Field redefinitions in Asymptotic Safety

Benjamin Knorr

2204.08564
2311.12097
2312.03831 w/ A. Baldazzi, K. Falls, Y. Kluth
2405.08860 w/ A. Platania

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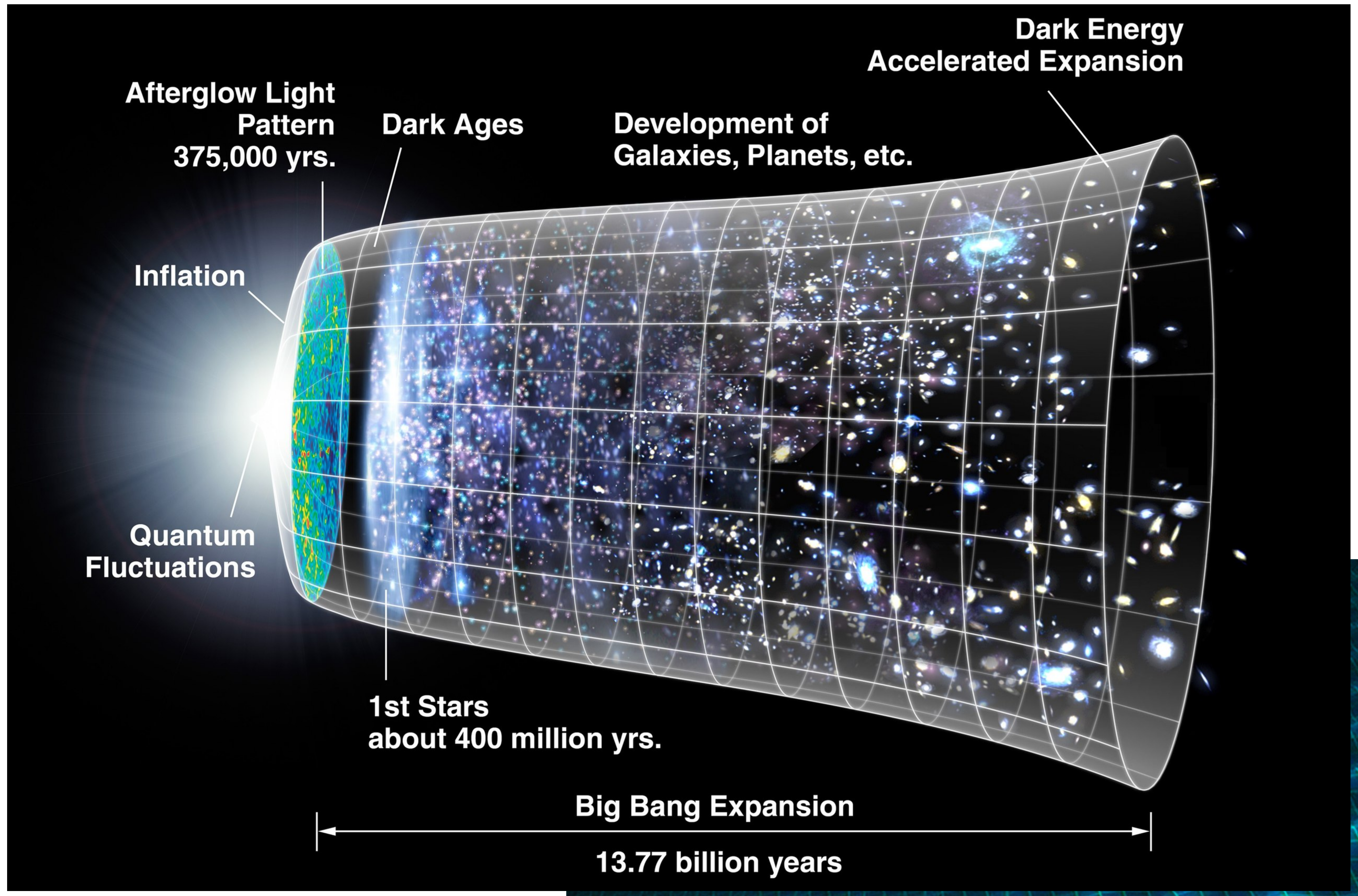
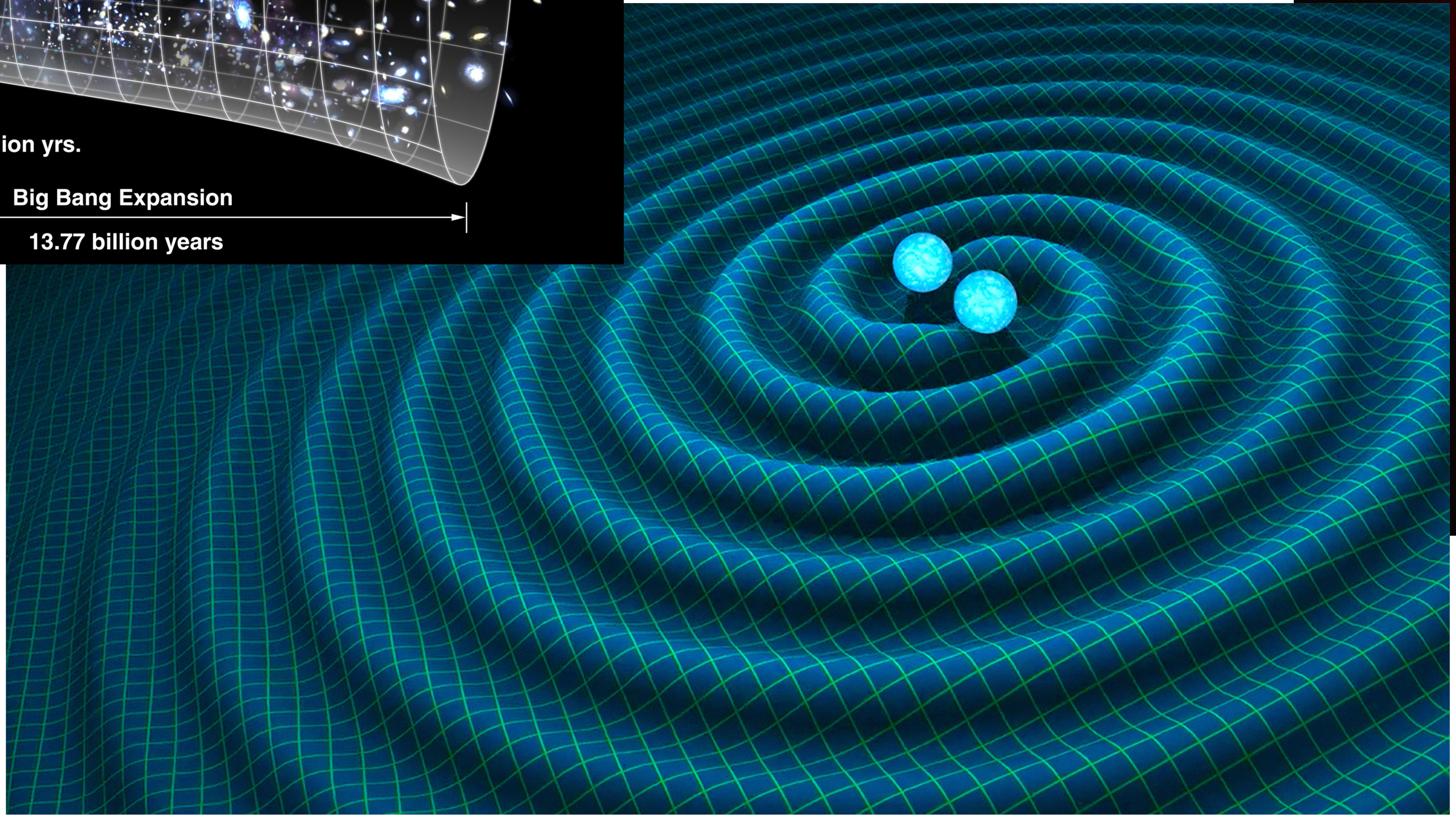
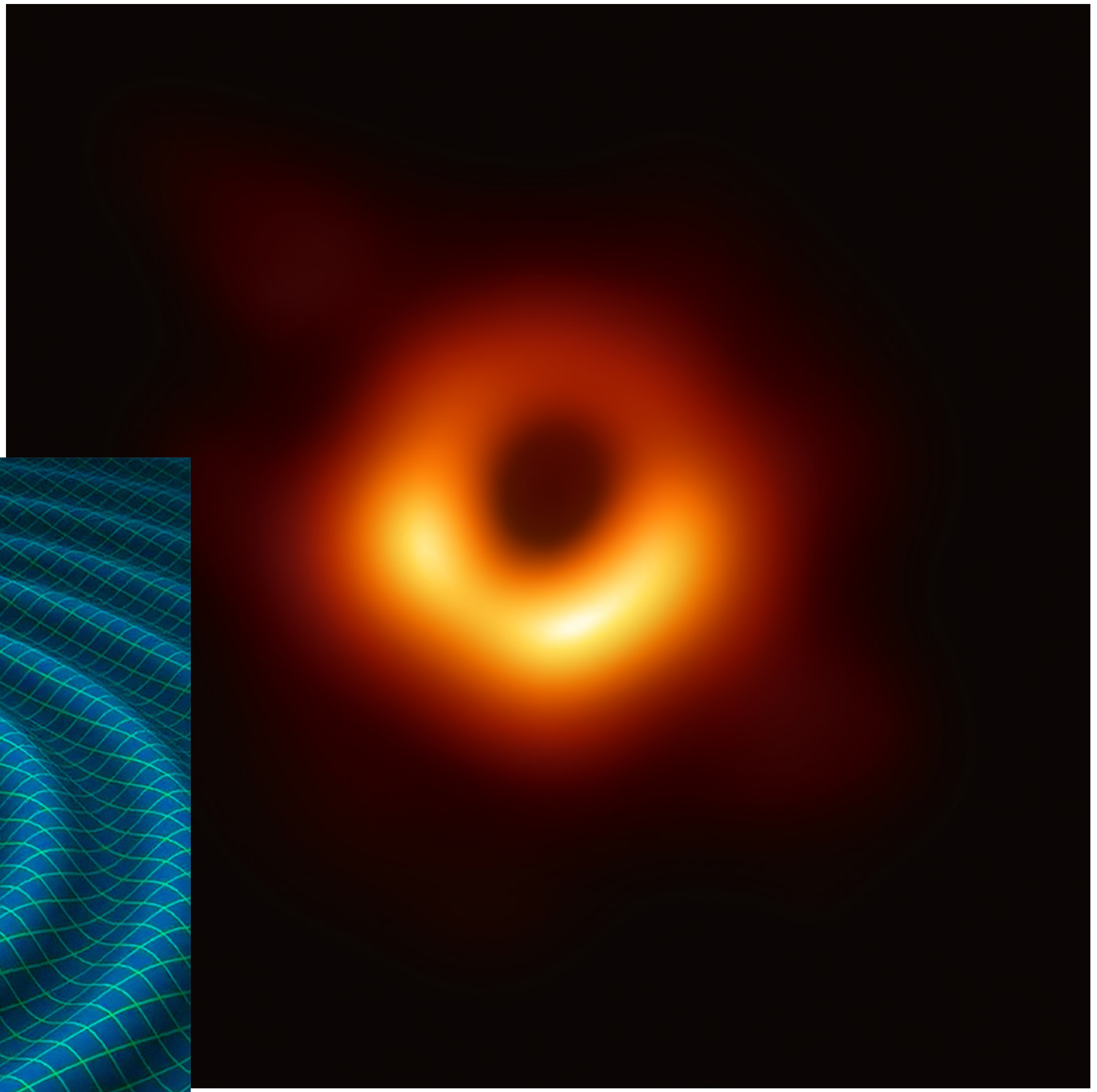
A Perspective on Quantum Gravity







A Perspective on Quantum Gravity

lack of smoking gun
quantum gravity experiments

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many big ideas
and even bigger claims
on QG

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**why trust any
approach in particular?**

A Perspective on Quantum Gravity

- recipe for a falsifiable and predictive quantum gravity theory:

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  1. set up quantum theory of gravity and matter (at least SM)

  2. confront the theory with as much available theory constraints (unitarity, causality, …) and experimental data (cosmological evolution, particle masses, …) as possible

  3. if consistent, only then move on to the “big questions”: black holes, big bang, …

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- less glamorous, more down-to-earth: fix starting point and see how far you get

A Perspective on Quantum Gravity

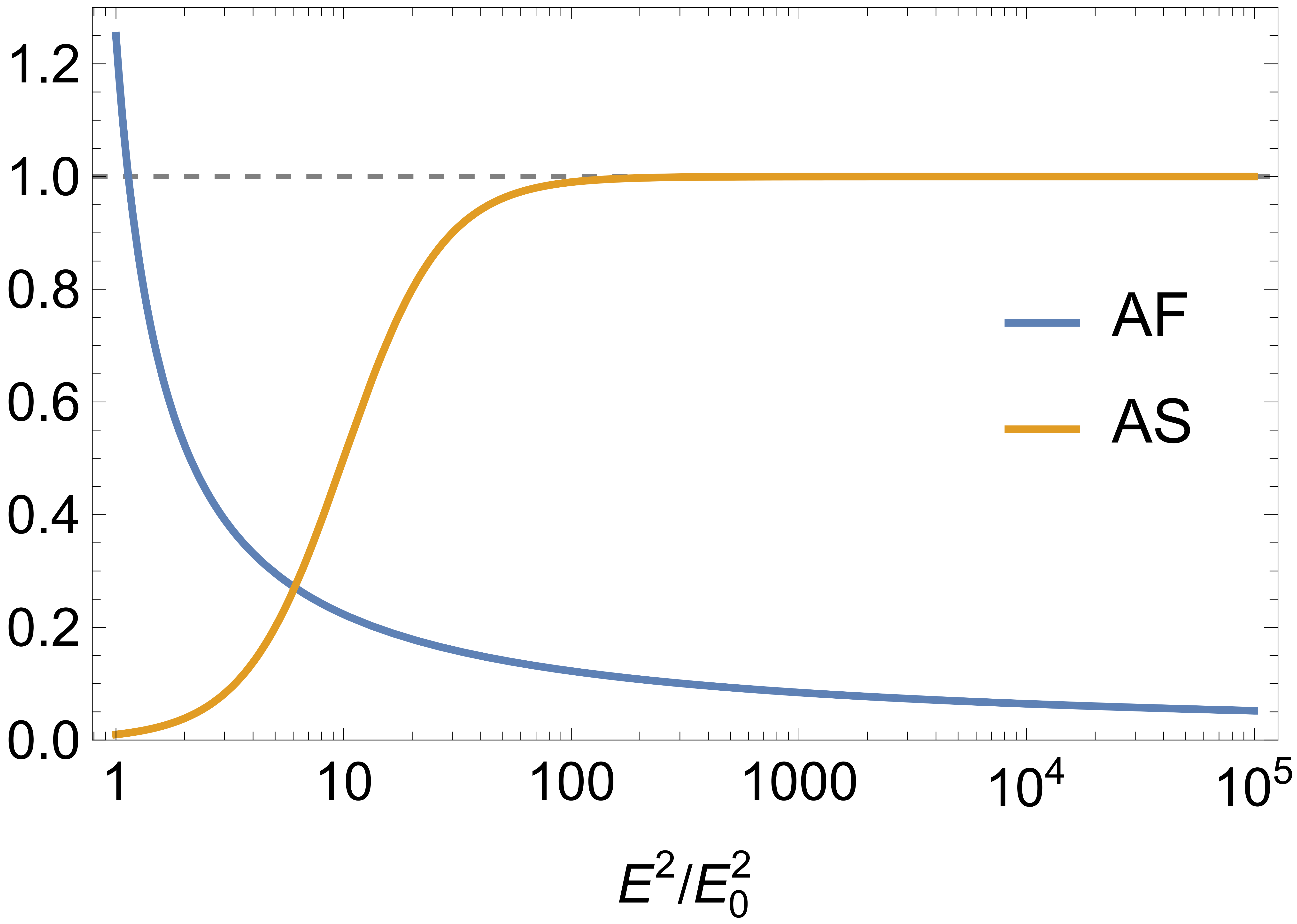
- working hypothesis: QFT works also for QG
  - access to standard QFT notions like **renormalisation group** and **scattering amplitudes**
  - connection to EFT: **positivity bounds**
- in this talk: work towards $(2 \to 2)$ amplitudes in Asymptotic Safety, but general ideas can be carried over to other approaches
- constraint: make my life as simple as possible

Outline

- Asymptotic Safety: lightning review
- Scattering amplitudes, field redefinitions and (in-)essential couplings
- Essential couplings in quantum gravity
- RG of essential QG - selected results
- Summary

Asymptotic Safety:
lightning review

Asymptotic Safety



Asymptotic Safety

- conditions:
  - all beta functions vanish at non-vanishing value of couplings (finiteness)
  - only finitely many couplings are relevant (predictivity)
- you also want unitarity, causality, compatibility with IR physics (observations/experiments), …
- tool: Functional Renormalisation Group (FRG)

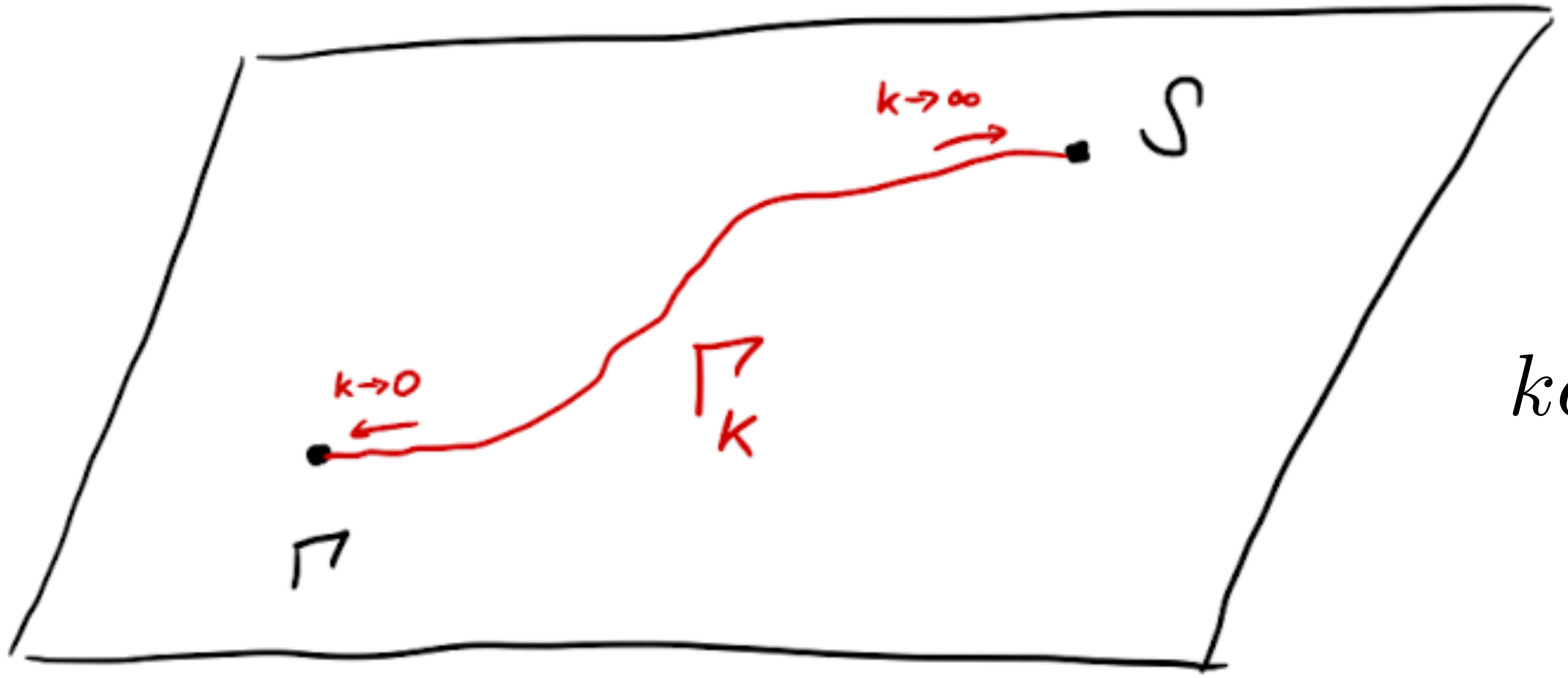
FRG

- effective average action Γ_k: interpolation between microscopic action S and effective action Γ

- **fiducial** scale k, physics recovered when $k \to 0$

recall Roberto's talk:
physical running vs. k-running

⇒ G_N and Λ do NOT have physical running!



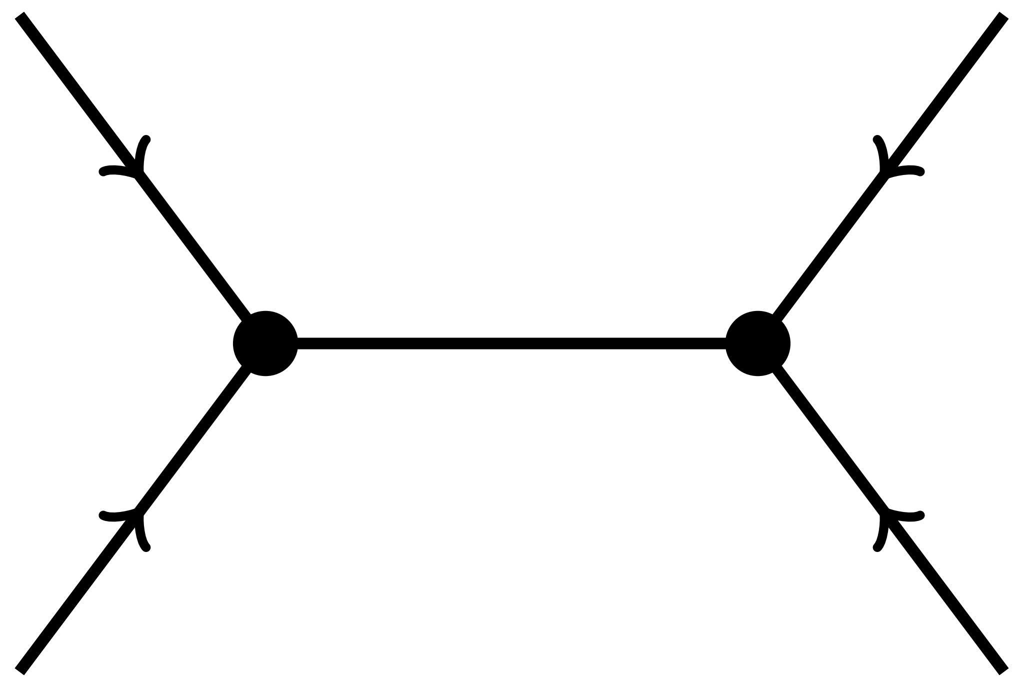
$$k\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr} \left[\left(\Gamma_k^{(2)} + \mathfrak{R}_k \right)^{-1} k\partial_k \mathfrak{R}_k \right]$$

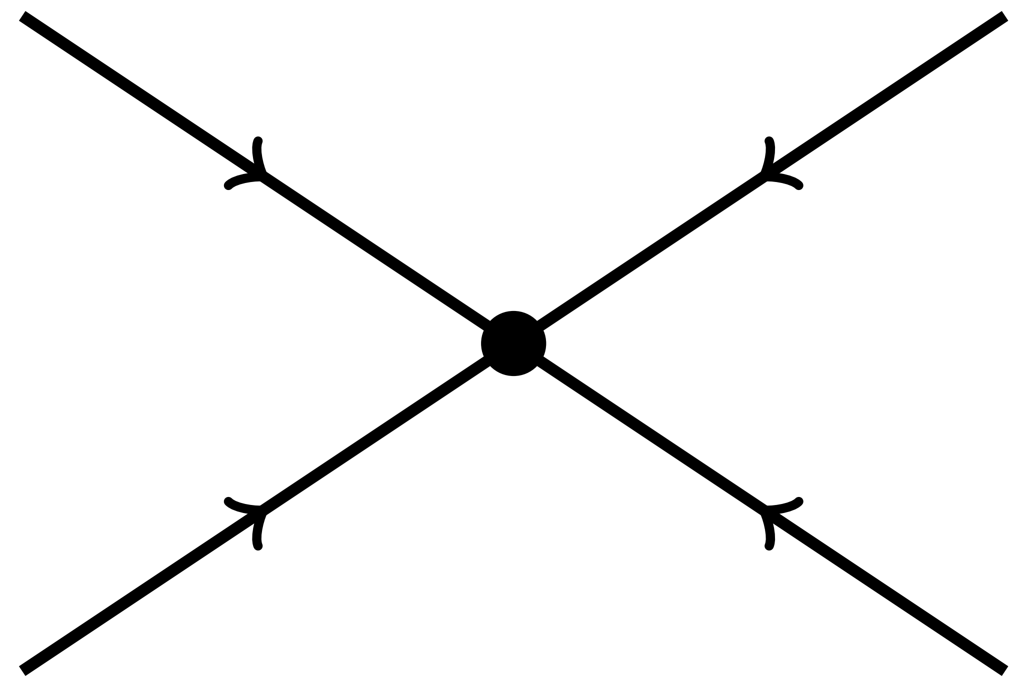
Wetterich '93

Scattering amplitudes, field redefinitions and (in-)essential couplings

Effective Action

- for now: talk about **exact** theory (later: approximations) in the "not too strongly" interacting regime
  - hierarchy in the importance of correlation functions
- recall: effective action Γ includes all quantum effects, full scattering amplitudes obtained from tree level diagrams





Scattering amplitudes

- within Asymptotic Safety: most investigations at level of off-shell RG flow
    - residual dependencies on parameterisation, gauge choice etc.
- scattering amplitudes are on-shell and related to observables - **independent** of such choices

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inessential couplings
multiply EoM in action

***conditions apply,**
read fine print for details

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Field redefinitions - some examples

- consider “standard” scalar field theory in Minkowski space
  - wave function renormalisation inessential: $\phi \mapsto Z(p^2)^{-1/2}\phi$
  - removes non-trivial momentum dependence of propagator by shifting it into interactions
  - caveat: needs $Z > 0$, cannot remove or add modes

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Field redefinitions - some examples

- consider Starobinsky-type theory, $R + R^2$
  - spectrum: EH + massive scalar
  - field redefinition can change value of R^2 coupling, but cannot be set to zero (removes mode)

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Classes and schemes

- in practice: have to **choose universality class** whose RG flow we are investigating beforehand
  - **fixed** spectrum
  - adapt regularisation, make specific choices for inessential couplings to simplify computations
  - how to treat branch cuts, (strong) IR non-localities, bound states, …?

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Classes and schemes

- in practice: have to **choose universality class** whose RG flow we are investigating beforehand

- in the following: discuss **[GR]** (+matter in **[Gauss]**)

    - only “standard” modes (no extra modes, ghostly or not)

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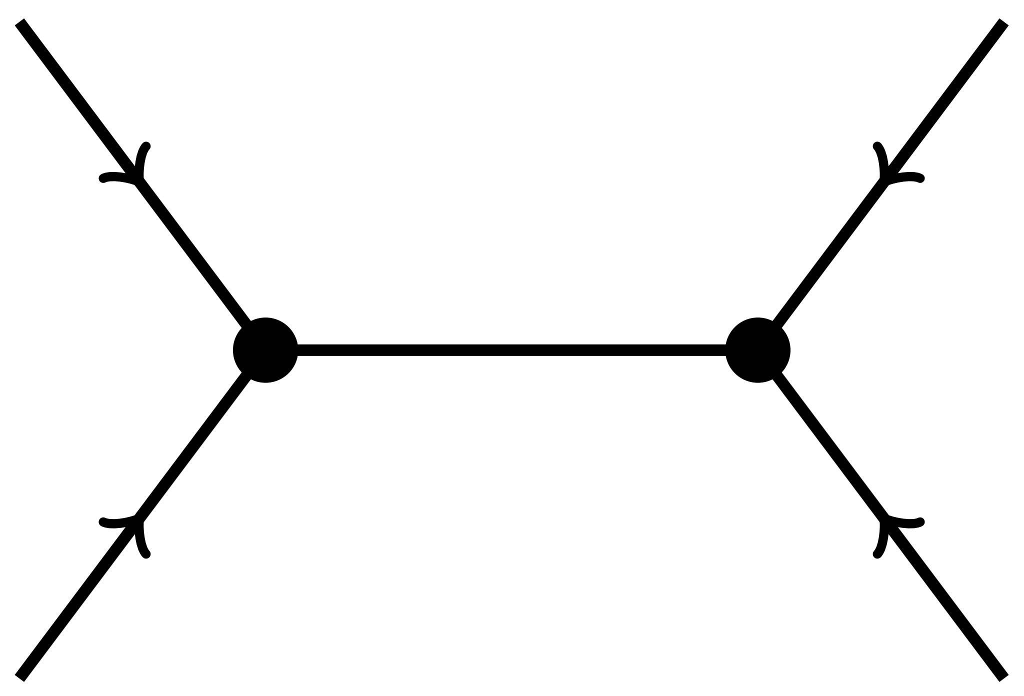
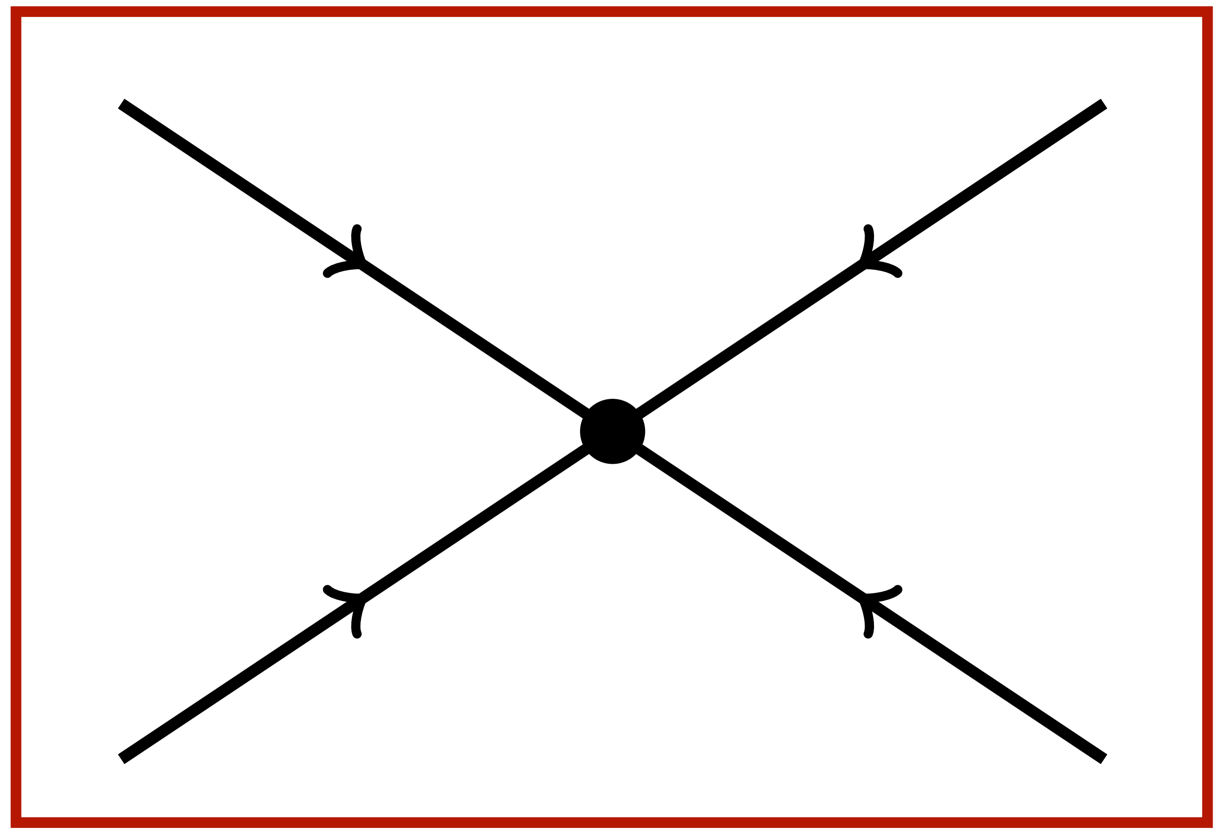
Classes and schemes

- in practice: have to **choose universality class** whose RG flow we are investigating beforehand
- in the following: discuss **[GR]** (+matter in **[Gauss]**)
- philosophy: any inessential couplings that can be set to zero should be set to zero ("minimal essential scheme")

Baldazzi, Ben Alì Zinati, Falls 2105.11482
Baldazzi, Falls 2107.00671





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Essential couplings in quantum gravity

Essential couplings in **[GR]**

Field redefinitions in QG

- general field redefinition of the metric:

$$g_{\mu\nu} \mapsto c_0 g_{\mu\nu} + c_1(\Box) R_{\mu\nu} + c_2(\Box) R \, g_{\mu\nu} + \dots$$

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(is this still a Lorentzian metric?)

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- curvature expansion of effective action of **[GR]** in minimal scheme:

$$\Gamma_{[\mathbf{GR}]} = \frac{1}{16\pi G_N} \int \mathrm{d}^d x \, \left[2\Lambda - R + \Theta \mathfrak{E} + G_{C^3} C^3 + \mathcal{O}(\mathcal{R}^4) \right]$$

NB: only one of G_N, Λ essential,
cannot set either to zero generically

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- quadratic terms inessential: multiply EoM

$$\left(R - \frac{2d}{d-2}\Lambda + \dots\right) f_R(\Box) \left(R - \frac{2d}{d-2}\Lambda + \dots\right)$$

$$\left(S_{\mu\nu} + \dots\right) f_S(\Box) \left(S^{\mu\nu} + \dots\right)$$

$$\Box C = \{DDS, DDR\}$$

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Field redefinitions in [GR]

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- quadratic terms inessential: multiply EoM
- cubic terms almost all inessential, except Goroff-Sagnotti term

no essential $D^2C^3, \ldots, D^{12}C^3$ terms
(Bianchi + partial integration)

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- essential higher order terms: **only** Weyl + covariant derivatives, no $\square C$

structurally, $f(s,t)\, C^4$

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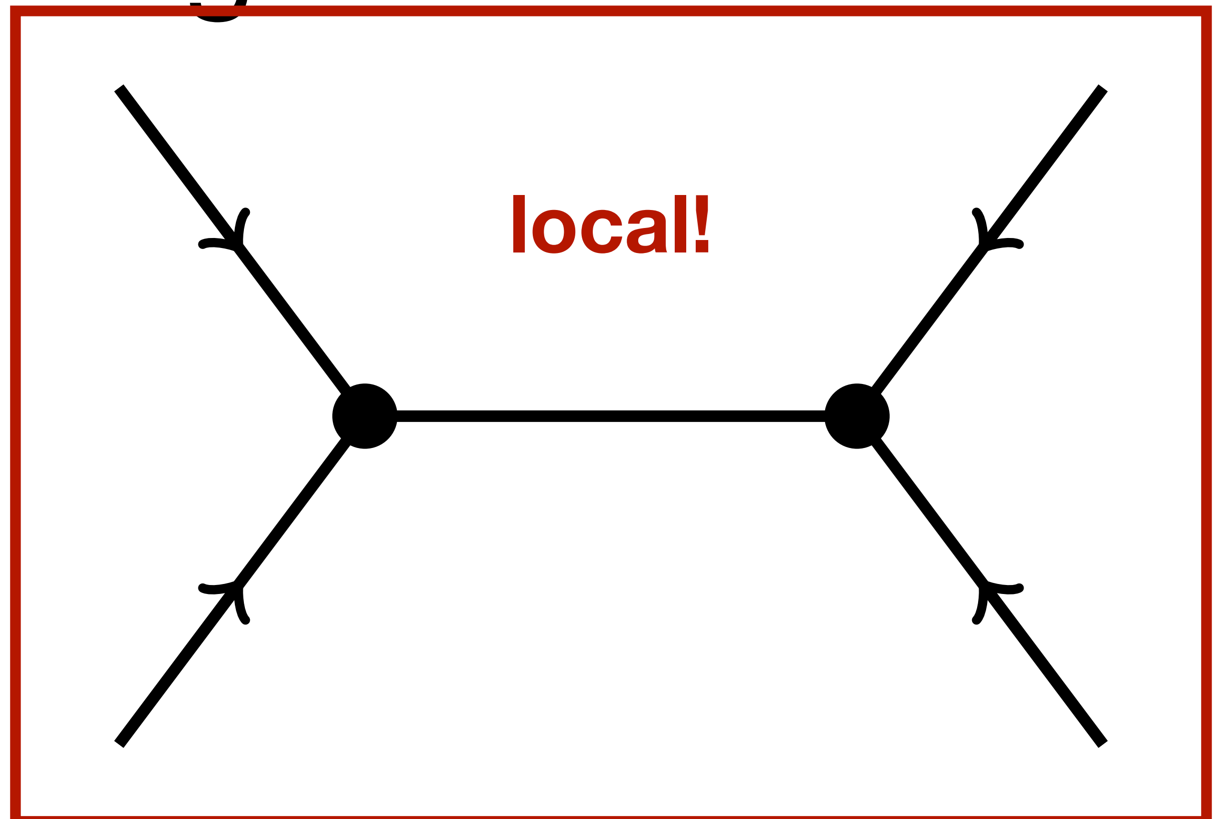

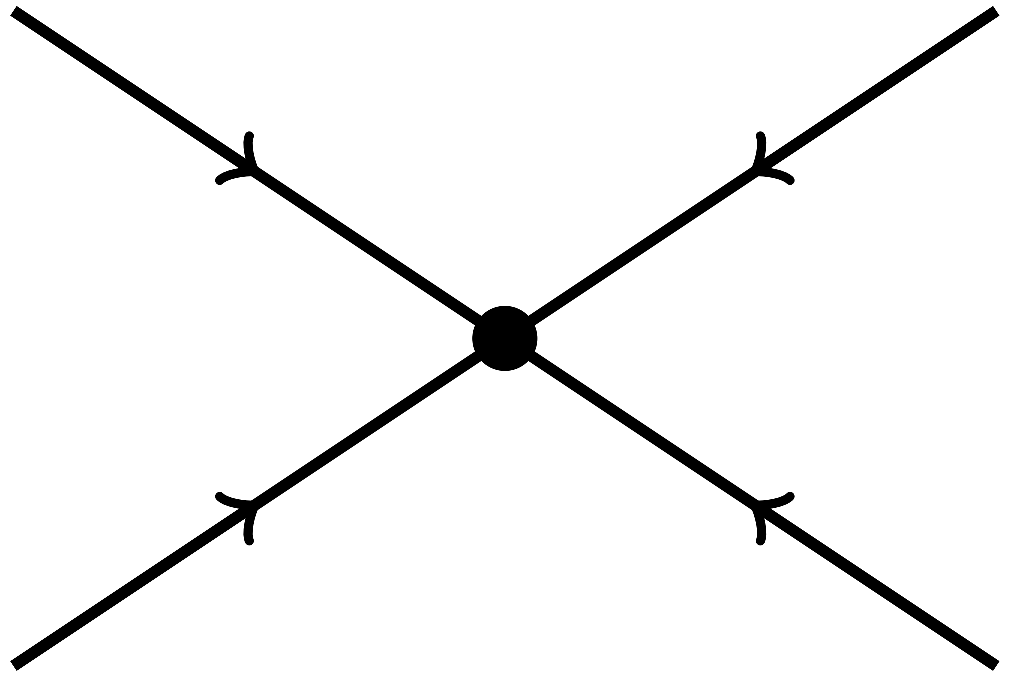

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- if you are in **[GR]**: **non-trivial** scattering physics starts at four-point function
- in FRG: have to impose field redefinition consistently along RG flow - reduction of complexity, but no free lunch
- non-minimal coupling to matter: only involves Weyl tensor

$$C^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

BK, Platania 2405.08860

BK 2311.12097

RG of essential QG selected results

FRG equation in essential scheme:
Baldazzi, Ben Alì Zinati, Falls ’21
Baldazzi, Falls ‘21

RG flow in [GR]

- consider derivative expansion up to 6th order (d=4):

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Baldazzi, Falls, Kluth, BK 2312.03831

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10 running field redefinitions!

Baldazzi, Falls, Kluth, BK 2312.03831

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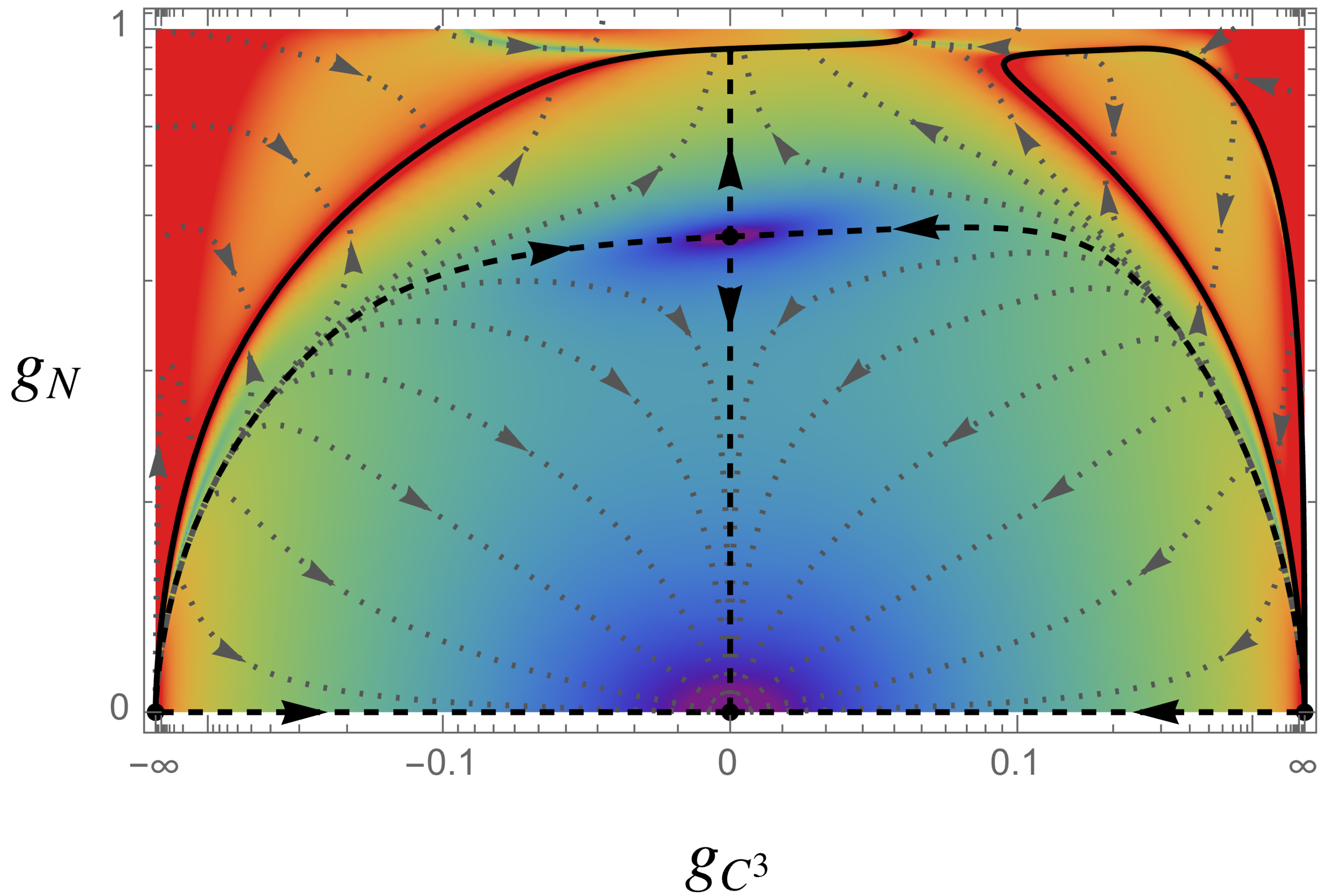

$$\theta_1 = 2.225$$
$$\theta_2 = -3.850$$

Baldazzi, Falls, Kluth, BK 2312.03831

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$$G_{C^3} = \left(\frac{G_N}{16\pi}\right)^2 \left(a - \frac{86}{315} \ln G_N k^2\right)$$

$$a = -0.5065$$

AS predicts the Goroff-Sagnotti coupling!

in line with earlier calculation: Gies, BK, Lippoldt, Saueressig 1601.01800

Baldazzi, Falls, Kluth, BK 2312.03831

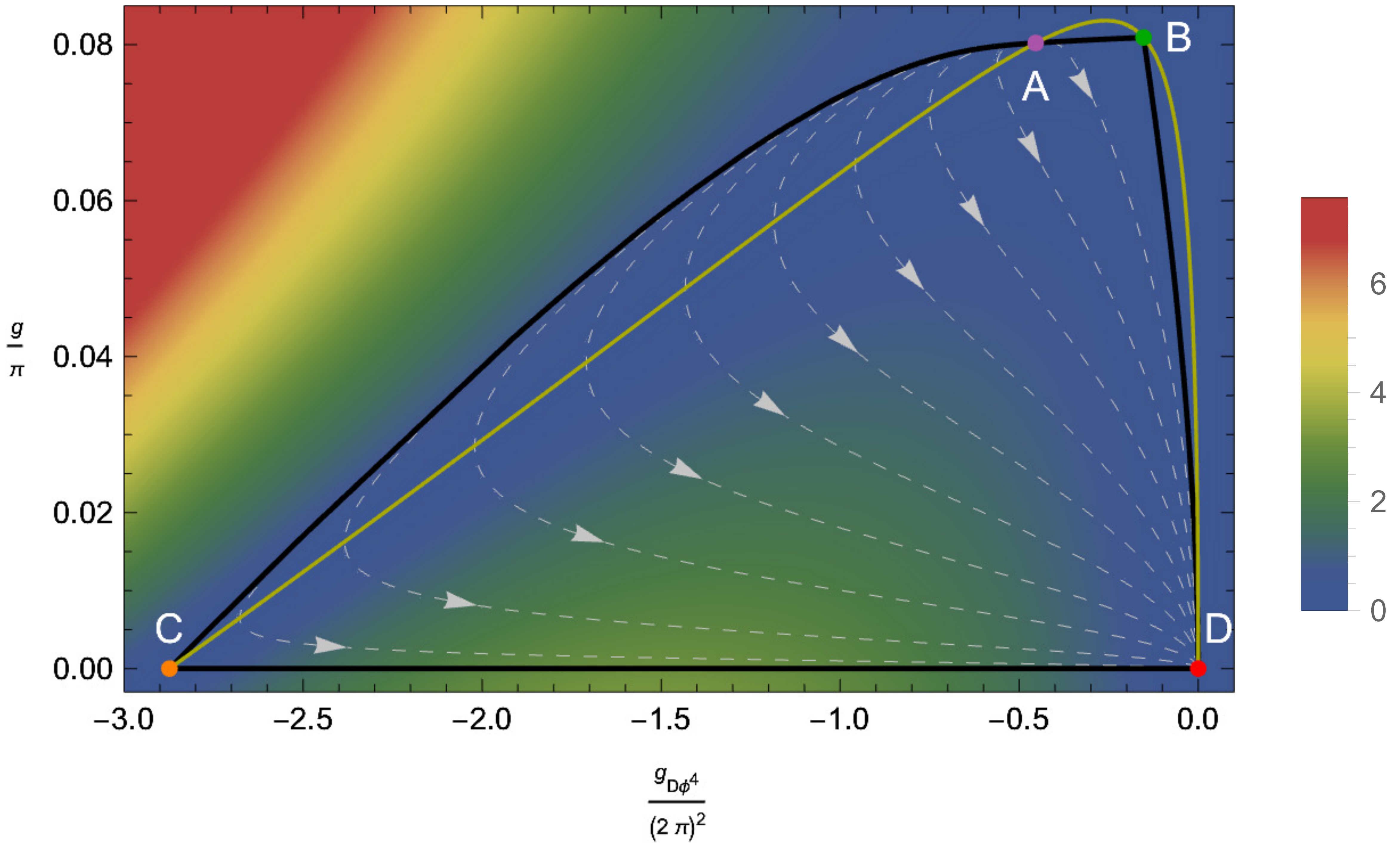
RG flow in [GR]

- other results:
  - propagator form factors in **[GR]** **BK 2311.12097**
- work in progress:
  - f(R) in **[GR]**, **[R2]** *BK, Sannestedt*
  - 6th order in general dimension *Felici, BK*

RG flow in [GR]+[Gauss]

- consider derivative expansion up to 4th order coupled to matter:

$$\Gamma_{[\mathbf{GR}]+[\phi]} = \int \mathrm{d}^4x \left(\frac{1}{16\pi G_N} \left[2\Lambda - R + \Theta \mathfrak{E}\right] + \frac{1}{2}(D_\mu \phi)^2 + G_{D\phi^4} \left(\frac{1}{2}(D_\mu \phi)^2 \right)^2 \right)$$



BK 2204.08564

$$G_{D\phi^4} = G_N^2 \left(a + \frac{203}{5} \ln\left[G_N k^2\right] \right)$$

$$a \lesssim 31.85$$

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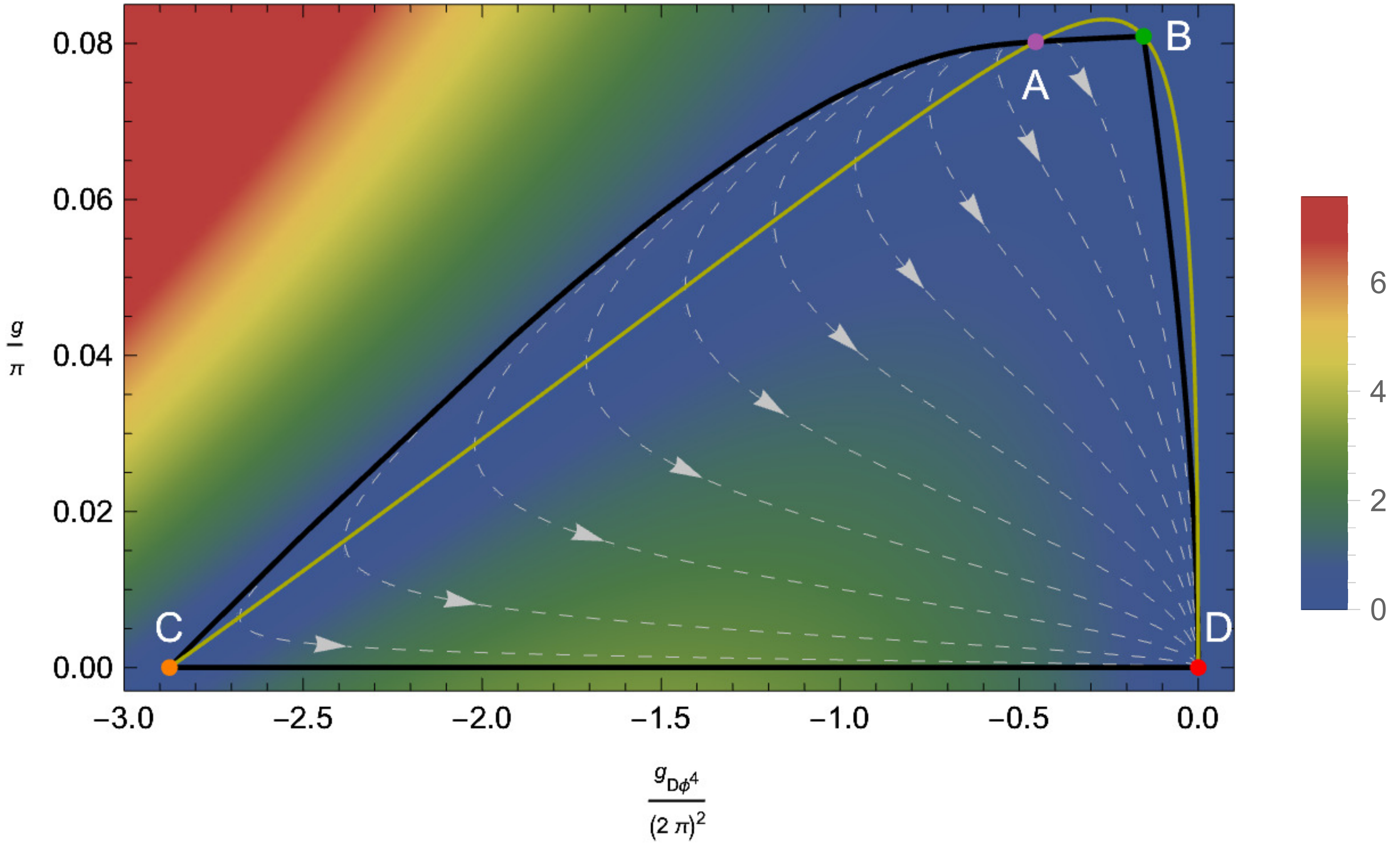

BK 2204.08564

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some issues with A+C, B+D stable
de Brito, BK, Schiffer 2302.10989

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BK 2204.08564

$$\Gamma_{[\mathbf{GR}]+[\gamma]} = \int \mathrm{d}^4x \left(\frac{1}{16\pi G_N} \left[2\Lambda - R + \Theta\mathfrak{E}\right] + \frac{1}{4}\mathrm{tr}F^2 + G_{\mathcal{F}^2} \left(\frac{1}{4}\mathrm{tr}F^2\right)^2 + \frac{G_{F^4}}{16}\mathrm{tr}F^4 + G_{CFF}C^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \right)$$

BK, Platania 2405.08860

intriguing relations to positivity/weak-gravity bounds
⇒ talk by Alessia

Summary

Summary

- falsifiability is at the heart of science, and it should also be at the heart of quantum gravity research

- scattering amplitudes are promising tool to probe quantum gravity

- field redefinitions allowed - all the difficulty starts at four-point function, strong results need high level of sophistication

- **[GR]** promising from perspective of Asymptotic Safety - maybe no free parameters? AS **[GR]** = string theorists' dreams come true?