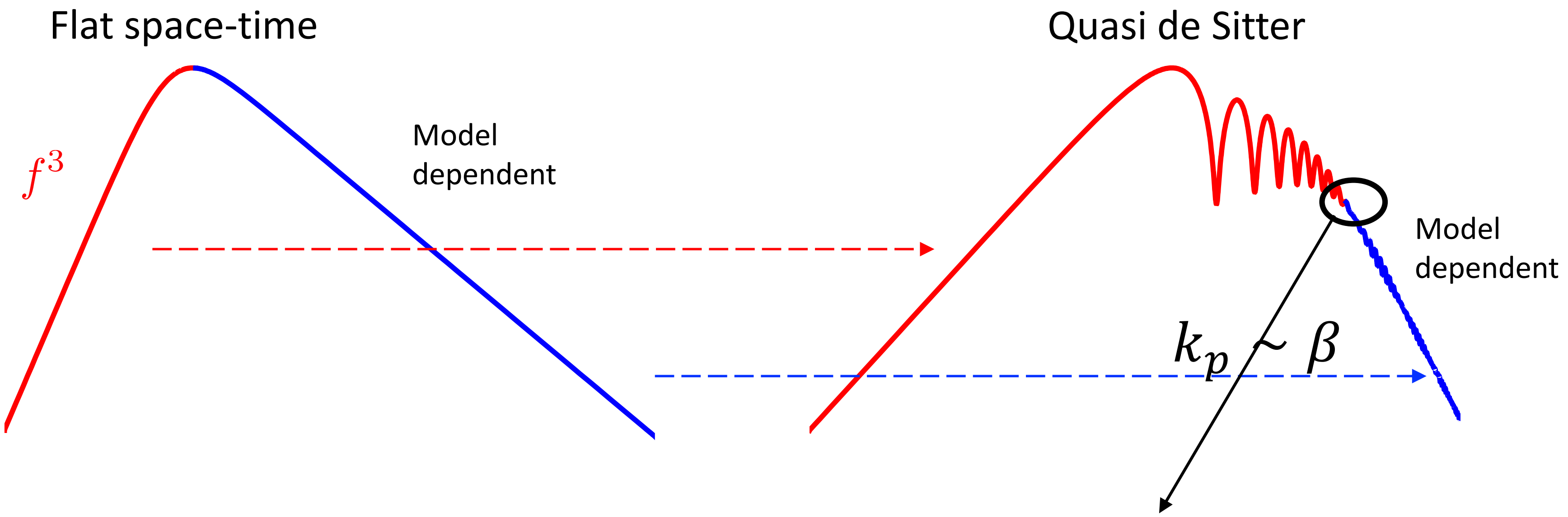


Redshifts of the GW signal



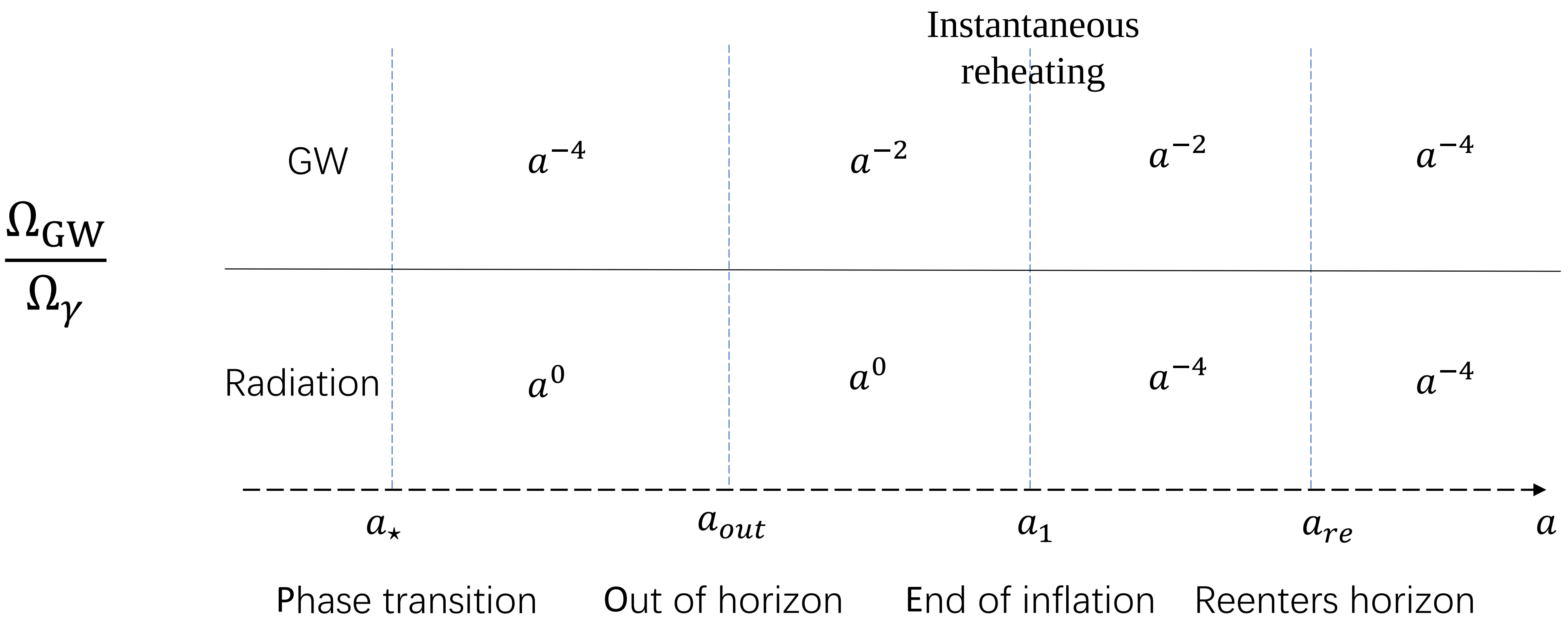
$$\frac{\Omega_{\mathrm{GW}}}{\Omega_\gamma} \sim \left(\frac{a_\star}{a_{\mathrm{out}}}\right)^4 \sim \left(\frac{H}{\beta}\right)^4$$

Spectrum distortion by inflation

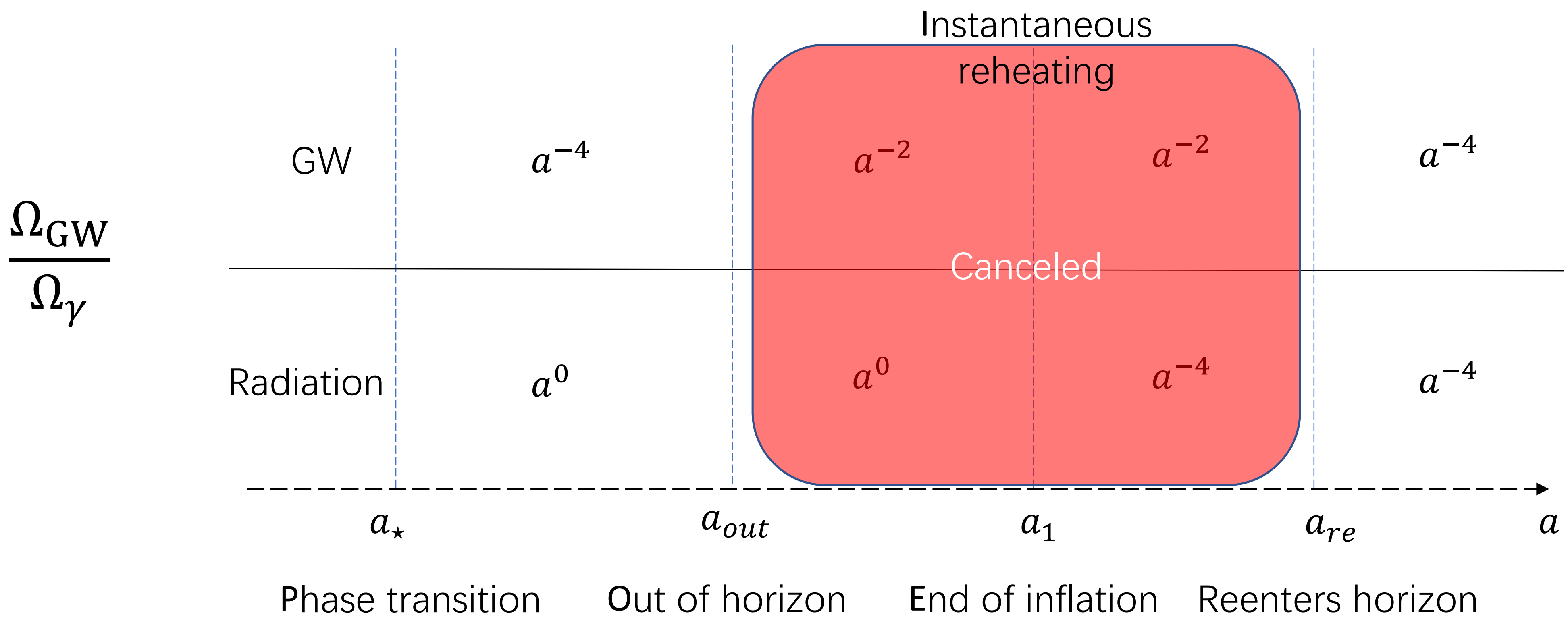


$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^6 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2$$

Redshifts of the GW signal

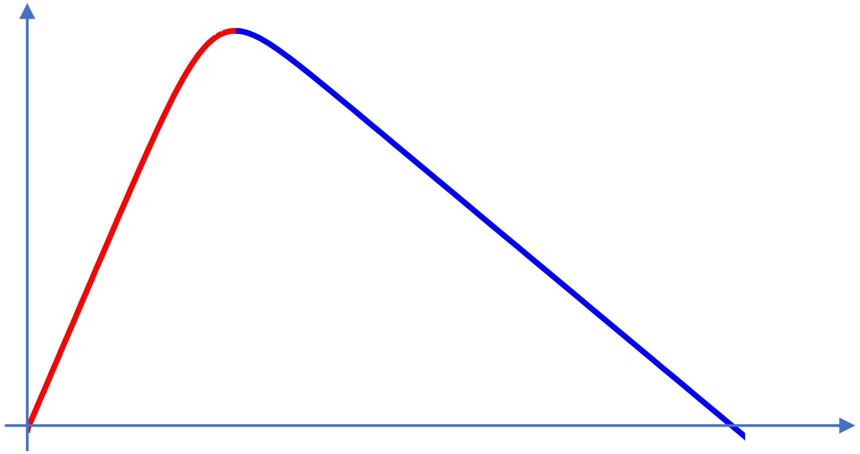


Redshifts of the GW signal



$$\frac{\Omega_{\rm GW}}{\Omega_\gamma} \sim \left(\frac{a_\star}{a_{\rm out}}\right)^4 \sim \left(\frac{H}{\beta}\right)^4$$

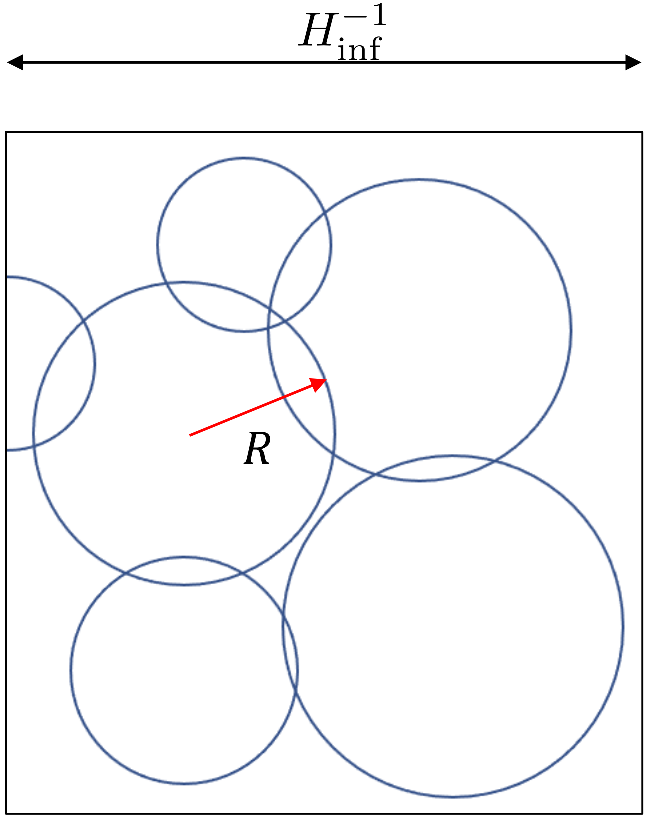
GWs produced in flat space-time



$$\frac{d\rho_{\rm GW}^{flat}}{\Delta\rho_{\rm vac} d\log k_p} \approx \left(\frac{H_{\rm inf}}{\beta}\right)^2 \times \frac{\beta\ k_p^{2.8}}{\beta^{3.8}+2.8k_p^{3.8}}$$

Huber and Konstandin, 0806.1828

$$\Omega_{\rm GW}^{(0)} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^2 \frac{\beta k_p^{2.8}}{\beta^{3.8}+2.8k_p^{3.8}}$$



First order phase transition during inflation

- $$\beta = \left|\frac{dS_4}{dt}\right| = \frac{dS_4}{d\log\mu_{\text{eff}}^2} \times \left|\frac{2\dot{\phi}}{\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)}\right|$$

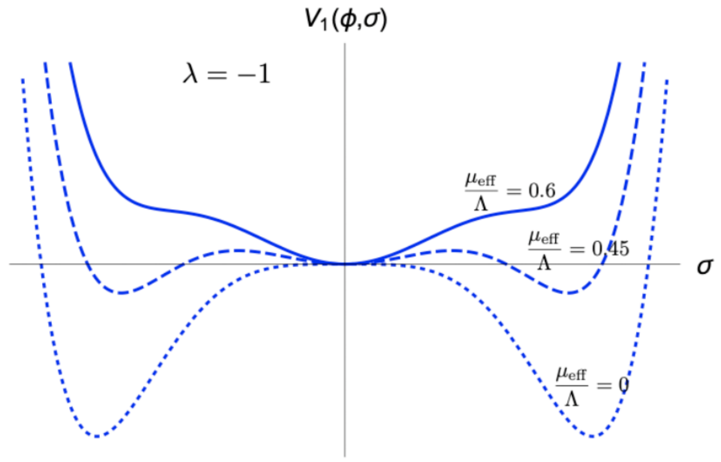
$$\Rightarrow \quad \frac{\beta}{H} = \left|\frac{dS_4}{d\log\mu_{\text{eff}}^2}\right| (2\epsilon)^{1/2} \times \frac{M_{\text{pl}}}{\left|\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)\right|}$$

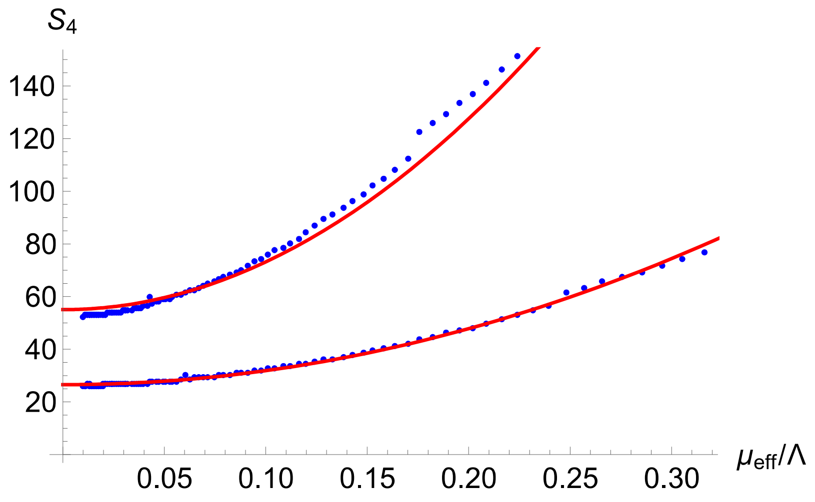
$$\boxed{\int_{\phi_{\text{end}}}^{\phi_{\text{PT}}} \frac{d\phi}{\sqrt{2\epsilon}M_{\text{pl}}} = N_{\text{e}}}$$

$$\sim \mu_{\text{eff}}^2/\Lambda^2$$

$$\frac{\beta}{H} \sim \left|\frac{dS_4}{d\log\mu_{\text{eff}}^2}\right| \times \frac{\Lambda^2}{\mu_{\text{eff}}^2} \times \frac{1}{N_e}$$

It is natural to have $\beta/H \sim O(10)$.

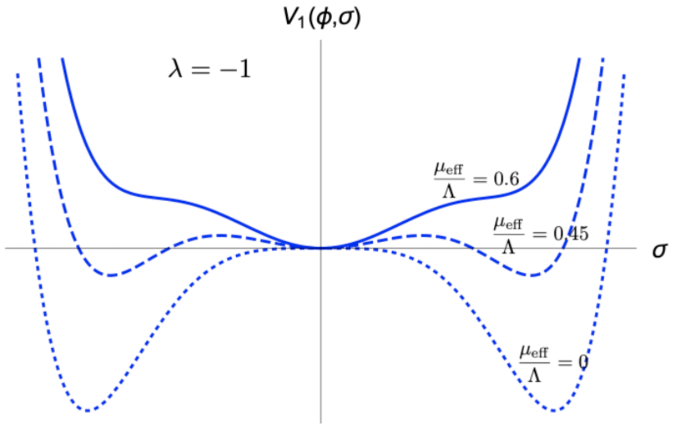




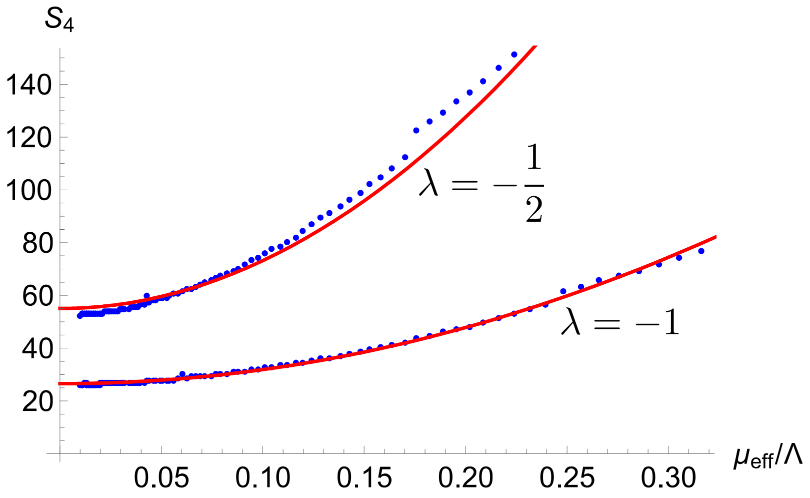
First order phase transition during inflation

- $$\beta = \left|\frac{dS_4}{dt}\right| = \frac{dS_4}{d\log\mu_{\rm eff}^2} \times \left|\frac{2\dot{\phi}}{\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)}\right| \qquad \mu_{\rm eff}^2 = -(\mu^2 - c^2\phi^2)$$

$$\Rightarrow \quad \frac{\beta}{H} = \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right| (2\epsilon)^{1/2} \times \frac{M_{\rm pl}}{\left|\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)\right|}$$



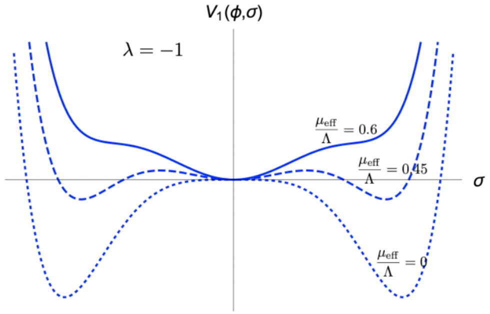
$$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$



CosmoTransitions

First order phase transition during inflation

- $$\beta = \left|\frac{dS_4}{dt}\right| = \frac{dS_4}{d\log\mu_{\rm eff}^2} \times \left|\frac{2\dot{\phi}}{\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)}\right|$$



$$\frac{\beta}{H} = \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right| (2\epsilon)^{1/2} \times \frac{M_{\rm pl}}{\left|\phi\left(1-\frac{\mu^2}{c^2\phi^2}\right)\right|}$$

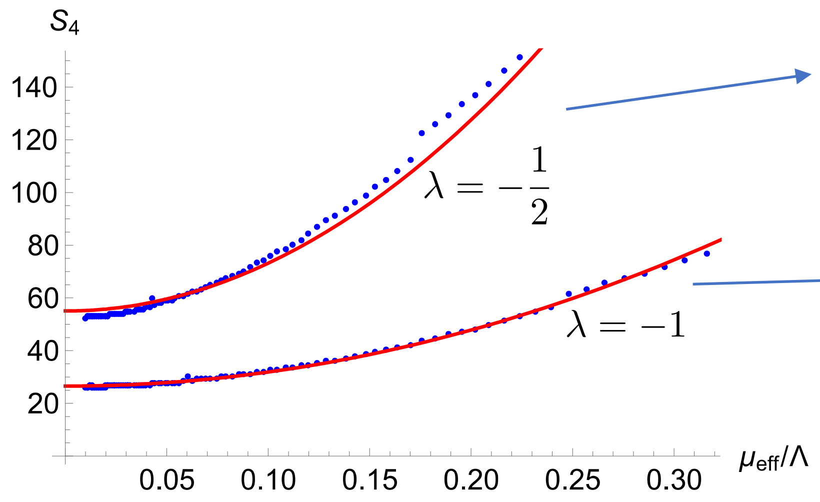
$$\boxed{\int_{\phi_{\rm end}}^{\phi_{\rm PT}} \frac{d\phi}{\sqrt{2\epsilon}M_{\rm pl}} = N_{\rm e}}$$

$$\sim \mu_{\rm eff}^2/\Lambda^2$$

$$\frac{\beta}{H} \sim \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right| \times \frac{\Lambda^2}{\mu_{\rm eff}^2} \times \frac{1}{N_e}$$

First order phase transition during inflation

- $$\frac{\beta}{H} \sim \left|\frac{dS_4}{d\log\mu_{\text{eff}}^2}\right| \times \frac{\Lambda^2}{\mu_{\text{eff}}^2} \times \frac{1}{N_e}$$



$$\frac{\beta}{H} \sim \frac{3800}{N_e}$$

$$\frac{\beta}{H} \sim \frac{500}{N_e}$$

N_e: e-folds before the end of inflation

$$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$

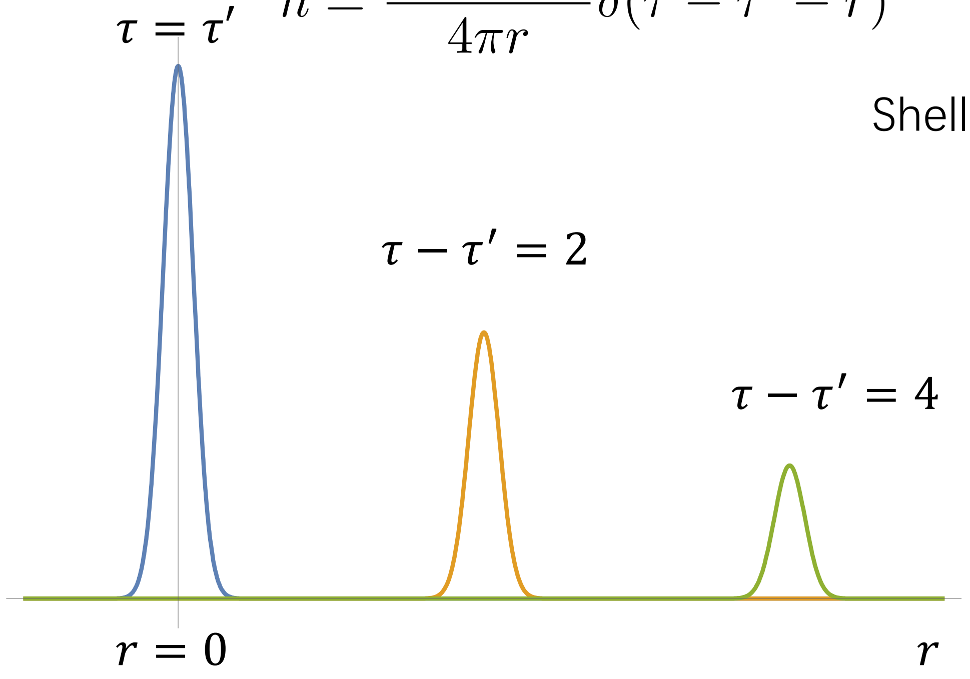
$$\frac{\beta}{H} \sim \mathcal{O}(10) - \mathcal{O}(100)$$

de Sitter inflation as an example

- What is the spatial configuration of h_{ij}?
- In Minkovski space

$$h = \frac{16\pi G_N T}{4\pi r}\delta(\tau - \tau' - r)$$

Shell with radius $|\tau - \tau'|$



de Sitter inflation as an example

- What is the spatial configuration of h_{ij}?
- In de Sitter space

$$h_{ij}(\tau, \mathbf{k}) = -16\pi G_N H T_{ij} \tau \Theta(\tau - \tau') \left[\frac{\sin k(\tau - \tau')}{k} + \left(\frac{1}{k^2\tau} - \frac{1}{k^2\tau'} \right) \cos k(\tau - \tau') + \frac{1}{k^3 \tau\tau'} \sin k(\tau - \tau') \right]$$

de Sitter inflation as an example

- What is the spatial configuration of h_{ij}?
- In de Sitter space

$$\frac{\tau}{4\pi x}\delta(\tau - \tau' - |\mathbf{x}|)$$

$$h_{ij}(\tau, \mathbf{k}) = -16\pi G_N H T_{ij} \tau \Theta(\tau - \tau') \left[\frac{\sin k(\tau - \tau')}{k} \right.$$

$$\underbrace{\left. + \left(\frac{1}{k^2 \tau} - \frac{1}{k^2 \tau'} \right) \cos k(\tau - \tau') + \frac{1}{k^3 \tau \tau'} \sin k(\tau - \tau') \right]}_{\frac{1}{4\pi}\Theta(\tau - \tau' - |\mathbf{x}|)}$$

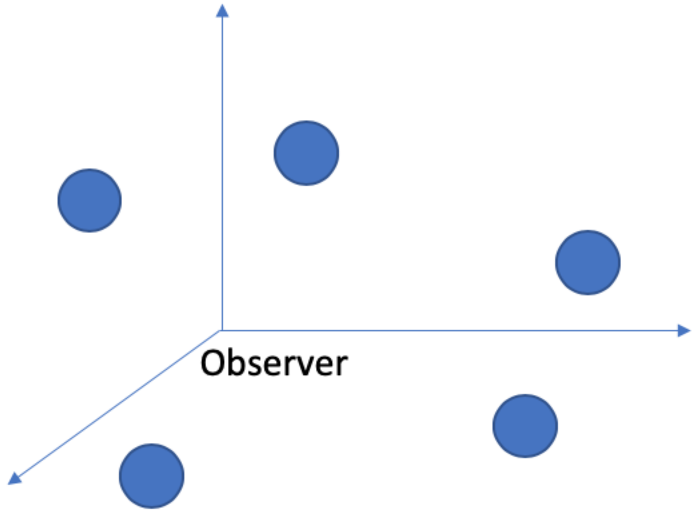
de Sitter inflation as an example

- What is the spatial configuration of h_{ij}?
- In de Sitter space

$$h(\tau, \mathbf{x}) \sim \underbrace{\frac{\tau}{4\pi x}\delta(\tau - \tau' - x)}_{\substack{\text{Similar to Minkovski} \\ \text{Decreases with both } x \text{ and } \tau \\ \text{Vanishes out of horizon}}} + \underbrace{\frac{1}{4\pi}\Theta(\tau - \tau' - x)}_{\substack{\text{Intrinsic in de Sitter} \\ \text{constant}}}$$

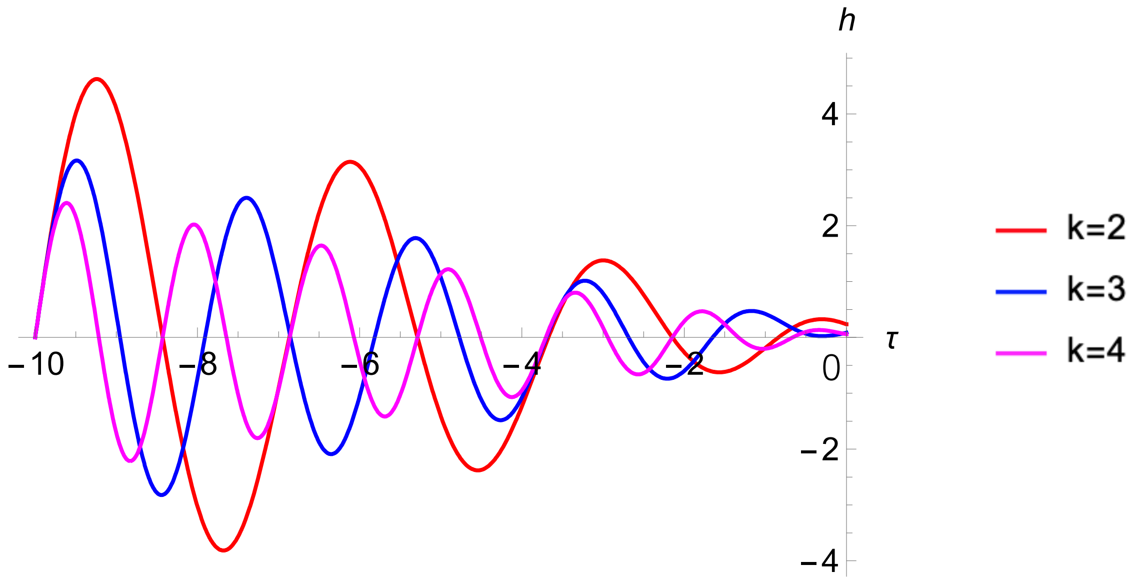
de Sitter inflation as an example

- At $\tau \to \mathbf{0}$ $\qquad h(\tau, \mathbf{x}) \sim \frac{1}{4\pi}\Theta(|\tau'| - x)$
- A ball of GW, with radius $|\tau'|$
- h uniformally distributed inside the GW balls.
- All the balls have the same radius.



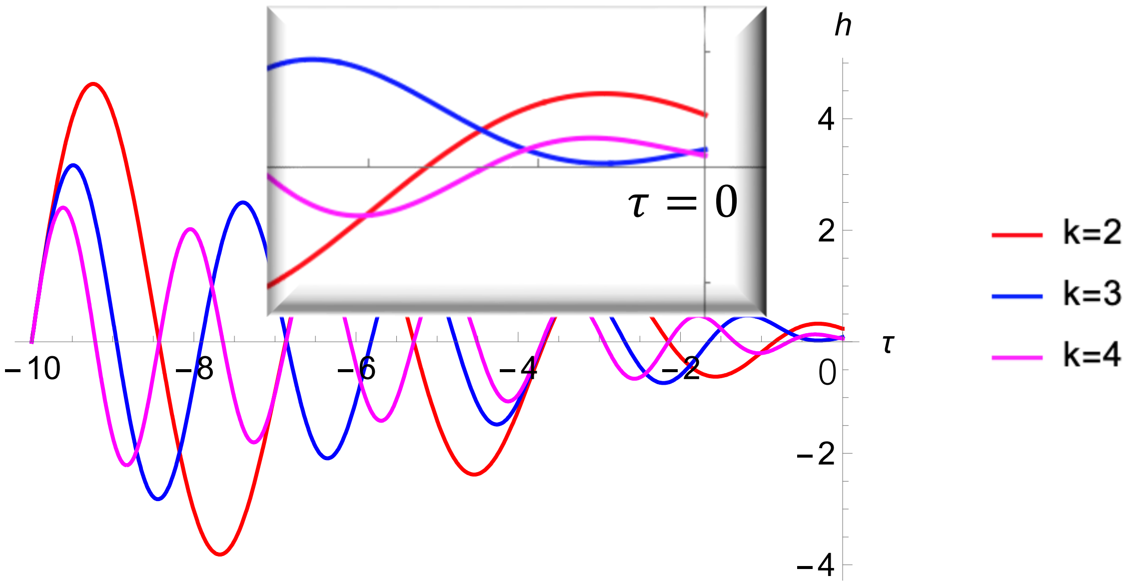
Quasi-de Sitter inflation as an example

- $$a = -\frac{1}{H\tau}$$
- $$h_{ij}(\tau, \mathbf{k}) = -\frac{16\pi G_N H T_{ij} \tau}{k}\left[\left(\frac{1}{k\tau} - \frac{1}{k\tau'}\right)\cos k(\tau - \tau') + \left(1 + \frac{1}{k^2\tau\tau'}\right)\sin k(\tau - \tau')\right]$$



De Sitter inflation as an example

- $$a = -\frac{1}{H\tau}$$
- $$h_{ij}(\tau, \mathbf{k}) = -\frac{16\pi G_N H T_{ij} \tau}{k}\left[\left(\frac{1}{k\tau} - \frac{1}{k\tau'}\right)\cos k(\tau - \tau') + \left(1 + \frac{1}{k^2\tau\tau'}\right)\sin k(\tau - \tau')\right]$$



De Sitter inflation as an example

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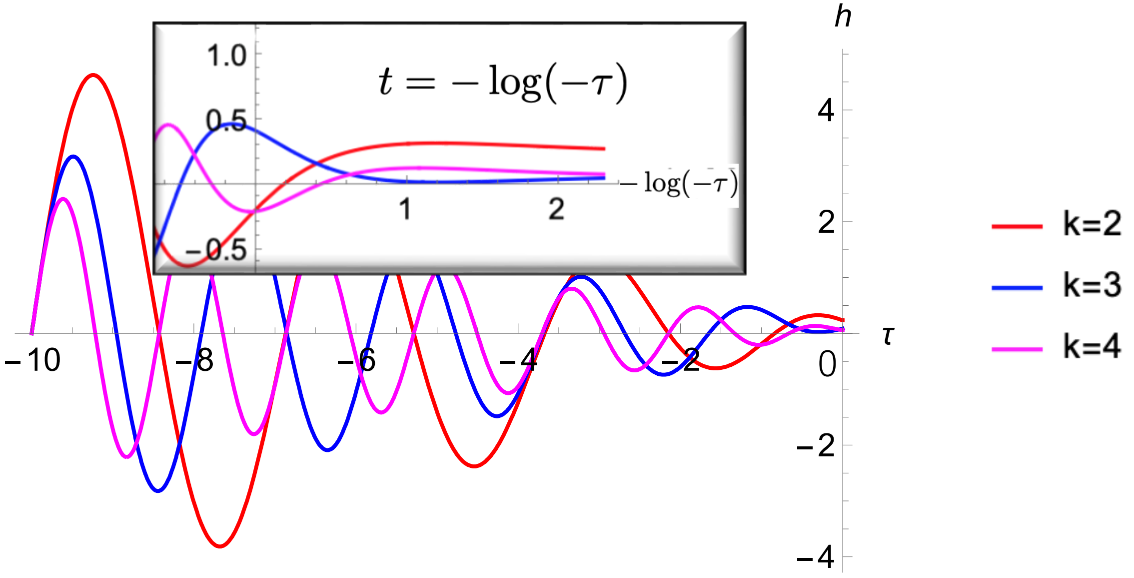
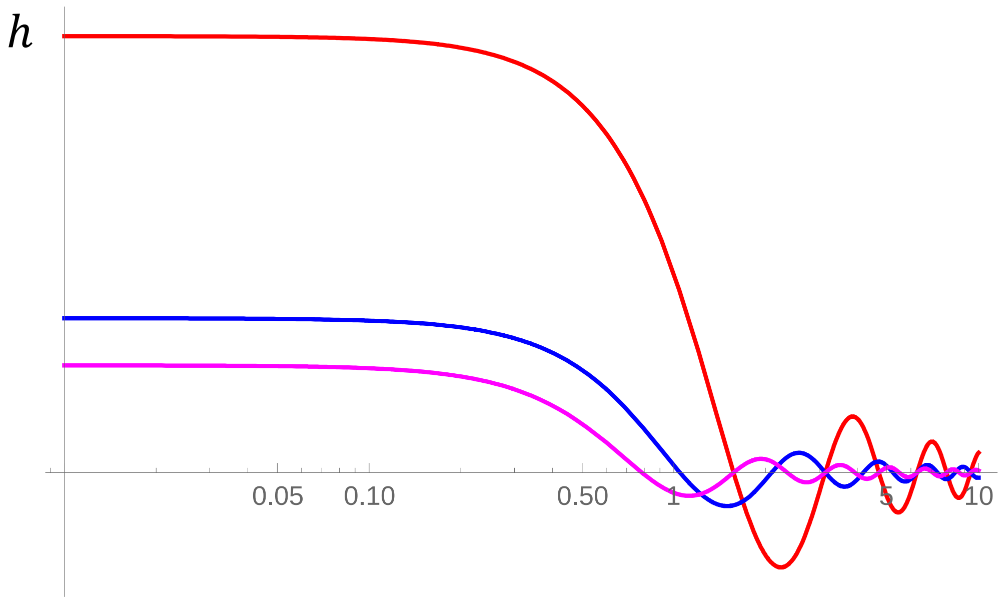


After inflation

- $h^f(k)$ is the initial amplitude for the GW oscillation after inflation.
- All the modes start to oscillate with the same phase.
- Example, in RD, the oscillation is $\sin k\tau / k\tau$



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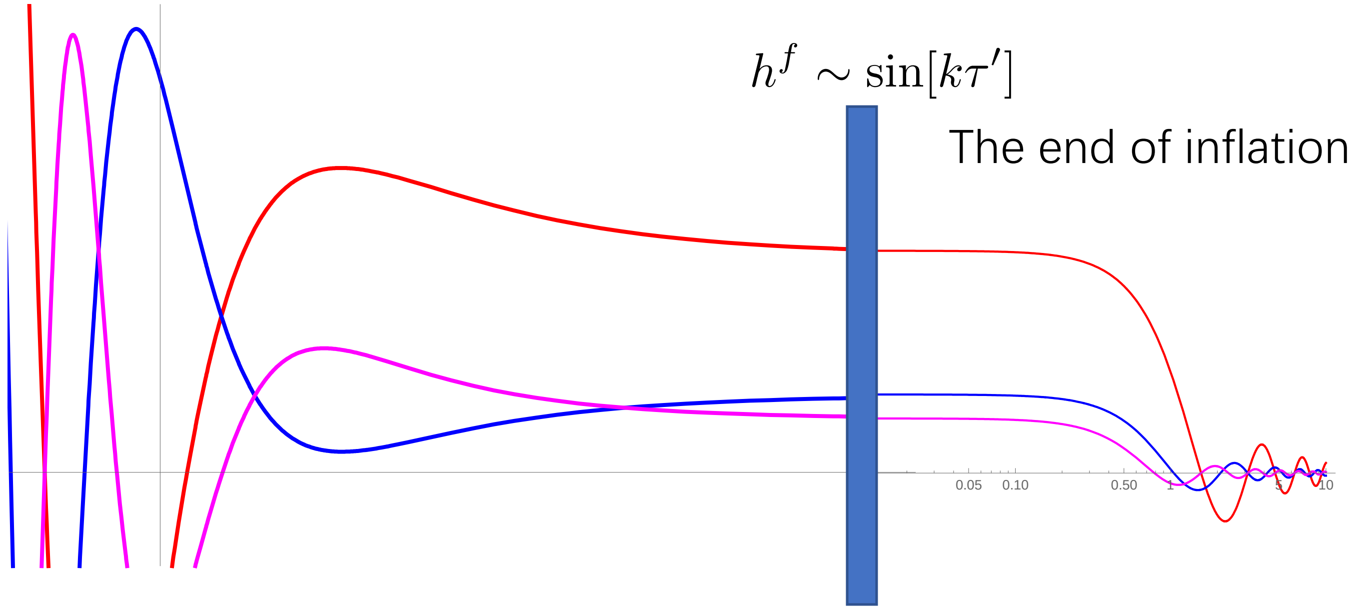
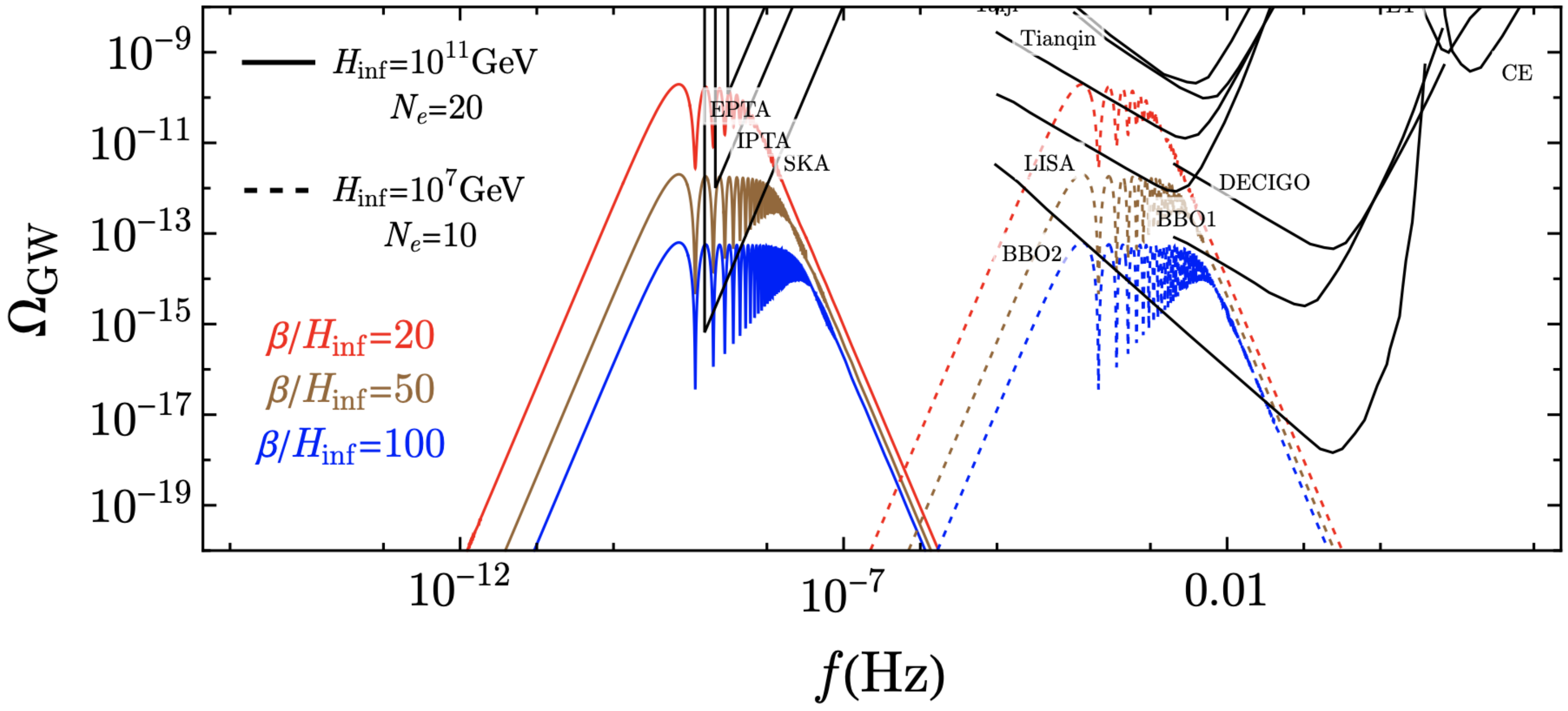


First order phase transition during inflation

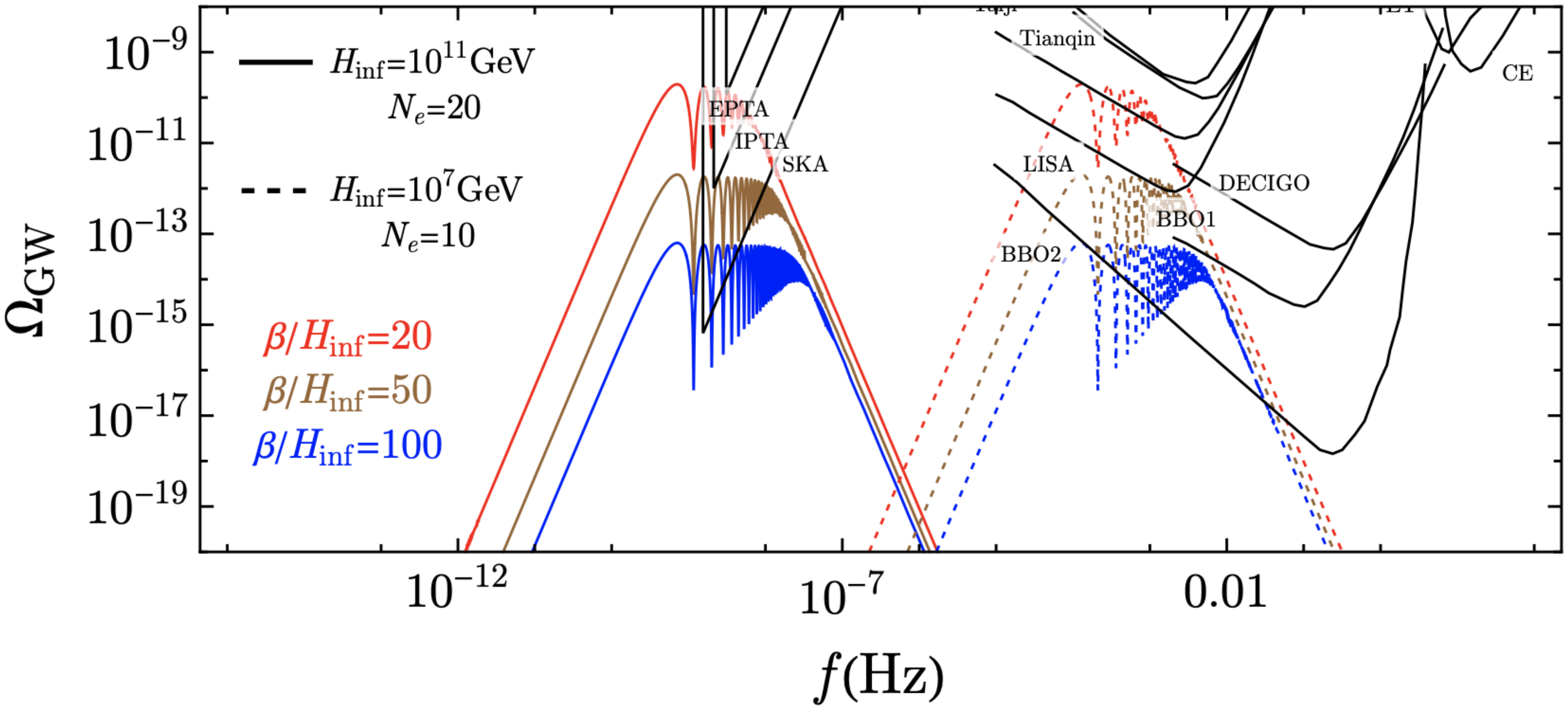
- Signal strength is also sensitive to intermediate stages

Kination domination



First order phase transition during inflation

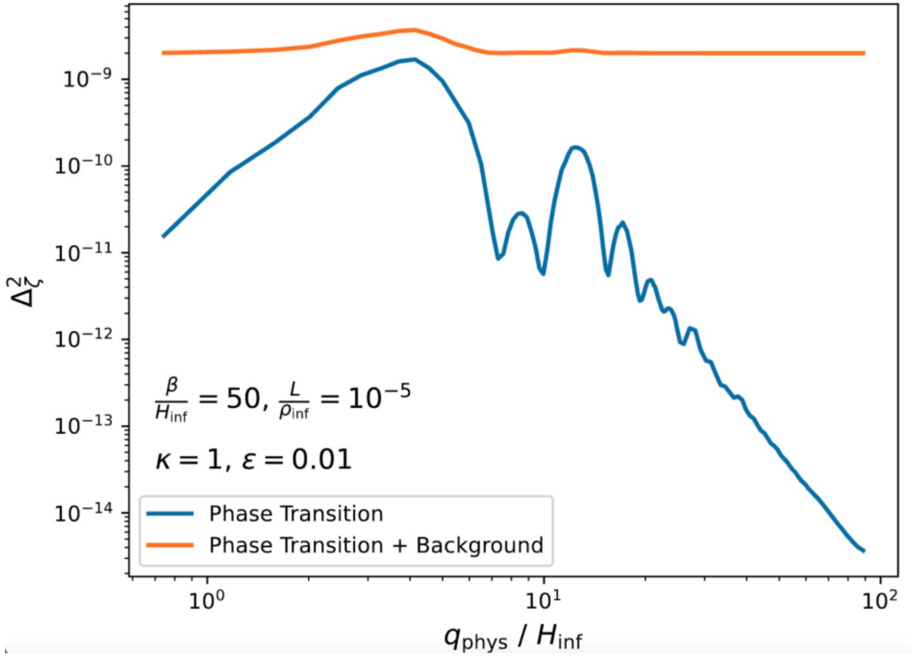
With kination domination intermediate stage



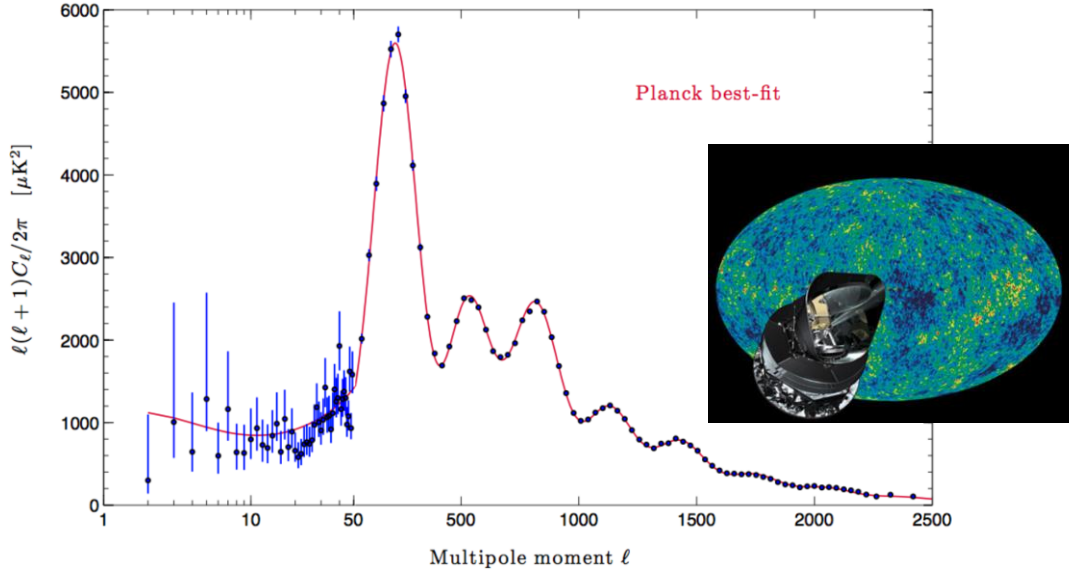
Power spectrum of ζ

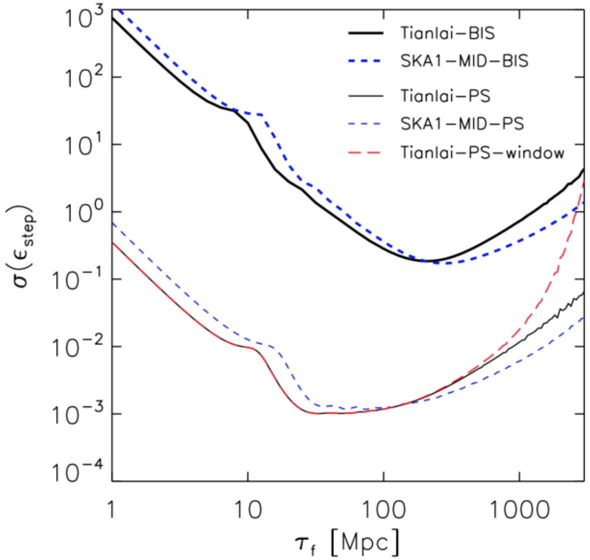
$$k_{\text{today}} = (2000\ \text{Mpc})^{-1} \times e^{60-N_e} \times \left(\frac{H_{\text{inf}}}{10^{14}\ \text{GeV}}\right)$$

$$A_{\text{ref}} = \frac{\mathcal{A}}{\epsilon}\left(\frac{M_{\text{pl}}}{\phi_0}\right)^2\left(\frac{H_{\text{inf}}}{\beta}\right)^3\left(\frac{L}{\rho_{\text{inf}}}\right)^2$$



HA, Boye Su, Yidong Xu, Chen Yang, work in progress.

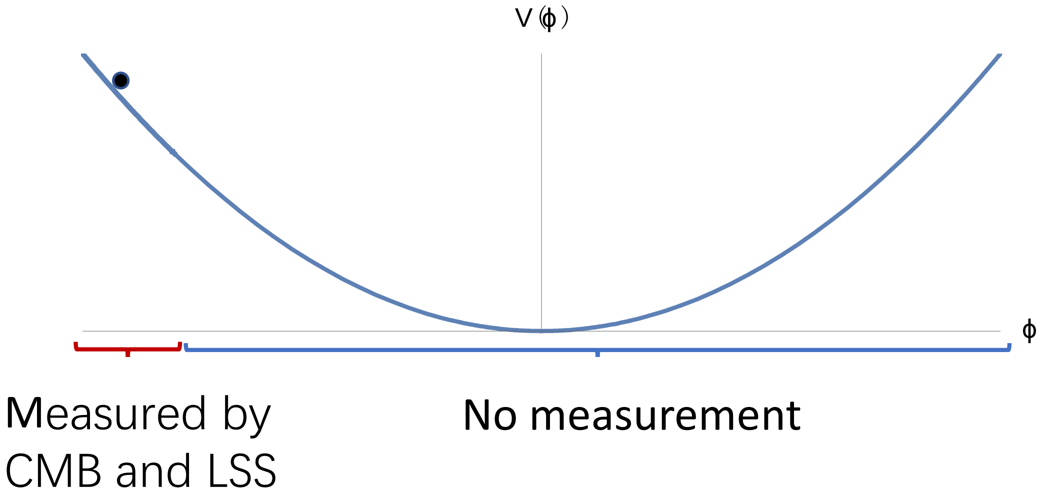




Xu, Hamann, Chen, 1607.00817

Slow roll models

- We usually assume a potential.
- Use **it to calculate** n_s, r ...



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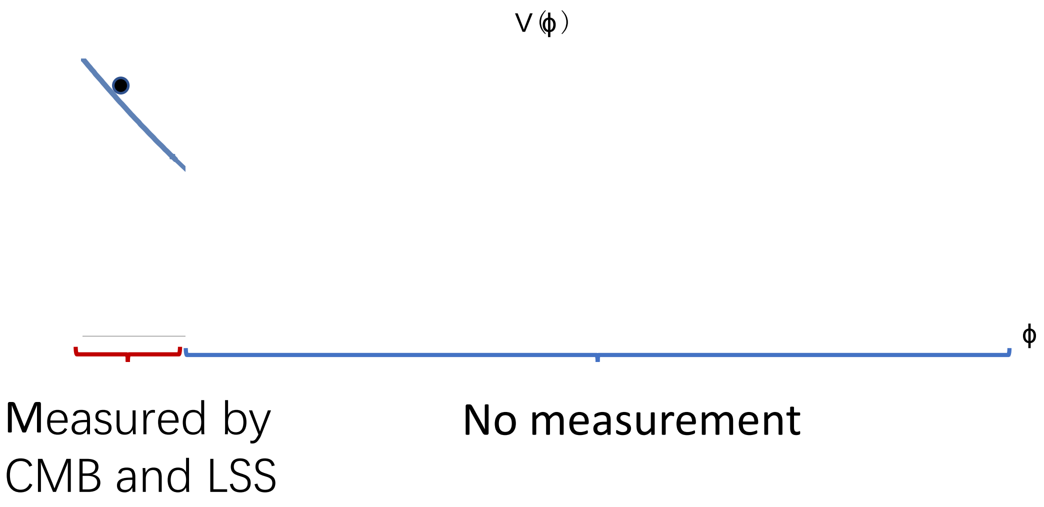

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