The scale(s) of quantum gravity
and integrable black holes

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Abstract and outline

It is common to assume that quantum gravity belongs at the Planck scale, but a possibly much larger size for the ground state emerges in the (non-perturbative) quantisation of the Oppenheimer-Snyder model of dust collapse that naturally recovers Bekenstein's area law. The effective geometry for such quantum black holes can then be obtained from coherent states which describe integrable singularities without inner horizons. The extension to quantum (differentially) rotating black holes with similar properties is also described.

Quantum (gravity) views at self-gravitating objects:

1. Macroscopic quantum width in (non-perturbative) gravitational collapse
2. Quantum (non-perturbative) coherent state for black hole geometry without Cauchy horizons

↓

3. Quantum (*integrable**) black holes

* V.N. Lukash, V.N. Strokov, IJMPA 28 (2013) 1350007 [arXiv:1301.5544]

Preamble - Units and scales

Units (of measure) are chosen for practical reasons + physics is a practical (observational) science
= units reflect *our* understanding of *nature*

INHUMAN

42 CHAPTER 4. UNITS AND NATURAL SCALES

4.3.4 Planck units—completely natural units

Define $c \equiv \hbar \equiv G \equiv 1$, so that $m_P \equiv \ell_P \equiv T_P \equiv 1$. This system of units serves to keep both classical relativists and particle physicists happy. This system also serves to keep both classical relativists and particle physicists confused since it is essentially impossible to use dimensional analysis to check results for consistency while using this system. This system works in the first place because for suitable exponents (α, β, γ) and (a, b, c) almost all interesting physical quantities X are dimensionless multiples of

$$[X] \equiv m_P^{\alpha}\, \ell_P^{\beta}\, T_P^{\gamma} \equiv \hbar^a\, c^b\, G^c. \qquad (4.20)$$

[Please, no niggling comments about thermodynamics (Boltzmann's constant), chemistry (moles), luminous intensity (candelas), etc.]

4.3 Units

4.3.1 Seminatural units

Most of this book will be written using "seminatural units". I shall define $c \equiv 1$, so that $m_P = \sqrt{\hbar/G}$; $\ell_P \equiv T_P \equiv \sqrt{\hbar G}$. Equivalently $G \equiv \hbar/m_P^2 \equiv \ell_P^2/\hbar \equiv \ell_P/m_P$. For pedagogical purposes this seems to me to be the best compromise between constantly keeping track of explicit factors of the fundamental constants and quickly getting totally confused about the dimensionality of various physical quantities.

Other conventions in common use are as follows:

4.3.2 Geometrodynamic units

Define $c \equiv G \equiv 1$, so that $m_P \equiv \ell_P \equiv T_P \equiv \sqrt{\hbar}$. These units are commonly used by classical relativists but are guaranteed to drive particle physicists rapidly up the wall.

4.3.3 Quantum units

Define $c \equiv \hbar \equiv 1$, so that $m_P \equiv 1/\sqrt{G}$, while $\ell_P \equiv T_P \equiv \sqrt{G}$. These units are commonly used by particle physicists but are guaranteed to drive classical relativists rapidly up the wall.

Strong (quantum) gravity = high compactness $X \equiv G\,M/R$ → $[G] = L/M$

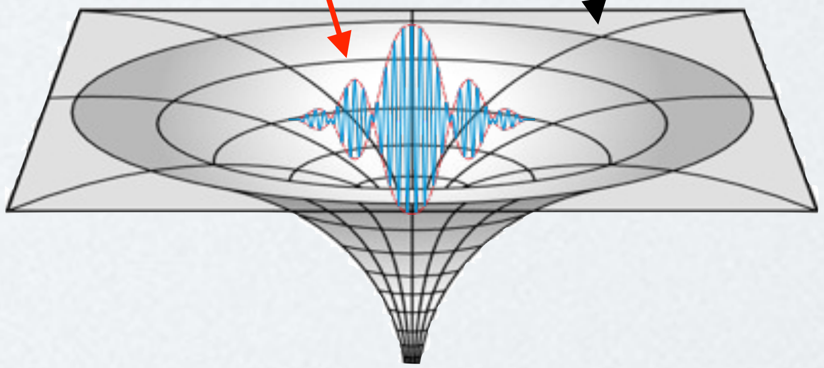
Preamble - QG and Planck scale

Planck scale:

$$m = m_{\rm p} \equiv \sqrt{\frac{c\,\hbar}{G_{\rm N}}} \sim 10^{-8}\,{\rm kg}$$

$$\lambda_{\rm C} = \ell_{\rm p} \equiv \sqrt{\frac{\hbar\, G_{\rm N}}{c^3}} \sim 10^{-35}\,{\rm m}$$

$$\frac{\hbar}{m\,c} \equiv \lambda_{\rm C} \sim R_{\rm H} \equiv \frac{2\,G_{\rm N}\,m}{c^2}$$

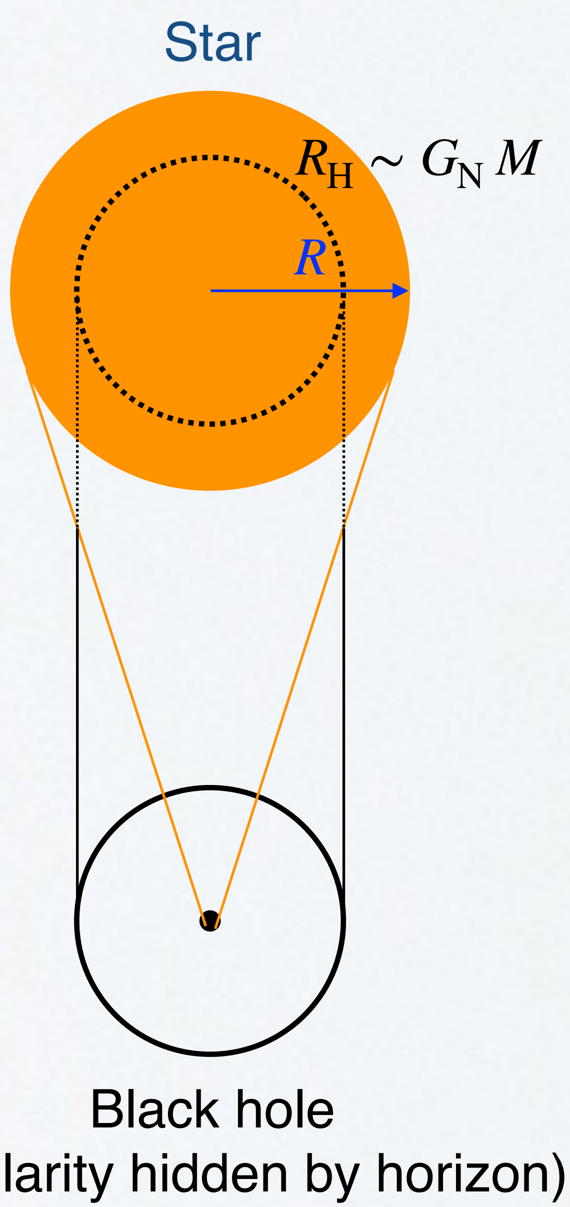


Is Quantum Gravity at the Planck scale $\left(\ell_{\rm p} \sim m_{\rm p}\right)$ or compactness $\frac{G_{\rm N}\,M}{R} = \frac{M\,\ell_{\rm p}}{m_{\rm p}\,R} \sim 1$?*

* Analogy: what is the scale of QED/SM? Electron Compton length << atom << BEC size << neutron star

Preamble - QG and gravitational collapse

- BH form from collapse - the classical view:



- $|\,\mathrm{matter}\rangle$ ~ very large number of SM particles ($M_\odot \sim 10^{57}$ neutrons)
- $|\,\mathrm{gravity}\rangle$ ~ very large number of gravitons* ($N_G \sim M_\odot^2/m_p^2 \sim 10^{76}$)

↓

Reduce and simplify**:

Lemaître-Tolman-Bondi-Oppenheimer-Snyder model

* J.D. Bekenstein, PRD 7 (1973) 2333

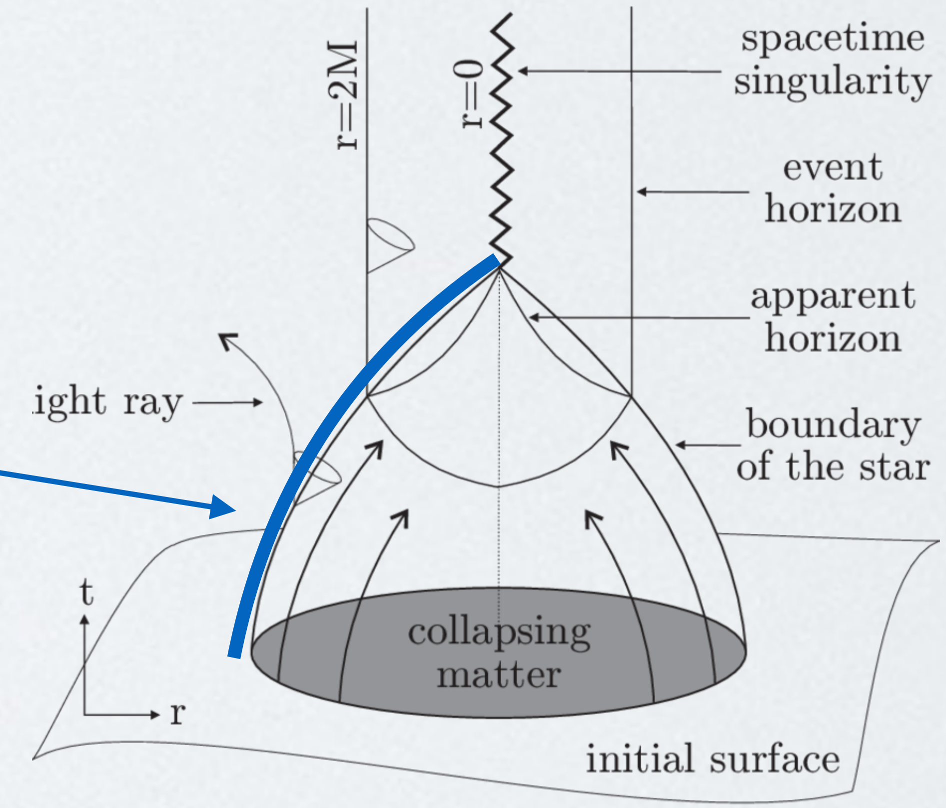
** Rovelli, Thiemann,… Relational approach: "Quantise only what you measure"… But this is just what physics is about

1 - Quantum dust core

- Collapsing ball of dust [1]

$$ds^2 = -\left(1 - \frac{2\,G_N\,M}{r}\right) dt^2 + \left(1 - \frac{2\,G_N\,M}{r}\right)^{-1} dr^2 + r^2\,d\Omega^2$$

$$\left(\frac{dR}{d\tau}\right)^2 + 1 - \frac{2\,G_N\,M}{R} \simeq \frac{E^2}{\mu^2}$$



- Effective one-body Hamiltonian [2,3]:

$$H \equiv \frac{P^2}{2\,\mu} - \frac{G_N\,\mu\,M}{R} = \frac{\mu}{2}\left(\frac{E^2}{\mu^2} - 1\right) \equiv \mathcal{E}$$

$$\hat{H}\,\Psi_n = \mathcal{E}_n\,\Psi_n$$

$$\frac{\mathcal{E}_n}{\mu} \simeq -\frac{G_N^2\,\mu^3\,M}{2\,\hbar^2\,n^2} \sim -\frac{\mu^3\,M}{m_p^4\,n^2}$$

$$\bar{R}_n \equiv \langle \Psi_n | \hat{R} | \Psi_n \rangle \simeq n^2\,\ell_p \left(\frac{m_p^3}{\mu^2\,M}\right)$$

Newtonian physics: $n \geq 1$ → $\bar{R}_1 \sim \ell_p \dfrac{m_p^3}{\mu^2\,M} \sim 0$

GR: $\dfrac{E_n^2}{\mu^2} = 1 + \dfrac{\mathcal{E}_n}{\mu} \geq 0$ → $n \geq N_M \sim \dfrac{\mu\,M}{m_p^2}$ → $\bar{R}_{N_M} \sim G_N\,M$

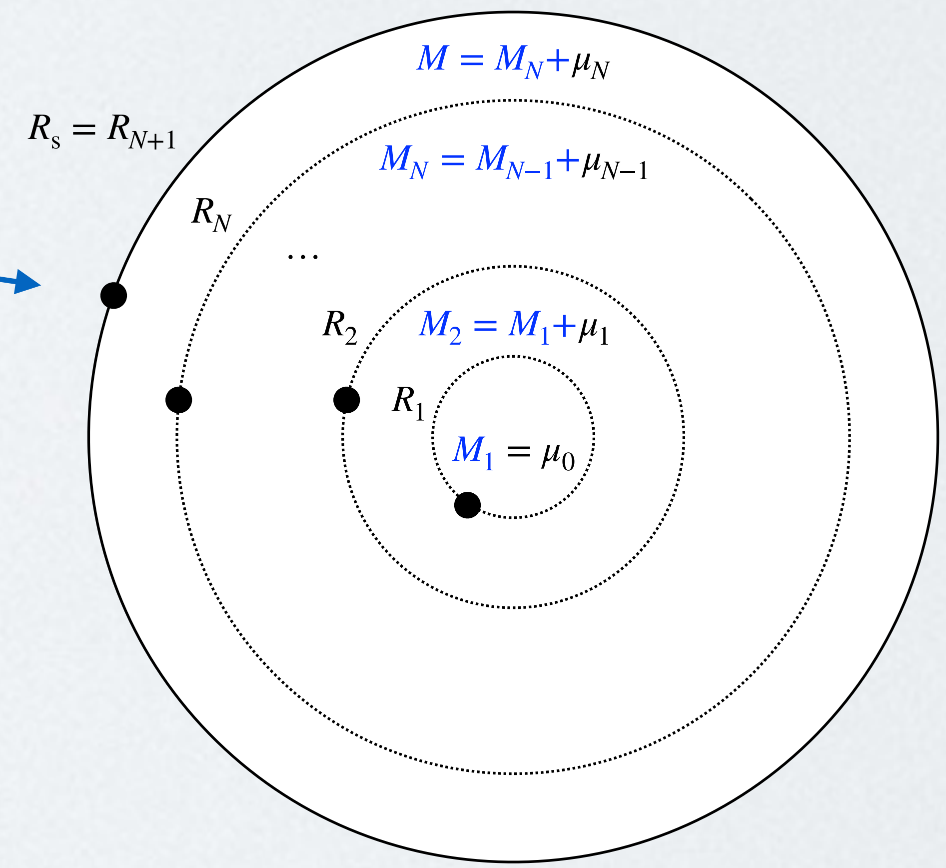
[1] R.C., *A quantum bound for the compactness*, EPJC 82 (2022) 1 [arXiv:2103.14582]

1 - Quantum dust core

- Collapsing ball of dust [2]

$$ds^2 = -\left(1 - \frac{2\,G_N\,M}{r}\right)dt^2 + \left(1 - \frac{2\,G_N\,M}{r}\right)^{-1} dr^2 + r^2\,d\Omega^2$$

$$\left(\frac{dR_i}{d\tau}\right)^2 + 1 - \frac{2\,G_N\,M_i}{R_i} \simeq \frac{E_i^2}{\mu^2}$$



- Dust particle Hamiltonians:

$$H_i \equiv \frac{P_i^2}{2\,\mu} - \frac{G_N\,\mu\,M_i}{R_i} = \frac{\mu}{2}\left(\frac{E_i^2}{\mu^2} - 1\right) \equiv \mathscr{E}_i$$

- Schrödinger equation: $\hat{H}_i\,\Psi_{n_i} = \mathscr{E}_{n_i}\,\Psi_{n_i}$

- Spectrum of bound states ($n_i \geq 1$):

"GR" [2]

$$\frac{\mathscr{E}_{n_i}}{\mu} \simeq -\frac{G_N^2\,\mu^3\,M_i}{2\,\hbar^2\,n_i^2} = -\frac{1}{2\,n_i^2}\left(\frac{\mu\,M_i}{m_p^2}\right)^2 = \frac{1}{2}\left(\frac{E_i^2}{\mu^2} - 1\right)$$

$$\bar{R}_{n_i} \equiv \langle \Psi_{n_i} | \hat{R}_i | \Psi_{n_i} \rangle \simeq n_i^2\,\ell_p \left(\frac{m_p^3}{\mu^2\,M_i}\right)$$

Newtonian spectrum

[2] R.C., *Quantum dust cores of black holes*, PLB 843 (2023) 138055 [arXiv:2304.06816]

1 - Quantum dust core

- Allowed spectrum * [1,2]: $0 \leq \frac{E_i^2}{\mu^2} \simeq 1 - \frac{1}{n_i^2}\left(\frac{\mu M_i}{m_\mathrm{p}^2}\right)^2$ → $n_i \geq N_i = \frac{\mu M_i}{m_\mathrm{p}^2}$ → $\bar{R}_{N_i} \simeq \frac{3}{2} G_\mathrm{N} M_i$

- Layer thickness [1]: $\frac{\overline{\Delta R}_{n_i}}{\bar{R}_{n_i}} = \frac{\sqrt{n_i^2+2}}{3\,n_i} \simeq \frac{1}{3}$ → $\bar{R}_{N_{i+1}} \simeq \bar{R}_{N_i} + \overline{\Delta R}_{N_i} \simeq \frac{4}{3}\bar{R}_{N_i}$ → $M_{i+1} \simeq \frac{4}{3} M_i$

- Bekenstein's area law: $N_\mathrm{G} = \frac{M}{\mu} N_N \simeq \frac{M^2}{m_\mathrm{p}^2}$

$$R_\mathrm{s} \simeq \frac{3}{2} G_\mathrm{N} M = \frac{3}{4} R_\mathrm{H}$$

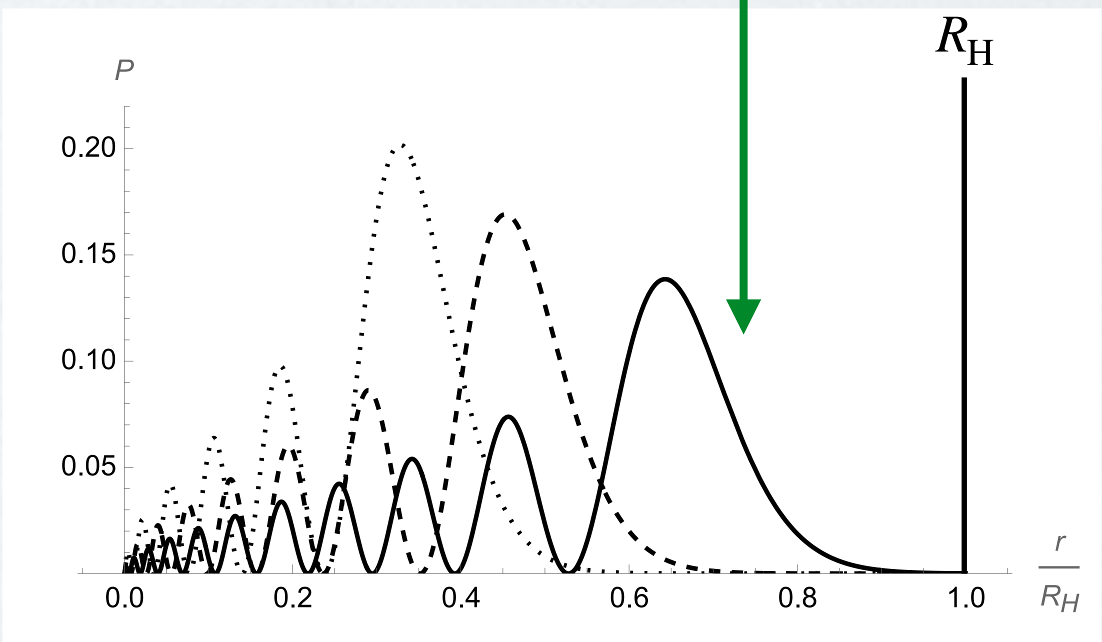
- Energy "spectra":

$$\delta H = |\mathscr{E}_{n+1} - \mathscr{E}_n| \sim m_\mathrm{p} \frac{m_\mathrm{p}}{M}$$ (Hawking temperature)

$$\delta E = |E_{n+1} - E_n| \sim m_\mathrm{p}$$ $n \geq N_M$

↓

(Dvali's) saturons



$$\bar{R}_{n=1} \sim \ell_\mathrm{p}\left(\frac{m_\mathrm{p}^3}{\mu^2 M}\right) \sim \ell_\mathrm{p}$$

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[2] R.C., *A quantum bound for the compactness*, EPJC 82 (2022) 1 [arXiv:2103.14582]

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$$R_s \simeq \frac{3}{2} G_N M = \frac{3}{4} R_H$$

- Bounded compactness

↕

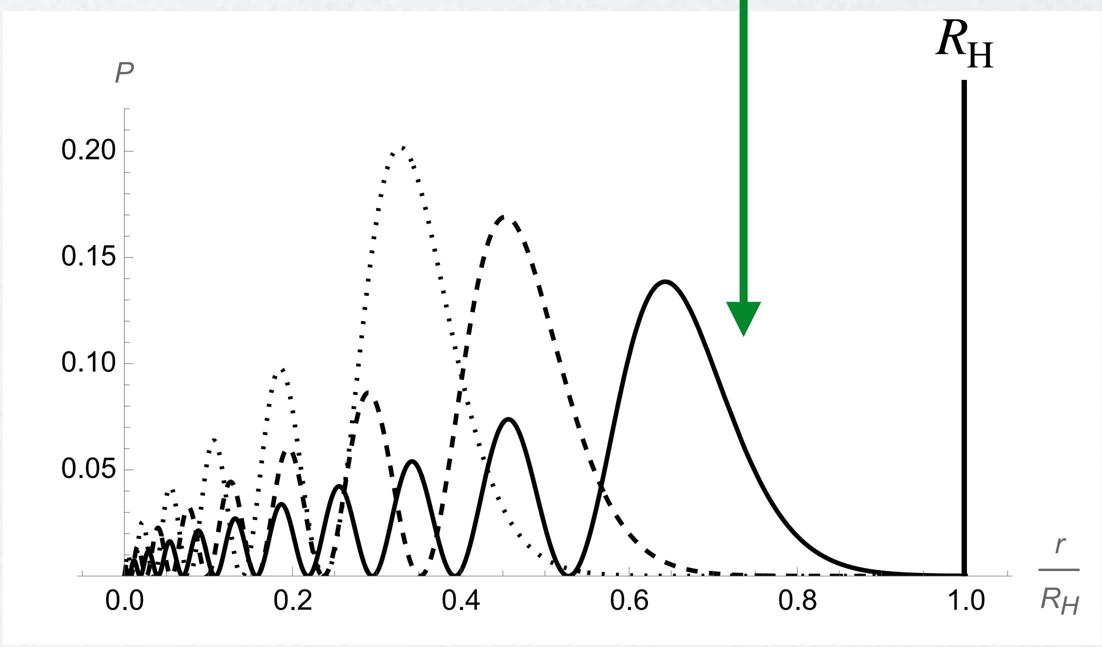
Semiclassical "bounce" (~ BH-to-WH transition)

↑

non-linearity

↓

* GUP in action ~ (Dvali's) classicalisation



$$\bar{R}_{n=1} \sim \ell_p \left(\frac{m_p^3}{\mu^2 M}\right) \sim \ell_p$$

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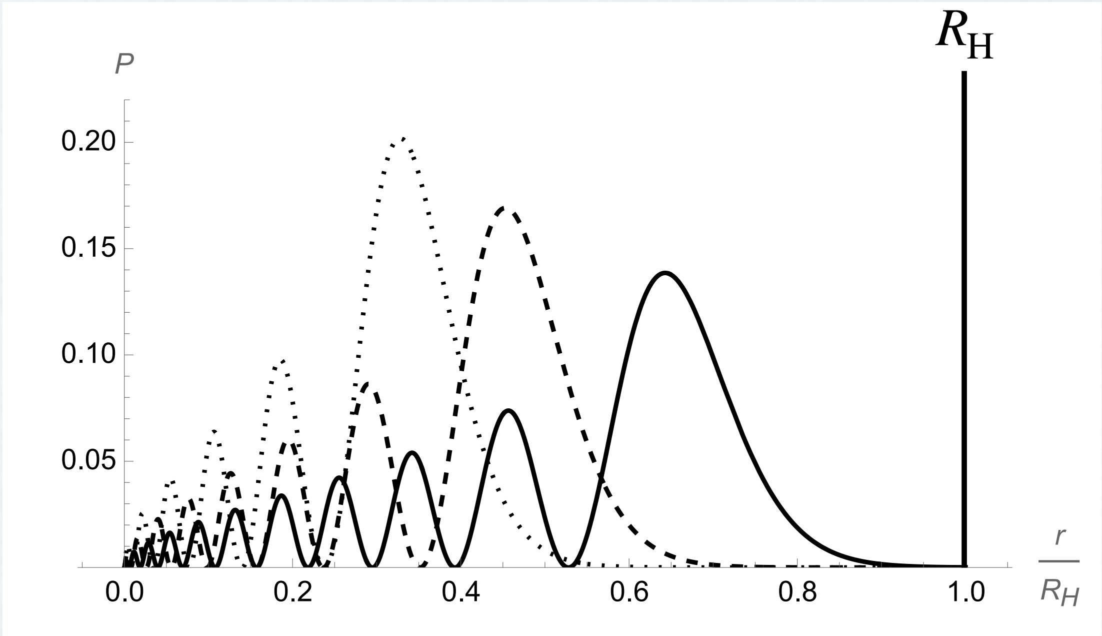
- Effective mass and energy density [1]*: $m(r) \sim r$

↓

- Integrable singularity: $\rho(r) \sim r^{-2}$

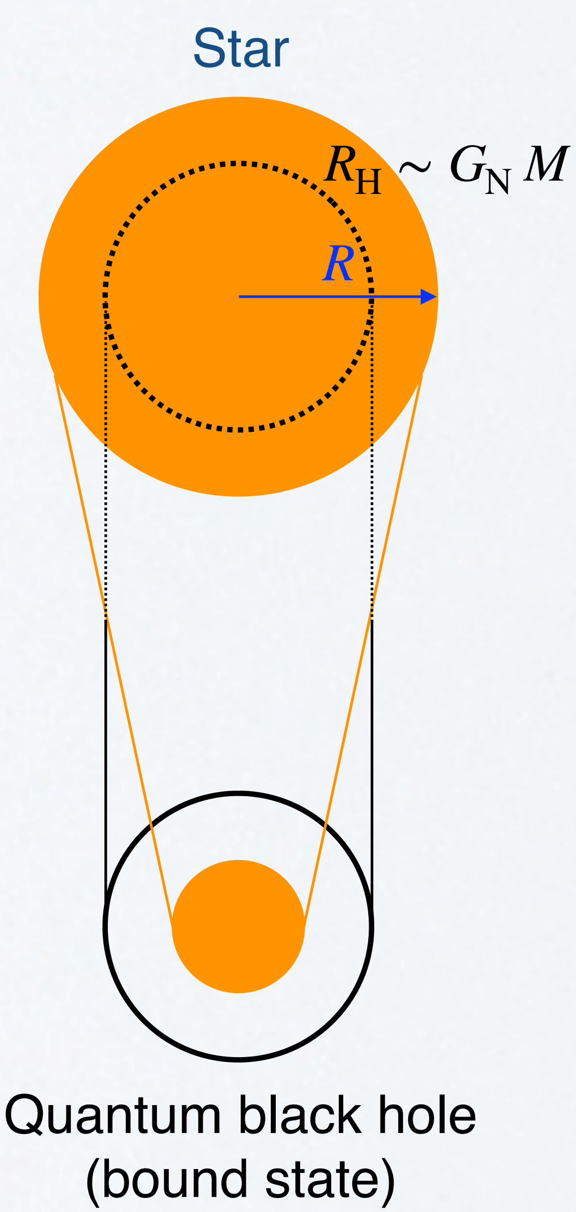


* Minkowski spacetime in disguise ($t \leftrightarrow r$) ~ asymptotic safety?

[1] R.C., *Quantum dust cores of black holes*, PLB 843 (2023) 138055 [arXiv:2304.06816]
[2] R.C., *A quantum bound for the compactness*, EPJC 82 (2022) 1 [arXiv:2103.14582]

Preamble - QG and gravitational collapse

- BH form from collapse - the quantum view:



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- $|\text{gravity}\rangle$ ~ very large number of gravitons* ($N_G \sim M_\odot^2 \sim 10^{76}$)

- $|\text{gravity}\rangle$ always entangled with $|\text{matter}\rangle$ $\Longleftrightarrow$ "quantum hair" [1,2]

Dynamics

$$|\mathbf{g}\ \phi\rangle = \sum_{ij} C_{ij} |\mathbf{g}_i\rangle |\phi_j\rangle \quad \not\Rightarrow \quad \left(\sum_{ab} c_{ab} |\mathbf{g}_a\rangle |\phi_b\rangle\right)\left(\sum_{AB} c_{AB} |\mathbf{g}_A\rangle |\phi_B\rangle\right)$$

$$\hat{H}^\mu |\mathbf{g}\ \phi\rangle = 0$$

BH interior BH exterior

[1] R.C., *A quantum bound for the compactness*, EPJC 82 (2022) 1 [arXiv:2103.14582]
[2] R.C., *Quantum dust cores of black holes*, PLB 843 (2023) 138055 [arXiv:2304.06816]

2 - Coherent state for classical geometry

- SdS geometry [1]:

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\,d\Omega^2$$

$$f = 1 + 2\,V_{\rm SdS} = 1 - \frac{2\,G_{\rm N}\,M}{r} - \frac{\Lambda\,r^2}{3}$$

$$V_{\rm SdS} = V_M + V_\Lambda = -\frac{G_{\rm N}\,M}{r} - \frac{\Lambda\,r^2}{6}$$

- Horizons ($f = 0 \leftrightarrow 2\,V = -1$):

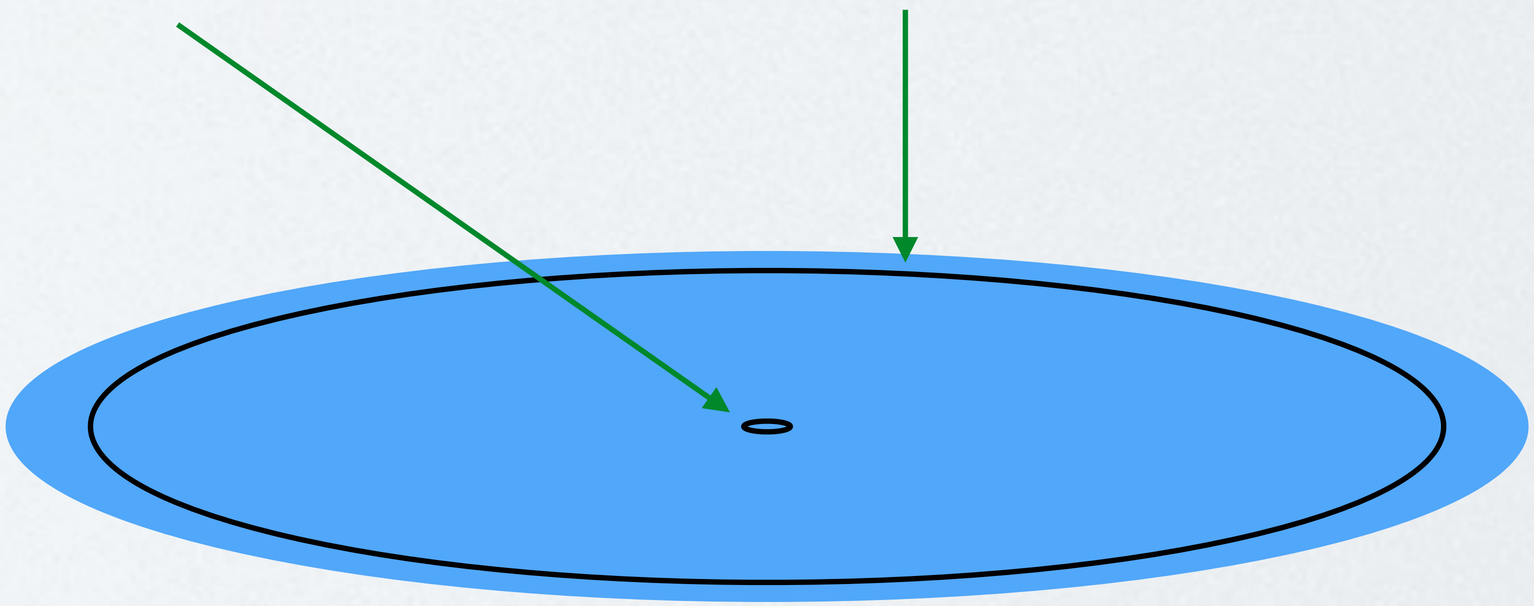
$1/\sqrt{\Lambda} \gg G_{\rm N}\,M$

Localised source gravitational radius

$$R_{\rm H} \simeq 2\,G_{\rm N}\,M$$

Cosmological horizon

$$H^{-1} = L \simeq \sqrt{\frac{3}{\Lambda}}$$



[1] A. Giusti, S. Buffa, L. Heisenberg, R.C., *A quantum state for the late* Universe, PLB 826 (2022) 136900 [arXiv:2108.05111]

2 - Coherent state for classical geometry

- "Effective" massless scalar field in Minkowski (~ true QG vacuum $\hat{a}_k\,|0\rangle = 0$ *):

$$\left\{-\frac{\partial^2}{\partial t^2} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\,\sin^2\theta}\left[\frac{\partial}{\partial\theta}\left(\sin^2\theta\,\frac{\partial}{\partial\theta}\right) + \frac{\partial^2}{\partial\varphi^2}\right] + \frac{\partial^2}{\partial y_1^2} + \ldots + \frac{\partial^2}{\partial y_n^2}\right\}\Phi = 0$$

↑ ON ↑ OFF

* Metric ~ causal structure ~ gravity emerges from "excitations" along with matter (~ LQG, Regge calculus, etc…)

2 - Coherent state for classical geometry

- "Effective" massless scalar field in Minkowski (~ true QG vacuum $\hat{a}_k\,|\,0\rangle = 0$ *):

$$\left[-\frac{\partial^2}{\partial t^2} + \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right)\right]\Phi(t,r) = 0$$

$$u_k(t,r) = e^{-i\,k\,t}\,j_0(k\,r)$$

$$4\,\pi\int_0^\infty r^2\,dr\,j_0(k\,r)\,j_0(p\,r) = \frac{2\,\pi^2}{k^2}\,\delta(k-p)$$

- Normal mode expansion of operators:

$$\hat{\Phi}(t,r) = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\sqrt{\frac{\hbar}{2\,k}}\left[\hat{a}_k\,u_k(t,r) + \hat{a}_k^\dagger\,u_k^*(t,r)\right]$$

$$\left[\hat{\Phi}(t,r),\hat{\Pi}(t,s)\right] = \frac{i\,\hbar}{4\,\pi\,r^2}\,\delta(r-s)$$

$$\hat{\Pi}(t,r) = i\int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\sqrt{\frac{\hbar\,k}{2}}\left[\hat{a}_k\,u_k(t,r) - \hat{a}_k^\dagger\,u_k^*(t,r)\right]$$

$$\left[\hat{a}_k,\hat{a}_p^\dagger\right] = \frac{2\,\pi^2}{k^2}\,\delta(k-p)$$

- Coherent state: $\hat{a}_k\,|\,g\rangle = g(k)\,e^{i\,\gamma_k(t)}\,|\,g\rangle$

$$\langle g\,|\,\hat{\Phi}(t,r)\,|\,g\rangle = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\sqrt{\frac{2\,\ell_p\,m_p}{k}}\,g(k)\,\cos[\gamma_k(t) - k\,t]\,j_0(k\,r)$$

* Metric ~ causal structure ~ gravity emerges from "excitations" along with matter (~ LQG, Regge calculus, etc…)

2 - Coherent state for classical geometry

- "Classical" coherent state: $$\sqrt{\frac{\ell_p}{m_p}}\langle g\,|\,\hat{\Phi}(t,r)\,|\,g\rangle = V(r) = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\,\tilde{V}(k)\,j_0(k\,r)$$

Single mode occupation number: $$g(k) = \sqrt{\frac{k}{2}}\,\frac{\tilde{V}(k)}{\ell_p}$$

$$\gamma_k = k\,t$$

← virtual non-propagating modes *

- Total occupation number (must be finite!) ~ distance from vacuum:

$$|\,g\rangle = e^{-N_G/2}\exp\left\{\int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\,g(k)\,\hat{a}_k^\dagger\right\}|\,0\rangle$$

$$N_G = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\,g^2(k) < \infty$$

$$\langle k\rangle = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\,k\,g^2(k) < \infty$$

* Potentials in QFT are generated by virtual / non propagating modes (same for spherical symmetry)

2 - Coherent state for classical geometry

- Localised source: $V_M = -\frac{G_N M}{r}$ $\tilde{V}_M = -4\pi G_N \frac{M}{k^2}$

Divergences

- Mass scaling [1]:

$$N_M = 4\frac{M^2}{m_p^2}\int_0^\infty \frac{dk}{k} \longrightarrow 4\frac{M^2}{m_p^2}\int_{k_{IR}}^{k_{UV}} \frac{dk}{k} = 4\frac{M^2}{m_p^2}\ln\left(\frac{k_{UV}}{k_{IR}}\right)$$

- Compton length scaling:

$$\langle k \rangle = 4\frac{M^2}{m_p^2}\int_0^\infty dk \longrightarrow 4\frac{M^2}{m_p^2}\int_{k_{IR}}^{k_{UV}} dk = 4\frac{M^2}{m_p^2}\left(k_{UV} - k_{IR}\right)$$

- Quantum core (BH = ground state): $k_{UV}^{-1} \simeq R_s \simeq G_N M$

* Cut-offs = existence condition for quantum state: $g(k < k_{IR}) = g(k > k_{UV}) \simeq 0$!

[1] R.C., *Geometry and thermodynamics of coherent quantum black holes,* IJMPD 31 (2022) 2250128 [arXiv:2103.00183]

2 - Coherent state for classical geometry

- Localised source: $V_M = -\dfrac{G_N M}{r}$

- Localised source in dS: $k_{UV}^{-1} \simeq R_s \simeq G_N M$

  $k_{IR}^{-1} \simeq L$

$$V_{QM} \equiv \sqrt{\frac{\ell_p}{m_p}} \langle g | \hat{\Phi}(t, r) | g \rangle \simeq V_M(r)$$

(*Observable?*) quantum hair

$$V_{QM} = V_M(r) \left\{ 1 - \left[1 - \frac{2}{\pi} \mathrm{Si}\left(\frac{r}{R_s} \right) \right] \right\}$$

$$\mathrm{Si}(x) = \int_0^x j_0(z)\, dz$$

- Excited (coherent) states $\Longleftrightarrow$ deformations (Love numbers…)

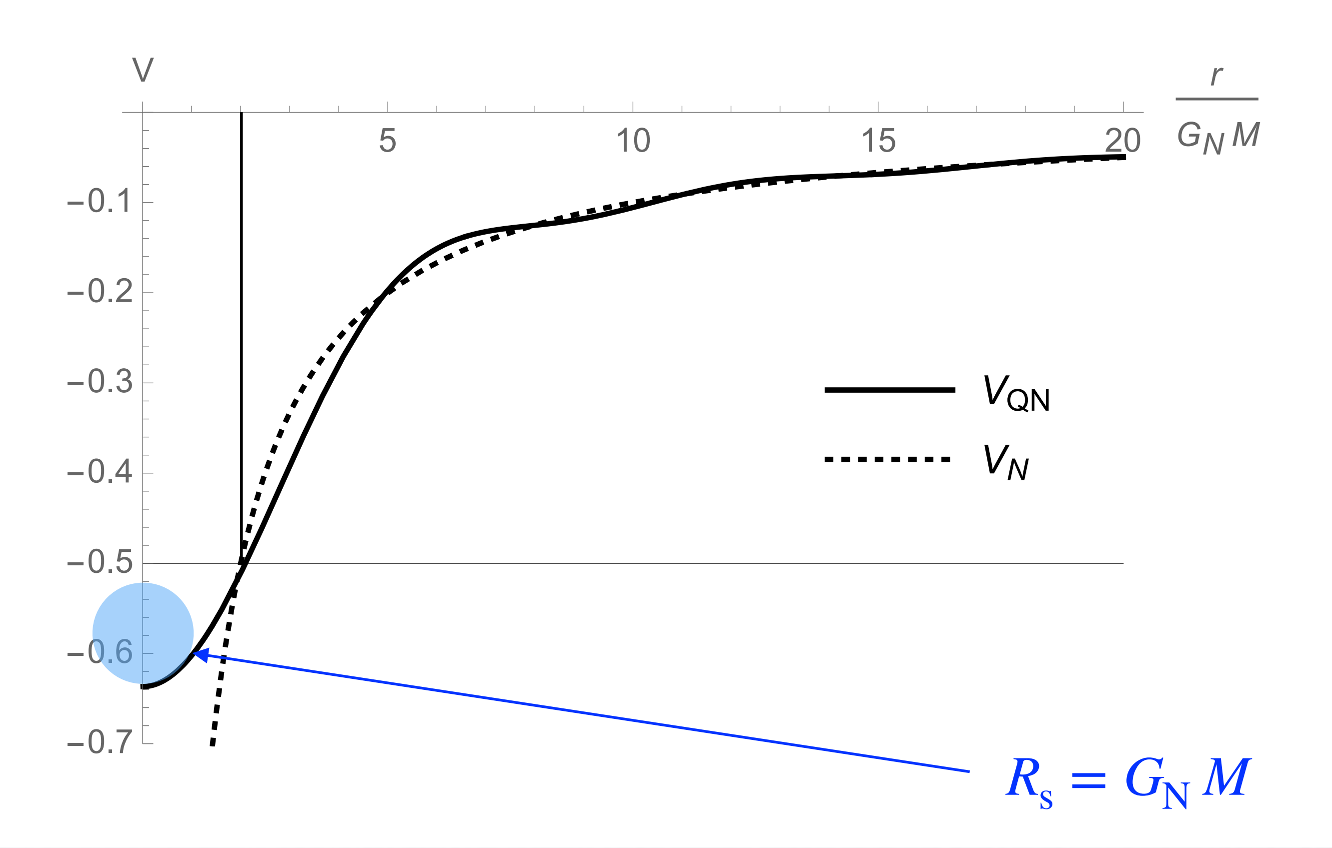
3 - Quantum integrable black holes

- Corpuscular scaling laws:

$$N_M \sim \frac{M^2}{m_p^2} \ln\left(\frac{R_\infty}{R_s}\right)$$

$$\lambda_M \simeq \frac{N_M}{\langle k \rangle} \sim R_H \sim G_N M$$

cutoffs → proper $g(k)$ ~ time-dependent "quantum hair"

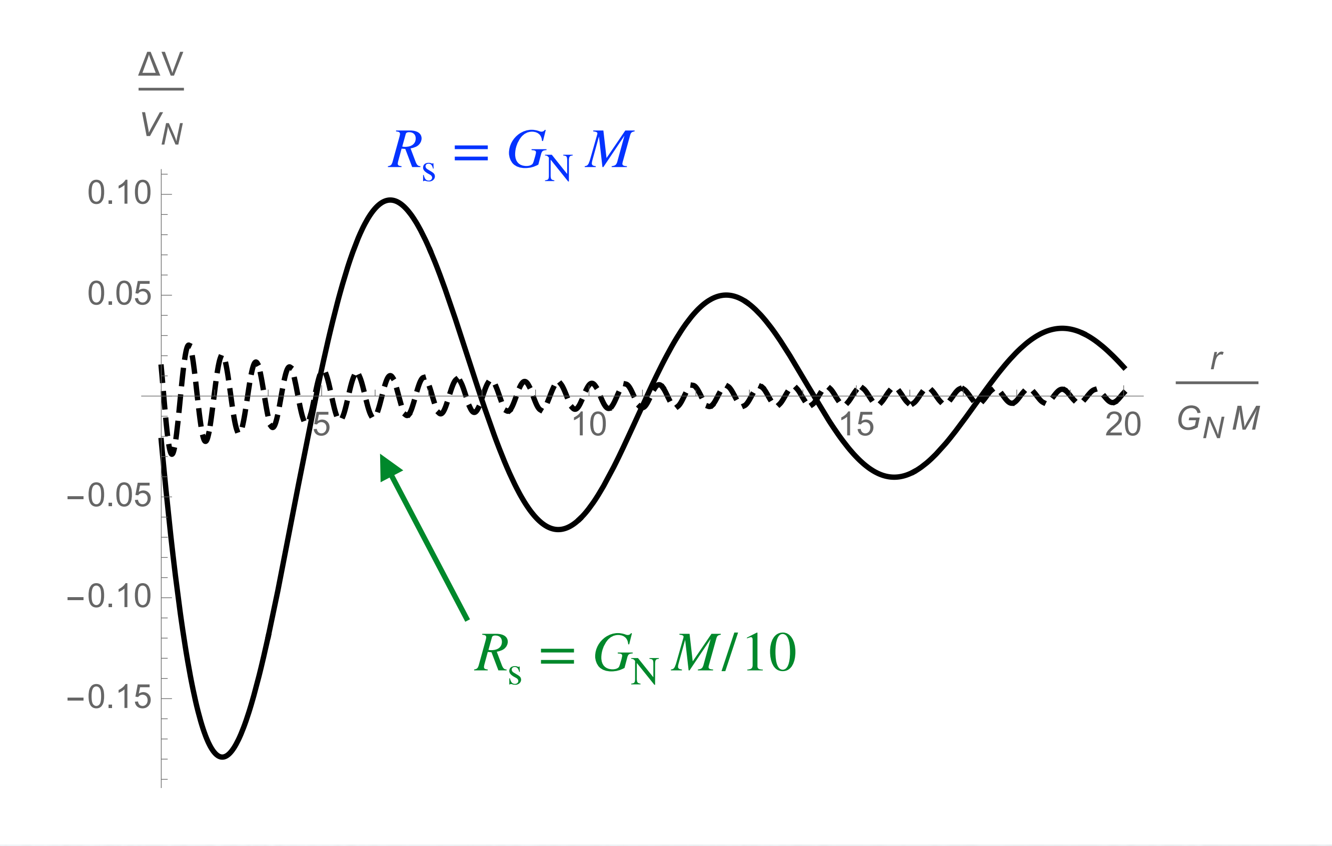


- "Quantum corrected" metric [1]:

$$ds^2 \simeq -\left(1 + 2\,V_{QM}\right) dt^2 + \frac{dr^2}{1 + 2\,V_{QM}} + r^2\,d\Omega^2$$

Integrable singularity without inner horizon

$$R^2 \sim R_{\mu\nu\alpha\beta}\,R^{\mu\nu\alpha\beta} \sim r^{-4}$$



[1] R.C., *Geometry and thermodynamics of coherent quantum black holes,* IJMPD 31 (2022) 2250128 [arXiv:2103.00183]

3 - Quantum integrable black holes

- Spherical *integrable singularity* without inner horizon [1,2]

$$\rho_{\text{eff}} \sim |\Psi^2(r)| \sim r^{-2} \qquad m(r) \sim \int_0^r \rho(x)\, x^2\, dx < \infty$$

$$p^r_{\text{eff}} \sim -\rho_{\text{eff}} \sim -r^{-2}$$
$$p^t_{\text{eff}} \sim r^0$$

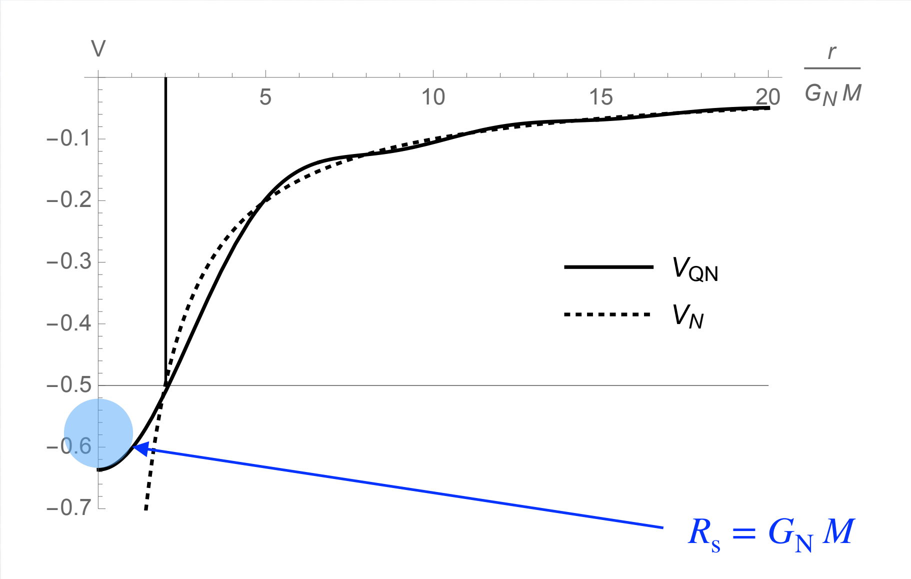
$m(r) \sim r$

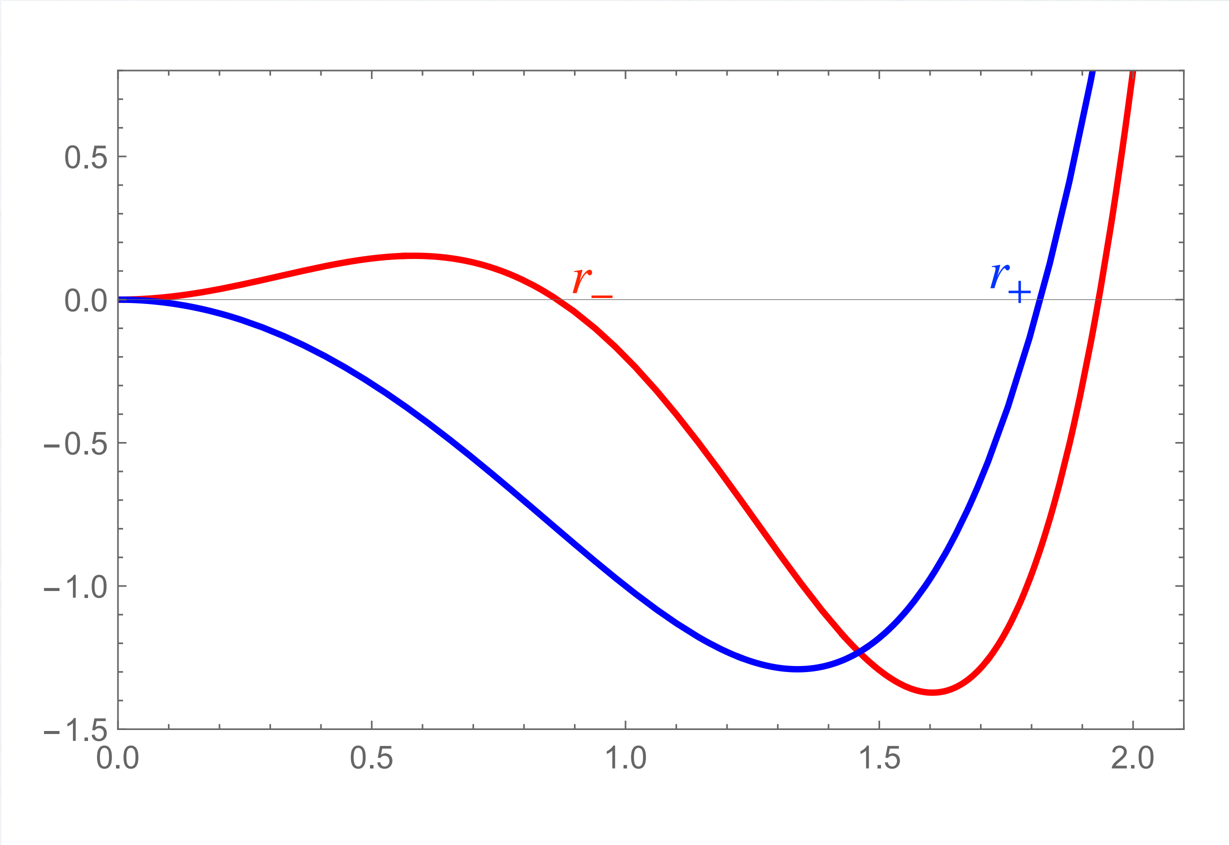
$$\Delta = r_\pm^2 - 2\, r_\pm\, m(r_\pm) = 0$$
$$\Delta(r \sim 0) \leq 0$$

Regular (classical) black holes:

$$\rho \sim r^0 \quad \Longrightarrow \quad m \sim r^3$$

$$\Delta = r_\pm^2 - 2\, r_\pm\, m(r_\pm) = 0$$
$$\Delta(r \sim 0) \sim r^2 \geq 0$$





[1] R.C., IJMPD 31 (2022) 2250128 [arXiv:2103.00183]
[6] R.C., A. Giusti, J. Ovalle, PRD 105 (2022) 124026 [arXiv:2203.03252]

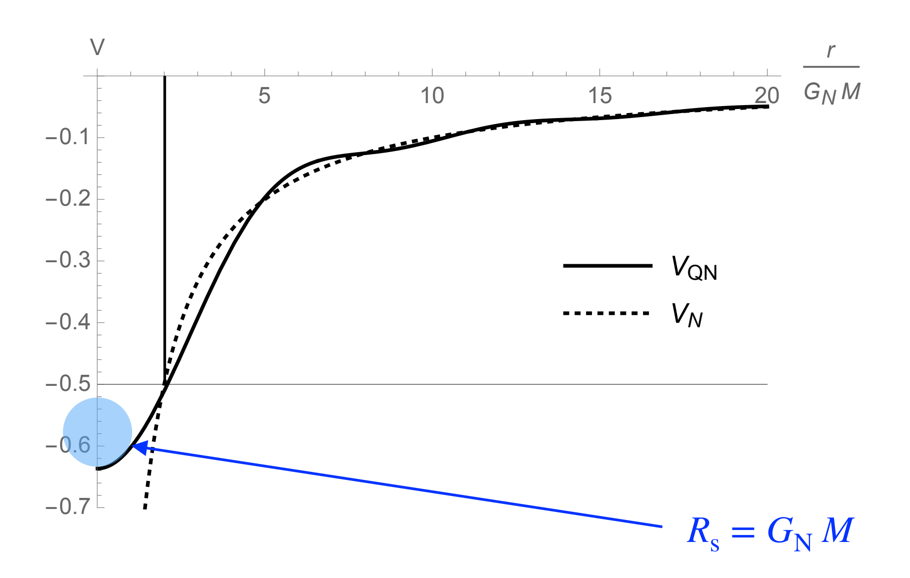
3 - Quantum integrable black holes

- Spherical *integrable singularity* without inner horizon [1,2]

$$\rho_{\text{eff}} \sim |\Psi^2(r)| \sim r^{-2} \qquad m(r) \sim \int_0^r \rho(x)\, x^2\, dx < \infty$$

$$p^r_{\text{eff}} \sim -\rho_{\text{eff}} \sim -r^{-2}$$

$$p^t_{\text{eff}} \sim r^0$$

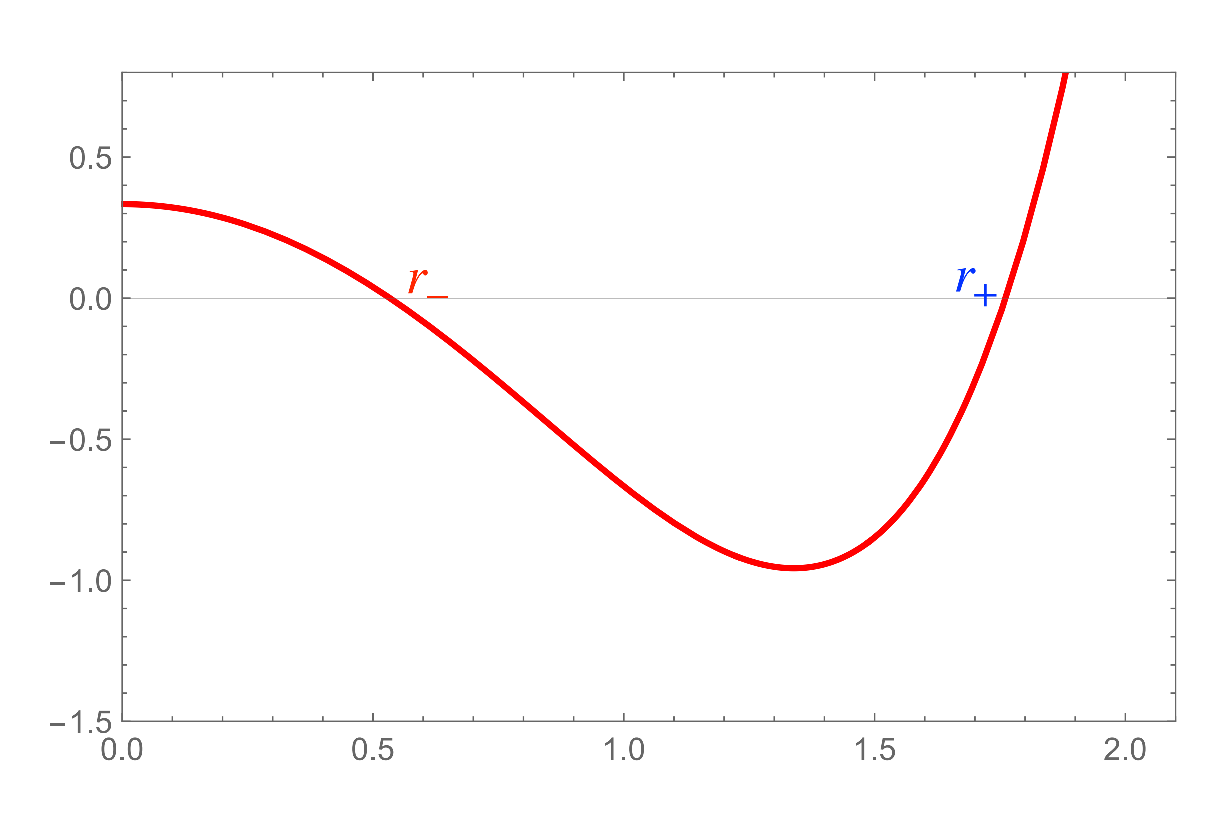


- Rotating *integrable singularity* without inner horizon [3]

$$m(r) \sim r$$

$$\Delta = a^2 - 2\, r_\pm\, m(r_\pm) + r_\pm^2 = 0$$

$$\Delta(0) = a^2 > 0$$



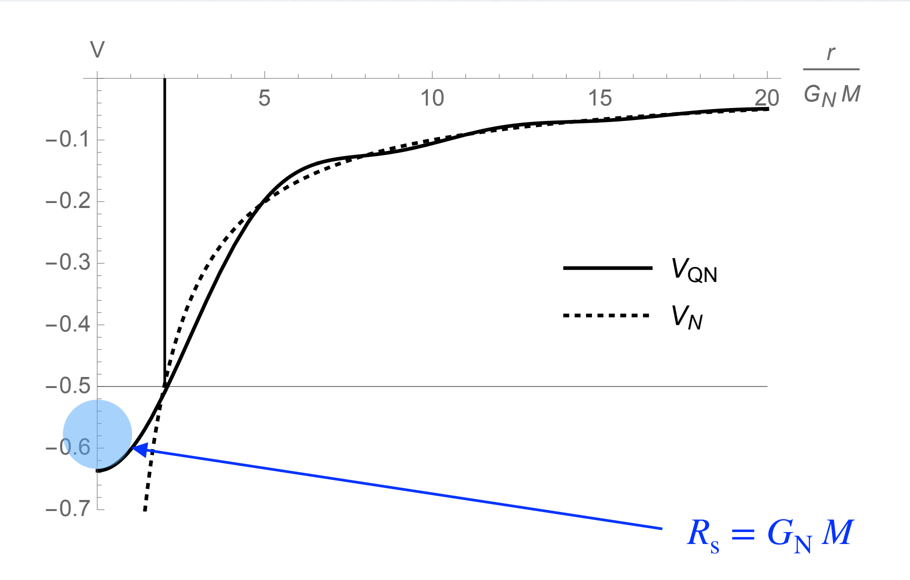
[1] R.C., IJMPD 31 (2022) 2250128 [arXiv:2103.00183]
[2] R.C., A. Giusti, J. Ovalle, PRD 105 (2022) 124026 [arXiv:2203.03252]
[3] R.C., A. Giusti, J. Ovalle, *Quantum rotating black holes*, JHEP 05 (2023) 118 [arXiv:2303.02713]

3 - Quantum integrable black holes

- Spherical *integrable singularity* without inner horizon [1,2]

$$\rho_{\rm eff} \sim |\Psi^2(r)| \sim r^{-2} \qquad\qquad m(r) \sim \int_0^r \rho(x)\, x^2\, dx < \infty$$

$$p^r_{\rm eff} \sim -\rho_{\rm eff} \sim -r^{-2}$$
$$p^t_{\rm eff} \sim r^0$$

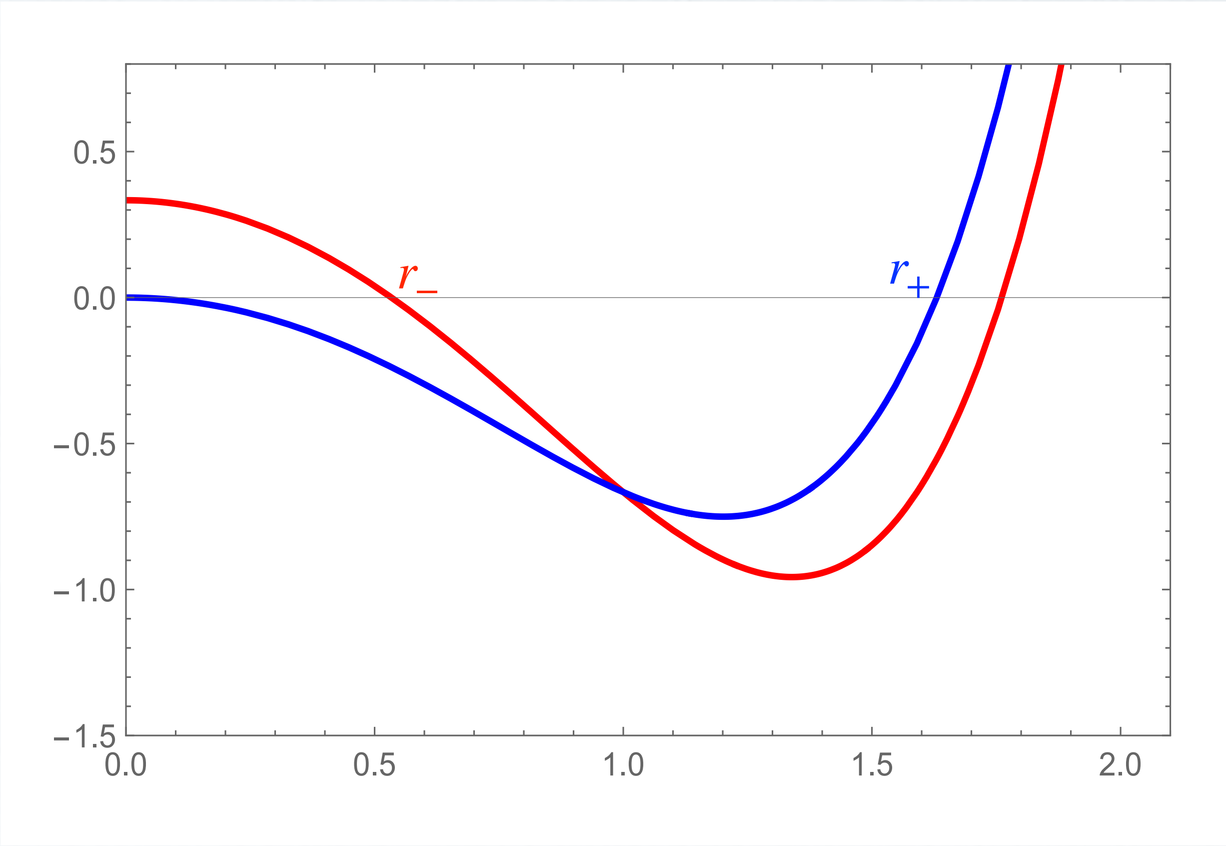


- Rotating *integrable singularity* without inner horizon [3]

$$m(r) \sim r$$

$$a(r) \sim r^\alpha \qquad \alpha \geq 1$$

$$\Delta = a^2(r_H) - 2\, r_H\, m(r_H) + r_H^2 = 0$$
$$\Delta(r \sim 0) \sim -(2\, m_1 - 1)\, r \leq 0$$



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4 - Coherent state for slowly rotating geometry

- Slowly rotating metric [1]: $$ds^2 \simeq -\left(1 + 2\,V_M + 2\,W_a\right) dt^2 + \frac{dr^2}{1 + 2\,V_M + 2\,W_a} - \frac{4\,G_N\,M\,a}{r}\sin^2\theta\,dt\,d\phi + r^2\,d\Omega^2$$

$$W_a = \frac{a^2}{2\,r^2}$$

$$\hbar \ll J = J^z = |\,a\,|\,M \ll G_N\,M^2$$

- Angular momentum modes: $$n_k\,|\,k\rangle \quad \Longrightarrow \quad n_{k\ell m}\,|\,k, \ell, m\rangle$$

$$J_{\ell m} = \hbar\sqrt{\ell\,(\ell+1)}\;n_{k\ell m}$$

$$J^z_{\ell m} = \hbar\,m\,n_{k\ell m}$$

- Entropy of Schwarzschild geometry from $\ell = m = 0$: $$\mathcal{N}_M \sim \sum_{n=0}^{N_M}\binom{N_M}{n} = \sum_{n=0}^{N_M}\frac{N_M!}{(N_M - n)!\,n!} = 2^{N_M}$$

$$\downarrow$$

$$S_M \propto \ln(\mathcal{N}_M) \sim \left(\frac{M}{m_p}\right)^2$$

$$\downarrow$$

- Entropy of Schwarzschild geometry from $|\,m\,| \ll \ell \lesssim \ell_c$: $$W_{qa} \sim \frac{G_N\,M}{r}\left(\frac{\ell_p}{r}\right)^2$$ ~ 1-loop corrections [2]

[1] W. Feng, R. da Rocha, R.C., *Quantum hair and entropy for slowly rotating quantum black holes,* EPJC 84 (2024) 586 [arXiv:2401.14540]
[2] M.B. Fröb., C. Rein, R. Verch, JHEP 01 (2022) 180 [arXiv:2401.14540]

Conclusions

- Black holes as (macroscopic) quantum states (*bound states* far from vacuum)
- Singularity is not resolved (integrable “fuzzy” geometry)
- Exterior quantum hair (from core size)
- No Cauchy horizon (also for electrically charged black holes)
- No Cauchy horizon for non-rigidly rotating black holes
- Effective cosmological DM
- Test the model*: perturbations $\Longrightarrow$ binary systems $\Longrightarrow$ GW
- Test the model**: geodesic motion $\Longrightarrow$ shadow

*R. Brustein, *Quantum Love numbers,* PRD 105 (2022) 024043 [arXiv:2008.02738]
**A. Urmanov, H. Chakrabarty, D. Malafarina, arXiv:2406.04813

Conclusions

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Thank you!

- No Cauchy horizon for non-rigidly rotating black holes
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**D. Malafarina et al, in preparation