中国科学技术大学
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Causality bounds on gravitational EFTs

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EFT space: "Not everything goes!"

high energy UV theory

unknown, but assume causality, unitarity, ...

UV

|

IR

mixing

Positivity bounds/
Causality bounds

a bootstrap approach

swampland idea

low energy EFT

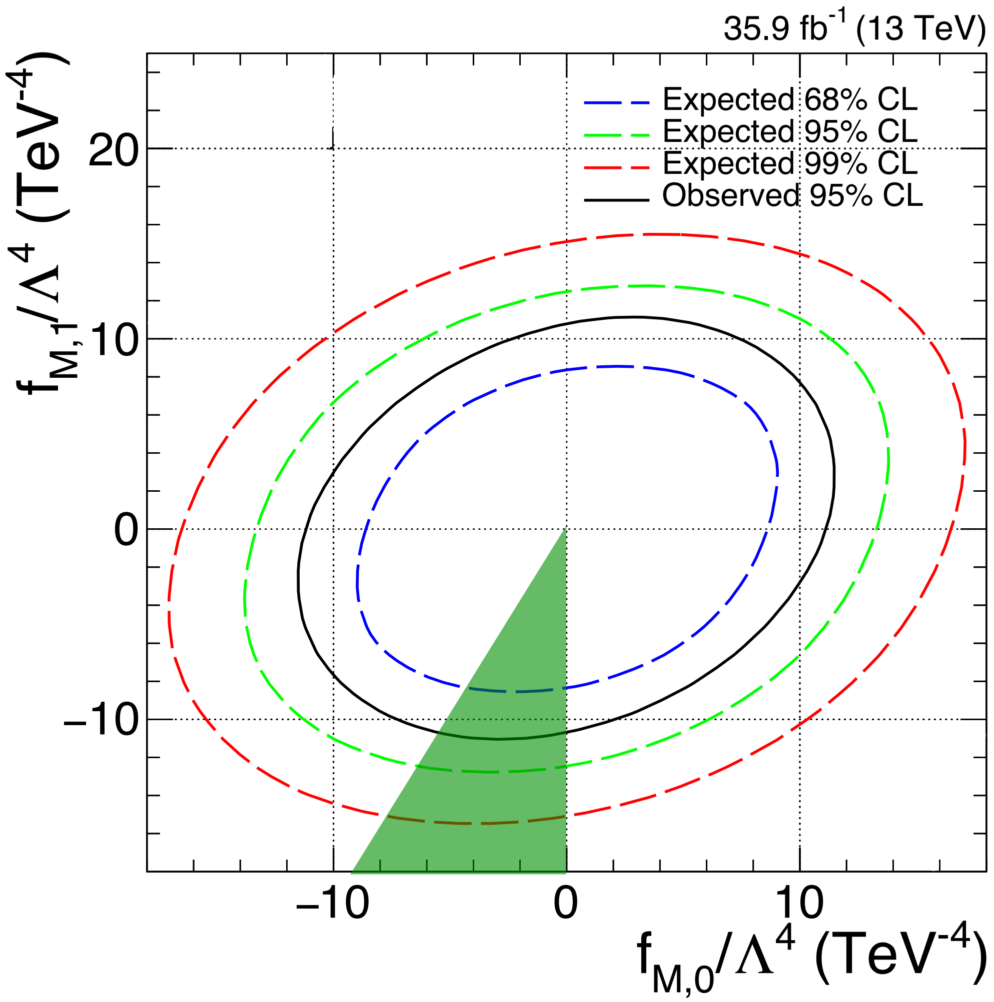
constraints on EFT coefficients

Snowmass White Paper: UV Constraints on IR Physics, de Rham, Kundu, Reece, Tolley & **SYZ**, 2203.06805

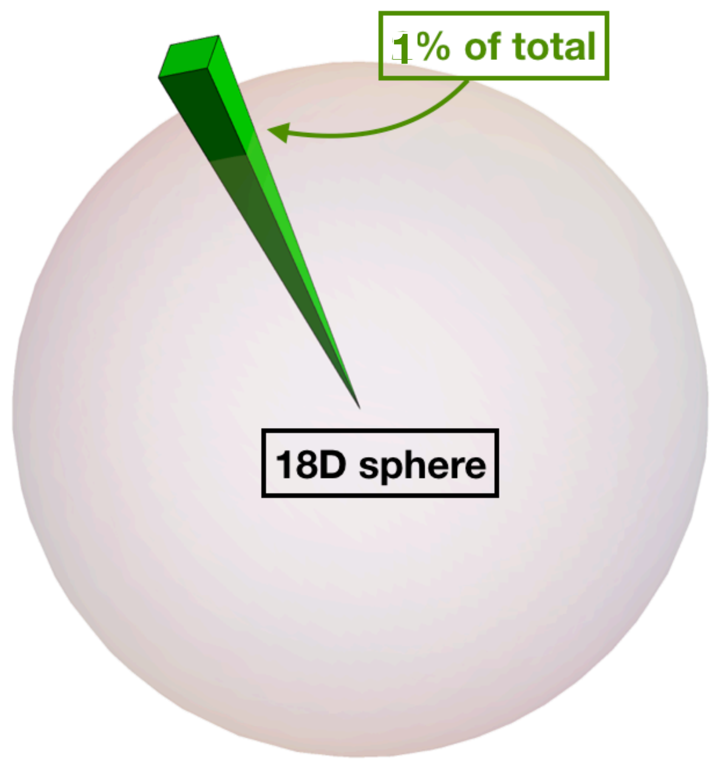
Example: Strong bounds on Standard Model EFT

Vector Boson Scattering: $V_1 + V_2 \to V_3 + V_4, \quad V_i \in \{Z, W^+, W^-, \gamma\}$

O_{M0} and O_{M1}



Space of 18 dim-8 Wilson coeff's



Only **<1%** of the total aQGC parameter space admits an analytic UV completion!

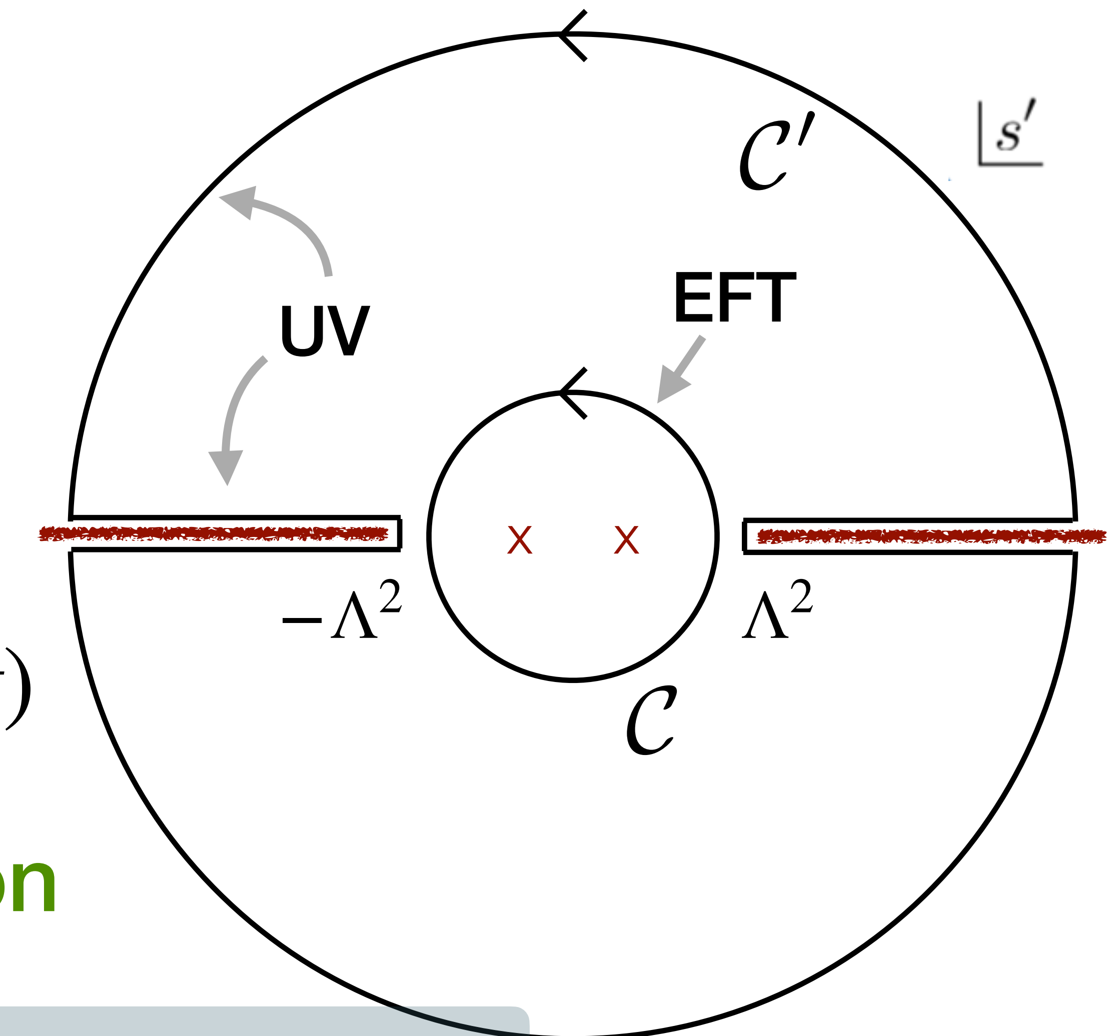
Zhang & **SYZ**,1808.00010

Causality ⇒ Analyticity ⇒ Dispersion relation

- Analyticity in complex s plane (fixed t)

$$A(s,t) = \frac{1}{2\pi i}\oint_{\mathcal{C}} ds' \, \frac{A(s',t)}{s'-s}$$

- Froissart bound $|A(s' \to \infty, t)| < s'^{2-\epsilon}$
- su crossing symmetry $A(s,t) = A(u,t)$



Twice subtracted dispersion relation

$$\boxed{A(s,t) \sim \int_{\Lambda^2}^{\infty} \frac{\mathrm{d}s'}{\pi s'^2}\left[\frac{s^2}{s'-s} + \frac{u^2}{s'-u}\right] \mathrm{Im}\, A(s',t)}$$

s': scale of UV particles

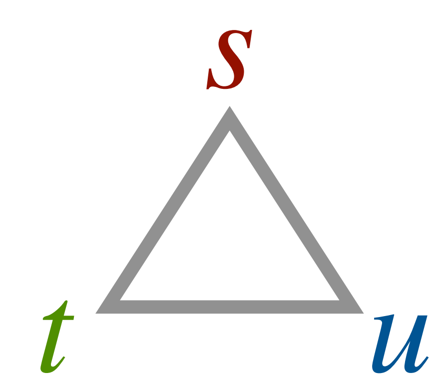
EFT amplitude

IR — UV connection

UV full amplitude

Dimensional analysis on firm footing

Identical scalar



$$A(u,t) = A(s,t) = A(t,s)$$

su dispersion relation

additional st crossing

Null constraints

$$\sum_{\ell} \int d\mu \frac{\mathrm{Im}\, a_\ell(\mu)}{\mu^{i+j}} \Gamma^{(n)}_{i,j}(\ell) = 0$$

Tolley, Wang & **SYZ**, 2011.02400
Caron-Huot & Duong, 2011.02957

weakly coupled IR

$$A(s,t) \sim c_{2,0} s^2 + c_{2,1} s^2 t + c_{2,2} s^2 t^2 + \cdots$$

All Wilson coefficients are parametrically $\lesssim O(1)$!

Two-sided bounds on cosmological scalars

In decoupling limit

Xu & **SYZ**, 2306.06639

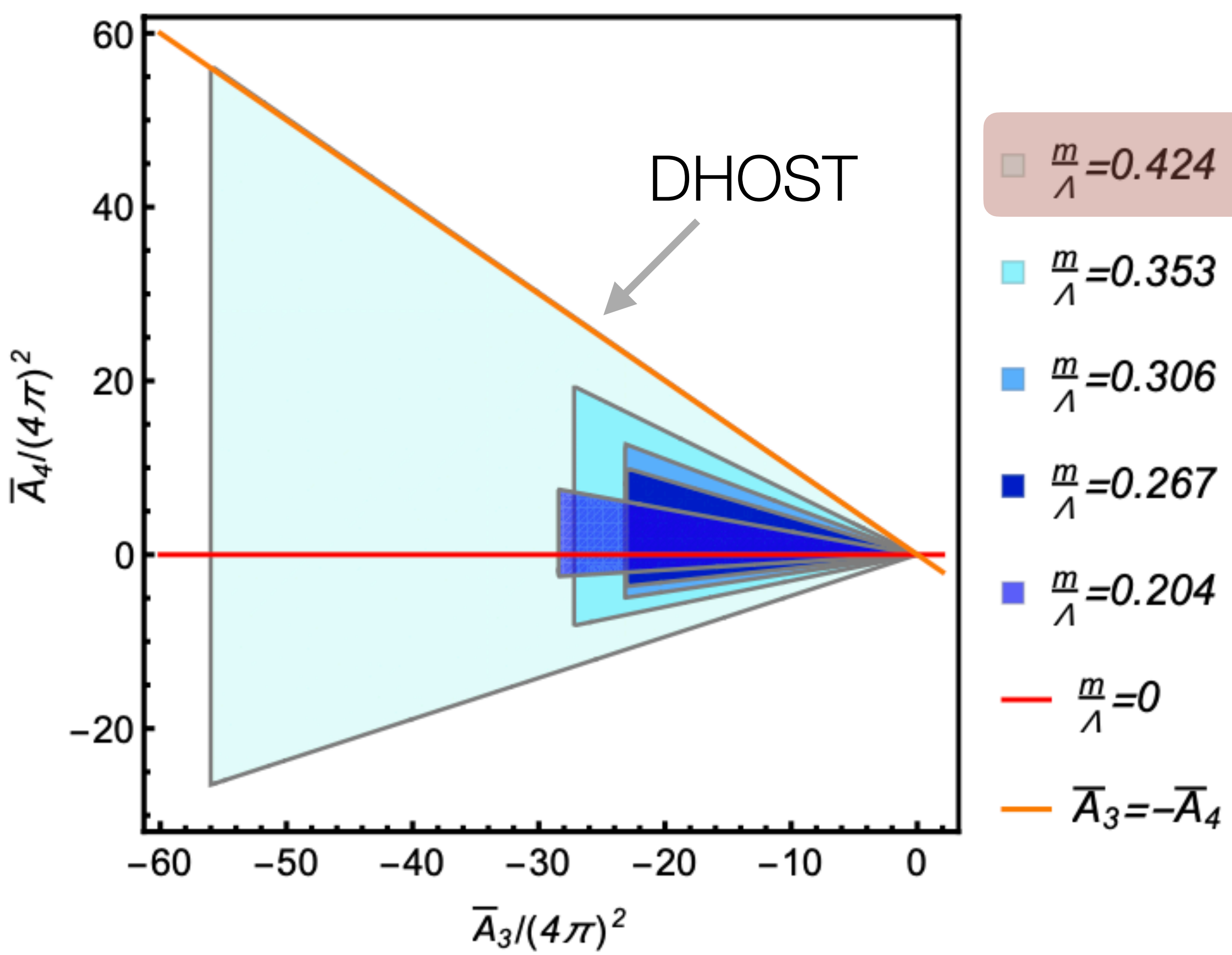
<table>
<tr><th rowspan="2"></th><th colspan="3" rowspan="2">Class</th><th colspan="2" rowspan="2">Positivity bounds</th><th colspan="2">Consistency</th></tr>
<tr><th>massless</th><th>massive</th></tr>
<tr><td rowspan="4">Horndeski (4.1-4.4)</td><td colspan="3">Generic</td><td colspan="2">(4.12-4.14)</td><td>✓</td><td>✓</td></tr>
<tr><td rowspan="3">$G_i = G_i(X)$</td><td colspan="2">K-essence ($G_3 = G_4 = G_5 = 0$)</td><td rowspan="3">(4.15) Fig 7-10</td><td>(4.17)</td><td>✓</td><td>✓</td></tr>
<tr><td colspan="2">Kinetic braiding ($G_4 = G_5 = 0$)</td><td>(4.19)</td><td>✓</td><td>✓</td></tr>
<tr><td colspan="2">Galileon (4.20)</td><td>(4.21)</td><td>$\times$</td><td>$\cancel{\checkmark}$</td></tr>
<tr><td rowspan="6">Beyond Horndeski</td><td rowspan="5">DHOST (5.1)</td><td rowspan="2">N-I (5.1)+(5.12)</td><td>Generic</td><td rowspan="5">(5.11)</td><td></td><td>✓</td><td>✓</td></tr>
<tr><td>GW constraints (5.13)</td><td>(5.14) Fig 11</td><td>$\times$</td><td>✗</td></tr>
<tr><td rowspan="2">M-I (5.1)+(5.18)</td><td>Generic</td><td>(5.19) Fig 12</td><td>$\times$</td><td>$\cancel{\checkmark}$</td></tr>
<tr><td>beyond Horndeski [34] +(5.20)</td><td>(5.21)</td><td>$\times$</td><td>✗</td></tr>
<tr><td colspan="2">M-II (5.1)+(5.22)</td><td>(5.23) Fig 13</td><td>$\times$</td><td>$\cancel{\checkmark}$</td></tr>
<tr><td colspan="3">Combining Horndeski & DHOST (5.28)</td><td colspan="2">(5.29), Fig 14</td><td>✓</td><td>✓</td></tr>
</table>

Example: rule out DHOST

Effectively rule out models due to $\Lambda \sim m$

Xu & **SYZ**, 2306.06639

GW speed constrained class



similar result for original beyond Horndeski model

Two-sided bounds on specific EFTs

- Identical scalar EFT
  - Fixed t dispersion relation — Tolley, Wang & **SYZ**, 2011.02400; Caron-Huot & Duong, 2011.02957
  - Moment problem approach — Chiang, Huang, Li, Rodina & Weng, 2105.02862
  - Fully symmetric dispersion relation — Sinha & Zahed, 2012.04877
- Multi-(scalar) field EFT — Du, Zhang & **SYZ**, 2111.01169
- Einstein EFT — Caron-Huot, Mazac, Rastelli, Simmons-Duffin, 2102.08951; Caron-Hot, Li, Parra-Martinez & Simmons-Duffin, 2201.06602; Chiang, Huang, Li, Rodina & Weng, 2201.07177
- Einstein-Maxwell EFT — Henriksson, McPeak, Russo & Vichi, 2203.08164
- Scalar-tensor EFT — Hong, Wang, **SYZ**, 2304.01259

Causality bounds on scalar-tensor EFTs

Motivation from phenomenology

cosmic acceleration universe	dark matter galaxy	solar system	strong gravity BH, NS, …	lab tests
$10^{-33}\mathrm{eV}$	$10^{-27}\mathrm{eV}$	$10^{-18}\mathrm{eV}$	$10^{-11}\mathrm{eV}$	$10^{-3}\mathrm{eV}$
				

Possible deviations from General Relativity in strong gravity regime?

Scalar-tensor EFT

being constrained in astrophysics (**GWs, EHT**…)

Light DoFs: $g_{\mu\nu}$ **+** scalar ϕ

$$S=\int \mathrm{d}^4x\sqrt{-g}\Big(\frac{M_P^2}{2}R-\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi-\frac{\lambda_3}{3!}\phi^3-\frac{\lambda_4}{4!}\phi^4+\frac{\alpha}{2}(\nabla_\mu\phi\nabla^\mu\phi)^2+\frac{\beta_1}{2!}\phi\mathcal{G}+\frac{\beta_2}{4}\phi^2\mathcal{G}$$
$$+\frac{\gamma_0}{3!}\mathcal{R}^{(3)}+\frac{\gamma_1}{3!}\phi\mathcal{R}^{(3)}+\frac{\gamma_2}{2}\nabla_\mu\phi\nabla^\mu\phi\mathcal{R}^{(2)}-\frac{4\gamma_3}{3}\nabla_\mu\phi\nabla_\rho\phi\nabla_\nu\nabla_\sigma\phi R^{\mu\nu\rho\sigma}+\dots\Big)$$

Hairy black holes

No-hair theorems   Ruffini & Wheeler, 1971

uniqueness of BHs even in presence of matter fields

scalar field case: a few no-go theorems

Hawking, 1972; Bekenstein, 1995; Sotiriou & Faraoni, 1109.6324; Hui & Nicolis, 1202.1296

But there are hairy BHs   Sotiriou & **SYZ**, 1312.3622

$$S = \frac{M_p^2}{2} \int d^4x \sqrt{-g} \left(R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \alpha \phi \mathcal{G} \right)$$

$\mathcal{G}$: Gauss-Bonnet invariant

from EFT viewpoint, easy to have hairy BHs: $\phi \mathcal{G}$ is leading n.m. coupling

Used as a fiducial model to

test deviations from GR in strong gravity regime (GWs, …)

Spontaneous scalarization

- GR solution with $\phi = 0$ in weak gravity
- Deviates from GR in strong gravity

Neutron stars

Damour & Esposito-Farese, 1995

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}[R - 2\nabla_\mu\varphi\nabla^\mu\varphi] + S_m\left[\Psi_m; \mathcal{A}^2(\varphi)g_{\mu\nu}\right]$$

non-minimal coupling

Black holes

Doneva & Yazadjiev, 1711.01187
Silva, Sakstein, Gualtieri, Sotiriou & Berti, 1711.02080

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}\nabla_\mu\varphi\nabla^\mu\varphi + f(\varphi)\mathcal{G}\right]$$

$f(\varphi) = a\varphi^2 + b\varphi^4 + \dots$ no linear term, so not always hairy

Both of them rely on tachyonic instability in scalar sector

$$\Box\,\delta\varphi + m^2(\mathcal{G}, T)\,\delta\varphi + \dots = 0$$

$m^2 > 0$ stay in GR solution

$m^2 < 0$ roll down to hairy solution

Fixed t dispersion relations with graviton

$$\delta_{k,2} a^{1234}_{k,-1} \frac{1}{t} + \sum_{n=0} a^{1234}_{k,n} t^n = \left\langle \frac{\partial_s^k}{k!} \left[\frac{s^2 d^{\ell,\mu,t}_{h_{12},h_{43}} c^{12}_{\ell,\mu} c^{*\bar{3}\bar{4}}_{\ell,\mu}}{\mu^2(\mu - s)} + \frac{(-s-t)^2 d^{\ell,\mu,t}_{h_{14},h_{23}} c^{14}_{\ell,\mu} c^{*,\bar{3}\bar{2}}_{\ell,\mu}}{\mu^2(\mu + s + t)} \right] \bigg|_{s\to 0} \right\rangle ,$$

t-channel pole s^2/t survives twice subtraction

$$\left\langle \cdots \right\rangle := 16\pi \sum_{\ell,X} (2\ell+1) \int_{\Lambda^2}^{\infty} \frac{\mathrm{d}\mu}{\pi} (\cdots)$$

st crossing symmetry

$$a^{1234}_{k,n} = a^{1324}_{n,k} , \quad n \geq 3$$

Thrice subtracted dispersion relations

Improved dispersion relations

$$\delta_{k,2} a^{1234}_{2,-1} \frac{1}{t} + a^{1234}_{k,0} + a^{1234}_{k,1} t + a^{1234}_{k,2} t^2 = \left\langle F^{1234}_{k,\ell}(\mu, t) \right\rangle$$

suitable to use even when $t \sim \Lambda$

More than 50 dispersion relations for leading few coefficients

$$-\frac{1}{M_P^2}+2\alpha t-\gamma_4 t^2=\left\langle F^{0000}_{1,\ell}(\mu,t)\right\rangle \tag{B.2}$$

$$-\frac{1}{M_P^2}\frac{1}{t}+2\alpha-\gamma_4 t+12g^S_{0,2}t^2=\left\langle F^{0000}_{2,\ell}(\mu,t)\right\rangle \tag{B.3}$$

$$8g^S_{0,2}t-4g^S_{1,1}t^2=\left\langle F^{0000}_{3,\ell}(\mu,t)\right\rangle \tag{B.4}$$

$$4g^S_{0,2}-2g^S_{1,1}t+\left(g^S_{2,0}+48g^S_{3,0}\right)t^2=\left\langle F^{0000}_{4,\ell}(\mu,t)\right\rangle \tag{B.5}$$

$$\frac{\beta_1}{M_P^3}t-\frac{\gamma_3}{M_P}t^2=\left\langle F^{+000}_{1,\ell}(\mu,t)\right\rangle \tag{B.6}$$

$$\frac{\beta_1}{M_P^3}-\frac{\gamma_3}{M_P}t=\left\langle F^{+000}_{2,\ell}(\mu,t)\right\rangle \tag{B.7}$$

$$-4g^{M_5}_{1,1}t^2=\left\langle F^{+000}_{3,\ell}(\mu,t)\right\rangle \tag{B.8}$$

$$-2g^{M_5}_{1,1}t+g^{M_5}_{2,0}t^2=\left\langle F^{+000}_{4,\ell}(\mu,t)\right\rangle \tag{B.9}$$

$$-\frac{\gamma_1}{M_P^3}t^2=\left\langle F^{+++0}_{1,\ell}(\mu,t)\right\rangle \tag{B.10}$$

$$-\frac{\gamma_1}{M_P^3}t=\left\langle F^{+++0}_{2,\ell}(\mu,t)\right\rangle \tag{B.11}$$

$$-4g^{M_1}_{1,1}t^2=\left\langle F^{+++0}_{3,\ell}(\mu,t)\right\rangle \tag{B.12}$$

$$-2g^{M_1}_{1,1}t+g^{M_1}_{2,0}t^2=\left\langle F^{+++0}_{4,\ell}(\mu,t)\right\rangle \tag{B.13}$$

$$-\frac{\gamma_0}{M_P^4}t^2=\left\langle F^{+++-}_{1,\ell}(\mu,t)\right\rangle \tag{B.14}$$

$$-\frac{\gamma_0}{M_P^4}t=\left\langle F^{+++-}_{2,\ell}(\mu,t)\right\rangle \tag{B.15}$$

$$0=\left\langle F^{+++-}_{3,\ell}(\mu,t)\right\rangle \tag{B.16}$$

$$g^{T_2}_{2,0}t^2=\left\langle F^{+++-}_{4,\ell}(\mu,t)\right\rangle \tag{B.17}$$

$$\left(-\frac{10\gamma_0}{M_P^4}+\frac{3\beta_1^2}{M_P^4}\right)t^2=\left\langle F^{++++}_{1,\ell}(\mu,t)\right\rangle \tag{B.18}$$

$$\left(-\frac{10\gamma_0}{M_P^4}+\frac{3\beta_1^2}{M_P^4}\right)t+12g^{T_3}_{0,2}t^2=\left\langle F^{++++}_{2,\ell}(\mu,t)\right\rangle \tag{B.19}$$

$$8g^{T_3}_{0,2}t-4g^{T_3}_{1,1}t^2=\left\langle F^{++++}_{3,\ell}(\mu,t)\right\rangle \tag{B.20}$$

$$4g^{T_3}_{0,2}-2g^{T_3}_{1,1}t+\left(g^{T_3}_{2,0}+48g^{T_3}_{0,3}\right)t^2=\left\langle F^{++++}_{4,\ell}(\mu,t)\right\rangle . \tag{B.21}$$

$$\frac{\beta_2}{M_P^2}-\frac{\gamma_0}{M_P^4}t-g^{M_3}_{2,1}t^2=\left\langle F^{++00}_{2,\ell}(\mu,t)\right\rangle \tag{B.22}$$

$$\frac{\gamma_2}{M_P^2}+\frac{\beta_1^2}{M_P^4}-g^{M_3}_{2,1}t-g^{M_3}_{3,1}t^2=\left\langle F^{++00}_{3,\ell}(\mu,t)\right\rangle \tag{B.23}$$

$$g^{M_3}_{4,0}-g^{M_3}_{3,1}t+(g^{M_3}_{2,2}-g^{M_3}_{4,1})t^2=\left\langle F^{++00}_{4,\ell}(\mu,t)\right\rangle \tag{B.24}$$

$$-\frac{\gamma_0}{M_P^4}t^2=\left\langle F^{+0+0}_{1,\ell}(\mu,t)\right\rangle \tag{B.25}$$

$$-\frac{\gamma_0}{M_P^4}t-g^{M_3}_{2,1}t^2=\left\langle F^{+0+0}_{2,\ell}(\mu,t)\right\rangle \tag{B.26}$$

$$0=\left\langle F^{+0+0}_{3,\ell}(\mu,t)\right\rangle \tag{B.27}$$

$$g^{M_3}_{2,2}t^2=\left\langle F^{+0+0}_{4,\ell}(\mu,t)\right\rangle \tag{B.28}$$

$$-\frac{\beta_1^2}{M_P^4}t+g^{M_4}_{0,2}t^2=\left\langle F^{+-00}_{2,\ell}(\mu,t)\right\rangle \tag{B.29}$$

$$g^{M_4}_{1,2}t^2=\left\langle F^{+-00}_{3,\ell}(\mu,t)\right\rangle \tag{B.30}$$

$$g^{M_4}_{2,2}t^2=\left\langle F^{+-00}_{4,\ell}(\mu,t)\right\rangle \tag{B.31}$$

$$-\frac{1}{M_P^2}-\frac{\beta_1^2}{M_P^4}t^2=\left\langle F^{+0-0}_{1,\ell}(\mu,t)\right\rangle \tag{B.32}$$

$$-\frac{1}{M_P^2}\frac{1}{t}-\frac{\beta_1^2}{M_P^4}t+g^{M_4}_{0,2}t^2=\left\langle F^{+0-0}_{2,\ell}(\mu,t)\right\rangle \tag{B.33}$$

$$2g^{M_4}_{0,2}t+2g^{M_4}_{1,2}t^2=\left\langle F^{+0-0}_{3,\ell}(\mu,t)\right\rangle \tag{B.34}$$

$$g^{M_4}_{0,2}+g^{M_4}_{1,2}t+\left(g^{M_4}_{2,2}-3g^{M_4}_{0,3}\right)t^2=\left\langle F^{+0-0}_{4,\ell}(\mu,t)\right\rangle \tag{B.35}$$

$$\frac{\beta_1}{M_P^3}+\frac{\beta_1\gamma_0}{M_P^5}t^2=\left\langle F^{0-++}_{2,\ell}(\mu,t)\right\rangle \tag{B.36}$$

$$\frac{\gamma_0\beta_1}{M_P^5}t-g^{M_2}_{3,1}t^2=\left\langle F^{0-++}_{3,\ell}(\mu,t)\right\rangle \tag{B.37}$$

$$-g^{M_2}_{3,1}t-g^{M_2}_{4,1}t^2=\left\langle F^{0-++}_{4,\ell}(\mu,t)\right\rangle \tag{B.38}$$

$$0=\left\langle F^{0+-+}_{1,\ell}(\mu,t)\right\rangle \tag{B.39}$$

$$\frac{\gamma_0\beta_1}{M_P^5}t^2=\left\langle F^{0+-+}_{2,\ell}(\mu,t)\right\rangle \tag{B.40}$$

$$0=\left\langle F^{0+-+}_{3,\ell}(\mu,t)\right\rangle \tag{B.41}$$

$$0=\left\langle F^{0+-+}_{4,\ell}(\mu,t)\right\rangle \tag{B.42}$$

$$0=\left\langle F^{+-+-}_{1,\ell}(\mu,t)\right\rangle \tag{B.43}$$

$$0=\left\langle F^{+-+-}_{2,\ell}(\mu,t)\right\rangle \tag{B.44}$$

$$0=\left\langle F^{+-+-}_{3,\ell}(\mu,t)\right\rangle \tag{B.45}$$

$$0=\left\langle F^{+-+-}_{4,\ell}(\mu,t)\right\rangle \tag{B.46}$$

$$0=\left\langle F^{+-+-}_{5,\ell}(\mu,t)\right\rangle \tag{B.47}$$

$$0=\left\langle F^{+-+-}_{6,\ell}(\mu,t)\right\rangle \tag{B.48}$$

$$-\frac{1}{M_P^2}\frac{1}{t}=\left\langle F^{++--}_{2,\ell}(\mu,t)\right\rangle \tag{B.49}$$

$$-\frac{\beta_1^2}{M_P^4}-\frac{\gamma_0^2}{M_P^6}t^2=\left\langle F^{++--}_{3,\ell}(\mu,t)\right\rangle \tag{B.50}$$

$$g^{T_1}_{4,0}-\frac{\gamma_0^2}{M_P^6}t-g^{T_1}_{4,1}t^2=\left\langle F^{++--}_{4,\ell}(\mu,t)\right\rangle \tag{B.51}$$

$$g^{T_1}_{5,0}-g^{T_1}_{4,1}t-g^{T_1}_{5,1}t^2=\left\langle F^{++--}_{5,\ell}(\mu,t)\right\rangle \tag{B.52}$$

$$g^{T_1}_{6,0}-g^{T_1}_{5,1}t+\left(g^{T_1}_{4,2}-g^{T_1}_{6,1}\right)t^2=\left\langle F^{++--}_{6,\ell}(\mu,t)\right\rangle . \tag{B.53}$$

+ higher order forward sum rules

$$F^{\mathbf{1234}}_{k,\ell}(\mu,t)=\frac{\partial_s^k}{k!}\left(\frac{s^2}{\mu^2(\mu-s)}d^{\ell,\mu,t}_{h_{12},h_{43}}c^{\mathbf{12}}_{\ell,\mu}c^{*\bar{\mathbf{3}}\bar{\mathbf{4}}}_{\ell,\mu}+\frac{(-s-t)^2}{\mu^2(\mu+s+t)}d^{\ell,\mu,t}_{h_{14},h_{23}}c^{\mathbf{14}}_{\ell,\mu}c^{*,\bar{\mathbf{3}}\bar{\mathbf{2}}}_{\ell,\mu}\right)\Bigg|_{s\to0}$$
$$-\frac{\partial_t^k}{k!}\left(\frac{s^3}{\mu^3(\mu-s)}d^{\ell,\mu,t}_{h_{13},h_{42}}c^{\mathbf{13}}_{\ell,\mu}c^{*\bar{\mathbf{2}}\bar{\mathbf{4}}}_{\ell,\mu}+\frac{(-s)^3}{(\mu+t)^3(\mu+s+t)}d^{\ell,\mu,t}_{h_{14},h_{32}}c^{\mathbf{14}}_{\ell,\mu}c^{*\bar{\mathbf{2}}\bar{\mathbf{3}}}_{\ell,\mu}\right)\Bigg|_{t\to0,s\to t}.$$

Hong, Wang, **SYZ**, 2304.01259

Graviton t-channel pole

Spin-2 pole s^2/t survives twice subtraction

$$\boxed{\frac{1}{M_P^2 t}} + (\cdots) \sim \int_{\Lambda^2}^{\infty} \frac{\mathrm{d}\mu}{\mu^3} \mathrm{Im}\, A(\mu, t)(\cdots)$$

Bounds are not strictly positive

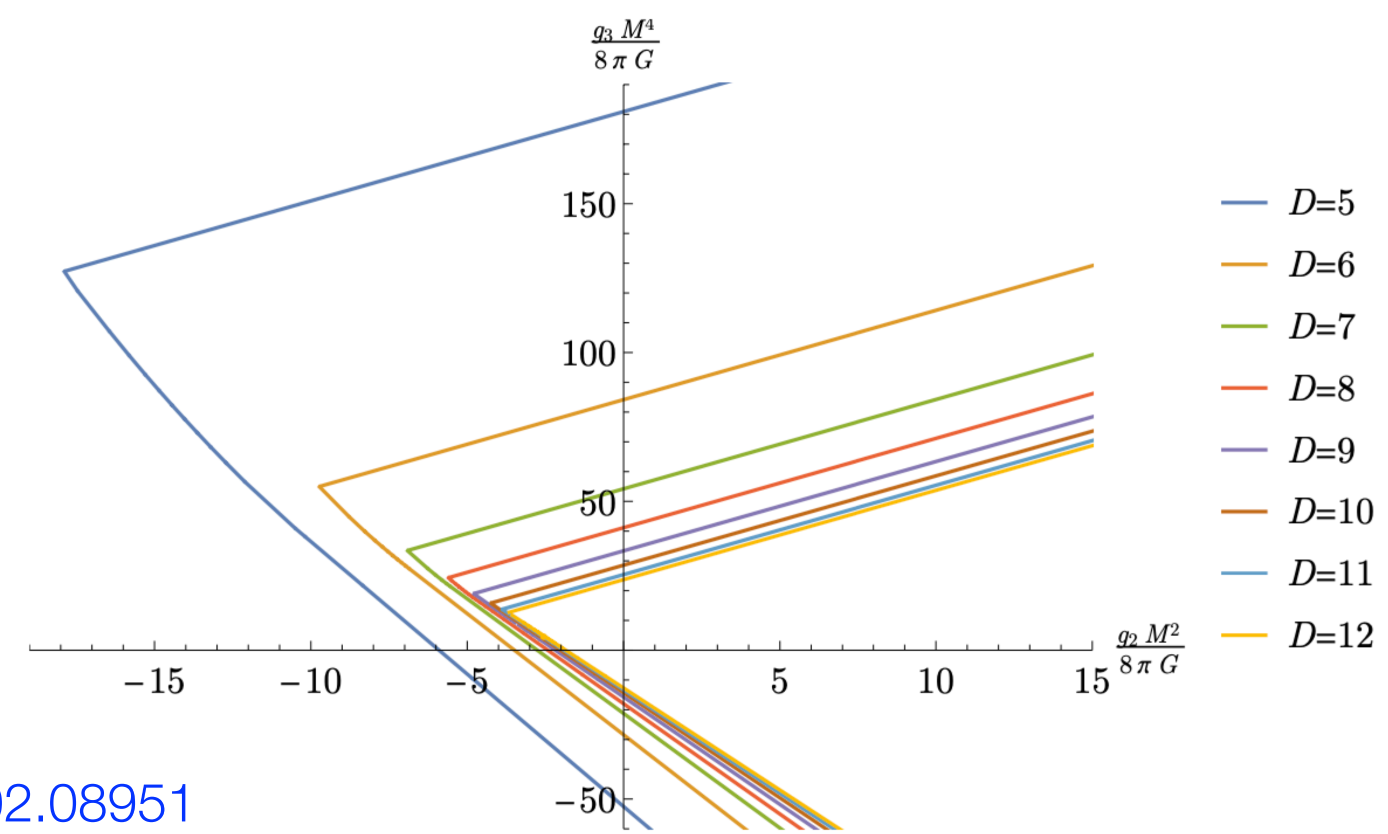
Alberte, de Rham, Jaitly & Tolley, 2007.12667
Tokuda, Aoki & Hirano, 2007.15009

$$a_{2,0} > -\frac{\Lambda^2}{M_{\mathrm{Pl}}^2} \times \mathcal{O}(1)$$

Functional optimization

Use impact parameter

$$t \;\to\; b = 2\ell/\mu^{1/2}$$



Caron-Huot, Mazac, Rastelli, Simmons-Duffin, 2102.08951

Functional optimization

Optimize against weight functions $\phi_k^{1234}(p)$ $t := -p^2$

$$\sum_{1234,k}\int_0^\Lambda \mathrm{d}p\,\phi_k^{1234}(p)\left[\delta_{k,2}a_{k,-1}^{1234}\frac{-1}{p^2}+a_{k,0}^{1234}+a_{k,1}^{1234}\left(-p^2\right)+a_{k,2}^{1234}p^4\right]=\left\langle\sum_{1234,k}\int_0^\Lambda \mathrm{d}p\,\phi_k^{1234}(p)F_{k,\ell}^{1234}(\mu,-p^2)\right\rangle$$

Wilson coefficients UV information

$$\sum\int\phi(p)a_{k,n}\geq 0 \longleftarrow \sum\int\phi(p)F\geq 0$$

Factor out UV spectral functions $c_{P_X,\ell,\mu}=\left(c_{P_X,\ell,\mu}^{00}\ c_{P_X,\ell,\mu}^{+0}\ c_{P_X,\ell,\mu}^{++}\ c_{P_X,\ell,\mu}^{+-}\right)^T$

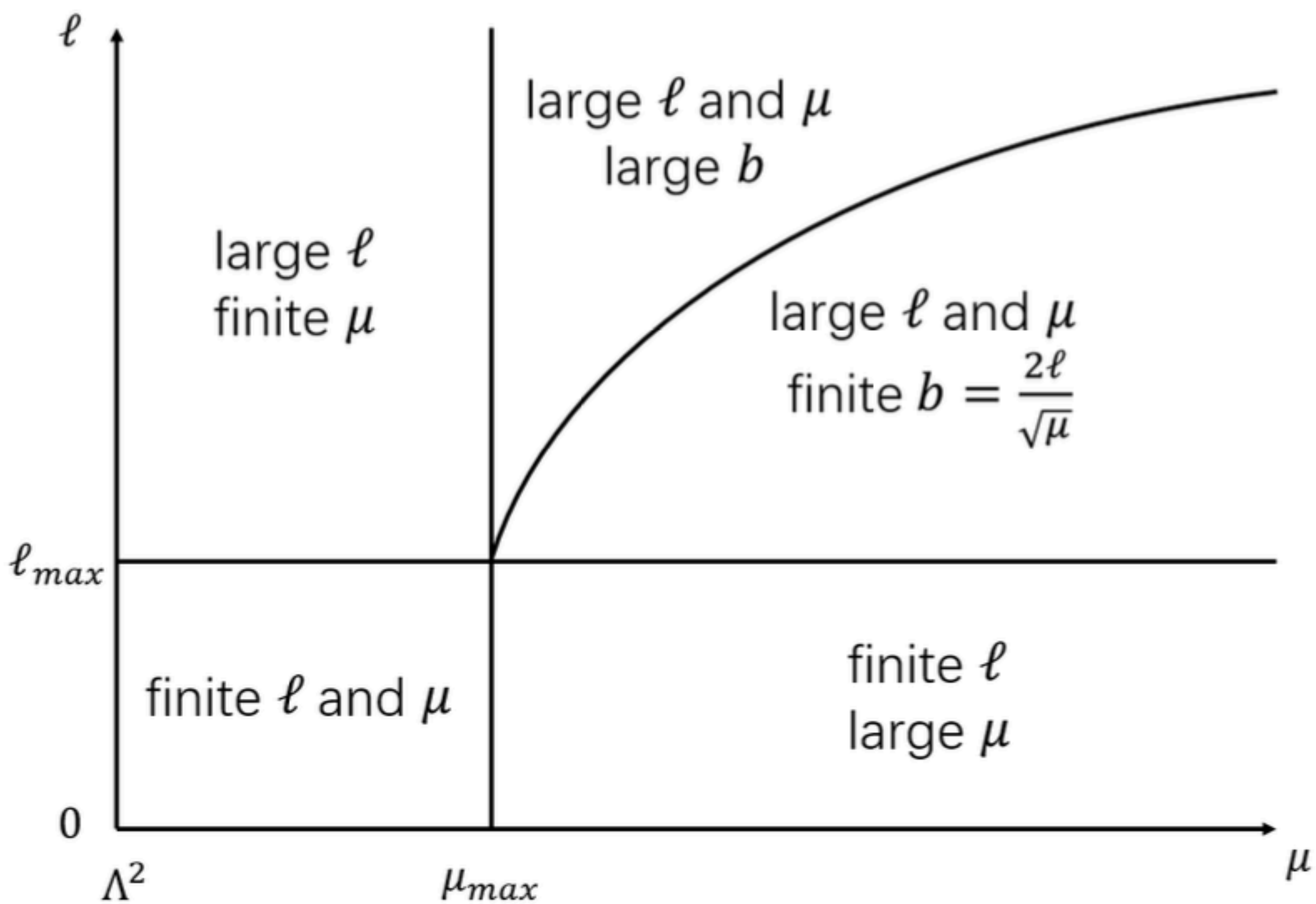
$$\sum_{1234,k}\int_0^\Lambda \mathrm{d}p\phi_k^{1234}(p)F_{k,\ell}^{1234}(\mu,-p^2):=\sum_{P_X=\pm1}\sum_{\mathbb{A},\mathbb{B}}B_{P_X,\ell}^{\mathbb{A},\mathbb{B}}(\mu)c_{P_X,\ell,\mu}^{\mathbb{A}}c_{P_X,\ell,\mu}^{\mathbb{B}}$$

$B_{P_X,\ell}(\mu)\succeq 0\,,$ for $P_X=\pm1$, all possible ℓ and all $\mu\geq\Lambda^2$

Numerical implementation

Sampling the UV constraint space

ℓ: UV spins μ: UV mass



SDP: ~400 decision variables
~16000 4X4 matrix constraints

- various approximations
- polynomials for $\phi_k^{\mathbb{1}\mathbb{2}\mathbb{3}\mathbb{4}}(p)$
- impact parameter sampling

$$\text{fixed } b = 2\ell/\sqrt{\mu}.$$

- in 4D, IR cutoff needed

$$\int_0^1 \mathrm{d}p \frac{\phi(p)}{p^2} = \infty \;\Rightarrow\; \int_{m_{\mathrm{IR}}}^1 \mathrm{d}p \frac{\phi(p)}{p^2} \sim \log\frac{\Lambda}{m_{\mathrm{IR}}}$$

eg, $\log\frac{\Lambda}{m_{\mathrm{IR}}} \geq \left\{\cdots\right\}\frac{\gamma_0^2}{M_P^4} + \left\{\cdots\right\}\frac{\beta_1^2}{M_P^2}$

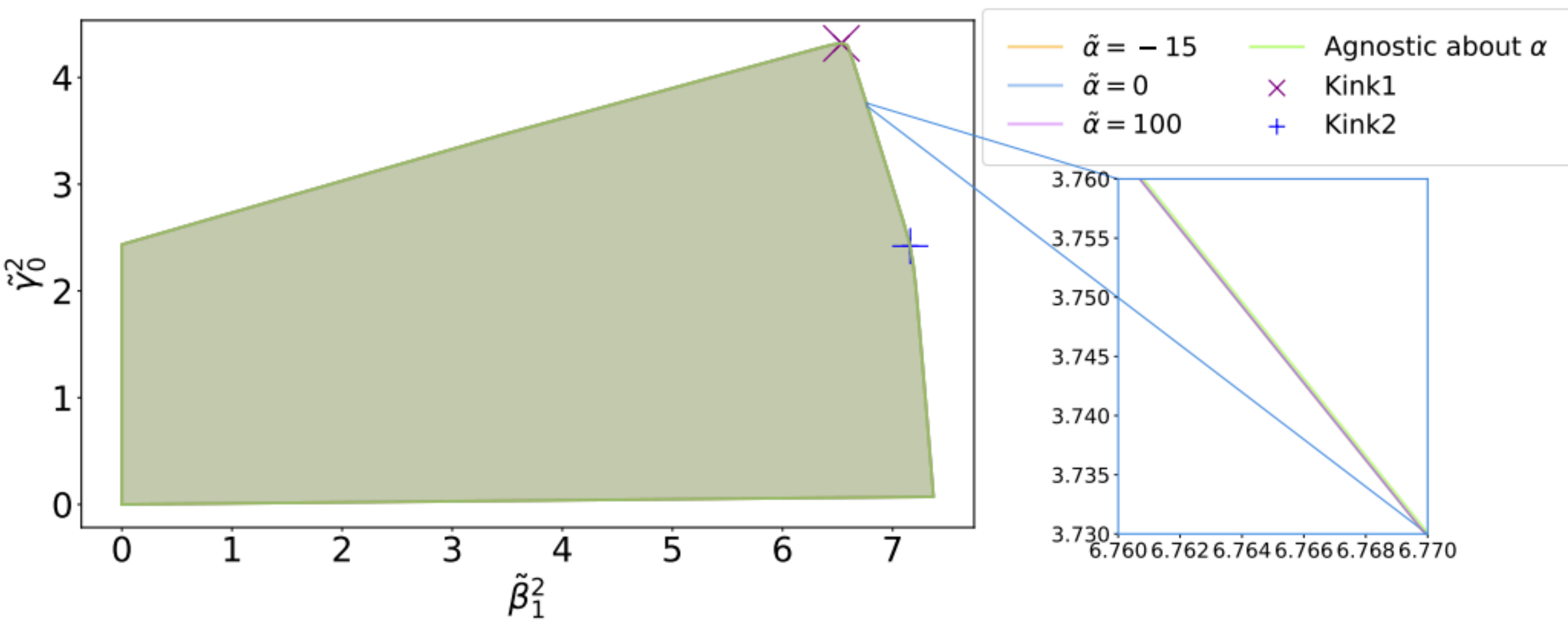
- add higher order forward sum rules

Results

Two-sided bounds on leading coefficients

$$S = \int \mathrm{d}^4x \sqrt{-g} \Big(\frac{M_P^2}{2} R - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - \frac{\lambda_3}{3!} \phi^3 - \frac{\lambda_4}{4!} \phi^4 + \frac{\alpha}{2} (\nabla_\mu \phi \nabla^\mu \phi)^2 + \boxed{\frac{\beta_1}{2!} \phi \mathcal{G}} + \frac{\beta_2}{4} \phi^2 \mathcal{G}$$
$$+ \boxed{\frac{\gamma_0}{3!} \mathcal{R}^{(3)}} + \frac{\gamma_1}{3!} \phi \mathcal{R}^{(3)} + \frac{\gamma_2}{2} \nabla_\mu \phi \nabla^\mu \phi \mathcal{R}^{(2)} - \frac{4\gamma_3}{3} \nabla_\mu \phi \nabla_\rho \phi \nabla_\nu \nabla_\sigma \phi R^{\mu\nu\rho\sigma} + \dots \Big)$$

leading corrections



Hong, Wang, **SYZ**, 2304.01259

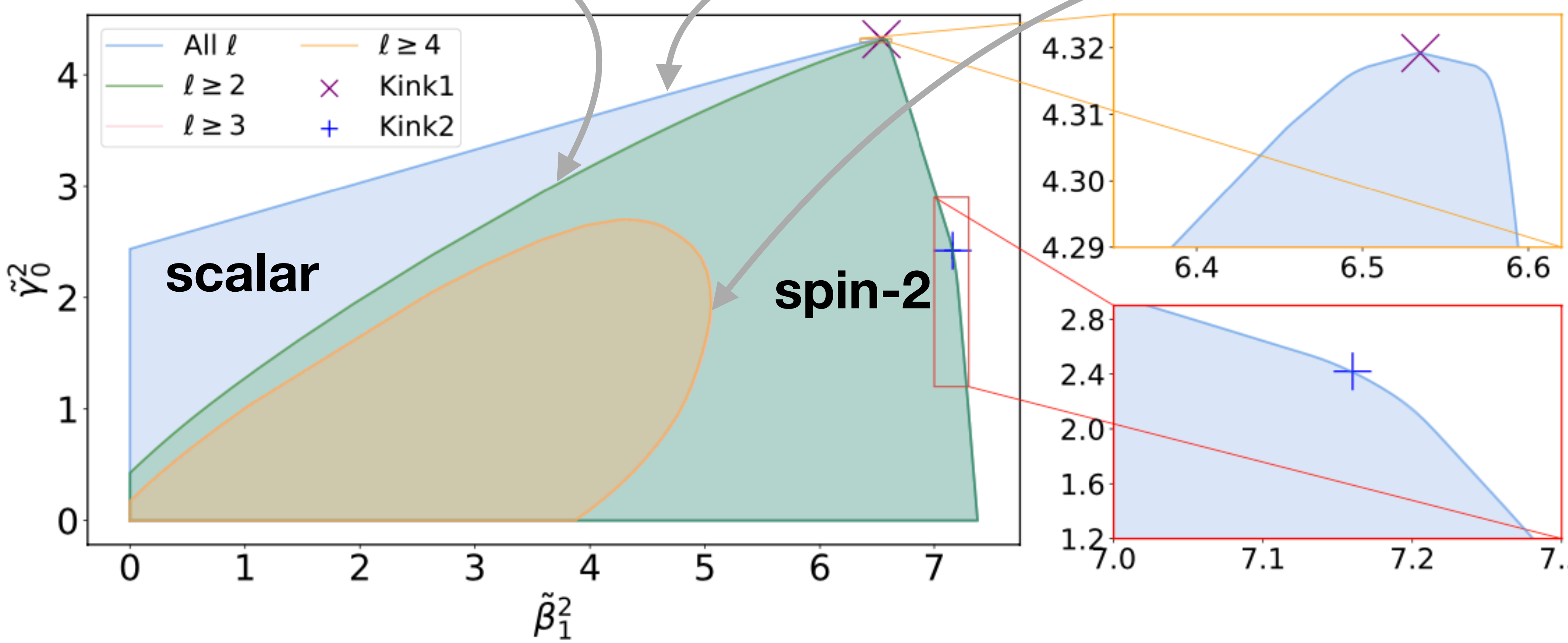
$$\tilde{\beta}_1^2 = \beta_1^2 \Lambda^4 / \big(M_P^2 \log(\Lambda/m_{\mathrm{IR}}) \big), \qquad \tilde{\gamma}_0^2 = \gamma_0^2 \Lambda^8 / \big(M_P^4 \log(\Lambda/m_{\mathrm{IR}}) \big)$$

Inferring UV spectrum

UV spins ≥ 2

all UV spins

UV spins ≥ 3



only with $\ell \geq 5$ is inconsistent

only with $\ell < \ell_{\mathrm{M}} \Rightarrow \gamma_0 = \beta_1 = 0$

Spin selection rules

Hong, Wang, **SYZ**, 2304.01259

$$\mathrm{Disc}\mathcal{M}^{1234}(\mu, t) \sim \sum_{\ell} d^{\ell}_{h_{12},h_{43}}(\arccos(1 + 2t/\mu))c^{12}_{\ell,\mu}(c^{\bar{3}\bar{4}}_{\ell,\mu})^*$$

$\ell = 1$: either $d^{\ell}_{h_{12},h_{43}} = 0$ or $c^{12}_{\ell,\mu} = 0$

$\ell = 3$: mostly either $d^{\ell}_{h_{12},h_{43}} = 0$ or $c^{12}_{\ell,\mu} = 0$

Fine-tuned EFTs

eg $\mathcal{L} = \sqrt{-g}\left(\frac{M_P^2}{2}R - \frac{1}{2}(\partial\phi)^2 + \left(\frac{\beta_1}{2!}\phi + \frac{\beta_2}{4}\phi^2 + \ldots\right)\mathcal{G}\right)$ set HO coeff's to 0

$$\left.\begin{aligned} \left\langle \frac{1}{\mu^5}\left(\left|c_{\ell,\mu}^{++}\right|^2 + \left|c_{\ell,\mu}^{+-}\right|^2\right)\right\rangle = 0 \\ -\frac{1}{M_P^2}\frac{1}{t} = \left\langle \frac{1}{\mu^3} d_{0,0}^{\ell}\left|c_{\ell,\mu}^{++}\right|^2 + \frac{1}{(\mu+t)^3} d_{4,4}^{\ell}\left|c_{\ell,\mu}^{+-}\right|^2\right\rangle \end{aligned}\right\}$$

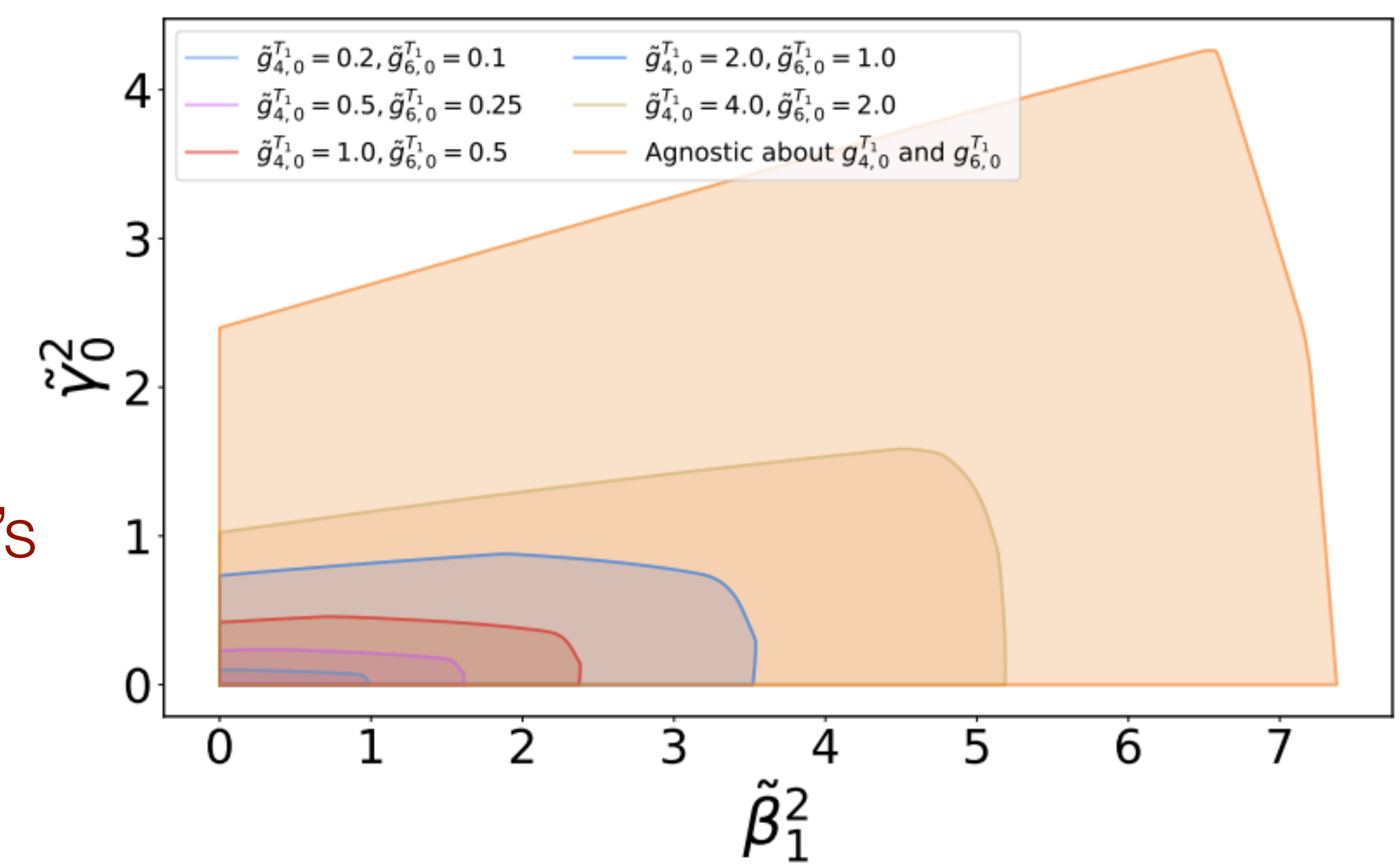
$M_P \to \infty \Rightarrow$ inconsistent

$$\boxed{\text{Avoid: } \begin{array}{l} c_{\ell,\mu}^{00} = 0 \ \text{ or } \ c_{\ell,\mu}^{+0} = 0 \\ \text{or } \ c_{\ell,\mu}^{++} = c_{\ell,\mu}^{+-} = 0 \end{array}}$$

Suppressed coeff's?

LO coeff's bounded by HO coeff's

Hong, Wang, **SYZ**, 2304.01259



Is scalarization natural?

Generically, causality bounds require

$$\mathcal{L} \supset M_P^2 \sqrt{-g}\left(\frac{\mathcal{O}(1)}{\Lambda^2} \boxed{\varphi \mathcal{G}} + \frac{\mathcal{O}(1) M_P}{\Lambda^3} \boxed{\varphi^2 \mathcal{G}} \right)$$

hairy BH

spontaneous scalarization

$\phi^2 \mathcal{G}$ term can be much bigger

Scalarization is natural!

Three EFT theorists

Two scales in Einstein EFT: $M_P, \ \Lambda$

How shall we do power counting?

Caron-Huot, Li, Parra-Martinez & Simmons-Duffin, 2201.06602

$$\sim \partial^2 h^2 + \frac{O(1)}{\Lambda^5}\partial^6 h^3$$

$$\mathcal{L} \sim M_P^2 R + \frac{O(1) M_P^3}{\Lambda^5} R^{(3)}$$

too relaxed

basically ignores M_P

$$\mathcal{L} \sim M_P^2 R + \frac{O(1)}{\Lambda^2} R^{(3)}$$

too restrictive

string theory violates it

$$\mathcal{L} \sim M_P^2 \left(R + \frac{O(1)}{\Lambda^4} R^{(3)} \right)$$

suggested positivity bounds!

correction < GR

What about adding matter fields?

Power counting via dispersion relations

$$\frac{\Lambda}{M_P} \Leftrightarrow \hat{c}^{++}_{\ell,\mu}, \hat{c}^{+-}_{\ell,\mu}, \hat{c}^{-+}_{\ell,\mu}, \hat{c}^{--}_{\ell,\mu} \qquad \frac{\Lambda}{M_P} \Leftrightarrow \hat{c}^{+0}_{\ell,\mu}, \hat{c}^{-0}_{\ell,\mu}, c^{0+}_{\ell,\mu}, \hat{c}^{0-}_{\ell,\mu} \qquad \begin{cases} 1 \Leftrightarrow \hat{c}^{00}_{\ell,\mu} \\ \frac{\Lambda}{M_P} \Leftrightarrow \hat{c}^{00}_{\ell,\mu} \end{cases}$$

$c^{++}_{\ell,\mu}, c^{+-}_{\ell,\mu}$: UV partial amplitude

Example

$$-\frac{\Lambda^6\gamma_1}{M_P^3} = \sum_{\ell,X}\int_1^\infty \mathrm{d}\hat{\mu}\left[\frac{(2\hat{\mu}-3\hat{t}\,)d^{\ell,\hat{\mu},\hat{t}}_{2,0}\,\hat{c}^{+0}_{\ell,\mu}\hat{c}^{*,--}_{\ell,\mu}}{\hat{t}\,\hat{\mu}^4} - \frac{\hat{t}\,\partial_{\hat{t}}\,d^{\ell,\hat{\mu},0}_{0,-2}\hat{c}^{++}_{\ell,\mu}\,\hat{c}^{*,-0}_{\ell,\mu}}{\hat{\mu}^3(\hat{\mu}-\hat{t}\,)} + \frac{\hat{t}\,\partial_{\hat{t}}\,d^{\ell,\hat{\mu},0}_{2,0}\,\hat{c}^{+0}_{\ell,\mu}\hat{c}^{*,--}_{\ell,\mu}}{\hat{\mu}^3(\hat{\mu}+\hat{t}\,)}\right] \quad \Rightarrow \quad \gamma_1 \sim \frac{M_P}{\Lambda^4}$$

For lowest few orders

$$\widehat{\mathcal{O}}_{\phi R} \sim M_P^2\Lambda^2\left[\frac{\nabla}{\Lambda}\right]^{N_\nabla}\left[\frac{R}{\Lambda^2}\right]^{N_R}\left[\frac{\phi}{M_P}\right]^{N_\phi}\left[\frac{M_P}{\Lambda}\right]^{\tilde{N}_\phi} \qquad \tilde{N}_\phi = \lfloor N_\phi/2 \rfloor$$

Hong, Wang, **SYZ**, 2304.01259

for higher orders, use the above correspondence rules

Summary

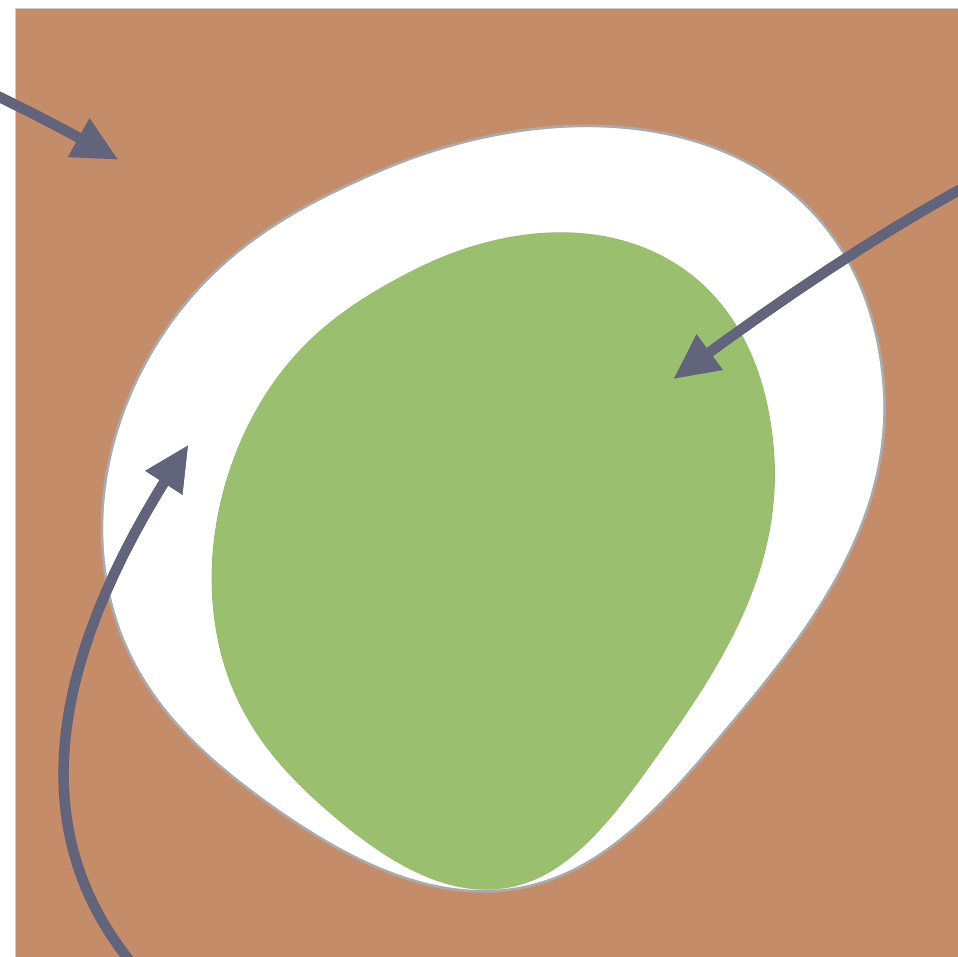
- Causality bounds are UV unitarity conditions passed down to IR by causality/analyticity/dispersion relations.

- Computed causality bounds on scalar-tensor EFTs
  - Another nontrivial multi-field example for two-sided bounds
    - not easy due to graviton t-channel pole
  - Relevant for relativistic astrophysics (hairy BH, scalarization, …)
    - parameter space for scalarization is natural

- Found simple way to power-count gravitational EFT operators via dispersion relations

Positivity bounds vs primal (S-matrix) bootstrap

QFT theory space



Positivity bounds
(*Dual bootstrap*)

- rule out unviable space
- <u>use dispersion relation</u>
- impose full crossing
- numerically efficient

Primal bootstrap

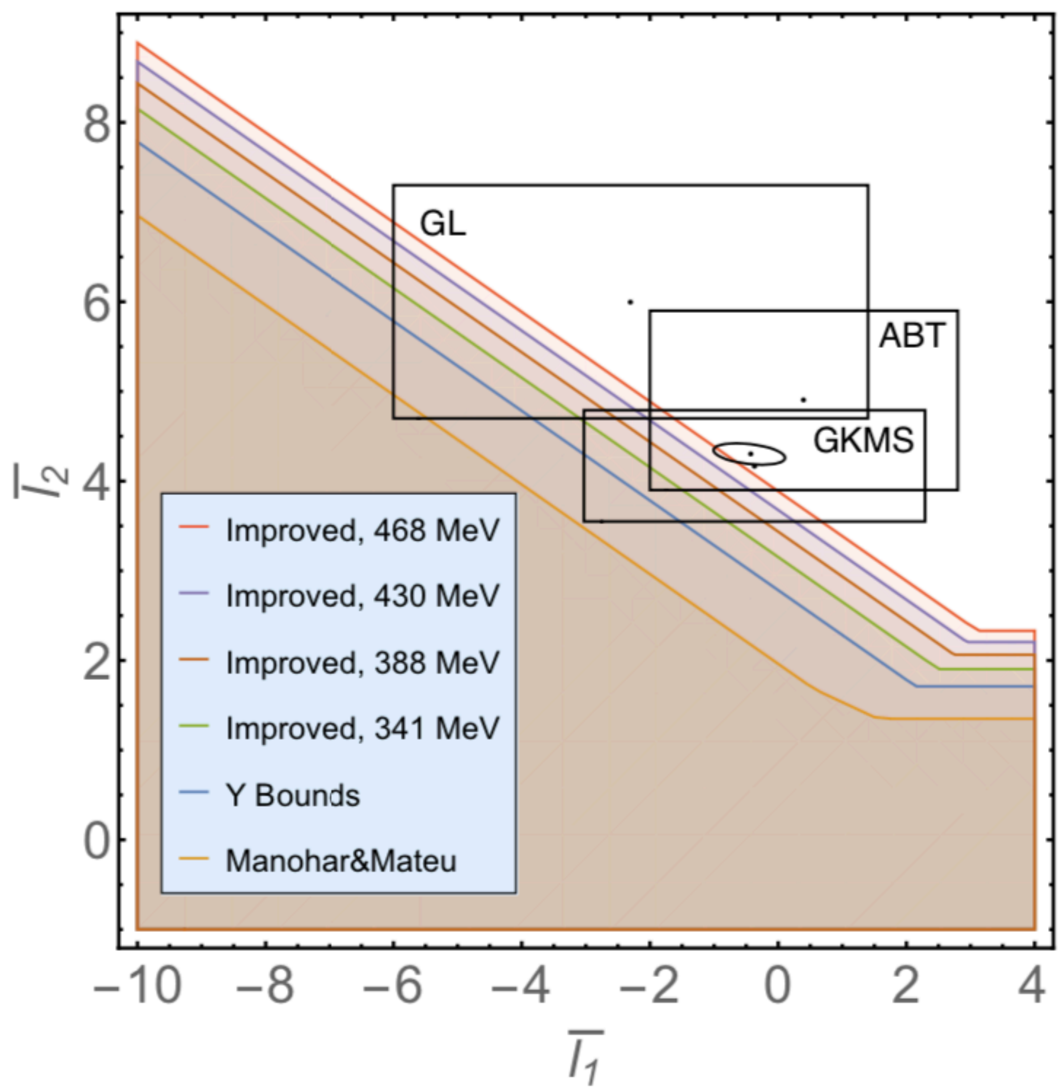
- rule in viable space
- <u>construct amplitudes</u>
- full crossing built-in
- may need extrapolation

Duality gap: ideally, it shrinks to zero
Many models worked out in both approaches

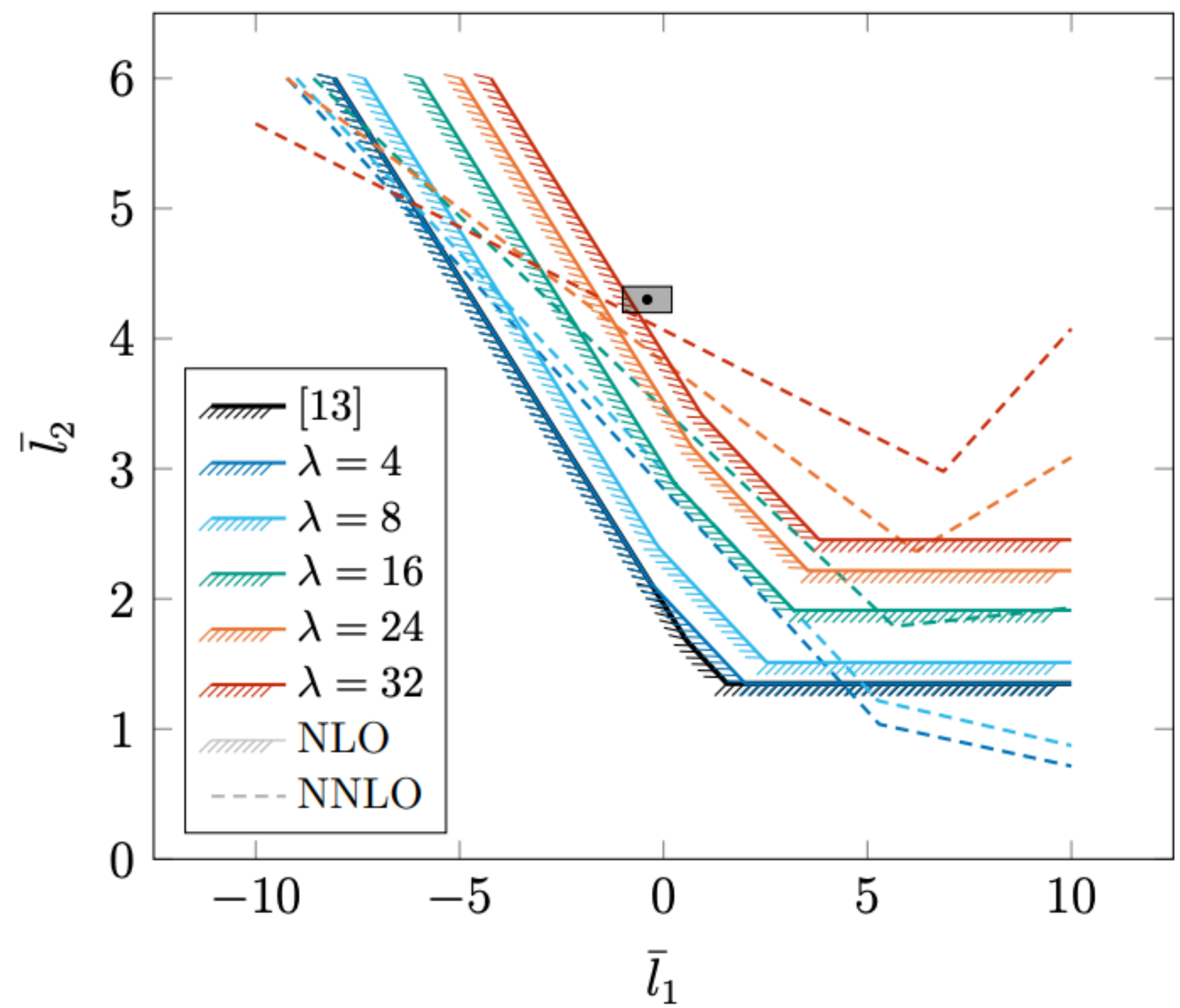
Primal S-matrix Bootstrap Snowmass white paper, Kruczenski, Penedones, van Rees, 2203.02421

Applications on Chiral PT

For example, bounds on $O(p^4)$ coefficients



Wang, Feng, Zhang & **SYZ,** 2004.03992



Alvarez, Bijnens, Sjo, 2112.04253

See also: Manohar & Mateu, 0801.3222; Du, Guo, Meibner & Yao, 1610.02963
Guerrieri, Penedones & Vieira, 2011.02802

Example: shift-symmetric Horndeski theory

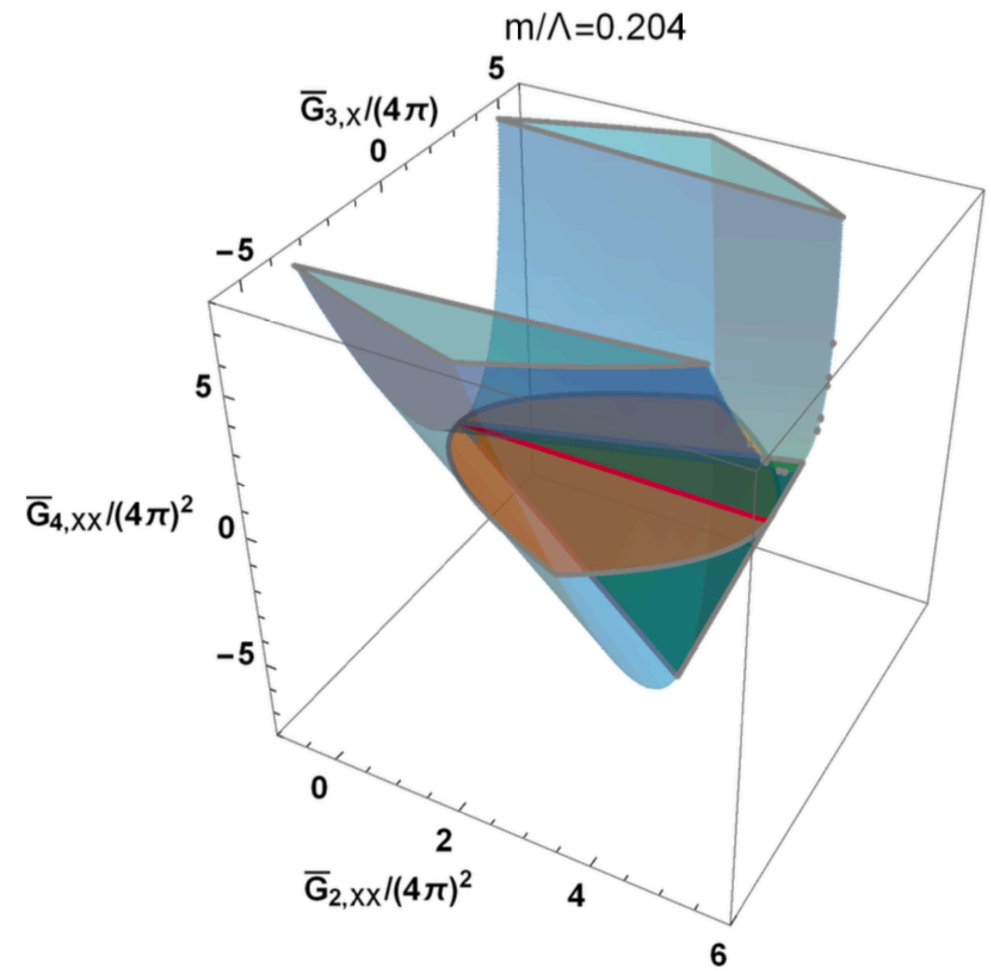
Horndeski theory

$$\mathcal{L}_2^{\mathrm{H}} = M_P^2 \Lambda^2 G_2(\phi, X),$$
$$\mathcal{L}_3^{\mathrm{H}} = \frac{M_P^2}{\Lambda} G_3(\phi, X)\Box\phi,$$
$$\mathcal{L}_4^{\mathrm{H}} = M_P^2 G_4(\phi, X) R + \frac{M_P^2}{\Lambda^4} G_{4,X}\left((\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2\right),$$
$$\mathcal{L}_5^{\mathrm{H}} = \frac{M_P^2}{\Lambda^3} G_5(\phi, X) G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{M_P^2 G_{5,X}}{6\Lambda^7}\left((\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right)$$

$$\frac{M_P^2}{\Lambda^2} G_i(\phi, X) = \left(\frac{M_P^2}{\Lambda^2}\right)^{\eta_i}\bar{G}_i + \frac{\bar{G}_{i,\phi}}{\Lambda}\phi + \frac{\bar{G}_{i,\phi\phi}}{2\Lambda^2}\phi^2 + \bar{G}_{i,X}X + \frac{\bar{G}_{i,\phi X}}{\Lambda}\phi X + \frac{\bar{G}_{i,\phi\phi X}}{\Lambda^2}\phi^2 X + \frac{1}{2}\bar{G}_{i,XX}X^2 + \dots$$

Optimized two-sided bounds:

$$\begin{cases} \dfrac{(3\bar{G}_{3,X}^2 - 6\bar{G}_{4,XX})}{4} - \left(\dfrac{\bar{G}_{2,XX}}{2} + \dfrac{1}{2}\left(\dfrac{m}{\Lambda}\right)^2(3\bar{G}_{3,X}^2 - 4\bar{G}_{4,XX})\right)\bar{c}_{\min}^{2,1}(m) \geq 0 \\ \dfrac{(3\bar{G}_{3,X}^2 - 6\bar{G}_{4,XX})}{4} - \left(\dfrac{\bar{G}_{2,XX}}{2} + \dfrac{1}{2}\left(\dfrac{m}{\Lambda}\right)^2(3\bar{G}_{3,X}^2 - 4\bar{G}_{4,XX})\right)\bar{c}_{\max}^{2,1}(m) \leq 0 \\ 0 \leq \dfrac{\bar{G}_{2,XX}}{2} + \dfrac{1}{2}\left(\dfrac{m}{\Lambda}\right)^2(3\bar{G}_{3,X}^2 - 4\bar{G}_{4,XX}) \leq \Lambda^4 c_{\max}^{2,0}(m). \end{cases}$$



Single field vs multiple fields

Optical theorem (for identical particle)

$$\mathrm{Im}\, a_\ell^{iiii} = \sum_X a_\ell^{ii\to X} \left(a_\ell^{ii\to X}\right)^* = \sum_X |a_\ell^{ii\to X}|^2 > 0$$

positive number

use **linear programing** to obtain optimal bounds

Du, Zhang & **SYZ**, 2111.01169

Generalized optical theorem (for multiple fields)

$$\mathrm{Im}\, a_\ell^{ijkl} = \sum_X a_\ell^{ij\to X} \left(a_\ell^{kl\to X}\right)^*$$

positive matrix

use **semi-definite programing** to obtain optimal bounds

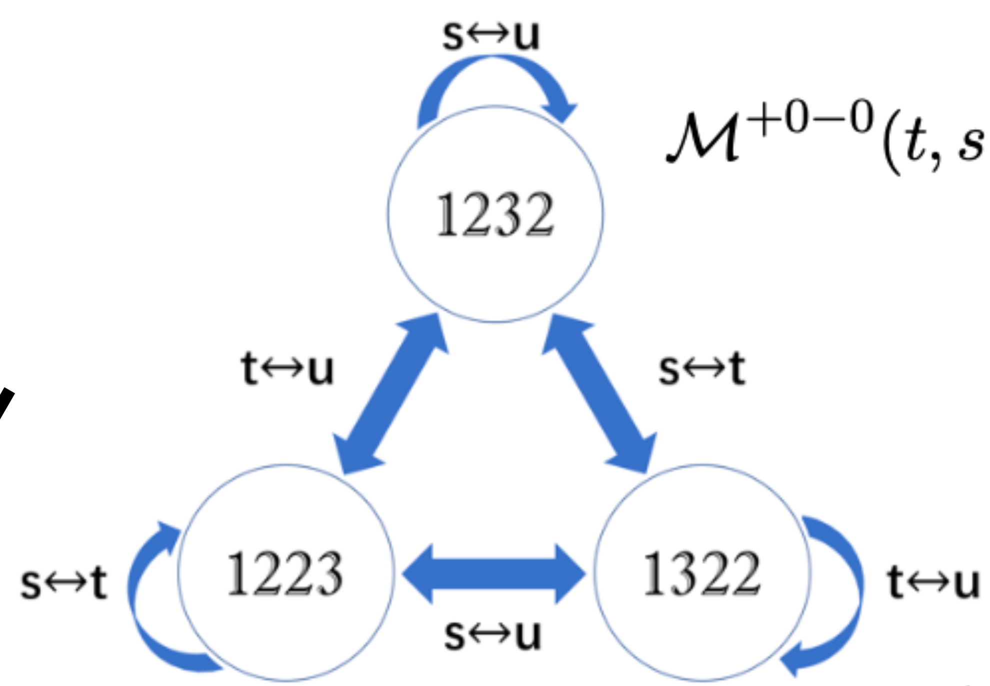
Crossing symmetries & relations

Full permutation symmetries (5 cases), eg,

$$\mathcal{M}^{0000}(s,t,u)=\mathcal{M}^{0000}(u,t,s)=\mathcal{M}^{0000}(t,s,u)\,,$$
$$\mathcal{M}^{+000}(s,t,u)=\mathcal{M}^{+000}(u,t,s)=\mathcal{M}^{+000}(t,s,u)\,,$$
$$\mathcal{M}^{+++0}(s,t,u)=\mathcal{M}^{+++0}(u,t,s)=\mathcal{M}^{+++0}(t,s,u)\,,$$

Only one symmetry + crossing links (4 groups), eg,

$$\mathcal{M}^{+0-0}(t,s,u)=\mathcal{M}^{+0-0}(u,s,t)$$



$$\mathcal{M}^{+00-}(u,t,s)=\mathcal{M}^{+00-}(t,u,s)$$

$$\mathcal{M}^{+-00}(s,t,u)=\mathcal{M}^{+-00}(s,u,t)$$

Fixed t dispersion relations with graviton

Partial wave expansion

$$\mathcal{M}^{\mathbf{1234}}(s,t,u) = 16\pi \sum_{\ell} (2\ell+1) d^{\ell}_{h_{12},h_{43}}\left(1+\frac{2t}{s}\right) A^{\mathbf{1234}}_{\ell}(s)$$

UV unitarity conditions

$$\mathrm{Abs}\, A^{\mathbf{1234}}_{\ell}(s) = \sum_{X} c^{\mathbf{12}\to X}_{\ell,s}\, c^{*\bar{\mathbf{3}}\bar{\mathbf{4}}\to X}_{\ell,s}$$

Froissart-like bounds

$$\begin{cases} \lim_{|s|\to\infty} \mathcal{M}(s,t)/s^2 = 0, & t<0, \\ \lim_{|s|\to\infty} \mathcal{M}(s,t)/s^3 = 0, & 0 \le t \le \xi, \end{cases}$$

Twice subtracted dispersion relations

$$\delta_{k,2} a^{\mathbf{1234}}_{k,-1} \frac{1}{t} + \sum_{n=0} a^{\mathbf{1234}}_{k,n} t^n = \left\langle \frac{\partial_s^k}{k!} \left[\frac{s^2 d^{\ell,\mu,t}_{h_{12},h_{43}} c^{\mathbf{12}}_{\ell,\mu} c^{*\bar{\mathbf{3}}\bar{\mathbf{4}}}_{\ell,\mu}}{\mu^2(\mu-s)} + \frac{(-s-t)^2 d^{\ell,\mu,t}_{h_{14},h_{23}} c^{\mathbf{14}}_{\ell,\mu} c^{*,\bar{\mathbf{3}}\bar{\mathbf{2}}}_{\ell,\mu}}{\mu^2(\mu+s+t)} \right]\Bigg|_{s\to 0} \right\rangle,$$

t-channel pole s^2/t
survives twice subtraction

$$\left\langle \cdots \right\rangle := 16\pi \sum_{\ell,X} (2\ell+1) \int_{\Lambda^2}^{\infty} \frac{\mathrm{d}\mu}{\pi} (\cdots)$$

Bounds on $(\partial\phi)^4$ coefficient

$$S = \int \mathrm{d}^4x\sqrt{-g}\Big(\frac{M_P^2}{2}R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{\lambda_3}{3!}\phi^3 - \frac{\lambda_4}{4!}\phi^4 + \frac{\alpha}{2}(\nabla_\mu\phi\nabla^\mu\phi)^2 + \frac{\beta_1}{2!}\phi\mathcal{G} + \frac{\beta_2}{4}\phi^2\mathcal{G}$$
$$+ \frac{\gamma_0}{3!}\mathcal{R}^{(3)} + \frac{\gamma_1}{3!}\phi\mathcal{R}^{(3)} + \frac{\gamma_2}{2}\nabla_\mu\phi\nabla^\mu\phi\mathcal{R}^{(2)} - \frac{4\gamma_3}{3}\nabla_\mu\phi\nabla_\rho\phi\nabla_\nu\nabla_\sigma\phi R^{\mu\nu\rho\sigma} + \dots\Big)$$

Without graviton

$$\alpha \geq 0$$

reason for name of positivity bounds

With graviton

$$\alpha \geq -16.091\frac{\log(\Lambda/m_{\mathrm{IR}})}{\Lambda^2 M_P^2}$$

$$\alpha \geq 0 \quad \text{as} \quad M_P \to \infty$$

Other coefficients' dependence on α

Hong, Wang, **SYZ**, 2304.01259

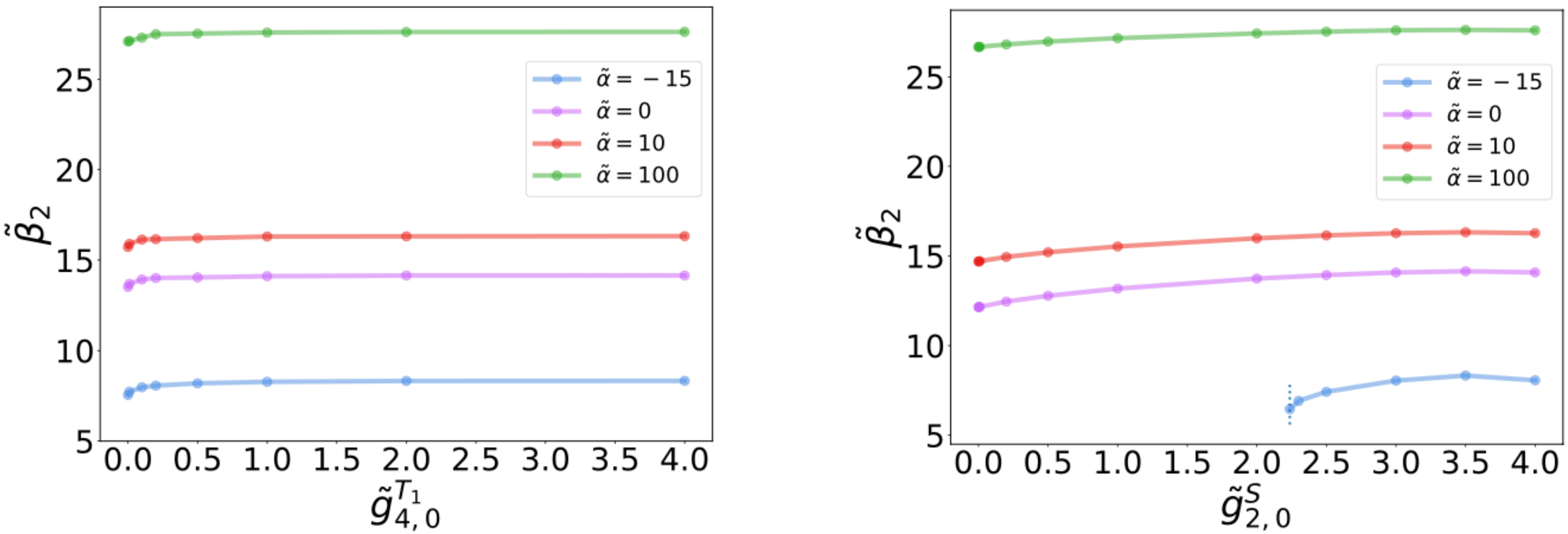
Insensitive to α: $\gamma_0 \sim \frac{M_P^2}{\Lambda^4}, \quad \gamma_1 \sim \frac{M_P}{\Lambda^4}, \quad \beta_1 \sim \frac{M_P}{\Lambda^2}, \quad \dots$

Sensitive to α:

$\gamma_2 \sim \frac{M_P}{\Lambda^5}, \quad \gamma_3 \sim \frac{1}{\Lambda^5}, \quad \gamma_4 \sim \frac{1}{\Lambda^6}, \quad \dots \qquad$ when $\quad \alpha \sim \frac{1}{\Lambda^4}$

$\gamma_2 \sim \frac{1}{\Lambda^4}, \quad \gamma_3 \sim \frac{1}{M_P\Lambda^4}, \quad \gamma_4 \sim \frac{1}{M_P^2\Lambda^4}, \quad \dots \qquad$ when $\quad \alpha \sim \frac{1}{M_P^2\Lambda^2}$

Insensitivity for some coefficients



Can be inspected from SDP

Hong, Wang, **SYZ**, 2304.01259

$$B_{P_X,\ell} \sim (\cdots) + y_* \mathcal{O}\left(\mu^{-n_*}\right) + y^{T_1}_{4,0} \mathcal{O}\left(\mu^{-5}\right) \succeq 0$$

$$[\cdots] + y_* \beta_* + y^{T_1}_{4,0} g^{T_1}_{4,0} \geq 0$$