(Late-time) 'Solutions' to H0 and S8 Tensions through Modified-Gravity

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- The history of Astronomy, Cosmology and Gravity is a history of tensions between theoretical predictions and observations

- Astrophysical cosmology has become a precision science with an incredibly huge amount of data

- New Tensions appear.

  Are we approaching New Physics?

Aristotle - 350 BC

- According to Aristotle heavier bodies fall faster.
- Bodies fall in order to com back to thei "initial state".

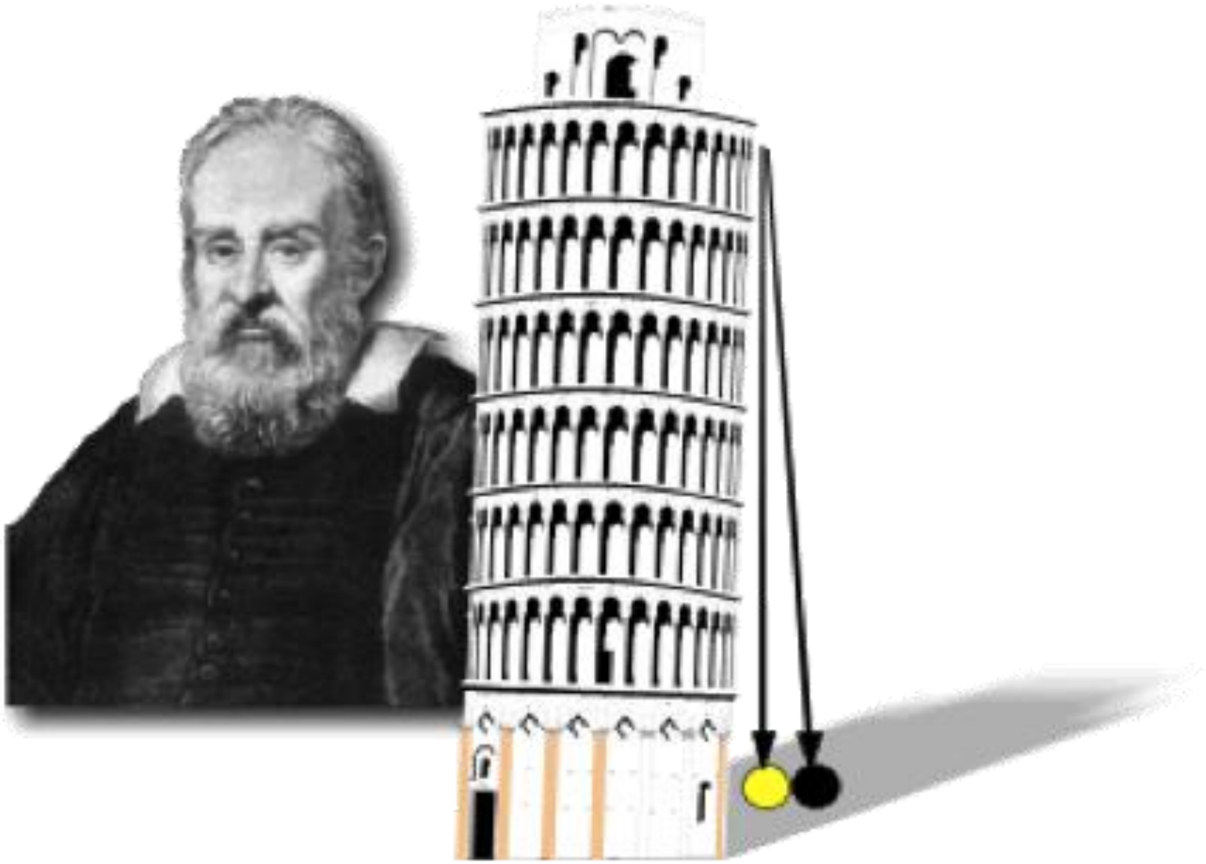
Schema huius præmissæ diuisionis Sphærarum.

Brahe, Kepler- 1600

- Heliocentrism, elliptical Orbits

Galileo - 1600

- Bodies fall with the same speed, independently from their weight.

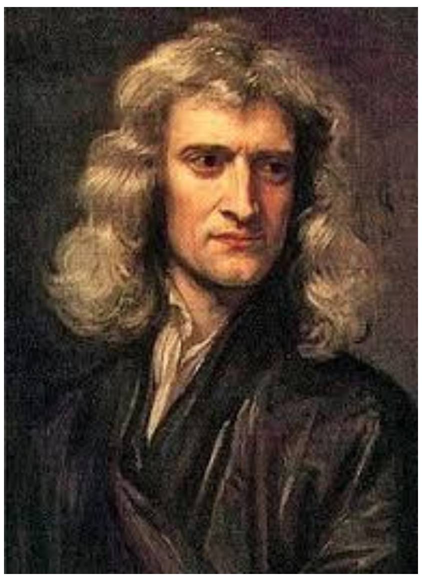


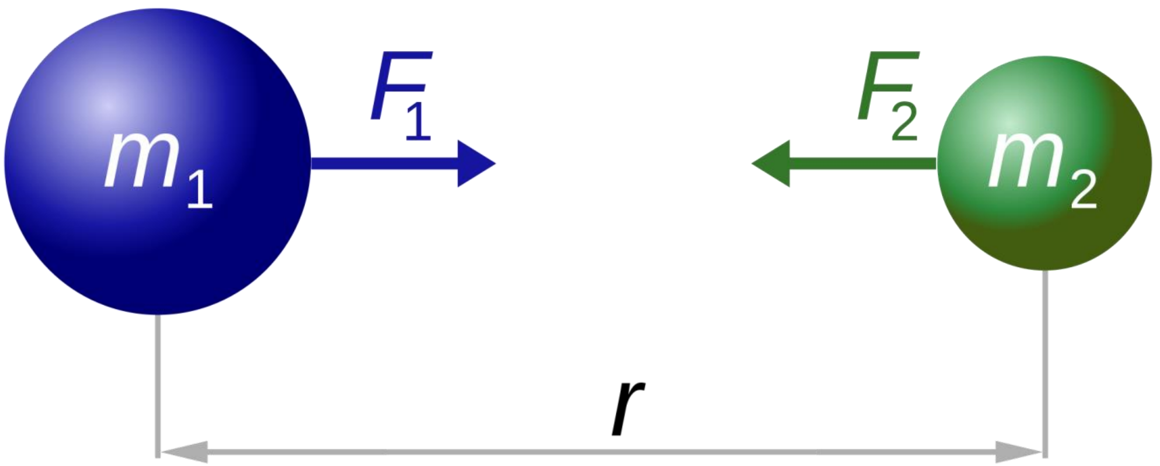
Newton - 1700

- Law of Universal Gravitation:

  All bodies (either apples or planets) attract mutually.

  First time that gravity is related to astronomy

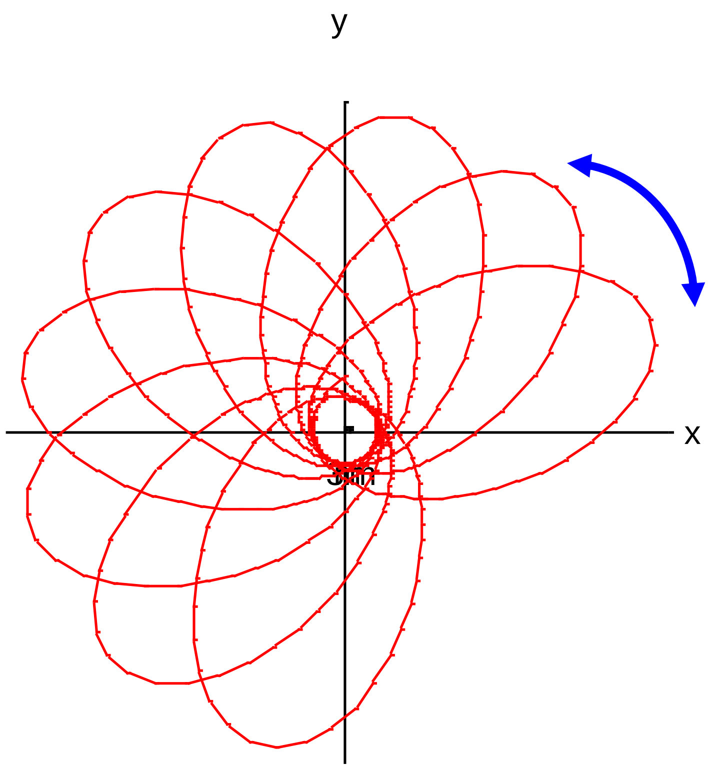




$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

Mercury periliheimum - 1859

- *The true orbits of planets, even if seen from the SUN are not ellipses. They are rather curves of this type:*

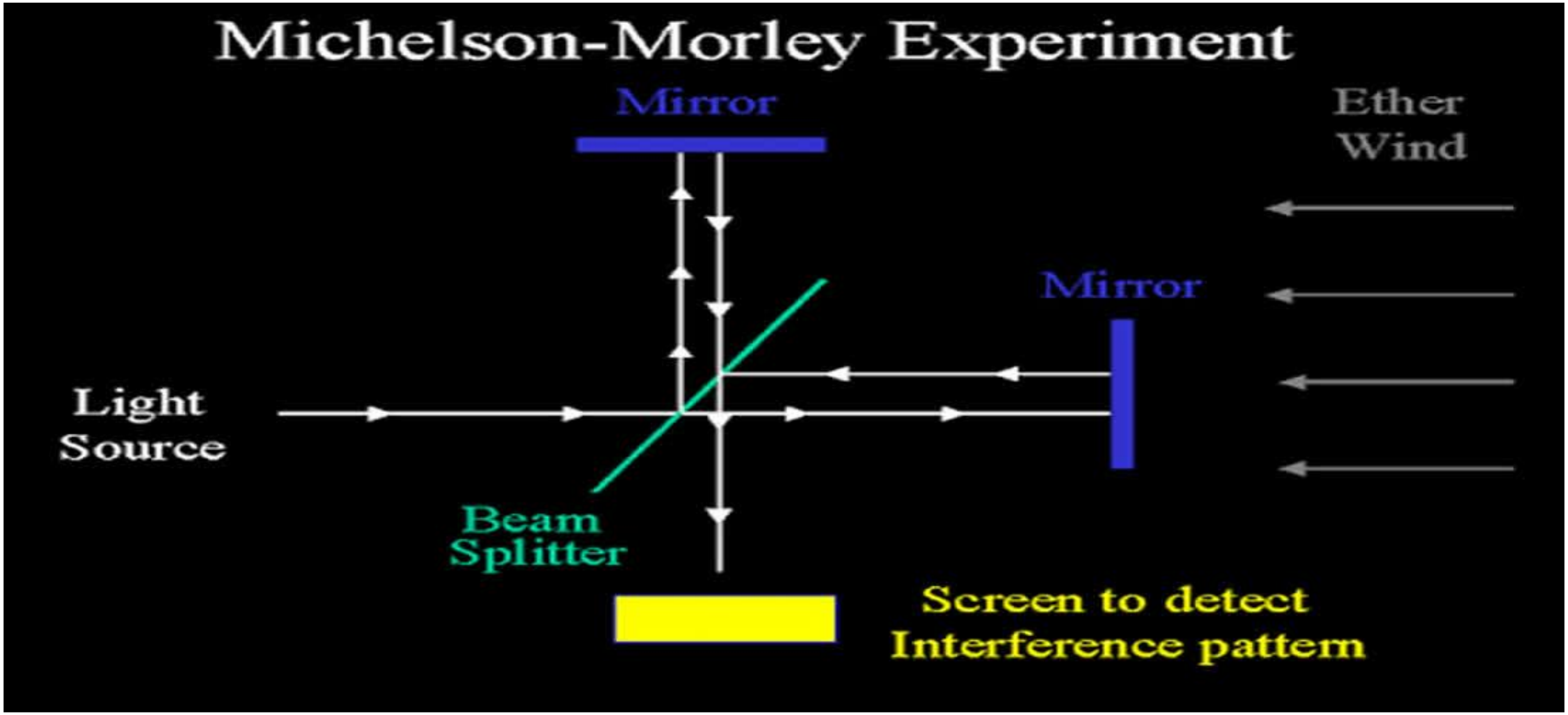


This angle is the perihelion advance, predicted by G.R.

For the planet Mercury it is

$$\Delta\varphi = 43'' \, of \;\; arc \;\; per \;\; century$$

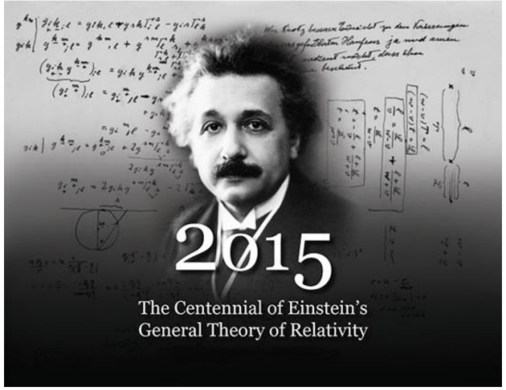
Michelson–Morley experiment - 1887



General Relativity

- Einstein 1915: **General Relativity**:



energy-momentum source of spacetime Curvature

$$\boxed{S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R-2\Lambda\right]+\int d^4x\, L_m\left(g_{\mu\nu},\psi\right)}$$

$$\Rightarrow \quad R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+g_{\mu\nu}\Lambda=8\pi G\, T_{\mu\nu}$$

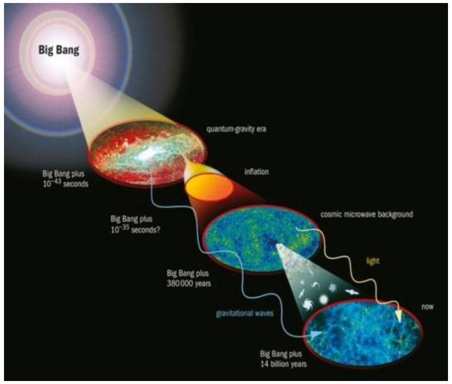
with $\quad T^{\mu\nu}\equiv\frac{2}{\sqrt{-g}}\frac{\delta L_m}{\delta g_{\mu\nu}}$

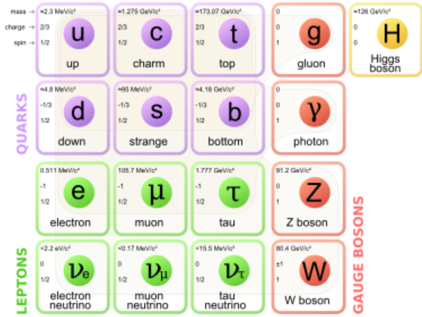
Modified Gravity before General Relativity

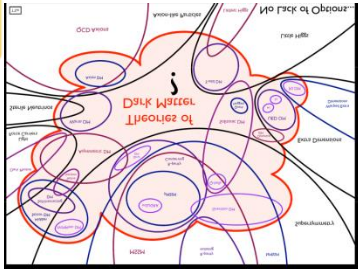
- Modifications to Newton's Law
- Inverse Cube Law.
- Extended Inverse-Square Law (Simon Newcomb -1880's)
- Lord Kelvin - theory of everything (end of 19th century)
- Hendrik Lorentz: gravity on the basis of his ether theory and Maxwell's equations. (1900)
- Nordström's theory of gravitation (1912 and 1913)
- Einstein's scalar theory of gravity (1913)

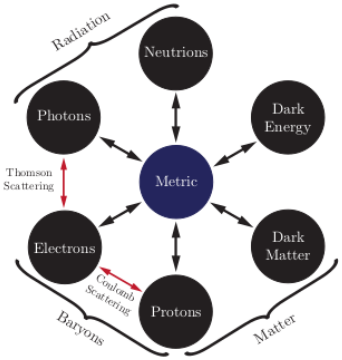
Summary of 20th century Observations

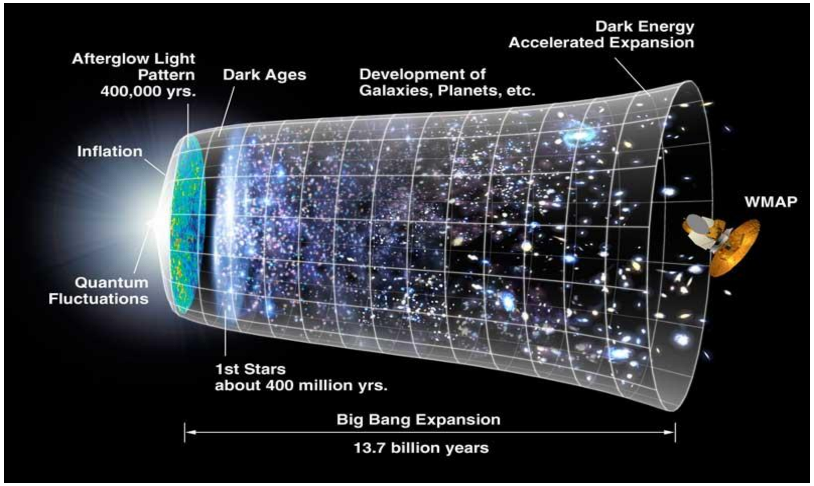
The Universe history:

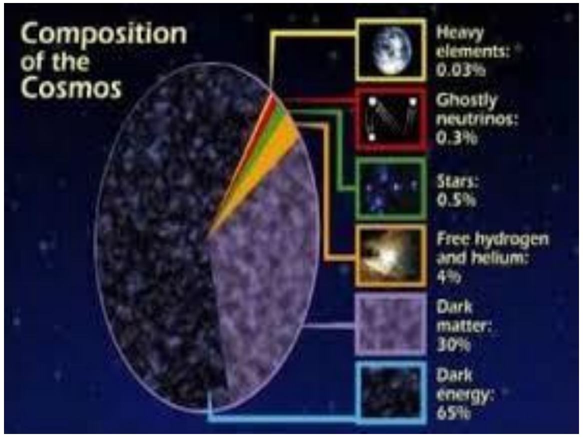












Standard Model of Cosmology

ΛCDM Paradigm + Inflation

$$\boxed{H(t)^2 + \frac{k}{a(t)^2} = \frac{8\pi G}{3}\left[\rho_{dm}(t) + \rho_b(t) + \rho_r(t)\right] + \frac{\Lambda}{3}}$$

$$\boxed{w_\Lambda \equiv \frac{p_\Lambda}{\rho_\Lambda} = -1}$$

$$\boxed{\dot{H}(t) - \frac{k}{a(t)^2} = -4\pi G\left[\rho_{dm}(t) + p_{dm}(t) + \rho_b(t) + p_b(t) + \rho_r(t) + p_r(t)\right]}$$

ΛCDM concordance model is almost perfect!

- Describes the thermal history of the Universe at the background level
- Epochs of inflation, radiation, matter, late-time acceleration

Cosmology-background

- Homogeneity and isotropy: $ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right)$

- Background evolution (Friedmann equations) in flat space

$$H^2 = \frac{8\pi G}{3}\left(\rho_m + \rho_{DE}\right)$$
$$\dot{H} = -4\pi G\left(\rho_m + p_m + \rho_{DE} + p_{DE}\right),$$

(the effective DE sector can be either Λ or any possible modification)

- One must obtain a H(z) and Ωm(z) and wDE(z) in agreement with observations (SNIa, BAO, CMB shift parameter, H(z) etc)

Cosmology-perturbations

- Perturbation evolution: $\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\rm eff}\,\rho\,\delta \approx 0$ where $\delta \equiv \delta\rho/\rho$

  where $G_{\rm eff}(z,k)$ is the effective Newton's constant, given by

$$\nabla^2\phi \approx 4\pi G_{\rm eff}\rho\,\delta$$

under the scalar metric perturbation $ds^2 = -(1+2\phi)dt^2 + a^2(1-2\psi)d\vec{x}^{\,2}$

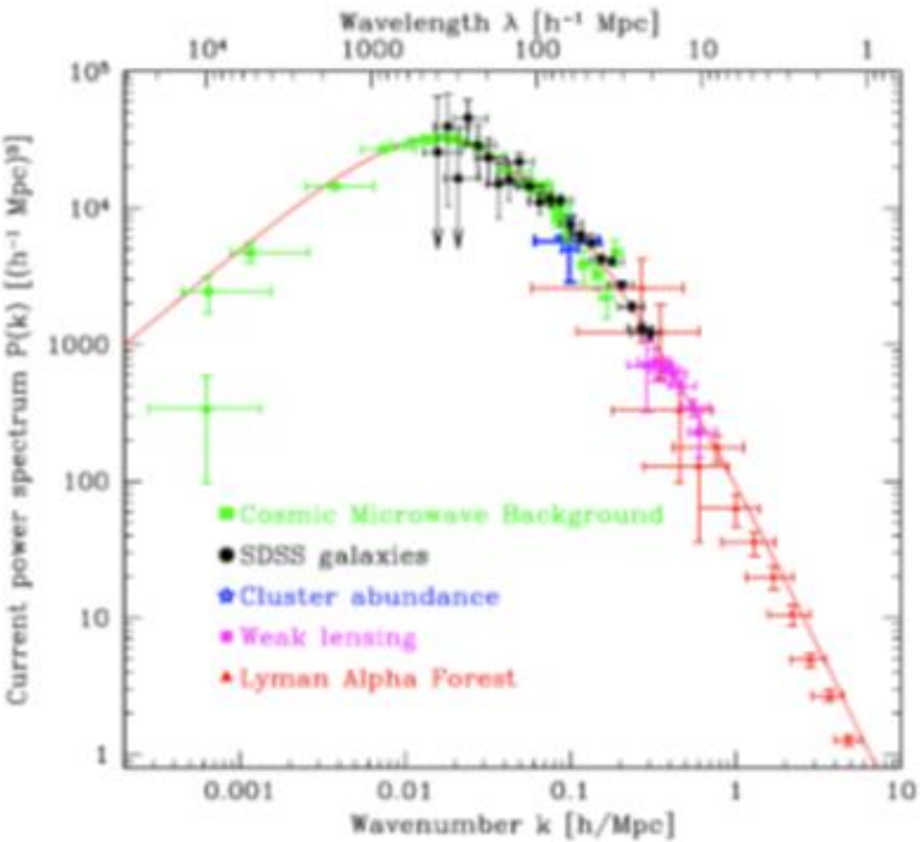
- Hence: $\delta'' + \left(\frac{(H^2)'}{2\,H^2} - \frac{1}{1+z}\right)\delta' \approx \frac{3}{2}(1+z)\frac{H_0^2}{H^2}\frac{G_{\rm eff}(z,k)}{G_N}\,\Omega_{0m}\delta$

  with $f(a) = \frac{d\ln\delta}{d\ln a}$ the growth rate, with $f(a) = \Omega_{\rm m}(a)^{\gamma(a)}$ and $\Omega_{\rm m}(a) \equiv \frac{\Omega_{0m}\,a^{-3}}{H(a)^2/H_0^2}$

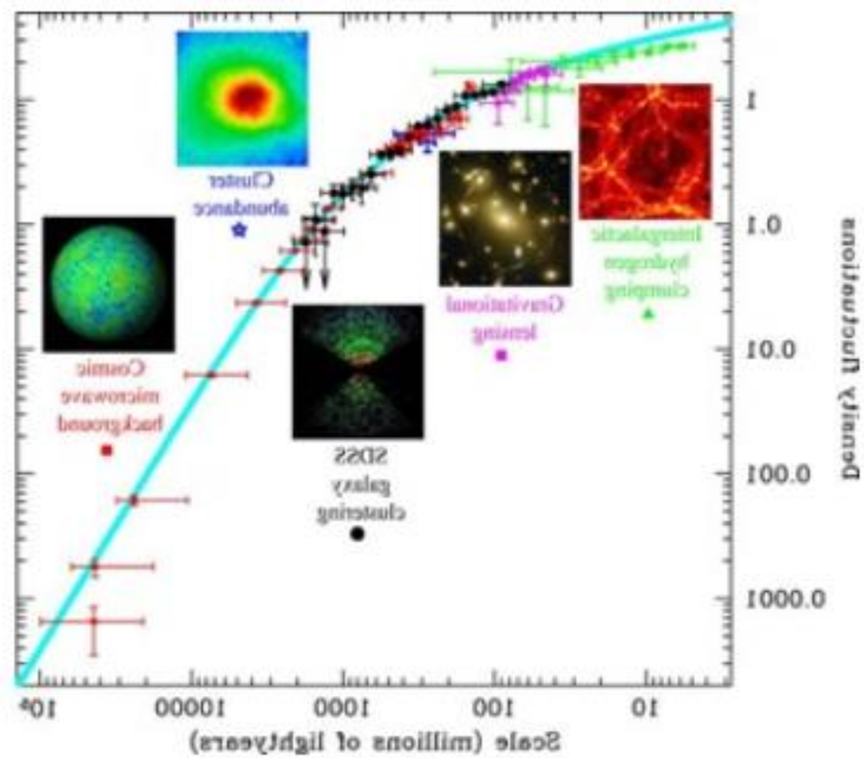
- One can define the observable: $\boxed{f\sigma_8(a) \equiv f(a)\cdot\sigma(a) = \frac{\sigma_8}{\delta(1)}\,a\,\delta'(a)}$

  with $\sigma(a) = \sigma_8\frac{\delta(a)}{\delta 1}$ the z-dependent rms fluctuations of the linear density field within spheres of radius $R = 8h^{-1}\mathrm{Mpc}$, and σ8 its value today.

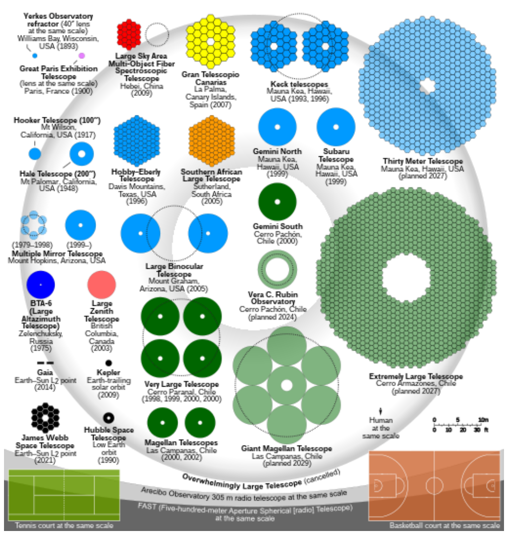
Matter Density Fluctuation Power Spectrum

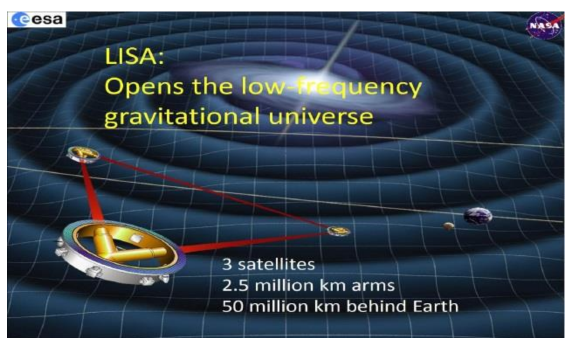


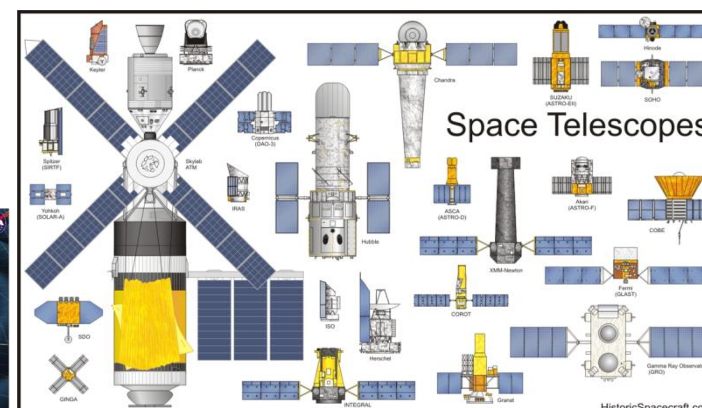
A different convention: plot $P(k)k^3$



Cosmology in the 21st century







Issues of ΛCDM Paradigm

- 1) General Relativity is non-renormalizable. It cannot get quantized.

  2) The cosmological-constant problem.

  3) How to describe primordial universe (inflation)

  4) Physics of Dark Matter

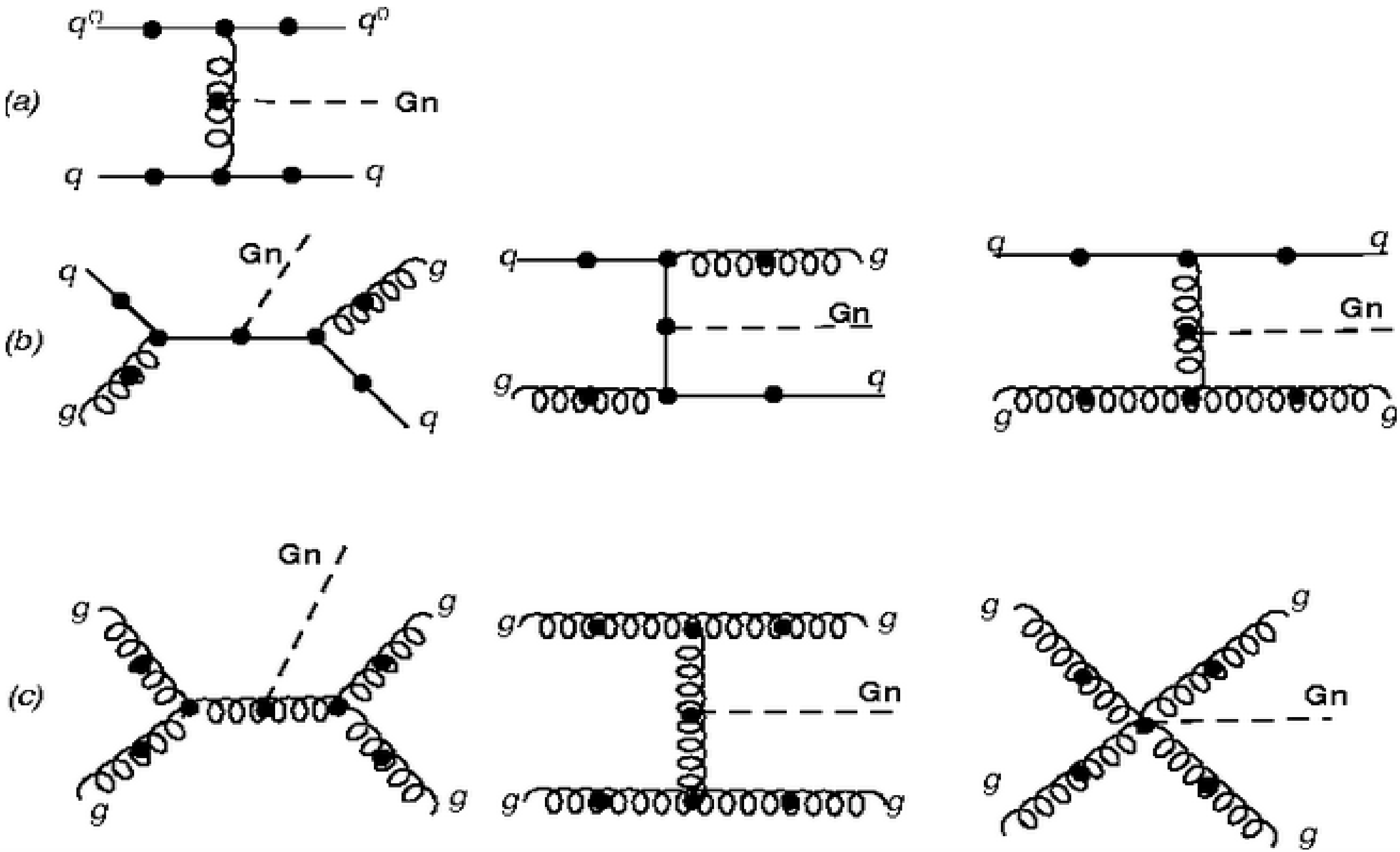
  5) A huge amount of accumulating data suggest possible tensions:

  H0, fσ8

Challenges for ΛCDM Beyond H_0 and S_8
A. The $A_{\rm lens}$ Anomaly in the CMB Angular Power Spectrum
B. Hints for a Closed Universe from *Planck* Data
C. Large-Angular-Scale Anomalies in the CMB Temperature and Polarization
  1. The Lack of Large-Angle CMB Temperature Correlations
  2. Hemispherical Power Asymmetry
  3. Quadrupole and Octopole Anomalies
  4. Point-Parity Anomaly
  5. Variation in Cosmological Parameters Over the Sky
  6. The Cold Spot
  7. Explaining the Large-Angle Anomalies
  8. Predictions and Future Testability
  9. Summary
D. Abnormal Oscillations of Best Fit Parameter Values
E. Anomalously Strong ISW Effect
F. Cosmic Dipoles
  1. The α Dipole
  2. Galaxy Cluster Anisotropies and Anomalous Bulk Flows
  3. Radio Galaxy Cosmic Dipole
  4. QSO Cosmic Dipole and Polarisation Alignments
  5. Dipole in SNIa
  6. Emergent Dipole in H_0
  7. CMB Dipole: Intrinsic Versus Kinematic?
G. The Ly-α Forest BAO and CMB Anomalies
  1. The Ly-α Forest BAO Anomaly
  2. Ly-α–*Planck* 2018 Tension in n_s–Ω_m
H. Parity Violating Rotation of CMB Linear Polarization
I. The Lithium Problem
J. Quasars Hubble Diagram Tension with Planck-ΛCDM
K. Oscillating Force Signals in Short Range Gravity Experiments
L. ΛCDM and the Dark Matter Phenomenon at Galactic Scales

[L. Perivolaropoulos , F. Scara,New Astron.Rev (2022), 2105.05208 [astro-ph.CO]]

Can General Relativity be quantized?



COSMOLOGICAL CONSTANT PROBLEM

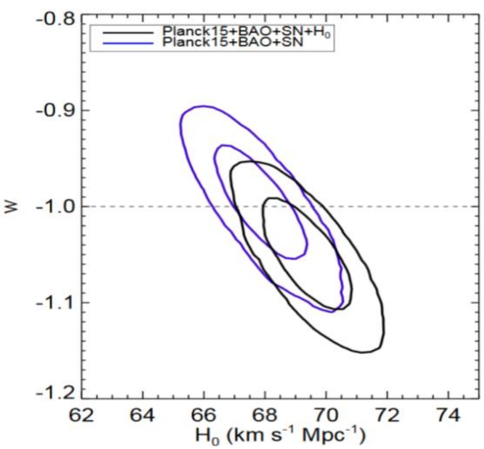
$$E_n \sim (n + 1/2) h\omega(k)$$

$$\rho_\Lambda(th) \sim M_p^4$$

$$\rho_\Lambda^0 \sim 10^{-120} \rho_\Lambda^{th}$$

H0 tension

- Tension (5σ!) between the data (direct measurements) and Planck/ΛCDM (indirect measurements). The data indicate a lack of "gravitational power".



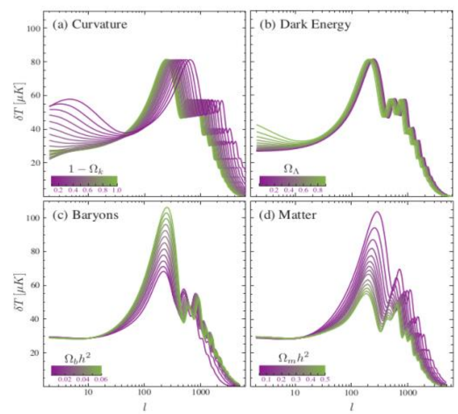
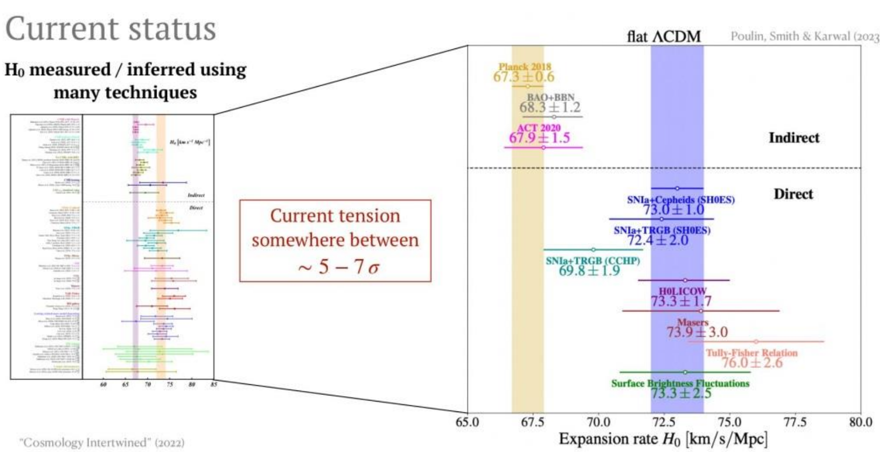
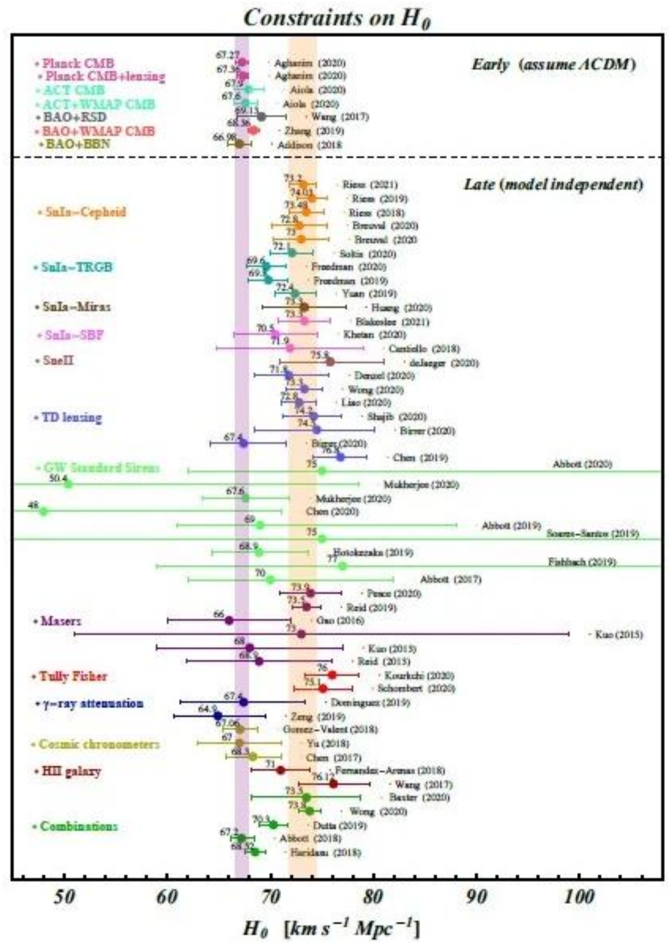


Figure 26. The CMB power spectrum as a function of cosmological parameters

[Riess et al, Astrophys.J 826]

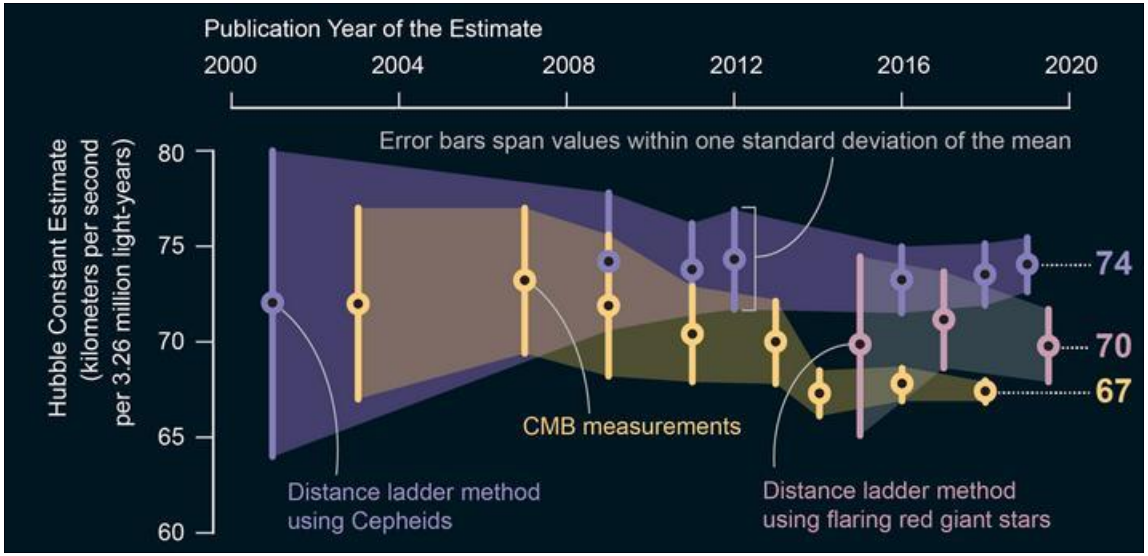




[Abdalla et al, JHEAp (2022)]

H0 tension

- Tension between the data (direct measurements) and Planck/ΛCDM (indirect measurements). This tension could be due to systematics.
- If not systematics then we may need changes in ΛCDM in early or late time behavior. 5σ seems to be very serious!



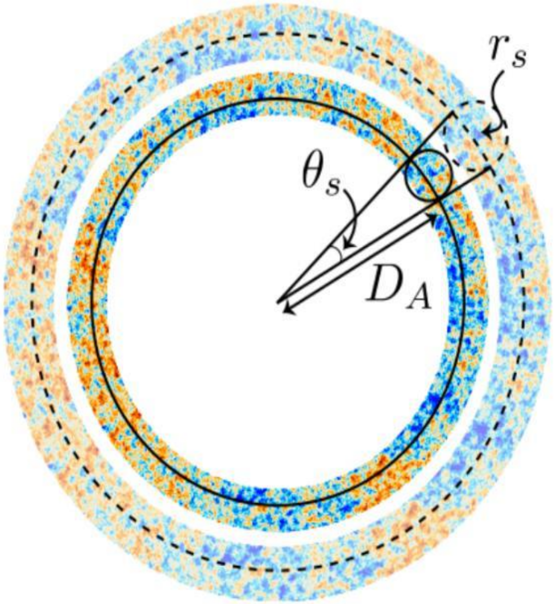
- Change early or late Universe physics. Higher number of effective relativistic species, dynamical dark energy, non-zero curvature, etc.
- The data indicate a lack of "gravitational power". Modified Gravity.

Restoring cosmological concordance

Is LCDM Wrong?

$$\theta_s = \frac{r_s}{D_A}$$

0.04% precision



$$r_s \propto \int_0^{t_{\rm recom}} dt \frac{c_s(t)}{\rho(t)}$$

$$D_A \propto \frac{1}{H_0} \int_{t_{\rm recom}}^{t_{\rm today}} dt \frac{1}{\rho(t)}$$

How do we increase H0?

Decrease sound horizon (r_s)

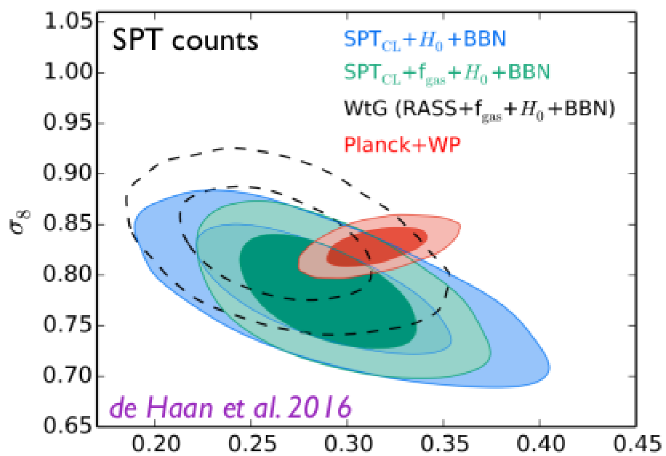
"Early time solutions"

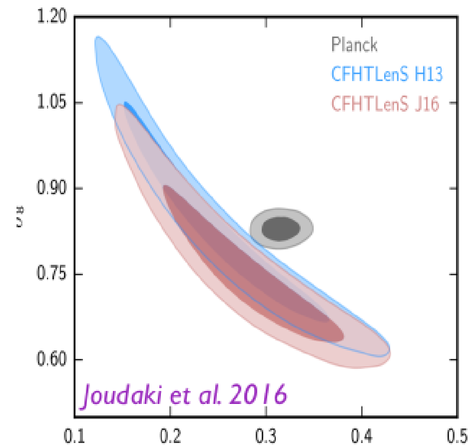
Increase integral in angular diameter distance (D_A)

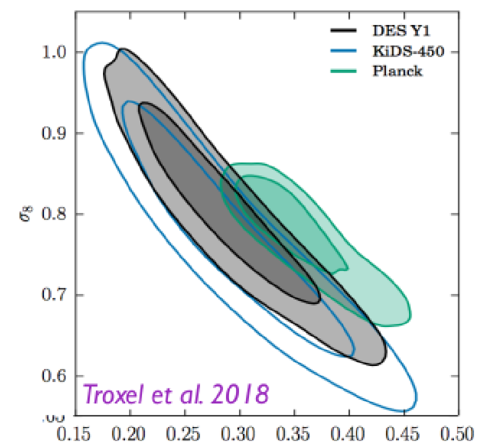
"Late time solutions"

S8 Tension

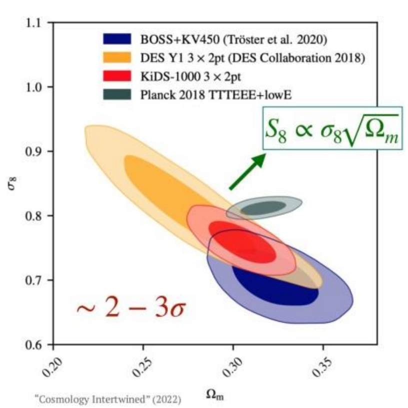
- Tension between direct data and Planck/ΛCDM estimation. The data indicate less matter clustering in structures at intermediate-small cosmological scales.

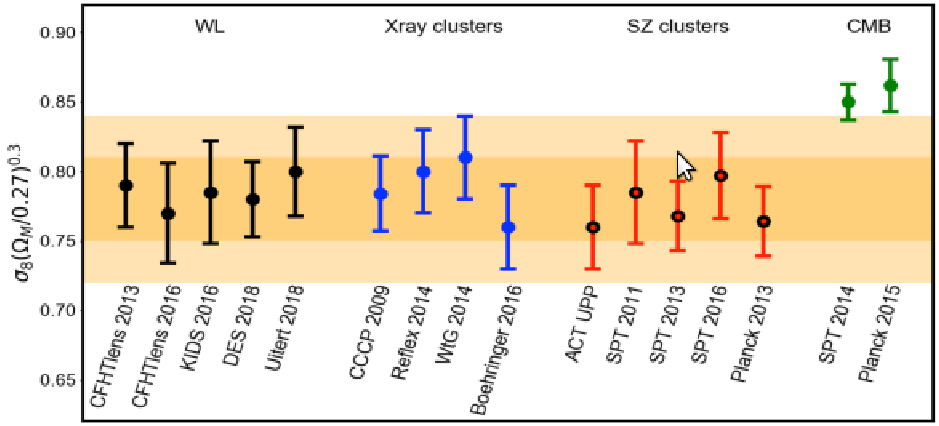






The S8 Tension





S8 Tension

TABLE II: A compilation of RSD data that we found published from 2006 since 2018

Index	Dataset	z	$f\sigma_8(z)$	Refs.	Year	Fiducial Cosmology
1	SDSS-LRG	0.35	0.440 ± 0.050	[75]	30 October 2006	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.25, 0, 0.756)$[76]
2	VVDS	0.77	0.490 ± 0.18	[75]	6 October 2009	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.25, 0, 0.78)$
3	2dFGRS	0.17	0.510 ± 0.060	[75]	6 October 2009	$(\Omega_{0m}, \Omega_K) = (0.3, 0, 0.9)$
4	2MRS	0.02	0.314 ± 0.048	[77], [78]	13 Novemver 2010	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.266, 0, 0.65)$
5	SnIa+IRAS	0.02	0.398 ± 0.065	[79], [78]	20 October 2011	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.3, 0, 0.814)$
6	SDSS-LRG-200	0.25	0.3512 ± 0.0583	[80]	9 December 2011	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.276, 0, 0.8)$
7	SDSS-LRG-200	0.37	0.4602 ± 0.0378	[80]	9 December 2011	
8	SDSS-LRG-60	0.25	0.3665 ± 0.0601	[80]	9 December 2011	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.276, 0, 0.8)$
9	SDSS-LRG-60	0.37	0.4031 ± 0.0586	[80]	9 December 2011	
10	WiggleZ	0.44	0.413 ± 0.080	[46]	12 June 2012	$(\Omega_{0m}, h, \sigma_8) = (0.27, 0.71, 0.8)$
11	WiggleZ	0.60	0.390 ± 0.063	[46]	12 June 2012	$C_{ij} = Eq.(3.3)$
12	WiggleZ	0.73	0.437 ± 0.072	[46]	12 June 2012	
13	6dFGS	0.067	0.423 ± 0.055	[81]	4 July 2012	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.27, 0, 0.76)$
14	SDSS-BOSS	0.30	0.407 ± 0.055	[82]	11 August 2012	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.25, 0, 0.804)$
15	SDSS-BOSS	0.40	0.419 ± 0.041	[82]	11 August 2012	
16	SDSS-BOSS	0.50	0.427 ± 0.043	[82]	11 August 2012	
17	SDSS-BOSS	0.60	0.433 ± 0.067	[82]	11 August 2012	
18	Vipers	0.80	0.470 ± 0.080	[83]	9 July 2013	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.25, 0, 0.82)$
19	SDSS-DR7-LRG	0.35	0.429 ± 0.089	[84]	8 August 2013	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.25, 0, 0.809)$[85]
20	GAMA	0.18	0.360 ± 0.090	[86]	22 September 2013	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.27, 0, 0.8)$
21	GAMA	0.38	0.440 ± 0.060	[86]	22 September 2013	
22	BOSS-LOWZ	0.32	0.384 ± 0.095	[87]	17 December 2013	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.274, 0, 0.8)$
23	SDSS DR10 and DR11	0.32	0.48 ± 0.10	[87]	17 December 2013	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.274, 0, 0.8)$[88]
24	SDSS DR10 and DR11	0.57	0.417 ± 0.045	[87]	17 December 2013	
25	SDSS-MGS	0.15	0.490 ± 0.145	[89]	30 January 2015	$(\Omega_{0m}, h, \sigma_8) = (0.31, 0.67, 0.83)$
26	SDSS-veloc	0.10	0.370 ± 0.130	[90]	16 June 2015	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.3, 0, 0.89)$[91]
27	FastSound	1.40	0.482 ± 0.116	[92]	25 November 2015	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.27, 0, 0.82)$[93]
28	SDSS-CMASS	0.59	0.488 ± 0.060	[94]	8 July 2016	$(\Omega_{0m}, h, \sigma_8) = (0.307115, 0.6777, 0.8288)$
29	BOSS DR12	0.38	0.497 ± 0.045	[2]	11 July 2016	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.31, 0, 0.8)$
30	BOSS DR12	0.51	0.458 ± 0.038	[2]	11 July 2016	
31	BOSS DR12	0.61	0.436 ± 0.034	[2]	11 July 2016	
32	BOSS DR12	0.38	0.477 ± 0.051	[95]	11 July 2016	$(\Omega_{0m}, h, \sigma_8) = (0.31, 0.676, 0.8)$
33	BOSS DR12	0.51	0.453 ± 0.050	[95]	11 July 2016	
34	BOSS DR12	0.61	0.410 ± 0.044	[95]	11 July 2016	
35	Vipers v7	0.76	0.440 ± 0.040	[55]	26 October 2016	$(\Omega_{0m}, \sigma_8) = (0.308, 0.8149)$
36	Vipers v7	1.05	0.280 ± 0.080	[55]	26 October 2016	
37	BOSS LOWZ	0.32	0.427 ± 0.056	[96]	26 October 2016	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.31, 0, 0.8475)$
38	BOSS CMASS	0.57	0.426 ± 0.029	[96]	26 October 2016	
39	Vipers	0.727	0.296 ± 0.0765	[97]	21 November 2016	$(\Omega_{0m}, \Omega_K, \sigma_8) = (0.31, 0, 0.7)$
40	6dFGS+SnIa	0.02	0.428 ± 0.0465	[98]	29 November 2016	$(\Omega_{0m}, h, \sigma_8) = (0.3, 0.683, 0.8)$
41	Vipers	0.6	0.48 ± 0.12	[99]	16 December 2016	$(\Omega_{0m}, \Omega_b, n_s, \sigma_8) = (0.3, 0.045, 0.96, 0.831)$[12]
42	Vipers	0.86	0.48 ± 0.10	[99]	16 December 2016	
43	Vipers PDR-2	0.60	0.550 ± 0.120	[100]	16 December 2016	$(\Omega_{0m}, \Omega_b, \sigma_8) = (0.3, 0.045, 0.823)$
44	Vipers PDR-2	0.86	0.400 ± 0.110	[100]	16 December 2016	
45	SDSS DR13	0.1	0.48 ± 0.16	[101]	22 December 2016	$(\Omega_{0m}, \sigma_8) = (0.25, 0.89)$[91]
46	2MTF	0.001	0.505 ± 0.085	[102]	16 June 2017	$(\Omega_{0m}, \sigma_8) = (0.3121, 0.815)$
47	Vipers PDR-2	0.85	0.45 ± 0.11	[103]	31 July 2017	$(\Omega_b, \Omega_{0m}, h) = (0.045, 0.30, 0.8)$
48	BOSS DR12	0.31	0.469 ± 0.098	[49]	15 September 2017	$(\Omega_{0m}, h, \sigma_8) = (0.307, 0.6777, 0.8288)$
49	BOSS DR12	0.36	0.474 ± 0.097	[49]	15 September 2017	
50	BOSS DR12	0.40	0.473 ± 0.086	[49]	15 September 2017	
51	BOSS DR12	0.44	0.481 ± 0.076	[49]	15 September 2017	
52	BOSS DR12	0.48	0.482 ± 0.067	[49]	15 September 2017	
53	BOSS DR12	0.52	0.488 ± 0.065	[49]	15 September 2017	
54	BOSS DR12	0.56	0.482 ± 0.067	[49]	15 September 2017	
55	BOSS DR12	0.59	0.481 ± 0.066	[49]	15 September 2017	
56	BOSS DR12	0.64	0.486 ± 0.070	[49]	15 September 2017	
57	SDSS DR7	0.1	0.376 ± 0.038	[104]	12 December 2017	$(\Omega_{0m}, \Omega_b, \sigma_8) = (0.282, 0.046, 0.817)$
58	SDSS-IV	1.52	0.420 ± 0.076	[105]	8 January 2018	$(\Omega_{0m}, \Omega_b h^2, \sigma_8) = (0.26479, 0.02258, 0.8)$
59	SDSS-IV	1.52	0.396 ± 0.079	[106]	8 January 2018	$(\Omega_{0m}, \Omega_b h^2, \sigma_8) = (0.31, 0.022, 0.8225)$
60	SDSS-IV	0.978	0.379 ± 0.176	[107]	9 January 2018	$(\Omega_{0m}, \sigma_8) = (0.31, 0.8)$
61	SDSS-IV	1.23	0.385 ± 0.099	[107]	9 January 2018	
62	SDSS-IV	1.526	0.342 ± 0.070	[107]	9 January 2018	
63	SDSS-IV	1.944	0.364 ± 0.106	[107]	9 January 2018	

- Model Dependence: Distance to galaxies is not measured directly, so a cosmological model is assumed in order to infer distances (ΛCDM with different parameters).
- Double counting: Some data points correspond to the same sample of galaxies analyzed by different groups/methods etc.

[Kazantzidis, Perivolaropoulos, PRD97]

Tension2 – $f\sigma 8$

- Tension between the data and Planck/ΛCDM.
- This tension could be due to systematics.

- If not systematics, the data less matter clustering in structures at intermediate-small cosmological scales (expressed as smaller Ωm at $z<0.6$, or smaller $\sigma 8$, or $wDE<-1$).

- It could be reconciled by a mechanism that reduces the rate of clustering between recombination and today: Hot Dark Matter, Dark Matter that clusters differently at small scales, or Modified Gravity.

Possible Solutions of H0 and S8 tensions

tension $\leq 1\sigma$ *"Excellent models"*	tension $\leq 2\sigma$ *"Good models"*	tension $\leq 3\sigma$ *"Promising models"*
Dark energy in extended parameter spaces [289]	Early Dark Energy [235]	Early Dark Energy [229]
Dynamical Dark Energy [309]	Phantom Dark Energy [11]	Decaying Warm DM [474]
Metastable Dark Energy [314]	Dynamical Dark Energy [11, 281, 309]	Neutrino-DM Interaction [506]
PEDE [392, 394]	GEDE [397]	Interacting dark radiation [517]
Elaborated Vacuum Metamorphosis [400–402]	Vacuum Metamorphosis [402]	Self-Interacting Neutrinos [700, 701]
IDE [314, 636, 637, 639, 652, 657, 661–663]	IDE [314, 653, 656, 661, 663, 670]	IDE [656]
Self-interacting sterile neutrinos [711]	Critically Emergent Dark Energy [997]	Unified Cosmologies [747]
Generalized Chaplygin gas model [744]	$f(\mathcal{T})$ gravity [814]	Scalar-tensor gravity [856]
Galileon gravity [876, 882]	Über-gravity [59]	Modified recombination [986]
Power Law Inflation [966]	Reconstructed PPS [978]	Super ΛCDM [1007]
$f(\mathcal{T})$ [818]		Coupled Dark Energy [650]

Early Dark Energy [228, 235, 240, 250]	Early Dark Energy [212, 229, 236, 263]	DE in extended parameter spaces [289]
Exponential Acoustic Dark Energy [259]	Rock 'n' Roll [242]	Dynamical Dark Energy [281, 309]
Phantom Crossing [315]	New Early Dark Energy [247]	Holographic Dark Energy [350]
Late Dark Energy Transition [317]	Acoustic Dark Energy [257]	Swampland Conjectures [370]
Metastable Dark Energy [314]	Dynamical Dark Energy [309]	MEDE [399]
PEDE [394]	Running vacuum model [332]	Coupled DM - Dark radiation [534]
Vacuum Metamorphosis [402]	Bulk viscous models [340, 341]	Decaying Ultralight Scalar [538]
Elaborated Vacuum Metamorphosis [401, 402]	Holographic Dark Energy [350]	BD-ΛCDM [852]
Sterile Neutrinos [433]	Phantom Braneworld DE [378]	Metastable Dark Energy [314]
Decaying Dark Matter [481]	PEDE [391, 392]	Self-Interacting Neutrinos [700]
Neutrino-Majoron Interactions [509]	Elaborated Vacuum Metamorphosis [401]	Dark Neutrino Interactions [716]
IDE [637, 639, 657, 661]	IDE [659, 670]	IDE [634–636, 653, 656, 663, 669]
DM - Photon Coupling [685]	Interacting Dark Radiation [517]	Scalar-tensor gravity [855, 856]
$f(\mathcal{T})$ gravity theory [812]	Decaying Dark Matter [471, 474]	Galileon gravity [877, 881]
BD-ΛCDM [851]	DM - Photon Coupling [686]	Nonlocal gravity [886]
Über-Gravity [59]	Self-interacting sterile neutrinos [711]	Modified recombination [986]
Galileon Gravity [875]	$f(\mathcal{T})$ gravity theory [817]	Effective Electron Rest Mass [989]
Unimodular Gravity [890]	Über-Gravity [871]	Super ΛCDM [1007]
Time Varying Electron Mass [990]	VCDM [893]	Axi-Higgs [991]
$\Lambda\lambda$CDM [995]	Primordial magnetic fields [992]	Self-Interacting Dark Matter [479]
Ginzburg-Landau theory [996]	Early modified gravity [859]	Primordial Black Holes [545]
Lorentzian Quintessential Inflation [979]	Bianchi type I spacetime [999]	
Holographic Dark Energy [351]	$f(\mathcal{T})$ [818]	

Possible Solutions of H0 and S8 tensions

Early-Time Alternative Proposed Models

1. Axion Monodromy
2. Early Dark Energy
3. Extra Relativistic Degrees of Freedom
4. Modified Recombination History
5. New Early Dark Energy

Late-Time Alternative Proposed Models

1. Bulk Viscous Models
2. Chameleon Dark Energy
3. Clustering Dark Energy
4. Diffusion Models
5. Dynamical Dark Energy
6. Emergent Dark Energy
7. Graduated Dark Energy - AdS to dS Transition in the Late Universe
8. Holographic Dark Energy
9. Interacting Dark Energy
10. Quintessence Models and their Various Extensions
11. Running Vacuum Models
12. Time-Varying Gravitational Constant
13. Vacuum Metamorphosis

Modified Gravity Models

1. Effective Field Theory Approach to Dark Energy and Modified Gravity
2. $f(T)$ Gravity
3. Horndeski Theory
4. Quantum Conformal Anomaly Effective Theory and Dynamical Vacuum Energy
5. Ultra-Late Time Gravitational Transitions

Beyond the FLRW Framework

1. Cosmological Fitting and Averaging Problems
2. Data Analysis in an Universe with Structure: Accounting for Regional Inhomogeneity and Anisotropy
3. Local Void Scenario

Specific Solutions Assuming FLRW

1. Active and Sterile Neutrinos
2. Cannibal Dark Matter
3. Decaying Dark Matter
4. Dynamical Dark Matter
5. Extended Parameter Spaces Involving $A_{\rm lens}$
6. Cosmological Scenario with Features in the Primordial Power Spectrum
7. Interacting Dark Matter
8. Quantum Landscape Multiverse
9. Quantum Fisher Cosmology
10. Quartessence
11. Scaling Symmetry and a Mirror Sector
12. Self-Interacting Neutrinos
13. Self-Interacting Sterile Neutrinos
14. Soft Cosmology
15. Two-Body Decaying Cold Dark Matter into Dark Radiation and Warm Dark Matter

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Cosmology Intertwined:
A Review of the Particle Physics, Astrophysics, and Cosmology Associated with the Cosmological Tensions and Anomalies

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[Abdalla et al, JHEAp (2022)]

10 commandments for Hubble hunters

1. I am $H_0 \approx 74$ thy Goal
2. Thou shalt not fail to fit key data (BAO, SNeIa, polarization)...
3. ...or include a local H_0 prior in vain
4. Remember to not just blow up the uncertainty on H_0...
5. ...honour its central value, and keep an eye on your $\Delta\chi^2$/Bayesian evidence
6. Thou shalt not murder σ_8/S_8...
7. ...but aim to solve this and other tensions/puzzles at the same time
8. Thy solution shall come from a compelling particle/gravity model...
9. ...which makes verifiable predictions...
10. ...which later better be verified!

Credits: Gustave Doré

Efficient model independent requirements to solve the tensions

- In general, to avoid the H_0 tension one needs a positive correction to the first Friedmann equation at late times that could yield an increase in H_0 compared to the ΛCDM scenario.

Efficient model independent requirements to solve the tensions

- For the σ_8 tension, we recall that in any cosmological model, at sub-Hubble scales and through matter epoch, the equation that governs the evolution of matter perturbations in the linear regime is

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G_{\rm eff}\rho_m\delta \ , \qquad (1)$$

  where $G_{\rm eff}$ is the effective gravitational coupling given by a generalized Poisson equation.

- Solving for $\delta(a)$ provides the observable quantity $f\sigma_8(a)$, following the definitions $f(a) \equiv d\ln\delta(a)/d\ln a$ and $\sigma(a) = \sigma_8\delta(1)/\delta(a=1)$. Hence, alleviation of the σ_8 tension may be obtained if $G_{\rm eff}$ becomes smaller than G_N during the growth of matter perturbations and/or if the "friction" term in (1) increases.

General Relativity
Assumptions and Considerations

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R - 2\Lambda\right] + \int d^4x\, L_m\left(g_{\mu\nu}, \psi\right)$$

- Diffeomorphism invariance
- Spacetime dimensionality=4
- Geometry=Curvature (connection=Levi Civita)
- Linear in Ricci scalar
- Metric compatibility (zero non-metricity)
- Minimal matter coupling
- Equivalence principle
- Lorentz invariance
- Locality

Standard Model vs General Relativity Lagrangians

1

$$-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu +$$
$$\frac{1}{2}ig_s^2(\bar q^\sigma_i \gamma^\mu q^\sigma_j)g^a_\mu + \bar G^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar G^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu -$$

2

$$M^2 W^+_\mu W^-_\mu - \frac12\partial_\nu Z^0_\mu\partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \frac12\partial_\mu A_\nu\partial_\mu A_\nu - \frac12\partial_\mu H\partial_\mu H -$$
$$\frac12 m_h^2 H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- - \frac12\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h\left[\frac{2M^2}{g^2} +\right.$$
$$\left.\frac{2M}{g}H + \frac12(H^2+\phi^0\phi^0+2\phi^+\phi^-)\right] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu -$$
$$W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu -$$
$$W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \frac12 g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu +$$
$$\frac12 g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) +$$
$$g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu -$$
$$W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] -$$
$$\frac18 g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2] -$$
$$gMW^+_\mu W^-_\mu H - \frac12 g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \frac12 ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) -$$
$$W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac12 g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ -$$
$$\phi^+\partial_\mu H)] + \frac12 g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) +$$
$$igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) +$$
$$igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac14 g^2W^+_\mu W^-_\mu[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] -$$
$$\frac14 g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + (\phi^0)^2 + 2(2s_w^2-1)^2\phi^+\phi^-] - \frac12 g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- +$$
$$W^-_\mu\phi^+) - \frac12 ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac12 g^2 s_wA_\mu\phi^0(W^+_\mu\phi^- +$$
$$W^-_\mu\phi^+) + \frac12 ig^2s_wA_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- -$$
$$g^1 s_w^2 A_\mu A_\mu\phi^+\phi^- - \bar e^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar\nu^\lambda\gamma\partial\nu^\lambda - \bar u^\lambda_j(\gamma\partial + m_u^\lambda)u^\lambda_j -$$

3

$$\bar d^\lambda_j(\gamma\partial + m_d^\lambda)d^\lambda_j + igs_wA_\mu\left[-(\bar e^\lambda\gamma^\mu e^\lambda) + \frac23(\bar u^\lambda_j\gamma^\mu u^\lambda_j) - \frac13(\bar d^\lambda_j\gamma^\mu d^\lambda_j)\right] +$$
$$\frac{ig}{4c_w}Z^0_\mu[(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar e^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar u^\lambda_j\gamma^\mu(\tfrac43 s_w^2 -$$
$$1 - \gamma^5)u^\lambda_j) + (\bar d^\lambda_j\gamma^\mu(1 - \tfrac83 s_w^2 - \gamma^5)d^\lambda_j)] + \frac{ig}{2\sqrt2}W^+_\mu[(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) +$$
$$(\bar u^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \frac{ig}{2\sqrt2}W^-_\mu[(\bar e^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar d^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1+$$
$$\gamma^5)u^\lambda_j)] + \frac{ig}{2\sqrt2}\frac{m_e^\lambda}{M}[-\phi^+(\bar\nu^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar e^\lambda(1+\gamma^5)\nu^\lambda)] -$$

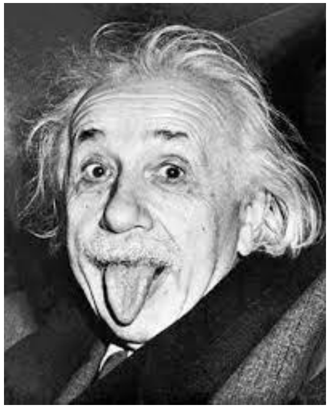
4

$$\frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar e^\lambda e^\lambda) + i\phi^0(\bar e^\lambda\gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt2}\phi^+[-m_d^\kappa(\bar u^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) +$$
$$m_u^\lambda(\bar u^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j] + \frac{ig}{2M\sqrt2}\phi^-[m_d^\lambda(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m_u^\kappa(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1-$$
$$\gamma^5)u^\kappa_j] - \frac g2\frac{m_u^\lambda}{M}H(\bar u^\lambda_j u^\lambda_j) - \frac g2\frac{m_d^\lambda}{M}H(\bar d^\lambda_j d^\lambda_j) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar u^\lambda_j\gamma^5 u^\lambda_j) -$$
$$\frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar d^\lambda_j\gamma^5 d^\lambda_j) + \bar X^+(\partial^2 - M^2)X^+ + \bar X^-(\partial^2 - M^2)X^- + \bar X^0(\partial^2 -$$

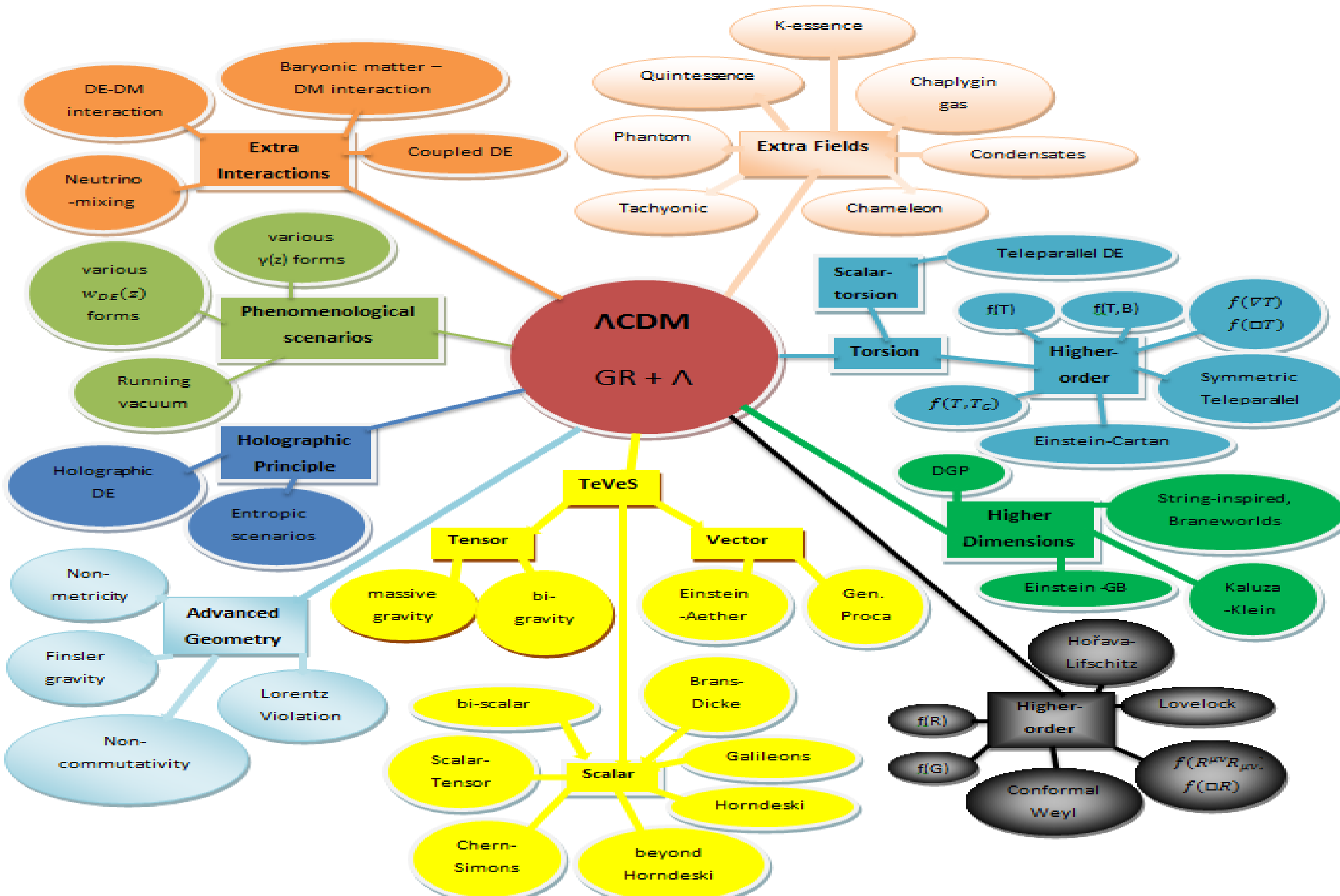
5

$$\frac{M^2}{c_w^2})X^0 + \bar Y\partial^2 Y + igc_wW^+_\mu(\partial_\mu\bar X^0 X^- - \partial_\mu\bar X^+ X^0) + igs_wW^+_\mu(\partial_\mu\bar Y X^- -$$
$$\partial_\mu\bar X^+ Y) + igc_wW^-_\mu(\partial_\mu\bar X^- X^0 - \partial_\mu\bar X^0 X^+) + igs_wW^-_\mu(\partial_\mu\bar X^- Y -$$
$$\partial_\mu\bar Y X^+) + igc_wZ^0_\mu(\partial_\mu\bar X^+ X^+ - \partial_\mu\bar X^- X^-) + igs_wA_\mu(\partial_\mu\bar X^+ X^+ -$$
$$\partial_\mu\bar X^- X^-) - \frac12 gM\left[\bar X^+X^+H + \bar X^-X^-H + \frac{1}{c_w^2}\bar X^0X^0H\right] +$$
$$\frac{1-2c_w^2}{2c_w}igM[\bar X^+X^0\phi^+ - \bar X^-X^0\phi^-] + \frac{1}{2c_w}igM[\bar X^0X^-\phi^+ - \bar X^0X^+\phi^-] +$$
$$igMs_w[\bar X^0X^-\phi^+ - \bar X^0X^+\phi^-] + \frac12 igM[\bar X^+X^+\phi^0 - \bar X^-X^-\phi^0]$$

$$S = -\frac{1}{16\pi G}\int \sqrt{-g}(R(g)+2\Lambda)\, d^4x$$



Modified Gravity



Scalar-Tensor Theories

- Field equations:

$$\phi G_{\mu\nu} + \left[\Diamond \phi + \frac{\omega}{2\phi} (\nabla \phi)^2 + V \right] g_{\mu\nu} - \nabla_\mu \nabla_\nu \phi - \frac{\omega}{\phi} \nabla_\mu \phi \nabla_\nu \phi = 8\pi \, T_{\mu\nu}$$

$$(2\omega + 3) \Box \phi + \omega' (\nabla \phi)^2 + 4V - 2\phi V' = 8\pi \, T$$

- For Brans-Dicke:

- PPN parameters: $\beta_{PPN} = 1, \ \gamma_{PPN} = \frac{1+\omega}{2+\omega} \ \Rightarrow \ \omega \geq 40000$
- Newton's constant: $G = \left(\frac{4+2\omega}{3+2\omega} \right) \frac{1}{\phi}$ with $\frac{\dot{G}}{G} \leq 1.7 \ 10^{-12} \ yr^{-1}$

Brans-Dicke Cosmology

- Friedmann-Robertson-Walker metric: $ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j$

- Friedmann equations:

$$H^2 = \frac{8\pi}{3\phi}\rho_m - H\frac{\dot{\phi}}{\phi} + \frac{\omega}{6}\frac{\dot{\phi}^2}{\phi^2} + \frac{V}{3\phi}$$

$$2\dot{H} + 3H^2 = -\frac{1}{\phi}\left(8\pi p_m + \frac{\omega}{2}\frac{\dot{\phi}^2}{\phi} + 2H\dot{\phi} + \ddot{\phi}\right) + \frac{V}{\phi}$$

- Scalar-field equation:

$$\ddot{\phi} + 3H\dot{\phi} - \frac{8\pi}{2\omega+3}\left(\rho_m - 3p_m\right) = 0 + \frac{2}{2\omega+3}\left(2V - \phi\frac{dV}{d\phi}\right)$$

- Matter equation: $\dot{\rho}_m + 3H\left(\rho_m + p_m\right) = 0$

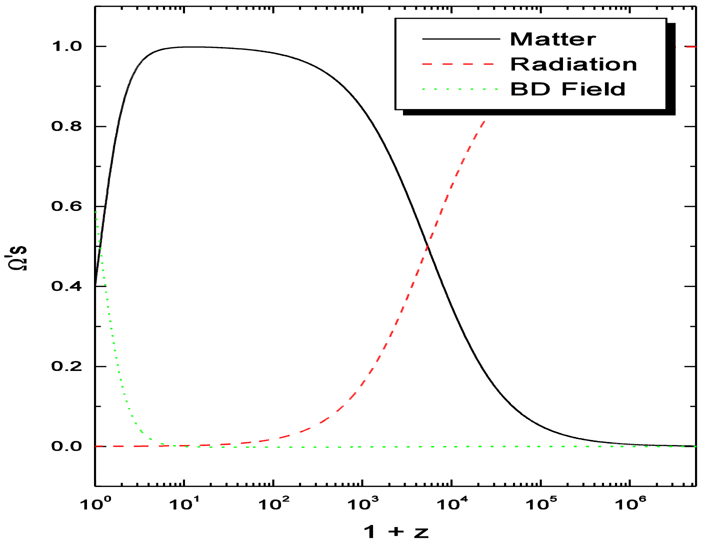
Dark Energy in Brans-Dicke Cosmology

- Effective Dark Energy sector:

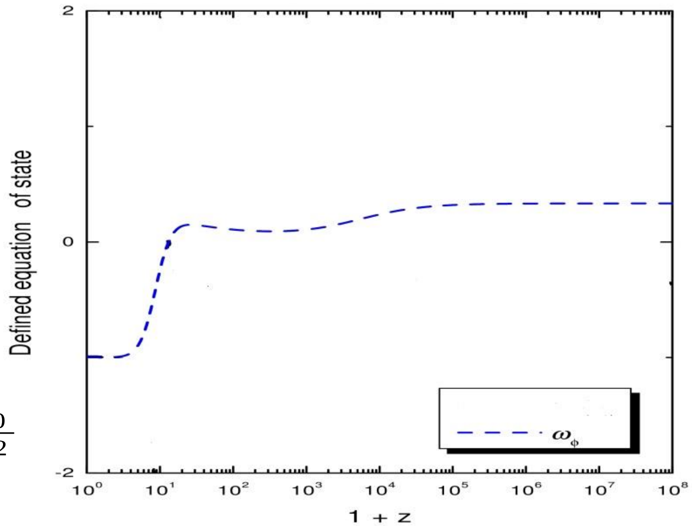
$$\rho_{DE} = \frac{3}{8\pi}\left(-H\dot{\phi} + \frac{\omega}{6}\frac{\dot{\phi}^2}{\phi}\right) + \frac{V}{8\pi}$$

$$p_{DE} = \frac{1}{8\pi}\left(\frac{\omega}{2}\frac{\dot{\phi}^2}{\phi} + 2H\dot{\phi} + \ddot{\phi}\right) - \frac{V}{8\pi}$$

$$\Rightarrow w_{DE} = \frac{p_{DE}}{\rho_{DE}}$$



$$V(\phi) = \frac{V_0}{\phi^2}$$



Scalar-Tensor Theories

- Most general 4D scalar-tensor theories having second-order field equations:

$$\boxed{L_H = \sum_{i=2}^{5} L_i}$$

$$L_2[K] = K(\phi, X)$$

$$L_3[G_3] = -G_3(\phi, X)\Diamond\phi$$

$$X = -\partial^\mu\phi\partial_\mu\phi/2$$

$$L_4[G_4] = G_4(\phi, X)R + G_{4,X}\left[(\Diamond\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right]$$

$$L_5[G_5] = G_5(\phi, X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}\left[(\Diamond\phi)^3 - 3(\Diamond\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)\right]$$

[G. Horndeski, Int. J. Theor. Phys. 10]

Horndeski Theories

- Most general 4D scalar-tensor theories having second-order field equations:

$$\boxed{L_H = \sum_{i=2}^{5} L_i}$$

$$L_2[K] = K(\phi, X)$$

$$L_3[G_3] = -G_3(\phi, X)\Diamond\phi \qquad X = -\partial^\mu\phi\partial_\mu\phi/2$$

$$L_4[G_4] = G_4(\phi, X)R + G_{4,X}\left[(\Diamond\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right]$$

$$L_5[G_5] = G_5(\phi, X)G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}\left[(\Diamond\phi)^3 - 3(\Diamond\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)(\nabla^\beta\nabla_\mu\phi)\right]$$

[G. Horndeski, Int. J. Theor. Phys. 10]

- Coincides with Generalized Galileon theories

$$\phi \to \phi + c, \quad \partial_\mu\phi \to \partial_\mu\phi + b_\mu$$

[Nicolis,Rattazzi,Trincherini, PRD 79]

Horndeski Cosmology (background)

- Field Equations: $L.H.S = R.H.S$
- In flat FRW:
- $$\boxed{\begin{aligned}&2XK_{,X}-K+6X\dot{\phi}HG_{3,X}-2XG_{3,\phi}-6H^2G_4+24H^2X(G_{4,X}+XG_{4,XX})-12HX\dot{\phi}G_{4,\phi X}-6H\dot{\phi}G_{4,\phi}\\&+2H^3X\dot{\phi}(5G_{5,X}+2XG_{5,XX})-6H^2X(3G_{5,\phi}+2XG_{5,\phi X})=-\rho_m\end{aligned}}$$
- $$\boxed{\begin{aligned}&K-2X(G_{3,\phi}+\ddot{\phi}G_{3,X})+2(3H^2+2\dot{H})G_4-12H^2XG_{4,X}-4H\dot{X}G_{4,X}-8\dot{H}XG_{4,X}-8HX\dot{X}G_{4,XX}\\&+2(\ddot{\phi}+2H\dot{\phi})G_{4,\phi}+4XG_{4,\phi\phi}+4X(\ddot{\phi}-2H\dot{\phi})G_{4,\phi X}-2X(2H^3\dot{\phi}+2H\dot{H}\dot{\phi}+3H^2\ddot{\phi})G_{5,X}-4H^2X^2\ddot{\phi}G_{5,XX}\\&+4HX(\dot{X}-HX)G_{5,\phi X}+2\left[2(\dot{H}X+H\dot{X})+3H^2X\right]G_{5,\phi}+4HX\dot{\phi}G_{5,\phi\phi}=-p_m\end{aligned}}$$
- $$\boxed{\frac{1}{a^3}\frac{d}{dt}(a^3J)=P_\phi}$$

with $J=\dot{\phi}K_{,X}+6HXG_{3,X}-2\dot{\phi}G_{3,\phi}+6H^2\dot{\phi}(G_{4,X}+2XG_{4,XX})-12HXG_{4,\phi X}+2H^3X(3G_{5,X}+2XG_{5,XX})-6H^2\dot{\phi}(G_{5,\phi}+XG_{5,\phi X})$

$P_\phi=K_{,\phi}-2X(G_{3,\phi\phi}+\ddot{\phi}G_{3,\phi X})+6(2H^2+\dot{H})G_{4,\phi}+6H(\dot{X}+2HX)G_{4,\phi X}-6H^2XG_{5,\phi\phi}+2H^3X\dot{\phi}G_{5,\phi X}$

[De Felice,Tsujikawa JCAP 1202]

Horndeski Cosmology (perturbations)

- Scalar perturbations: $ds^2=-(1+2\psi)dt^2+a^2(1-2\phi)\delta_{ij}dx^i dx^j$ $\quad\Rightarrow L.H.S=R.H.S$
- No-ghost condition: $\boxed{Q_S\equiv\frac{w_1\left(4w_1w_3+9w_2^2\right)}{3w_2^2}>0}$
- No Laplacian instabilities condition: $\boxed{c_S^2\equiv\frac{3\left(2w_1^2w_2H-4w_2^2w_4+4w_1w_2\dot{w}_1-2w_1^2\dot{w}_2\right)-6w_1^2(\rho_m+p_m)}{w_1\left(4w_1w_3+9w_2^2\right)}>0}$

with $\quad w_1\equiv2\left(G_4-2XG_{4,X}\right)-2X\left(G_{5,X}\dot{\phi}H-G_{5,\phi}\right)$

$$w_2\equiv-2G_{3,X}X\dot{\phi}+4G_4H-16X^2G_{4,XX}H+4\left(\dot{\phi}G_{4,\phi X}-4HG_{4,X}\right)X+2G_{4,\phi}\dot{\phi}$$
$$+8X^2G_{5,\phi X}H+2HX\left(6G_{5,\phi}-5HG_{5,X}\dot{\phi}\right)-4G_{5,XX}\dot{\phi}X^2H^2$$

$$w_3\equiv3X\left(K_{,X}+2XK_{,XX}\right)+6X\left(3X\dot{\phi}HG_{3,XX}-G_{3,\phi X}X-G_{3,\phi}+6\dot{\phi}HG_{3,X}\right)$$
$$+18H\left(4HX^3G_{4,XXX}-HG_4-5X\dot{\phi}G_{4,\phi X}-G_{4,\phi}\dot{\phi}+7HG_{4,X}X+16HX^2G_{4,XX}-2X^2\dot{\phi}G_{4,X\phi X}\right)$$
$$+6H^2X\left(2H\dot{\phi}G_{5,XXX}X^2-6X^2G_{5,\phi XX}+13XH\dot{\phi}G_{5,XX}-27G_{5,\phi X}X+15H\dot{\phi}G_{5,X}-18G_{5,\phi}\right)$$

$$w_4\equiv2G_4-2XG_{5,\phi}-2XG_{5,X}\ddot{\phi}$$

Beyond Horndeski Theories

- Beyond Horndeski, free from Ostrogradski instabilities but still propagating 2+1 dof's: $\boxed{L_{BH} = \sum_{i=2}^{5} L_i}$

$$L_2 = L_2^H[A_2]$$

$$L_3 = L_3^H[C_3 + 2XC_{3,X}] + L_2^H[XC_{3,\phi}]$$

$$L_4 = L_4^H[B_4] + L_3^H[C_4 + 2XC_{4,X}] + L_2^H[XC_{4,\phi}] - \frac{B_4 + A_4 - 2XB_{4,X}}{X^2} L^{gal1}$$

$$L_5 = L_5^H[G_4] + L_4^H[C_5] + L_3^H[D_5 + 2XD_{5,X}] + L_2^H[XD_{5,\phi}] + \frac{XB_{5,X} + 3A_5}{3(-X)^{5/2}} L^{gal2}$$

$$X = -\partial^\mu\phi\partial_\mu\phi/2$$

$$A_i = A_i(\phi, X)$$

$$B_i = B_i(\phi, X)$$

with

$$L^{gal1} = X\left[(\Diamond\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right] - 2\left[(\nabla^\mu\phi\nabla^\nu\phi)(\nabla_\mu\nabla_\nu\phi)(\Diamond\phi) - (\nabla^\mu\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla_\lambda\phi)(\nabla^\lambda\nabla^\nu\phi)\right]$$

$$L^{gal2} = X\left[(\Diamond\phi)^3 - 3(\Diamond\phi)(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla_\mu\nabla_\nu\phi)(\nabla^\nu\nabla^\rho\phi)(\nabla^\mu\nabla_\rho\phi)\right]$$
$$-3\begin{bmatrix}(\Diamond\phi)^2(\nabla_\mu\phi)(\nabla^\mu\nabla^\nu\phi)(\nabla_\nu\phi) - 2(\Diamond\phi)(\nabla_\mu\phi)(\nabla^\mu\nabla^\nu\phi)(\nabla_\nu\nabla_\rho\phi)(\nabla^\rho\phi)\\ -(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)(\nabla_\rho\phi)(\nabla^\rho\nabla^\lambda\phi)(\nabla_\lambda\phi) + 2(\nabla_\mu\phi)(\nabla^\mu\nabla^\nu\phi)(\nabla_\nu\nabla_\rho\phi)(\nabla^\rho\nabla^\lambda\phi)(\nabla_\lambda\phi)\end{bmatrix}$$

$$C_3 = \frac{1}{2}\int A_3(-X)^{-3/2}dX \qquad C_5 = -\frac{1}{4}X\int B_{5,\phi}(-X)^{-3/2}dX$$

$$C_4 = -\int B_{4,\phi}(-X)^{-1/2}dX \qquad D_5 = -\int C_{5,\phi}(-X)^{-1/2}dX \qquad G_5 = -\int B_{5,X}(-X)^{-1/2}dX$$

- Primary constraint prevents the propagation of extra degrees of freedom

[Gleyzes,Langlois,Piazza,Vernizzi, PRL 114], [Crisostomi,Hull,Koyama,Tasinato, JCAP 1603]

Solving H0 tensions in Horndeski Gravity

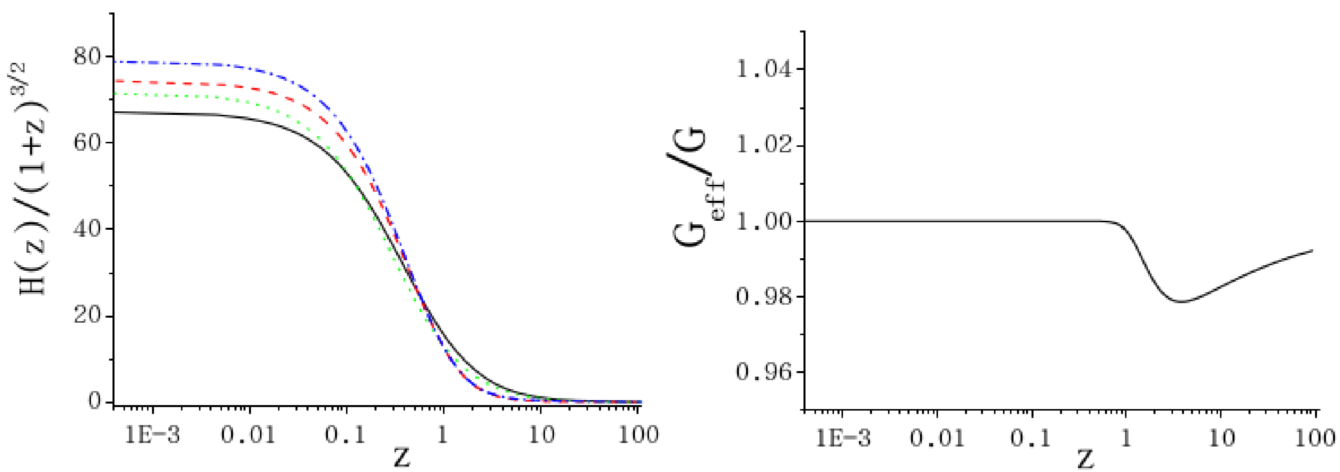
$G_4 = 1/(16\pi G)$ and $G_3 = 0$, $\quad K = -V(\phi) + X$

$\rho_{DE} = 2X - K + 2H^3 X\dot{\phi}(5G_{5,X} + 2XG_{5,XX}),$

$p_{DE} = K - 2XG_{5,X}\left(2H^3\dot{\phi} + 2H\dot{H}\dot{\phi} + 3H^2\ddot{\phi}\right) - 4H^2X^2\ddot{\phi}G_{5,XX}$

$$\frac{G_{eff}}{G} = \frac{1}{2}\left(G_4 - 2XG_{4,X} + XG_{5,\phi} - \dot{\phi}HXG_{5,X}\right)^{-1}$$

- Model I: $G_5(X) = \xi X^2$



[M. Petronikolou, S. Basilakos, E.N.Saridakis, PRD 106]

Solving H0 tensions in Horndeski Gravity

$$S=\int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R+G_2(X)+G_3(X)\Box\phi\right]$$

$$G_2(X)=-c_2M_2^{4(1-p)}(-X/2)^p\,,\qquad G_3(X)=-c_3M_3^{1-4p_3}(-X/2)^{p_3}$$

$$H_0=72^{+8}_{-5}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}\ \text{at 95\% CL,}$$

[N. Frusciante, S. Peirone, L. Atayde, A. De Felice, PRD 101]

$$G_2(X)=a_1X+a_2X^2\,,\qquad G_3(X)=3a_3X$$

$$H_0=\left(69.3^{+3.6}_{-3.0}\right)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}\ \text{at 95\%}$$

[S. Peirone, G. Benevento, N. Frusciante, S. Tsujikawa, PRD 100]

Bi-scalar Theories

- Modified gravity propagating 2+2 dof's $$\boxed{S=\int d^4x\sqrt{-g}\, f\left(R,(\nabla R)^2,\Diamond R\right)}$$
- For $f\left(R,(\nabla R)^2,\Diamond R\right)=K\left(R,(\nabla R)^2\right)+Q\left(R,(\nabla R)^2\right)\Diamond R$

[Naruko,Yoshida,Mukohyama CQG 33]

$$\boxed{\Rightarrow S=\int d^4x\sqrt{-g}\left[\frac{1}{2}\hat{R}-\frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-\frac{1}{\sqrt{6}}e^{-\sqrt{2/3}\chi}\hat{g}^{\mu\nu}Q\partial_\mu\chi\partial_\nu\phi+\frac{1}{4}e^{-2\sqrt{2/3}\chi}K+\frac{1}{2}e^{-\sqrt{2/3}\chi}Q\hat{\Diamond}\phi-\frac{1}{4}e^{-\sqrt{2/3}\chi}\phi\right]}$$

$$K=K(\phi,B),\ G=G(\phi,B),\ B=2e^{\sqrt{2/3}\chi}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$$

Bi-scalar Theories

- Modified gravity propagating 2+2 dof's $$\boxed{S=\int d^4x\sqrt{-g}\, f\left(R,(\nabla R)^2,\Box R\right)}$$

- For $f\left(R,(\nabla R)^2,\Box R\right)=K\left(R,(\nabla R)^2\right)+Q\left(R,(\nabla R)^2\right)\Box R$ [Naruko,Yoshida,Mukohyama CQG 33]

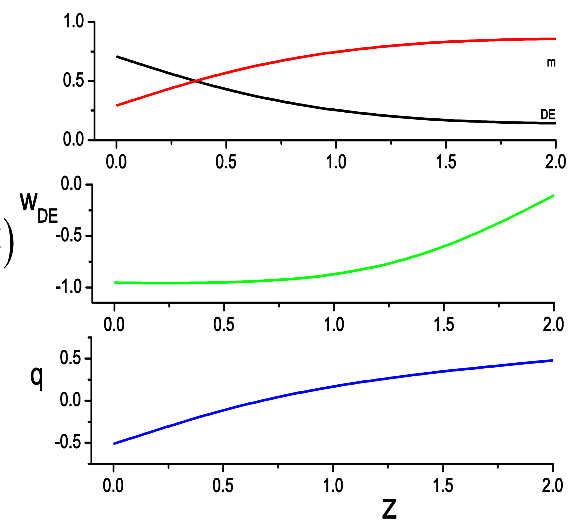
$$\boxed{\Rightarrow S=\int d^4x\sqrt{-g}\left[\frac{1}{2}\hat{R}-\frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-\frac{1}{\sqrt{6}}e^{-\sqrt{2/3}\chi}\hat{g}^{\mu\nu}Q\partial_\mu\chi\partial_\nu\phi+\frac{1}{4}e^{-2\sqrt{2/3}\chi}K+\frac{1}{2}e^{-\sqrt{2/3}\chi}Q\hat{\Box}\phi-\frac{1}{4}e^{-\sqrt{2/3}\chi}\phi\right]}$$

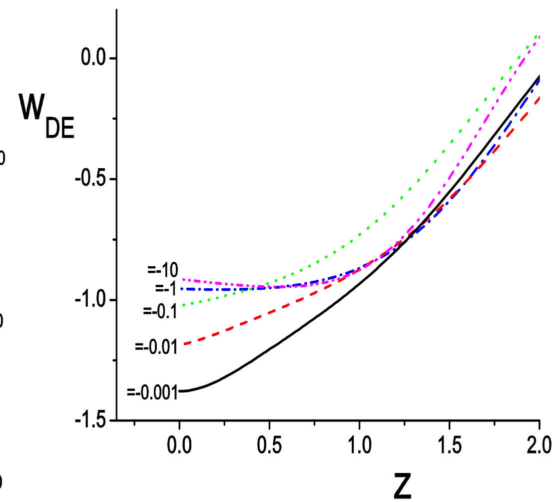
$$K=K(\phi,B),\ G=G(\phi,B),\ B=2e^{\sqrt{2/3}\chi}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$$

- eg.: $K(\phi,B)=\dfrac{\phi}{2},\ G(\phi,B)=\xi B$

$$\rho_{DE}=\frac{1}{2}\dot{\chi}^2-\frac{1}{8}e^{-2\sqrt{2/3}\chi}\left(1-2e^{\sqrt{2/3}\chi}\right)\phi-\xi\dot{\phi}^3\left(\sqrt{6}\dot{\chi}-6H\right)$$

$$p_{DE}=\frac{1}{2}\dot{\chi}^2+\frac{1}{8}e^{-2\sqrt{2/3}\chi}\left(1-2e^{\sqrt{2/3}\chi}\right)\phi-\frac{1}{3}\xi\dot{\phi}^2\left(\sqrt{6}\dot{\phi}\dot{\chi}+6\ddot{\phi}\right)$$

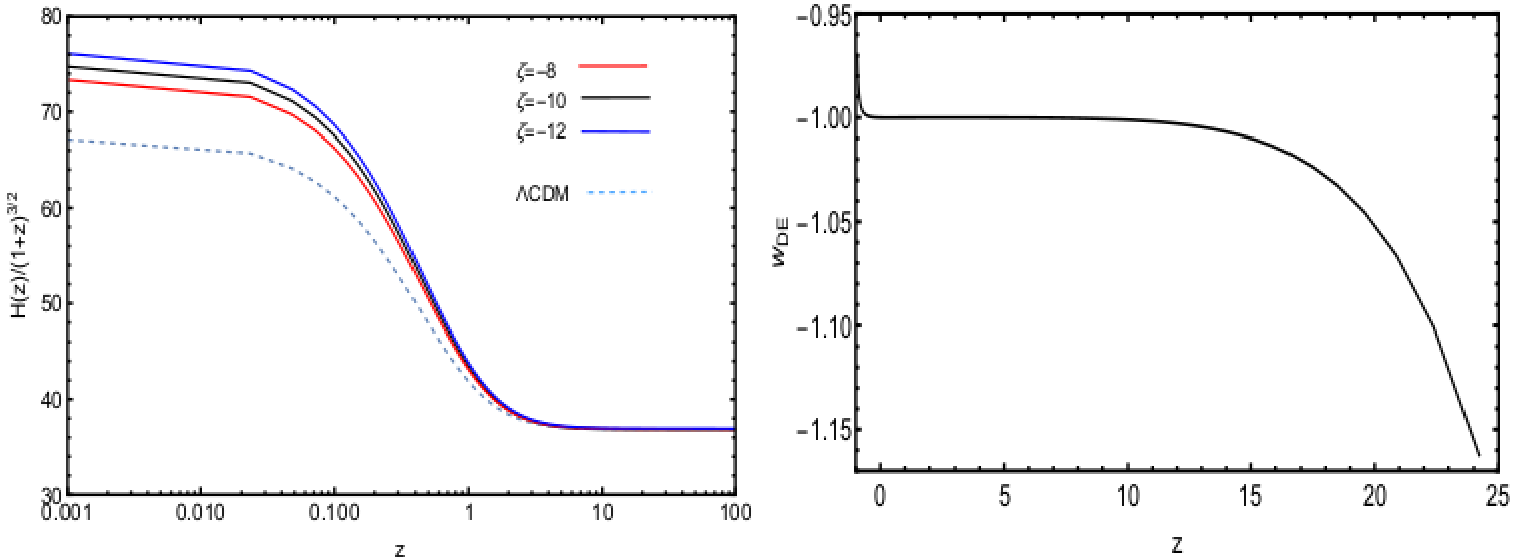




[Saridakis,Tsoukalas PRD 93]

Solving H0 tensions in Bi-scalar Gravity

- Model I: $\mathscr{K}(\phi,B)=\frac{1}{2}\phi-\frac{\zeta}{2}B \quad \text{and} \quad \mathscr{G}(\phi,B)=0$



[M. Petronikolou, E.N.Saridakis, Universe 9]

Running Vacuum

- Upgrade the cosmological constant Λ (vacuum energy) to a running vacuum:

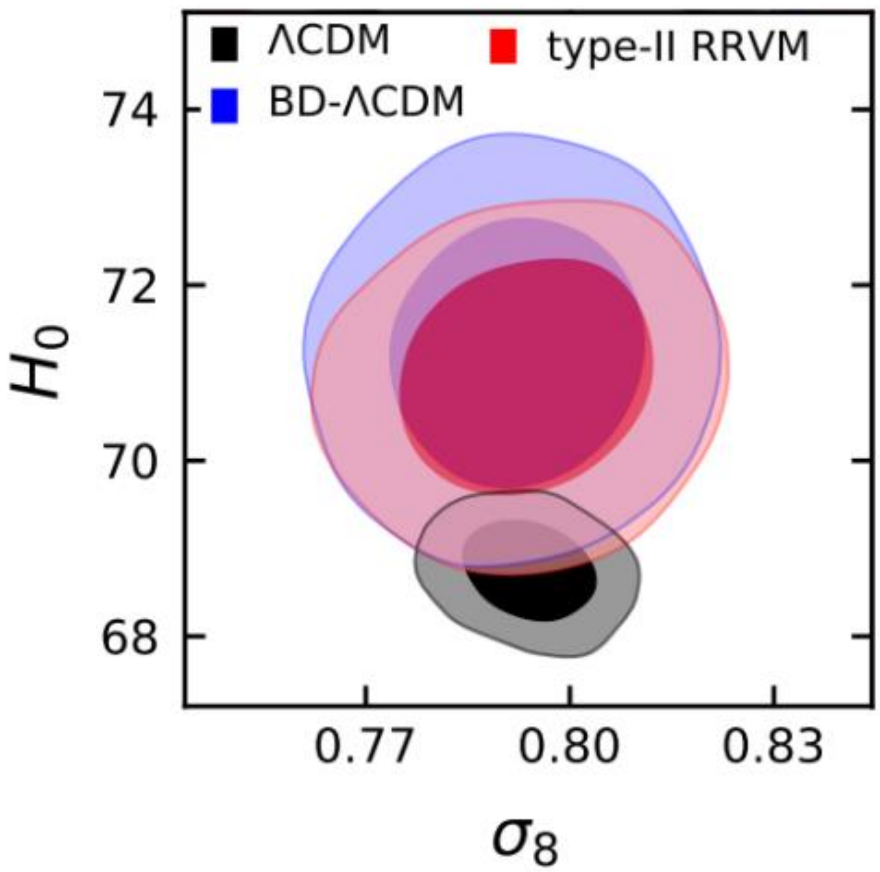
$$\boxed{\begin{aligned} 3H^2 &= 8\pi\, G(H)\,(\rho_m + \rho_r + \rho_\Lambda(H)) \\ 3H^2 + 2\dot{H} &= -8\pi\, G(H)\,(p_r - \rho_\Lambda(H)) \end{aligned}}$$

$$p_{\rm RVM}^{\rm vac} = -\rho_{\rm RVM}^{\rm vac}$$

$$\rho_\Lambda(H;\nu,\alpha) = \frac{3}{8\pi G}\left(c_0 + \nu H^2 + \frac{2}{3}\alpha\,\dot{H}\right) + \mathcal{O}(H^4)$$

[Sola, Gomez-Valent, Perez Astrophys. J 836] [Basilakos, Polarski, Sola PRD86]

Solving the tensions in Running Vacuum



[J. Sola, A. Gomez-Valent, J. de Cruz Perez, C. MorenoPulido CQG 37]

“Those that do not know geometry are not allowed to enter”.
Front Door of Plato’s Academy

Descriptions of Gravity

- Einstein 1916: **General Relativity**:
  energy-momentum source of spacetime Curvature
  Levi-Civita connection: Zero Torsion

- Einstein 1928: **Teleparallel Equivalent of GR:**
  Weitzenbock connection: Zero Curvature

[Cai, Capozziello, De Laurentis, Saridakis, Rept.Prog.Phys. 79]

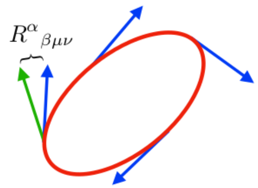
$$\left\{ {\alpha \atop \mu\nu} \right\} = \frac{1}{2} g^{\alpha\lambda} \left(g_{\lambda\nu,\mu} + g_{\mu\lambda,\nu} - g_{\mu\nu,\lambda} \right). \tag{1.3}$$

The corresponding covariant derivative will be denoted by $\mathcal{D}$ so that we will have $\mathcal{D}_\alpha g_{\mu\nu} = 0$. A general connection $\Gamma^\alpha{}_{\mu\nu}$ then admits the following convenient decomposition:

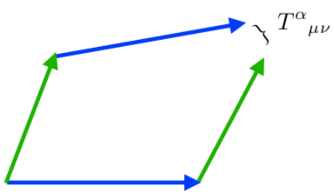
$$\Gamma^\alpha{}_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\} + K^\alpha{}_{\mu\nu} + L^\alpha{}_{\mu\nu} \tag{1.4}$$

with

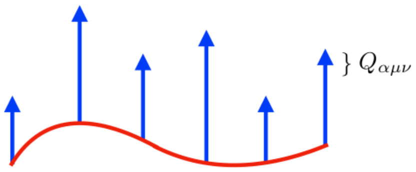
$$K^\alpha{}_{\mu\nu} = \frac{1}{2} T^\alpha{}_{\mu\nu} + T_{(\mu}{}^\alpha{}_{\nu)}\,, \quad L^\alpha{}_{\mu\nu} = \frac{1}{2} Q^\alpha{}_{\mu\nu} - Q_{(\mu}{}^\alpha{}_{\nu)} \tag{1.5}$$



The rotation of a vector transported along a closed curve is given by the curvature: General Relativity.



The non-closure of parallelograms formed when two vectors are transported along each other is given by the torsion: Teleparallel Equivalent of General Relativity.



The variation of the length of a vector as it is transported is given by the non-metricity: Symmetric Teleparallel Equivalent of General Relativity.

Metric-Affine Modified Gravity

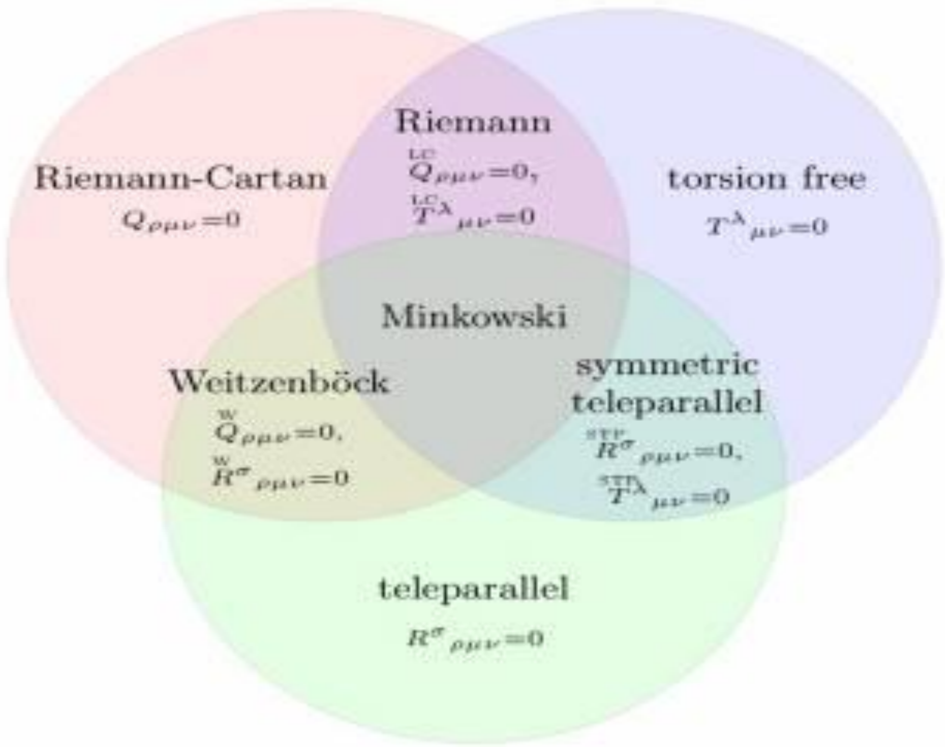


FIG. 1. Subclasses of metric-affine geometry, depending on the properties of connection.

$$S_{\rm GR} = \frac{1}{2\kappa^2}\int \left\{ g^{\mu\nu}\hat{R}_{\mu\nu} + \lambda^{\mu\nu\lambda}_{(1)}T_{\mu\nu\lambda} + \lambda^{\mu\nu\lambda}_{(2)}Q_{\mu\nu\lambda}\right\}\sqrt{-g}\,d^4x\,,$$
$$S_{\rm total} = S_{\rm GR} + S_{\rm matter}\,,$$

Curvature and Torsion

- Vierbeins e_A^{μ} : four linearly independent fields in the tangent space
  $g_{\mu\nu}(x) = \eta_{AB}\ e_{\mu}^{A}(x)\ e_{\nu}^{B}(x)$
- Connection: ω_{ABC}
- Curvature tensor: $R^{A}_{B\mu\nu} = \omega^{A}_{B\nu,\mu} - \omega^{A}_{B\mu,\nu} + \omega^{A}_{C\mu}\omega^{C}_{B\nu} - \omega^{A}_{C\nu}\omega^{C}_{B\mu}$
- Torsion tensor: $T^{A}_{\mu\nu} = e^{A}_{\nu,\mu} - e^{A}_{\mu,\nu} + \omega^{A}_{B\mu}e^{B}_{\nu} - \omega^{A}_{B\nu}e^{B}_{\mu}$

Curvature and Torsion

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  $g_{\mu\nu}(x) = \eta_{AB}\, e_\mu^A(x)\, e_\nu^B(x)$
- Connection: ω_{ABC}
- Curvature tensor: $R^A_{B\mu\nu} = \omega^A_{B\nu,\mu} - \omega^A_{B\mu,\nu} + \omega^A_{C\mu}\omega^C_{B\nu} - \omega^A_{C\nu}\omega^C_{B\mu}$
- Torsion tensor: $T^A_{\mu\nu} = e^A_{\nu,\mu} - e^A_{\mu,\nu} + \omega^A_{B\mu}e^B_\nu - \omega^A_{B\nu}e^B_\mu$
- Levi-Civita connection and Contortion tensor: $\omega_{ABC} = \Gamma_{ABC} + K_{ABC}$

$$K_{ABC} = \frac{1}{2}\left(T_{CAB} - T_{BCA} - T_{ABC}\right) = -K_{BAC}$$

- Curvature and Torsion Scalars: $\boxed{R = \overline{R} + T - 2\left(T_\nu^{\ \nu\mu}\right)_{;\mu}}$

$$R = g^{\mu\nu}R_{\mu\nu} = g^{\mu\nu}R^\rho_{\mu\rho\nu} \qquad T = \frac{1}{4}T^{\rho\mu\nu}T_{\rho\mu\nu} + \frac{1}{2}T^{\rho\mu\nu}T_{\nu\mu\rho} - \mathrm{T}^{\rho}_{\ \rho\mu}\mathrm{T}^{\nu\mu}_{\ \ \nu}$$

Additional motivation

- **Gauge Principle**: global symmetries replaced by local ones:

  The group generators give rise to the compensating fields

  It works perfect for the standard model of strong, weak and E/M interactions

$$SU(3) \times SU(2) \times U(1)$$

- Can we apply this to gravity?

Additional motivation

- Formulating the gauge theory of gravity (mainly after 1960):
- Start from Special Relativity

$\Longrightarrow$ Apply (Weyl-Yang-Mills) gauge principle to its Poincaré-group symmetries

$\Longrightarrow$ Get Poinaré gauge theory:
Both curvature and torsion appear as field strengths

- Torsion is the field strength of the translational group

(Teleparallel and Einstein-Cartan theories are subcases of Poincaré theory)

Additional motivation

- One could extend the gravity gauge group (SUSY, conformal, scale, metric affine transformations) obtaining SUGRA, conformal, Weyl, metric affine gauge theories of gravity

- In all of them torsion is always related to the gauge structure.
- Thus, a possible way towards gravity quantization would need to bring torsion into gravity description.

Teleparallel Equivalent of General Relativity (TEGR)

- Let's start from the simplest tosion-based gravity formulation, namely TEGR:
- Vierbeins e_A^{μ} : four linearly independent fields in the tangent space

$$g_{\mu\nu}(x) = \eta_{AB}\, e_\mu^A(x)\, e_\nu^B(x)$$

- Use curvature-less Weitzenböck connection instead of torsion-less Levi-Civita one: $\Gamma^{\lambda\{W\}}_{\nu\mu} = e_A^{\lambda}\partial_\mu e_\nu^A$
- Torsion tensor:

$$T^{\lambda}_{\mu\nu} = \Gamma^{\lambda\{W\}}_{\nu\mu} - \Gamma^{\lambda\{W\}}_{\mu\nu} = e_A^{\lambda}\left(\partial_\mu e_\nu^A - \partial_\nu e_\mu^A\right)$$

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- Lagrangian (imposing coordinate, Lorentz, parity invariance, and up to 2nd order in torsion tensor)

$$\boxed{L \equiv T = \frac{1}{4}T^{\rho\mu\nu}T_{\rho\mu\nu} + \frac{1}{2}T^{\rho\mu\nu}T_{\nu\mu\rho} - T_{\rho\mu}^{\rho}T_{\nu}^{\nu\mu}}$$

- **Completely equivalent** with GR at the level of equations

f(T) Gravity and f(T) Cosmology

- f(T) Gravity: Simplest torsion-based modified gravity
- Generalize T to f(T) (inspired by f(R))

$$S = \frac{1}{16\pi G}\int d^4x\, e\, \left[T + f(T)\right] + S_m$$

- Equations of motion:

$$e^{-1}\partial_\mu\left(e e_A^\rho S_\rho^{\mu\nu}\right)\left(1+f_T\right) - e_A^\lambda T^\rho_{\mu\lambda} S_\rho^{\nu\mu} + e_A^\rho S_\rho^{\mu\nu}\partial_\mu(T) f_{TT} - \frac{1}{4} e_A^\nu [T + f(T)] = 4\pi G e_A^\rho \mathrm{T}_\rho^{\nu\{\mathrm{EM}\}}$$

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- f(T) Cosmology: Apply in FRW geometry:

$$e_\mu^A = diag(1,a,a,a) \;\Longrightarrow\; ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j$$ (not unique choice)

- Friedmann equations:

$$\boxed{H^2 = \frac{8\pi G}{3}\rho_m - \frac{f(T)}{6} - 2 f_T H^2}$$

$$\boxed{\dot{H} = -\frac{4\pi G(\rho_m + p_m)}{1 + f_T - 12H^2 f_{TT}}}$$

- Find easily

$$T = -6H^2$$

f(T) Cosmology: Background

- Effective Dark Energy sector:

$$\rho_{DE} \equiv \frac{3}{8\pi G_N}\left[-\frac{f}{6}+\frac{Tf_T}{3}\right],$$

$$P_{DE} \equiv \frac{1}{16\pi G_N}\left[\frac{f-f_T T+2T^2 f_{TT}}{1+f_T+2Tf_{TT}}\right]$$

$$w_{DE} = -\frac{f - Tf_T + 2T^2 f_{TT}}{[1+f_T+2Tf_{TT}][f-2Tf_T]}$$

[Linder PRD 82]

- Interesting cosmological behavior: Late-time acceleration, Inflation etc

[Cai, Capozziello, De Laurentis, Saridakis, Rept.Prog.Phys. 79]

f(T) Cosmology: Background

- Re-write Firedmann Equation as:

$$E^2(z, \mathbf{r}) = \Omega_{m0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \Omega_{F0}\, y(z, \mathbf{r})$$

with $E^2(z) \equiv \frac{H^2(z)}{H_0^2} = \frac{T(z)}{T_0}$ and $\Omega_{F0} = 1 - \Omega_{m0} - \Omega_{r0}$;

- $\boxed{y(z, \mathbf{r}) = \frac{1}{T_0 \Omega_{F0}} \left[f - 2T f_T \right]}$ quantifies the deviation from ΛCDM (for f=const. we obtain ΛCDM)

f(T) Cosmology: Perturbations

- For scalar perturbations:

$$e_\mu^0 = \delta_\mu^0 (1+\psi) \ , \ e_\mu^\alpha = \delta_\mu^\alpha a (1-\phi)$$

$$\Rightarrow \ ds^2 = (1+2\psi) dt^2 - a^2 (1-2\phi) \delta_{ij} dx^i dx^j$$

- Obtain Perturbation Equations.

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\rm eff}\, \rho\, \delta \approx 0$$

$$Q(a) = \frac{G_{\rm eff}(a)}{G_N} = \frac{1}{1+f_T}$$

$$\begin{aligned}
\delta \overset{\mathbf{em}}{T}{}_0{}^0 &= -\delta\rho_m \\
\delta \overset{\mathbf{em}}{T}{}_0{}^i &= a^{-2}(\rho_m + p_m)(-\partial_i \delta u) \\
\delta \overset{\mathbf{em}}{T}{}_i{}^0 &= (\rho_m + p_m)(\partial_i \delta u) \\
\delta \overset{\mathbf{em}}{T}{}_i{}^j &= \delta_{ij}\delta p_m + \partial_i \partial_j \pi^S .
\end{aligned}$$

$$\begin{aligned}
E_0^0 \equiv \ & (1+f_0')(\nabla^2\phi) + 6(1+f_0')H\dot{\phi} \\
& +6(1+f_0')H^2\psi - 3f_1'H^2 \\
& -\frac{T_1+f_1}{4} = -4\pi G\delta\rho_m, \\
E_0^i \equiv \ & (1+f_0')\partial_i\dot{\phi} + (1+f_0')H\partial_i\psi \\
& -12H\dot{H}f_0''\partial_i\phi = -4\pi G(\rho_m+p_m)\partial_i\delta u, \\
E_a^0 \equiv \ & 12H^2\partial_i\delta_a^i(\dot{\phi}+H\psi)f_0'' - (1+f_0')\partial_i\delta_a^i(\dot{\phi}+H\psi) \\
& = 4\pi G(\rho_m+p_m)\partial_i\delta_a^i\delta u,
\end{aligned}$$

$$\begin{aligned}
E_a^i\delta_i^a \equiv \ & \frac{f_1'}{a}\left(-3H^2 - \dot{H}\right) + \frac{f_1''}{a}\left(12H^2\dot{H}\right) \\
& -\frac{(1+f_0')}{2a}\sum_{b\neq a}\partial^j\delta_j^b\partial_i\delta_b^i(\psi-\phi) \\
& -\frac{\phi(T_0+f_0)}{4a} - \frac{T_1+f_1}{4a} \\
& +\frac{(1+f_0')}{a}[6H\dot{\phi} + 6H^2\psi - 3H^2\phi \\
& \qquad +\ddot{\phi} + \dot{H}(2\psi-\phi) + H\dot{\psi}] \\
& +\frac{f_0''}{a}(-24H\dot{H}\dot{\phi} - 48\psi H^2\dot{H} - 12H^2\ddot{\phi} \\
& \qquad -12H^3\dot{\psi} + 12H^2\dot{H}\phi) \\
& \qquad = \frac{4\pi G}{a}(p_m\phi + \delta p_m), \\
E^i_{b;\, b\neq a}\delta_i^a \equiv \ & \frac{(1+f_0')}{2}\partial_j\delta_b^j\partial^i\delta_i^a(\phi-\psi) \\
& = 4\pi G a^2 \partial_j\delta_b^j\partial^i\delta_i^a\pi^S
\end{aligned}$$

[Chen, Dent, Dutta, Saridakis PRD 83],
[Dent, Dutta, Saridakis JCAP 1101]

Viable f(T) models

- 1) Power-law model (f1CDM)

$$\boxed{f(T) = \alpha(-T)^b} \qquad \alpha = (6H_0^2)^{1-b}\frac{\Omega_{F0}}{2b-1}$$

$$y(z,b) = E^{2b}(z,b) \qquad G_{\rm eff}(z) = \frac{G_N}{1+\frac{b\Omega_{F0}}{(1-2b)E^{2(1-b)}}}$$

- 2) The Linder model (f2CDM)

$$\boxed{f(T) = \alpha T_0(1-e^{-p\sqrt{T/T_0}})} \qquad \alpha = \frac{\Omega_{F0}}{1-(1+p)e^{-p}}$$

$$y(z,p) = \frac{1-(1+pE)e^{-pE}}{1-(1+p)e^{-p}} \qquad G_{\rm eff}(z) = \frac{G_N}{1+\frac{\Omega_{F0}p\ e^{-pE}}{2E[1-(1+p)e^{-p}]}}$$

- 3) The exponential model (f3CDM)

$$\boxed{f(T) = \alpha T_0(1-e^{-pT/T_0})} \qquad \alpha = \frac{\Omega_{F0}}{1-(1+2p)e^{-p}}$$

$$y(z,p) = \frac{1-(1+2pE^2)e^{-pE^2}}{1-(1+2p)e^{-p}} \qquad G_{\rm eff}(z) = \frac{G_N}{1+\frac{\Omega_{F0}p\ e^{-pE^2}}{1-(1+2p)e^{-p}}}$$

[Nesseris, Basilakos, Saridakis, Perivolaropoulos, PRD 88]

Efficient model independent requirements to solve the tensions

We consider a correction in the first Friedmann equation of the form

$$H(z) = -\frac{d(z)}{4} + \sqrt{\frac{d^2(z)}{16} + H^2_{\Lambda\mathrm{CDM}}(z)}\,, \qquad (2)$$

where $H_{\Lambda\mathrm{CDM}}(z) \equiv H_0\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}$ is the Hubble rate in ΛCDM, with $\Omega_m = \rho_m/(3M_p^2 H^2)$ the matter density parameter and primes denote derivatives with respect to z.

- Îf $d < 0$ and is suitably chosen, one can have $H(z \to z_{\mathrm{CMB}}) \approx H_{\Lambda\mathrm{CDM}}(z \to z_{\mathrm{CMB}})$ but $H(z \to 0) > H_{\Lambda\mathrm{CDM}}(z \to 0)$; i.e., the H_0 tension is solved [one should choose $|d(z)| < H(z)$, and thus, since $H(z)$ decreases for smaller z, the deviation from ΛCDM will be significant only at low redshift].

- Since the friction term in (1) increases, the growth of structure gets damped, and therefore, the σ_8 tension is also solved.

Solving H0 and S8 tensions in f(T) Gravity

- We consider the following ansatz:

$$f(T) = -[T + 6H_0^2(1 - \Omega_{m0}) + F(T)], \qquad (9)$$

where $F(T)$ describes the deviation from GR
The first Friedmann equation becomes

$$T(z) + 2\frac{F'(z)}{T'(z)}T(z) - F(z) = 6H^2_{\Lambda CDM}(z)\,. \qquad (10)$$

- In order to solve the H_0 tension, we need
$T(0) = 6H_0^2 \simeq 6(H_0^{CC})^2$, with $H_0^{CC} = 74.03\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$,
while in the early era of $z \gtrsim 1100$ we require the Universe expansion to evolve as in ΛCDM, namely
$H(z \gtrsim 1100) \simeq H_{\Lambda CDM}(z \gtrsim 1100)$
This implies $F(z)|_{z\gtrsim 1100} \simeq cT^{1/2}(z)$ (the value $c = 0$ corresponds to standard GR, while for $c \neq 0$ we obtain ΛCDM too).

[S-F Yan, P. Zhang, J_W Chen, X_Z Zhang, Y-F Cai, E.N. Saridakis, PRD 101]

Solving H0 and S8 tensions in f(T) Gravity

The effective gravitational coupling is given by

$$G_{\rm eff} = \frac{G_N}{1 + F_T} \, . \qquad (11)$$

Therefore, the perturbation equation becomes

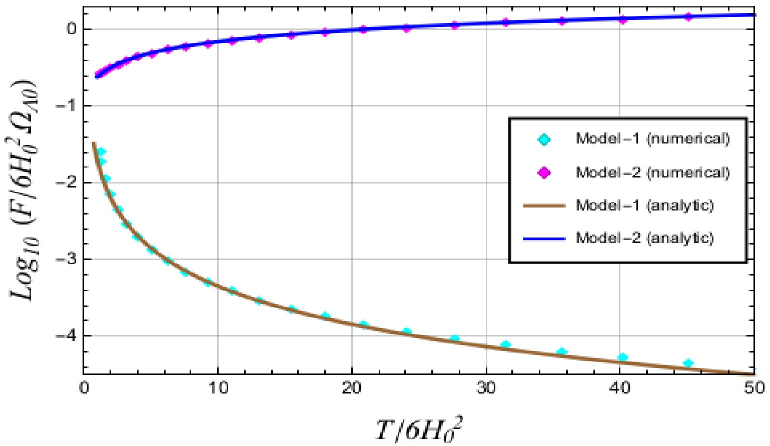
$$\delta'' + \left[\frac{T'(z)}{2T(z)} - \frac{1}{1+z}\right]\delta' = \frac{9H_0^2\Omega_{m0}(1+z)}{[1 + F'(z)/T'(z)]T(z)}\delta \, . \qquad (12)$$

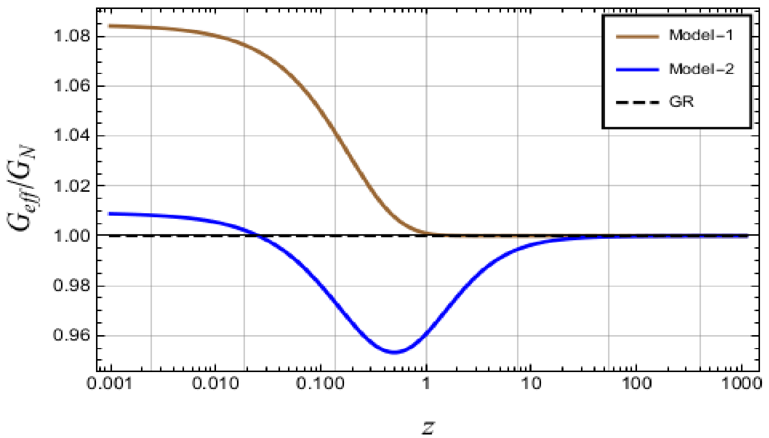
Since around the last scattering moment $z \gtrsim 1100$ the Universe should be matter-dominated, we impose $\delta'(z)|_{z\gtrsim 1100} \simeq -\frac{1}{1+z}\delta(z)$, while at late times we look for $\delta(z)$ that leads to an $f\sigma_8$ in agreement with redshift survey observations.

[S-F Yan, P. Zhang, J_W Chen, X_Z Zhang, Y-F Cai, E.N. Saridakis, PRD 101]

Solving H0 and S8 tensions in f(T) Gravity

By solving (10) and (12) with initial and boundary conditions at $z \sim 0$ and $z \sim 1100$, we can find the functional forms for the free functions of the $f(T)$ gravity that we consider, namely, $T(z)$ and $F(z)$, that can alleviate both H_0 and σ_8 tensions.



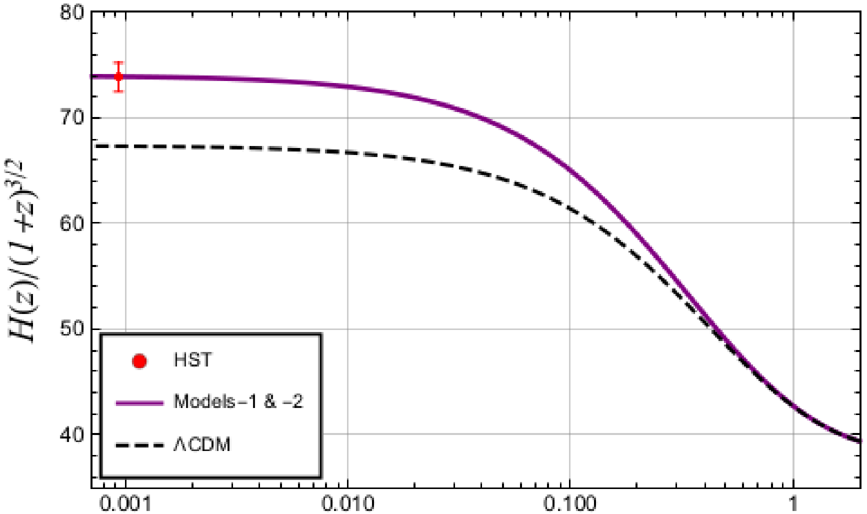


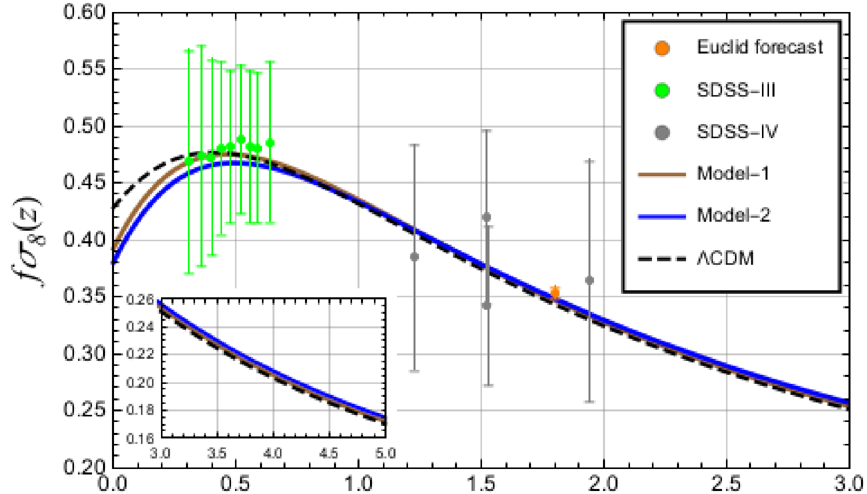
$$\text{Model-1: } F(T) \approx 375.47\left(\frac{T}{6H_0^2}\right)^{-1.65}$$

$$\text{Model-2: } F(T) \approx 375.47\left(\frac{T}{6H_0^2}\right)^{-1.65} + 25\,T^{1/2}.$$

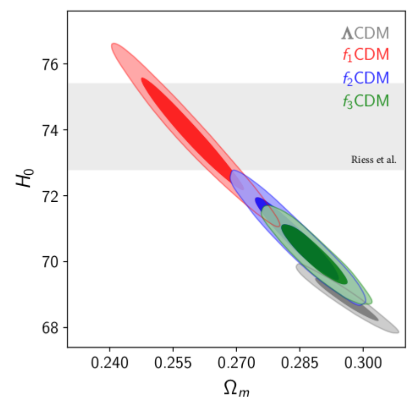
[S-F Yan, P. Zhang, J_W Chen, X_Z Zhang, Y-F Cai, E.N. Saridakis, PRD 101]

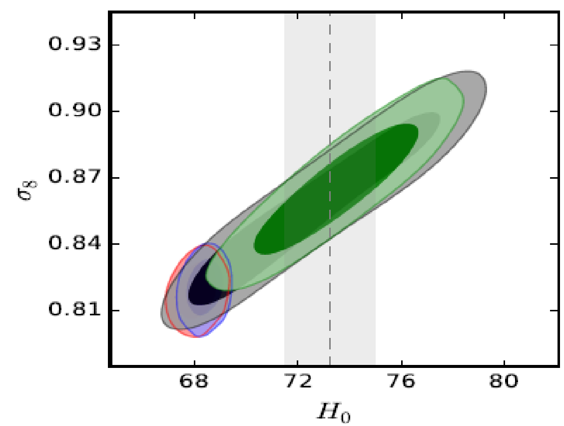
Solving H0 and S8 tensions in f(T) Gravity





Parameter	CMB + BAO	CMB + BAO + H_0
$10^2\omega_b$	$2.235^{+0.013}_{-0.013}$	$2.235^{+0.013}_{-0.013}$
ω_{cdm}	$0.1181^{+0.001}_{-0.001}$	$0.118^{+0.001}_{-0.001}$
$100\theta_s$	$1.041^{+0.00027}_{-0.00027}$	$1.041^{+0.00030}_{-0.00027}$
$\ln 10^{10}A_s$	$3.078^{+0.023}_{-0.023}$	$3.08^{+0.022}_{-0.022}$
n_s	$0.9678^{+0.0039}_{-0.0039}$	$0.9684^{+0.0039}_{-0.0044}$
τ_{reio}	$0.073^{+0.012}_{-0.013}$	$0.075^{+0.012}_{-0.012}$
n	$0.0043^{+0.0033}_{-0.0039}$	$0.0054^{+0.0020}_{-0.0020}$
$\log\alpha$	$10.00^{+0.081}_{-0.12}$	$10.03^{+0.06}_{-0.06}$
Ω_{F0}	$0.73^{+0.021}_{-0.028}$	$0.738^{+0.015}_{-0.015}$
H_0	$72.4^{+3.3}_{-4.1}$	$73.5^{+2.1}_{-2.1}$
σ_8	$0.855^{+0.023}_{-0.033}$	$0.866^{+0.02}_{-0.02}$
$\chi^2_{min}/2$	6480.48	6482.27



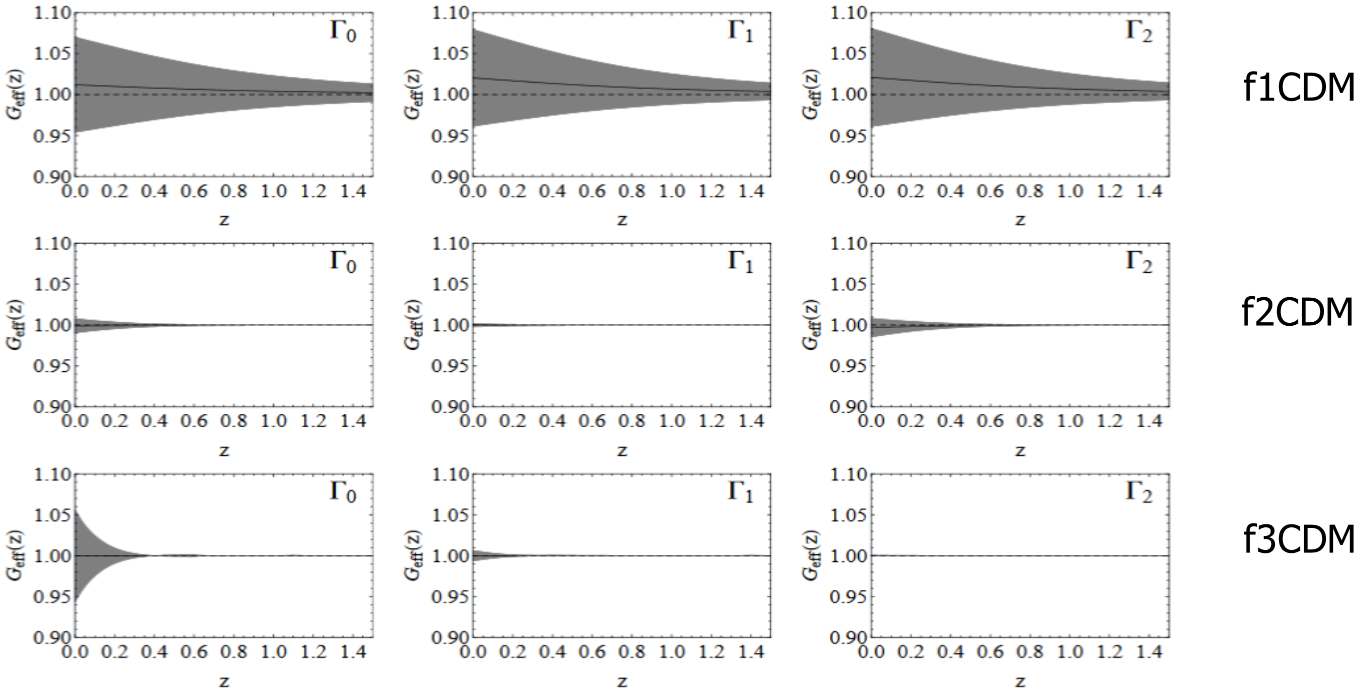


[S-F Yan, P. Zhang, J-W Chen, X_Z Zhang, Y-F Cai, E.N. Saridakis, PRD 101]

[J-W Chen, W. Luo, Y-F Cai, E.N. Saridakis, PRD 102]

[S. Basilakos, S. Nesseris, F. Anagnostopoulos, E.N.Saridakis, JCAP 2019]

Viable f(T) models



- In f(T) gravity we can indeed obtain $G_{\rm eff}/G_N$ <1 for z<2, without affecting the background evolution.
- fσ8 tension may be alleviated. [Nesseris, Basilakos, Saridakis, Perivolaropoulos, PRD 88]

In other modified gravities: Not possible

- This behavior is not possible in other modified gravities. e.g.:

$$S=\int d^4x\sqrt{-g}\left(\frac{1}{2}f(R,\phi,X)+\mathcal{L}_m\right)$$

$$X=-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$G_{\rm eff}(a,k)/G_{\rm N}=\frac{1}{F}\frac{f_{,X}+4\left(f_{,X}\frac{k^2}{a^2}\frac{F_{,R}}{F}+\frac{F_{,\phi}^2}{F}\right)}{f_{,X}+3\left(f_{,X}\frac{k^2}{a^2}\frac{F_{,R}}{F}+\frac{F_{,\phi}^2}{F}\right)}$$

$$F=F(R,\phi,X)=\partial_R f(R,\phi,X)$$

- $G_{\rm eff}/G_{\rm N}$ >1 for all models that do not have ghosts (i.e. with fR,fRR>0).

- On the contrary, f(T) gravity has second-order field equations and moreover perturbations are stable in a large part of the parameter phase.

f(Q) gravity

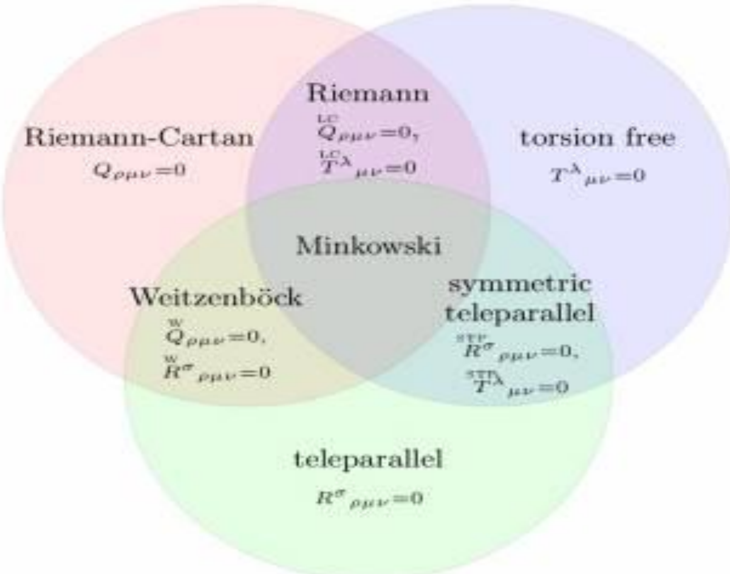


FIG. 1. Subclasses of metric-affine geometry, depending on the properties of connection.

affine connection $\Gamma^{\alpha}_{\mu\nu}$ can be decomposed as

$$\Gamma^{\alpha}_{\mu\nu} = \hat{\Gamma}^{\alpha}_{\mu\nu} + K^{\alpha}{}_{\mu\nu} + L^{\alpha}{}_{\mu\nu}, \tag{1}$$

where $\hat{\Gamma}^{\alpha}_{\mu\nu}$ is the Levi-Civita connection,

$$K^{\alpha}{}_{\mu\nu} = \frac{1}{2}T^{\alpha}{}_{\mu\nu} + T_{(\mu}{}^{\alpha}{}_{\nu)} \tag{2}$$

is the contortion tensor with $T^{\alpha}{}_{\mu\nu}$ the torsion tensor, and

$$L^{\alpha}{}_{\mu\nu} = \frac{1}{2}Q^{\alpha}{}_{\mu\nu} - Q_{(\mu}{}^{\alpha}{}_{\nu)} \tag{3}$$

is the disformation tensor arising from the non-metricity

$$Q_{\alpha\mu\nu} \equiv \nabla_{\alpha} g_{\mu\nu}, \tag{4}$$

f(Q) gravity

$$T^{\lambda}{}_{\mu\nu} \equiv \Gamma^{\lambda}{}_{\mu\nu} - \Gamma^{\lambda}{}_{\nu\mu}$$
$$R^{\sigma}{}_{\rho\mu\nu} \equiv \partial_{\mu}\Gamma^{\sigma}{}_{\nu\rho} - \partial_{\nu}\Gamma^{\sigma}{}_{\mu\rho} + \Gamma^{\alpha}{}_{\nu\rho}\Gamma^{\sigma}{}_{\mu\alpha} - \Gamma^{\alpha}{}_{\mu\rho}\Gamma^{\sigma}{}_{\nu\alpha} \quad (5)$$

while the nonmetricity can be expressed as

$$Q_{\rho\mu\nu} \equiv \nabla_{\rho} g_{\mu\nu} = \partial_{\rho} g_{\mu\nu} - \Gamma^{\beta}{}_{\rho\mu} g_{\beta\nu} - \Gamma^{\beta}{}_{\rho\nu} g_{\mu\beta} \,. \quad (6)$$

$$Q = -\frac{1}{4} Q_{\alpha\beta\gamma} Q^{\alpha\beta\gamma} + \frac{1}{2} Q_{\alpha\beta\gamma} Q^{\gamma\beta\alpha} + \frac{1}{4} Q_{\alpha} Q^{\alpha} - \frac{1}{2} Q_{\alpha} \tilde{Q}^{\alpha} \,, \quad (7)$$

where $Q_{\alpha} \equiv Q_{\alpha}{}^{\mu}{}_{\mu}$, and $\tilde{Q}^{\alpha} \equiv Q_{\mu}{}^{\mu\alpha}$.

f(Q) gravity

$$S=-\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}f(Q). \tag{8}$$

$$\begin{aligned}
&\frac{2}{\sqrt{-g}}\nabla_\alpha\left\{\sqrt{-g}g_{\beta\nu}f_Q\left[-\frac{1}{2}L^{\alpha\mu\beta}+\frac{1}{4}g^{\mu\beta}\left(Q^\alpha-\tilde{Q}^\alpha\right)\right.\right.\\
&\qquad\qquad\left.\left.-\frac{1}{8}\left(g^{\alpha\mu}Q^\beta+g^{\alpha\beta}Q^\mu\right)\right]\right\}\\
&+f_Q\left[-\frac{1}{2}L^{\mu\alpha\beta}-\frac{1}{8}\left(g^{\mu\alpha}Q^\beta+g^{\mu\beta}Q^\alpha\right)\right.\\
&\qquad\left.+\frac{1}{4}g^{\alpha\beta}\left(Q^\mu-\tilde{Q}^\mu\right)\right]Q_{\nu\alpha\beta}+\frac{1}{2}\delta^\mu_\nu f=T^\mu{}_\nu\,,
\end{aligned} \tag{9}$$

with $f_Q=\partial f/\partial Q$.

f(Q) cosmology

- Background:

$$
\begin{aligned}
6 f_Q H^2 - \frac{1}{2} f &= \rho_m , \\
\left(12 H^2 f_{QQ} + f_Q\right) \dot{H} &= -\frac{1}{2}\left(\rho_m + p_m\right) . \qquad (11)
\end{aligned}
$$

$$
Q = 6H^2 , \qquad (12)
$$

f(Q) cosmology

- Perturbations:

$$-a^2\delta\rho = 6\left(f_Q + 12a^{-2}\mathcal{H}^2 f_{QQ}\right)\mathcal{H}\left(\mathcal{H}\phi + \varphi'\right) + 2f_Q k^2\psi$$
$$-2\left[f_Q + 3a^{-2}f_{QQ}\left(\mathcal{H}' + \mathcal{H}^2\right)\right]\mathcal{H}k^2 B\,. \qquad (19)$$

$$\frac{1}{2}a^2\left(\rho + p\right)v = \left[f_Q + 3a^{-2}f_{QQ}\left(\mathcal{H}' + \mathcal{H}^2\right)\right]\mathcal{H}\phi$$
$$+6a^{-2}f_{QQ}\mathcal{H}^2\varphi' - 9a^{-2}f_{QQ}\left(\mathcal{H}' - \mathcal{H}^2\right)\mathcal{H}\varphi$$
$$+f_Q\psi' - a^{-2}f_{QQ}\mathcal{H}^2k^2B\,, \qquad (20)$$

$$\frac{1}{2}a^2\delta p = \left(f_Q + 12a^{-2}f_{QQ}\mathcal{H}^2\right)\left(\mathcal{H}\phi' + \varphi''\right) + \left[f_Q\left(\mathcal{H}' + 2\mathcal{H}^2 - \frac{1}{3}k^2\right) + 12a^{-2}f_{QQ}\mathcal{H}^2\left(4\mathcal{H}' - \mathcal{H}^2\right) + 12a^{-2}\frac{df_{QQ}}{d\tau}\mathcal{H}^3\right]\phi$$
$$+\ 2\left[f_Q + 6a^{-2}f_{QQ}\left(3\mathcal{H}' - \mathcal{H}^2\right) + 6a^{-2}\frac{df_{QQ}}{d\tau}\mathcal{H}\right]\mathcal{H}\varphi' + \frac{1}{3}f_Q k^2\psi$$
$$-\ \frac{1}{3}\left(f_Q + 6a^{-2}f_{QQ}\mathcal{H}^2\right)k^2B' - \frac{1}{3}\left[2f_Q + 3a^{-2}f_{QQ}\left(5\mathcal{H} - \mathcal{H}^2\right) + 6a^{-2}\frac{df_{QQ}}{d\tau}\mathcal{H}\right]\mathcal{H}k^2B\,, \qquad (21)$$

f(Q) cosmology

- Perturbations:

$$\delta' = (1+w)\left(-k^2 v - k^2 B + 3\varphi'\right) + 3\mathcal{H}\left(w\rho - \frac{\delta p}{\rho}\right), \quad (22)$$

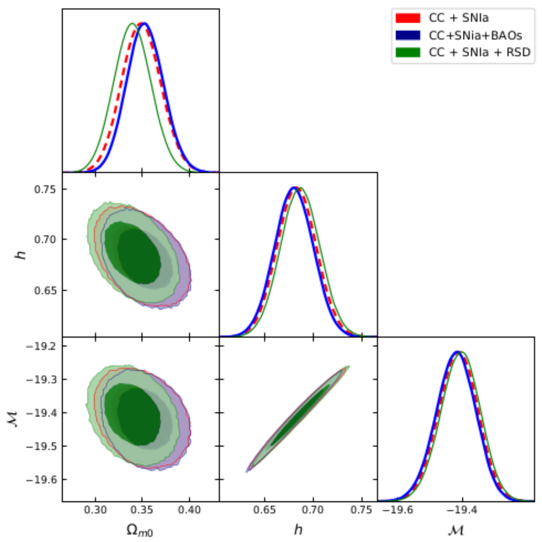
$$v' = -\mathcal{H}\left(1 - c_s^2\right) v + \frac{\delta p}{\rho + p} + \phi\,. \quad (23)$$

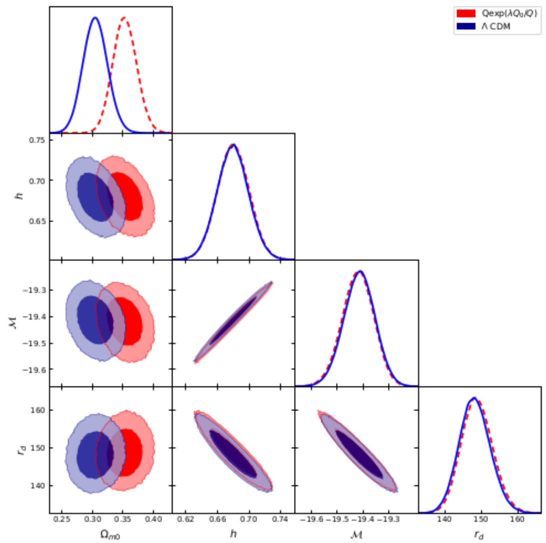
$$\begin{aligned} &- f_{QQ}\mathcal{H}\left[2\mathcal{H}\varphi' + \left(\mathcal{H}' + \mathcal{H}^2\right)\phi + \left(\mathcal{H}' - \mathcal{H}^2\right)(\psi - B')\right] \\ &- \left[f_{QQ}\left(\mathcal{H}'^2 + \mathcal{H}\mathcal{H}'' - 3\mathcal{H}^2\mathcal{H}' - \frac{1}{3}\mathcal{H}^2 k^2\right)\right. \\ &+ \left.\frac{df_{QQ}}{d\tau}(\mathcal{H}' - \mathcal{H}^2)\mathcal{H}\right] B = 0\,, \end{aligned} \quad (24)$$

$$\delta'' + \mathcal{H}\delta' = \frac{4\pi G\rho}{f_Q}\delta\,, \quad (30)$$

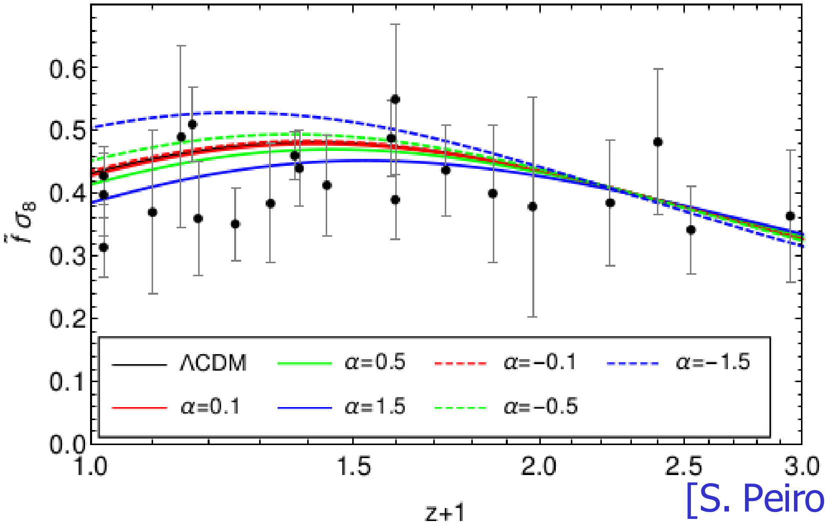
$$G_{eff} \equiv \frac{G}{f_Q}\,, \quad (31)$$

Solving the tensions in f(Q) gravity





[F. Anagnostopoulos, S. Basilakos, E.N.Saridakis, JCAP 2019]



[S. Peirone, G. Benevento, N. Frusciante, S. Tsujikawa, PRD 100]

Conclusions

- i) Astrophysics and Cosmology have become precision sciences.
- ii) A huge amount of accumulating data suggest possible tensions with theoretical predictions of ΛCDM paradigm.
- iii) New Physics or paradigm shift may be the way out
- iv) We can modify the Universe content, the interactions, or/and the gravitational theory. Historically, modified gravity has ben proven to be the solution quite often.

E.Di Valentino, E. N. Saridakis, A. Riess, Nature Astronomy (2022) 2211.05248 [astro-ph.CO]

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THANK YOU!