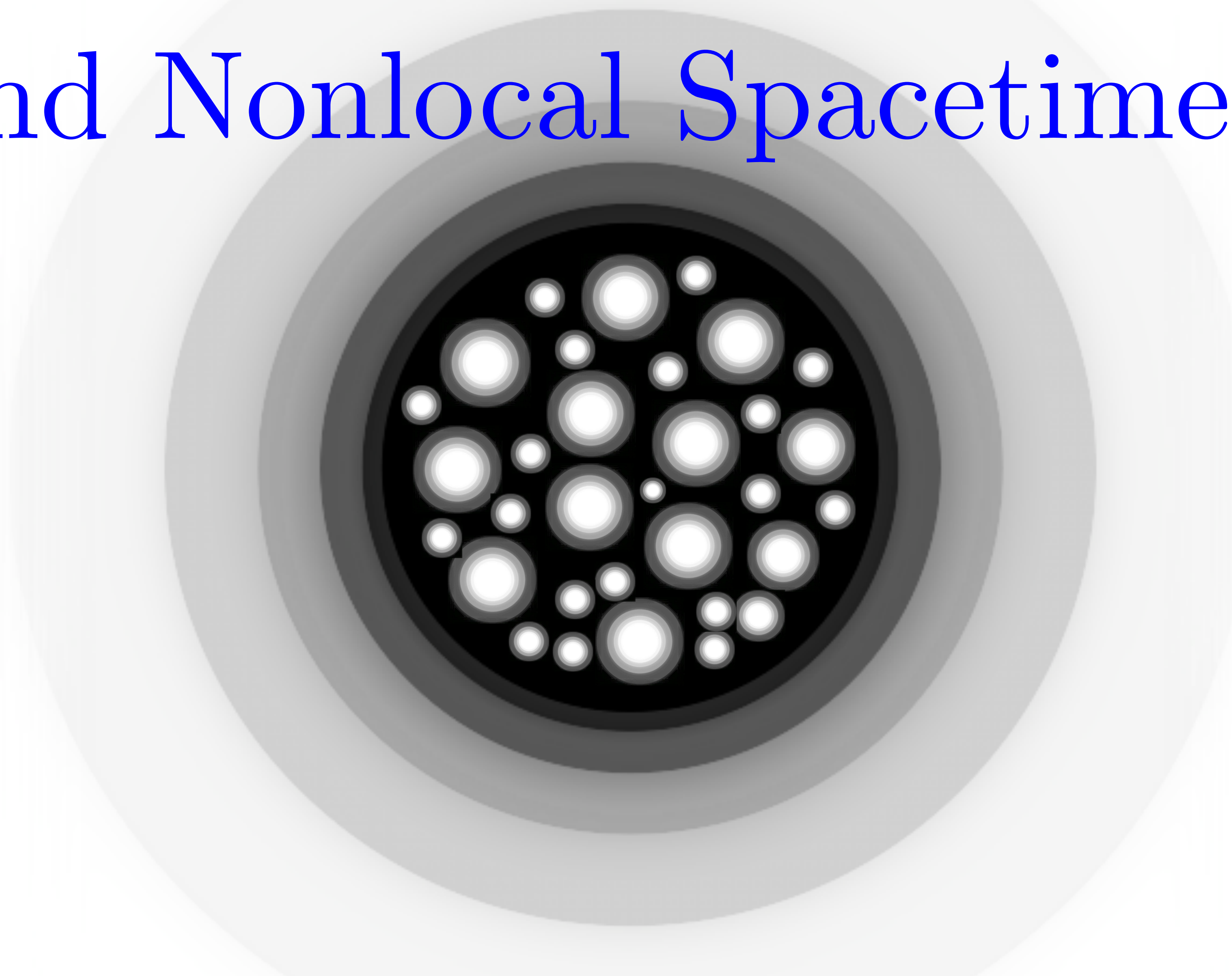
Quantum Gravity and Cosmology 2024

ShanghaiTech University, Shanghai, China, July 1-5, 2024

Local and Nonlocal Spacetime-Matter



Leonardo Modesto

Università degli Studi di Cagliari

Outline Theory

- Local & Nonlocal Gravitational Theories.
  - Unitarity: no ghosts, complex pairs, fakions.
  - (super-)renormalizability.

- Finite Quantum Gravity :
  - Finiteness in Odd Dimension.
  - Finiteness in Even Dimension.
  - Conformal invariant quantum gravity.

- Classical & quantum scattering amplitudes, causality.
- Fundamental Confinement.
- Nonlocal Particle Physics.
- Spacetime Singularities:
  black holes, galactic rotation curves, etc.

Local Gravitational Theories

Local Gravitational Theories

- $$\mathcal{L}_{\rm EH} = \frac{2}{\kappa^2} R\,,$$
  second order diff. EoM, d.o.f $= 2$.
  NON-REN.
- $$\mathcal{L}_{\rm Starobinsky} = \frac{2}{\kappa^2}\,(R - 2\Lambda) + \epsilon R^2\,,$$
  forth order diff. EoM, d.o.f $= 3$, ghost-free .
  NON-REN.
- $$\mathcal{L}_{\rm Stelle} = \frac{2}{\kappa^2}\,(R - 2\Lambda) + \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu}\,,$$
  forth order diff. EoM, d.o.f $= 8$, 1 ghost of spin=2.
  REN. (Unitary *alla* Lee-Wick (Donoghue), or with the Anselmi-Piva prescription).
  (PT-Symmetry-Unitarity, Mannheim.)

Asorey, Lopez, Shapiro (1996)

- $$\mathcal{L}_{\rm ALS} = \frac{2}{\kappa^2}\,(R - 2\Lambda) + \sum_{n=0}^{\rm N} \omega_{{\rm Ric},n}\, R_{\mu\nu}\,\Box^n R^{\mu\nu} + \sum_{n=0}^{\rm N} \omega_{{\rm R},n}\, R\,\Box^n R + \mathcal{V}(\mathcal{R})\,,$$

  2N + 4 order diff. EoM,

  number of initial conditions: 2N + 4,

  d.o.f $\leqslant 10 \times (N+2)$, multiple ghosts.

  SUPER-REN. (Unitary: LM, Shapiro, and Anselmi-Piva prescription).

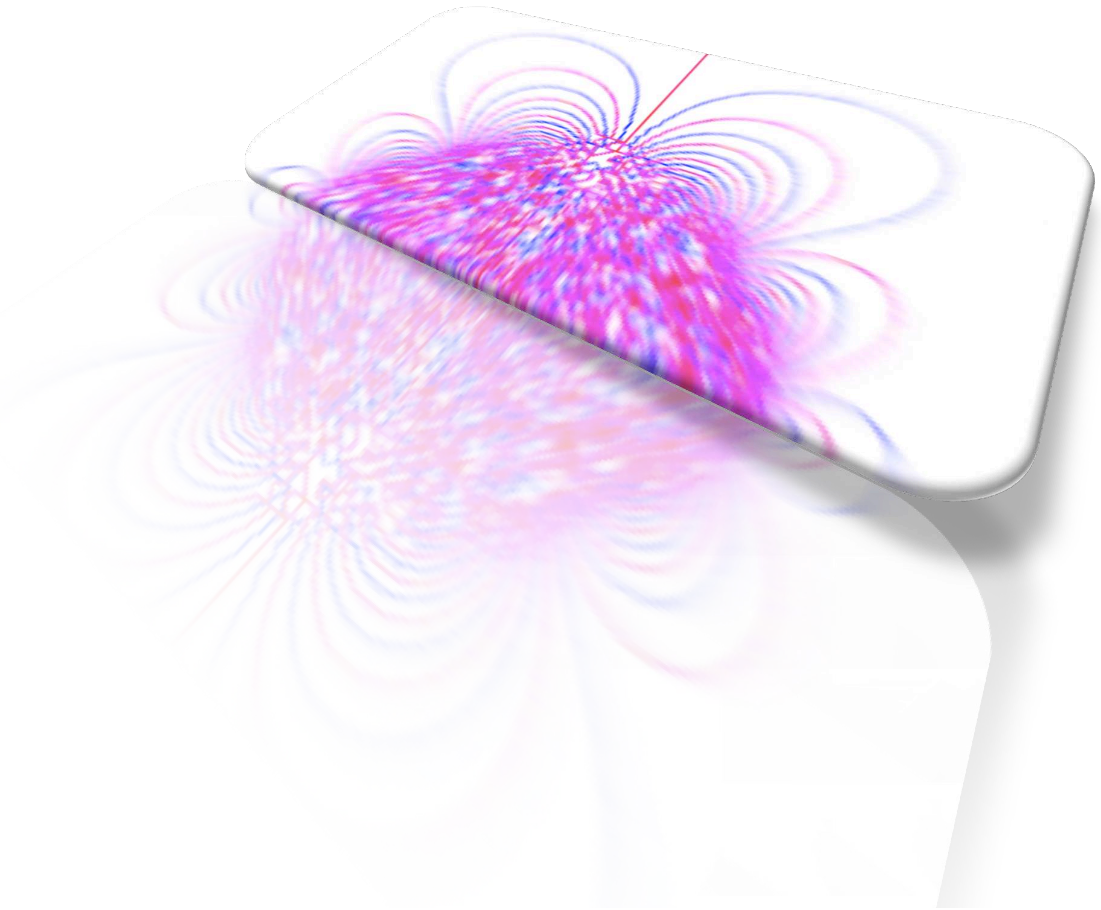
- $$\mathcal{L}_{\rm ghost-free} = \frac{2}{\kappa^2}\,(R - 2\Lambda) + \epsilon R^2 + \mathcal{R}^3 + \cdots + \mathcal{R}^n\,,$$

  4th order diff. EoM,

  number of initial conditions: 4,

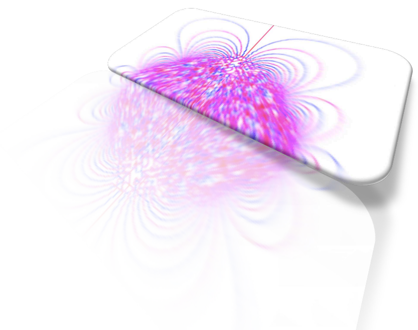
  d.o.f $\leqslant$ 10×4 ghost-free.

  NON-REN.



Nonlocal Gravitational Theories

$D = 4$



Krasnikov (1988), Kuzmin (1989), Tomboulis (1997), L.M.

$$\mathcal{L}_4 = \frac{2}{\kappa_4^2} \left(R + G_{\mu\nu} \frac{e^{H(-\Box_\Lambda)} - 1}{\Box} R^{\mu\nu} \right) .$$

$$e^{H(z)} = e^{\gamma_E + \Gamma[0,p(z)] + \log[p(z)]} \equiv e^{\gamma_E + \Gamma[0,p(z)]}\, p(z) \, ,$$

$$p(z) = a_1 z + a_2 z + \cdots + a_{n+1} z^{n+1} \, , \quad n \in \mathbb{N} \, .$$

$$e^{H(z)} \quad \rightarrow \quad e^{\gamma_E} p(z) + e^{\gamma_E} e^{-p(z)} + \dots \quad \text{for} \quad z \gg 1 \, .$$

Unitarity

&

Super-renormalizability

Unitarity

Propagator and Unitarity

Tree-level Unitarity

$$\mathcal{L}_D = \frac{2}{\kappa_D^2}\left(R + G_{\mu\nu}\frac{e^{H(-\Box_\Lambda)}-1}{\Box}R^{\mu\nu}\right) .$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa_D\, h_{\mu\nu}$$

$$\mathcal{O}^{-1}(k) = \frac{e^{-H(k^2/\Lambda^2)}}{-k^2}\left(P^{(2)} - \frac{P^{(0)}}{D-2}\right) .$$

Tree-level Unitarity

$$\langle f|S^{(2)}|i\rangle = \langle f|\mathrm{i}\,\mathcal{T}|i\rangle = (2\pi)^D\delta^D(p_i-p_f)\,\mathrm{i}\,\mathcal{M}_{if}$$
$$= (2\pi)^D\delta^D(p_i-p_f)\,\mathrm{i}^2\,T^{\mu\nu}\,\mathrm{i}\mathcal{O}^{-1}_{\mu\nu\rho\sigma}\,T^{\rho\sigma}\,,$$
$$S = 1+\mathrm{i}\mathcal{T} \simeq 1+S^{(2)}\,, \qquad S^\dagger S = 1 \quad \Longrightarrow \quad 2\,\mathrm{Im}\mathcal{M}_{if} > 0\,.$$

$$\mathrm{i}\mathcal{O}^{-1}(k) = \frac{\mathrm{i}}{-k^2+\mathrm{i}\epsilon}\left[\frac{P^{(2)}}{e^{H_2}} - \frac{P^{(0)}}{(D-2)\,e^{H_0}}\right]\,.$$

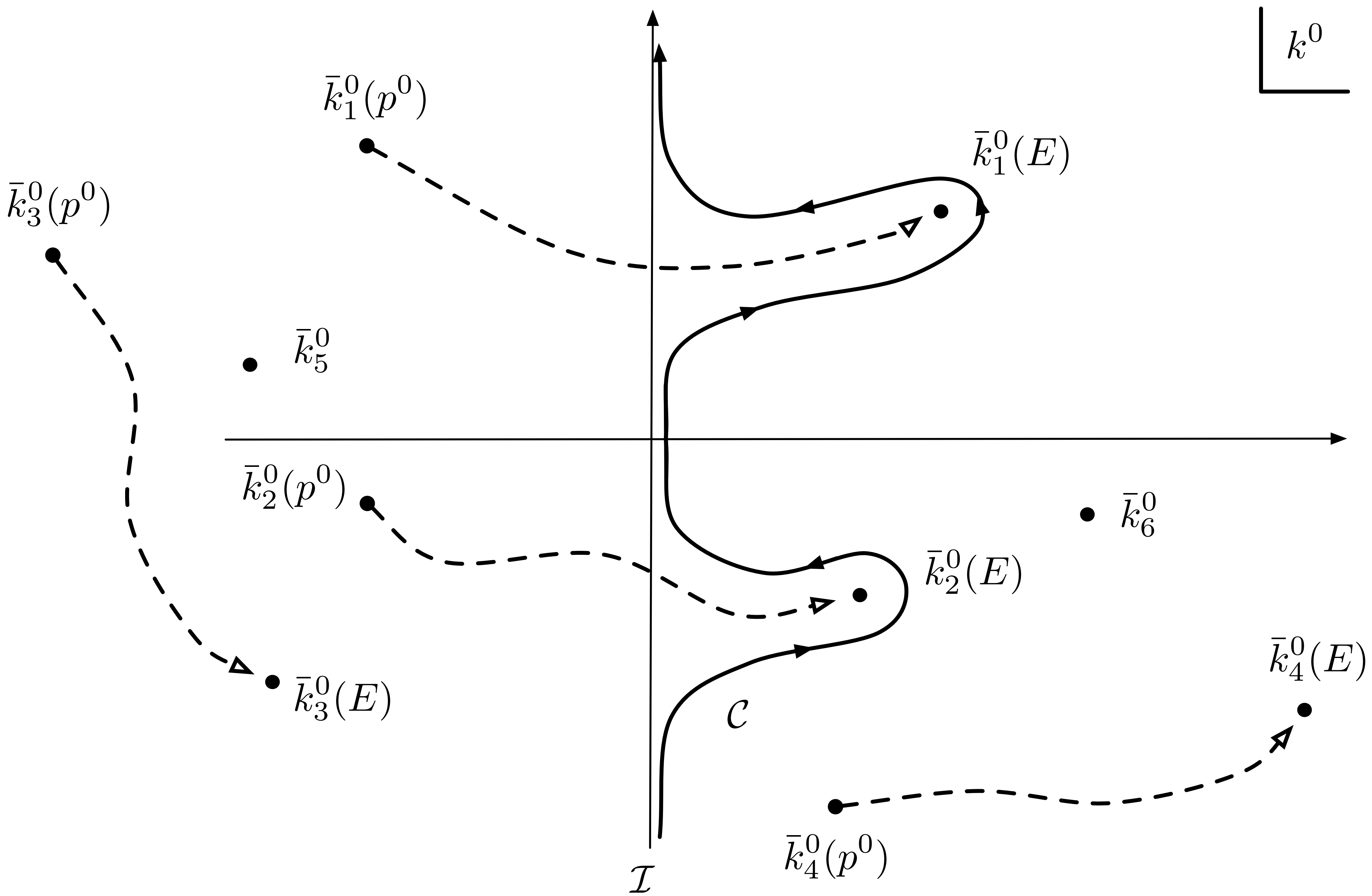
$$2\,\mathrm{Im}\,\mathcal{M}_{if} = \lim_{\epsilon\to 0^+} 2\,\mathrm{Im}\left(\mathrm{i}^2\left\{\left[T_{\mu\nu}T^{\mu\nu} - \frac{1}{D-1}(T^\mu_\mu)^2\right]e^{-H_2(k^2)} - \frac{T^{\mu\,2}_{\mu}}{(D-1)(D-2)}e^{-H_0(k^2)}\right\}\left(\frac{-k^2-\mathrm{i}\epsilon}{k^4+\epsilon^2}\right)\right)$$
$$= \lim_{\epsilon\to 0^+} 2\left\{\left[T_{\mu\nu}T^{\mu\nu} - \frac{1}{D-1}(T^\mu_\mu)^2\right]e^{-H_2(k^2)} - \frac{(T^\mu_\mu)^2}{(D-1)(D-2)}e^{-H_0(k^2)}\right\}\frac{\epsilon}{k^4+\epsilon^2}$$

$$\boxed{2\,\mathrm{Im}\,\mathcal{M}_{if} = 2\left[T_{\mu\nu}T^{\mu\nu} - \frac{(T^\mu_\mu)^2}{D-2}\right]\pi\,\delta(k^2)},$$

$$2\,\mathrm{Im}\left[\mathrm{i}^2\,T^{\mu\nu}(k)\mathcal{O}^{-1}_{\mu\nu\rho\sigma}T^{\rho\sigma}(k)\right] = 2\pi\,\mathrm{Res}\left[T^{\mu\nu}(k)\mathcal{O}^{-1}_{\mu\nu\rho\sigma}T^{\rho\sigma}(k)\right]\Big|_{k^2=0} = 2\pi\mathrm{Res}\,(\mathcal{A})\,\Big|_{k^2=0} > 0.$$

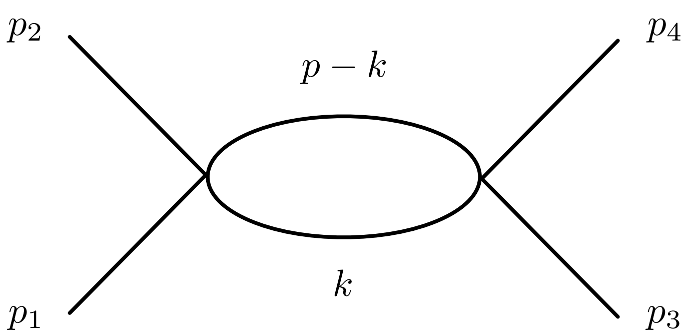
Analytic continuation of the Euclidean amplitudes.

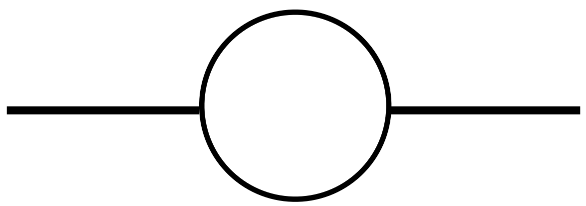
(F. Briscese, Li Qiang, Jiangfan Liu, LM., Spallucci, Pius, Sen, Efimov.)



Example $\lambda\phi^4$ theory at $1-$loop.

$(\lambda_3\phi^3)$

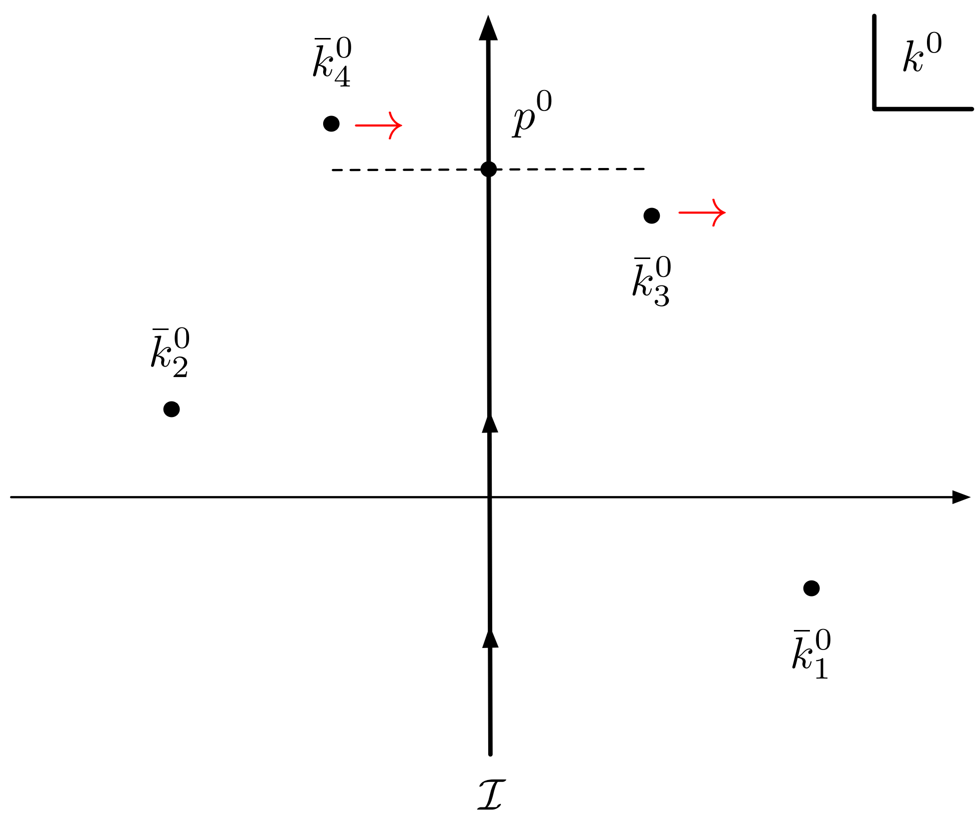


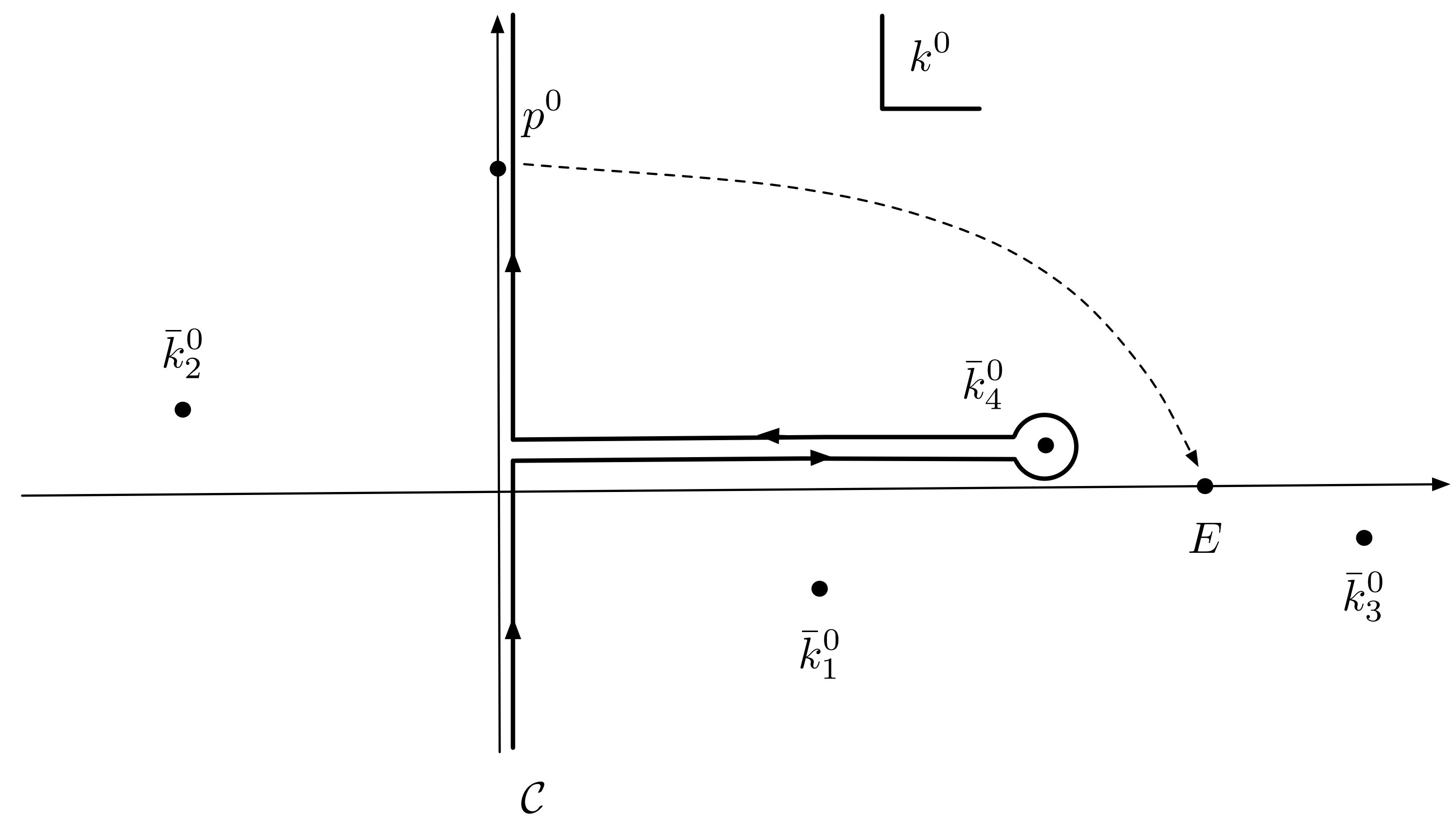


$$\mathcal{M}(p_h,\epsilon) = -\frac{\lambda^2}{2}\int_{(\mathcal{I}\times\mathbb{R}^3)} \frac{i\,d^4k}{(2\pi)^4}\,\frac{e^{-H(k^2-m^2)}}{k^2-m^2+i\epsilon}\,\frac{e^{-H((k-p)^2-m^2)}}{(k-p)^2-m^2+i\epsilon}$$

$$\rightarrow \quad \mathcal{M}(p_h,\epsilon) = -\frac{\lambda^2}{2}\int_{(\mathcal{C}\times\mathbb{R}^3)} \frac{i\,d^4k}{(2\pi)^4}\,\frac{e^{-H(k^2-m^2)}}{k^2-m^2+i\epsilon}\,\frac{e^{-H((k-p)^2-m^2)}}{(k-p)^2-m^2+i\epsilon},$$

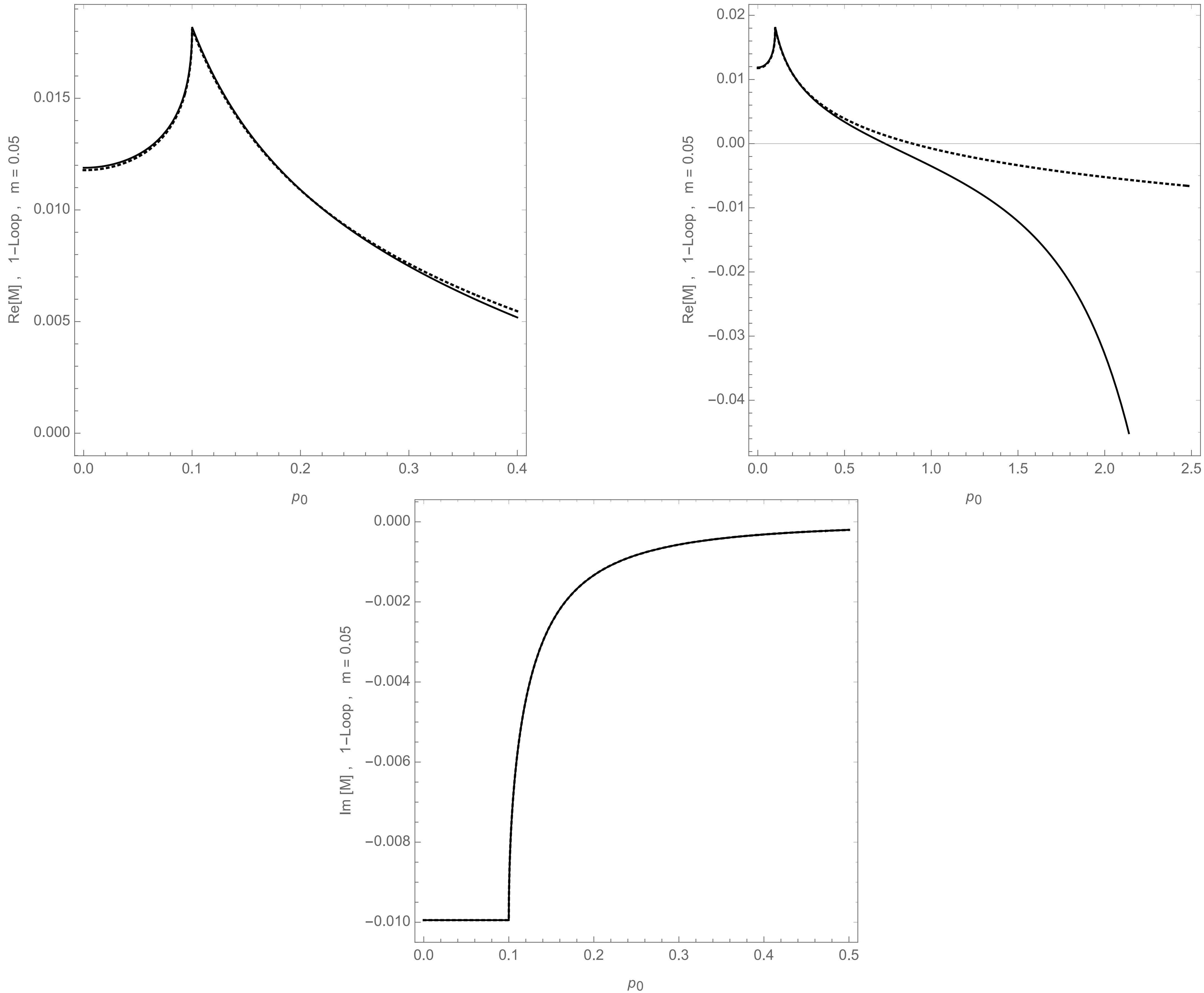
$$\bar{k}^0_{1,2} = \pm\sqrt{\vec{k}^2+m^2-i\epsilon}\,, \qquad \bar{k}^0_{3,4} = p^0 \pm \sqrt{(\vec{k}-\vec{p})^2+m^2-i\epsilon}.$$





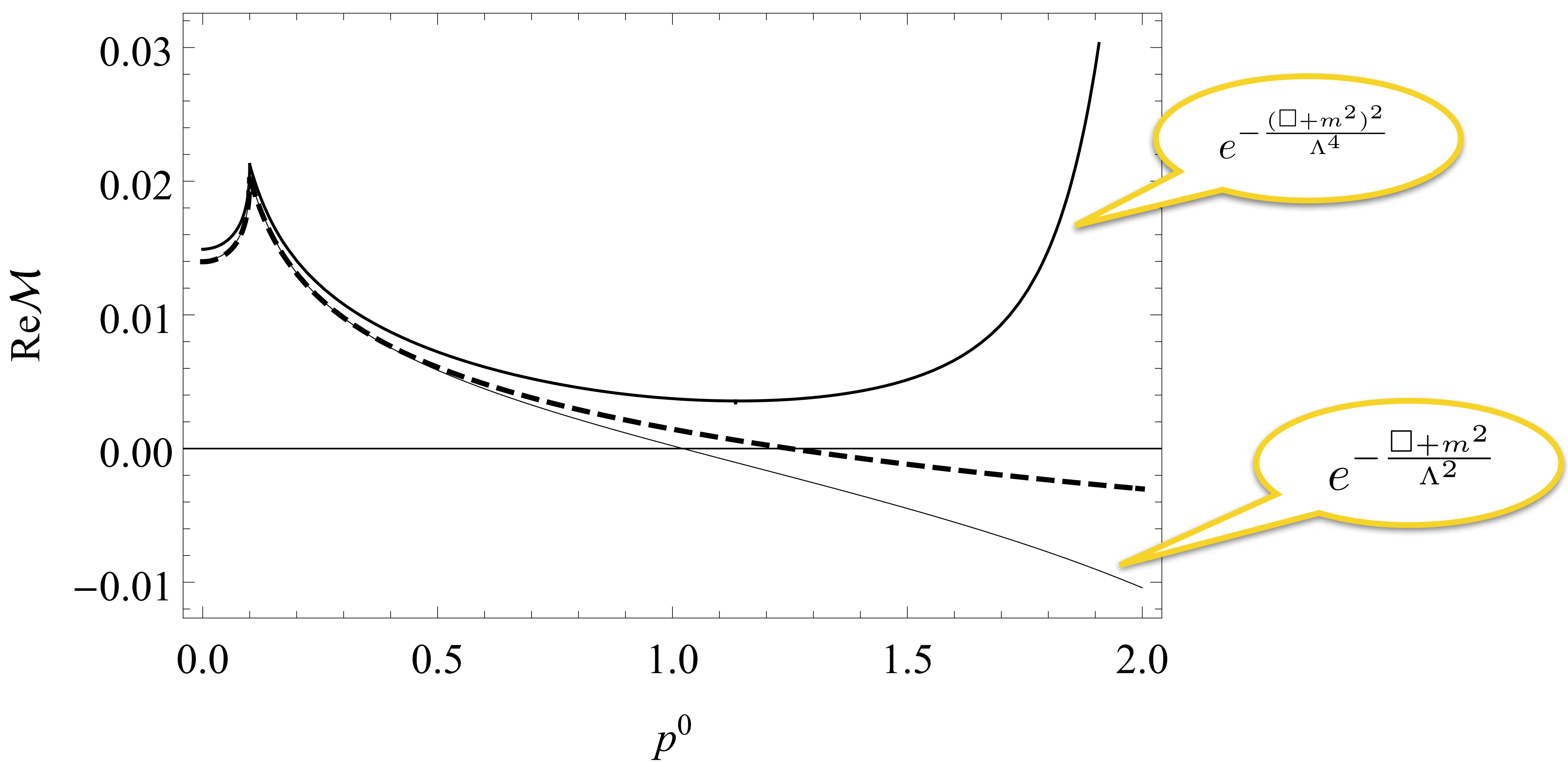
$$\lim_{\epsilon\to 0}\{\mathcal{M}(p_h,\epsilon)-\mathcal{M}(p_h,\epsilon)^*\} = -\frac{\lambda^2}{2}\int_{(\mathbb{R}^4)} \frac{i\,d^4k}{(2\pi)^4}\,(-2\pi i)\,\sigma(k^0)\,\delta((k-p)^2-m^2)\,(-2\pi i)\,\sigma(p^0)\,\delta(k^2-m^2)\,.$$

$$e^{-H} = e^{-\frac{\Box + m^2}{\Lambda^2}} \quad :$$



$$e^{-\frac{(\Box+m^2)^2}{\Lambda^4}}:$$

$$\mathcal{M}_{\rm Res}(p_0) = -\frac{\lambda^2}{2}\int_m^{p_0}\frac{dE}{(2\pi)^2}\frac{\sqrt{E^2-m^2}}{p_0}\frac{e^{-\frac{(p_0^2-2p_0E)^2}{\Lambda^4}}}{p_0-2E} + \frac{i\,\lambda^2}{32\pi}\sqrt{1-\frac{4m^2}{p_0^2}}\,\sigma(p_0-2m)$$



Perturbative Unitarity

F. Briscese, L.M., R. Pius, A. Sen

Analytic continuation of the Euclidean theory.

$$S = 1 + i(2\pi)^4 \mathcal{M}_{ab}\, \delta^{(4)}\Big(\sum_i p_i - \sum_f p_f\Big)\,,$$

$$S^\dagger S = 1 \quad \Longrightarrow \quad \mathcal{M}_{ba} - \mathcal{M}^*_{ab} = i \sum_c \mathcal{M}^*_{cb}\mathcal{M}_{ca}\ ,$$

$$\lim_{\epsilon\to 0} \{\mathcal{M}(E_h,\epsilon) - \mathcal{M}^*(E_h,\epsilon)\} = \lim_{\epsilon\to 0} \{\mathcal{M}(E_h,\epsilon) - \mathcal{M}(E_h,-\epsilon)\} \equiv \mathrm{Disc}\mathcal{M}_{ab}\ ,$$

$$\mathrm{Disc}\mathcal{M}_{ab} = -\frac{\lambda^V}{S_\#}\sum \int_{\Omega_1}\cdots\int_{\Omega_L} \prod_{i=1}^{L}\frac{i\, d^4k_i}{(2\pi)^4}\prod_{k=1}^{N}(-2\pi i)\,\delta(Q_k^2 - m^2)\sigma(Q_k^0)\prod_{j=1}^{I-N}\frac{1}{Q_j^2 - m^2 + i\epsilon}\, B(k_i,p_h)\,.$$

Anomalous Thresholds (local theory) :

$$\int_{\Omega_1}\cdots\int_{\Omega_L} \prod_{i=1}^{L}\frac{i\, d^4k_i}{(2\pi)^4}\prod_{k=1}^{N}(-2\pi i)\,\delta(Q_k^2 - m^2)\sigma(Q_k^0)\prod_{j=1}^{I-N}\frac{1}{Q_j^2 - m^2 + i\epsilon} = 0 \quad \Longleftrightarrow \quad \prod_{k=1}^{N}\delta(Q_k^2 - m^2)\sigma(Q_k^0) = 0\,.$$

Anomalous Thresholds (nonlocal theory): same of the local theory,

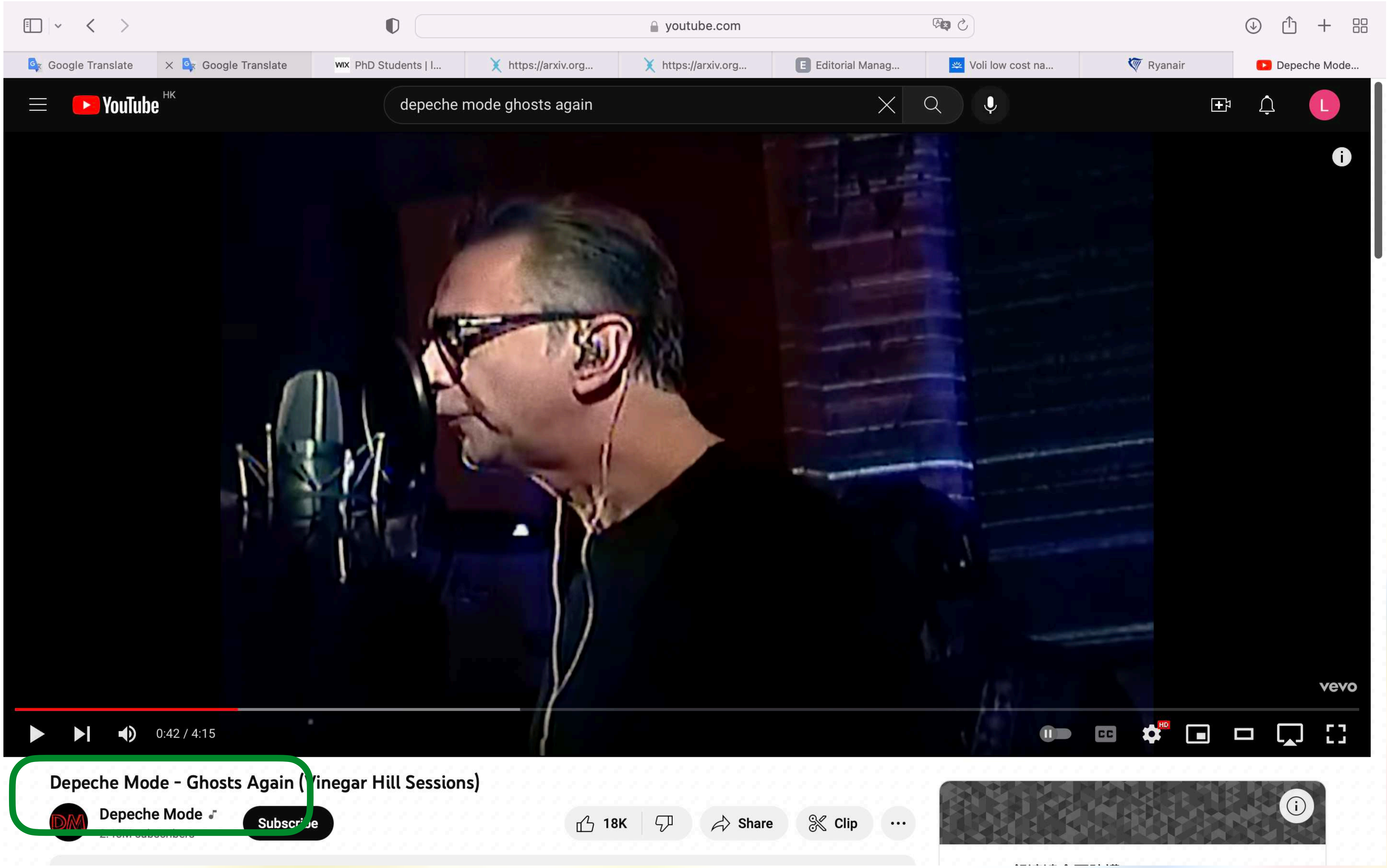
$$\int_{\Omega_1}\cdots\int_{\Omega_L} \prod_{I=1}^{L}\frac{I\, d^4k_i}{(2\pi)^4}\prod_{k=1}^{N}(-2\pi i)\,\delta(Q_k^2 - m^2)\sigma(Q_k^0)\prod_{j=1}^{I-N}\frac{1}{Q_j^2 - m^2 + i\epsilon}\, B(k_i,p_h) = 0$$

$$\Longleftrightarrow \quad \prod_{k=1}^{N}\delta(Q_k^2 - m^2)\sigma(Q_k^0) = 0\,.$$

If the diagram is cut in less then 2 parts or more then 2 parts: the contribution to the discontinuity is zero.

Back to Local Quantum Gravity

Ghosts Again



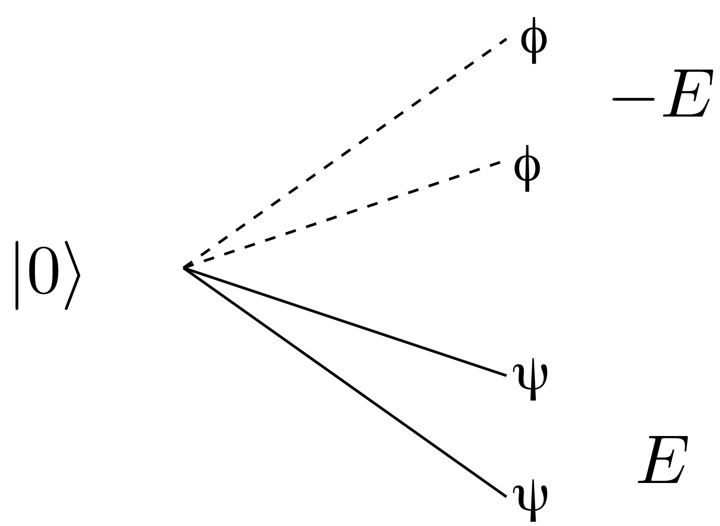
Vacuum Instability

(J. M. Cline, S. Jeon, G. D. Moore. M. Jaccard M. Maggiore, E. Mitsou. J. Garriga, A. Vilenkin.)

$$S = \int d^4x \left[\frac{1}{2}(-\partial_\mu \psi \partial^\mu \psi - m_\psi^2 \psi^2) + \frac{1}{2}(\partial_\mu \phi \partial^\mu \phi - m_\phi^2 \phi^2) + \frac{\lambda}{4}\phi^2\psi^2 \right].$$

$$i\mathcal{M}_{fi} = i\lambda(2\pi)^4 \delta^{(4)}(p_1 + p_2 + k_1 + k_2).$$

Energy conservation, non-zero interaction:



$$\Gamma(\text{dacay probability}) = +\infty \quad \Longrightarrow \quad \tau(\text{lifetime } |0\rangle) \equiv 0.$$

Regardless of the ghost's mass !!

Six-derivative Gravity

(LM, I. L. Shapiro.)

$$\mathcal{L}_{\text{super-ren}} = R - 2\Lambda + \sigma_0 G_{\mu\nu}\Box R^{\mu\nu} + \sum_i (\mathbf{Riem}^3)_i \,,$$

Tree-level Unitarity

$$\mathcal{O}^{-1}(k) = -\frac{1}{k^2 p(k)} \underbrace{\left(P^{(2)} - \frac{P^{(0)}}{D-2}\right)}_{\mathcal{P}} = -\left(\frac{1}{k^2 - \mathrm{i}\epsilon} + \frac{c}{k^2+\eta^2} + \frac{c^*}{k^2+\eta^{2*}}\right)\mathcal{P}\,,$$

$$\mathrm{i}\mathcal{O}^{-1}(k) = -\mathrm{i}\left[\frac{1}{k^2-\mathrm{i}\epsilon} + \sum_n \left(\frac{c_n}{k^2+\eta_n^2} + \frac{c_n^*}{k^2+\eta_n^{2*}}\right)\right]\mathcal{P}.$$

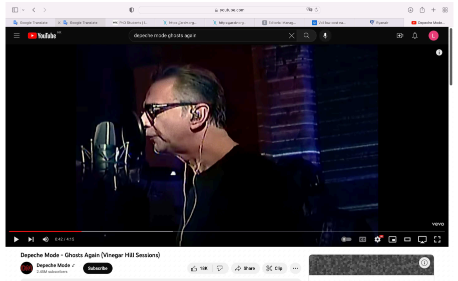
$$\lim_{\epsilon\to 0} \mathrm{Im}[\mathcal{O}^{-1}(k)] = -\lim_{\epsilon\to 0^+} \frac{\epsilon}{k^4+\epsilon^2} = -\pi\,\delta(k^2)\,.$$

$$\mathcal{M}_{if} = -\lim_{\epsilon\to 0^+} \mathrm{i}^2\, T^{\mu\nu}\left[\frac{1}{k^2-\mathrm{i}\epsilon} + \sum_n \left(\frac{c_n}{k^2+\eta_n^2} + \frac{c_n^*}{k^2+\eta_n^{2*}}\right)\right]\mathcal{P}_{\mu\nu\rho\sigma} T^{\rho\sigma}.$$

Perturbative Unitarity

Quantum field theory with ghost pairs

Jiangfan Liu,[a] Leonardo Modesto[a] and Gianluca Calcagni[b,1]



Theory

$$\mathcal{L}_\phi = -\frac{1}{2}\phi\left[\left(\frac{\Box}{M^2}\right)^2+1\right]\left(\Box+m^2\right)\phi-\lambda\sum_{n=4}^{N}\frac{c_n}{n!}\phi^n\,,\qquad N\in\mathbb{N}\,,$$

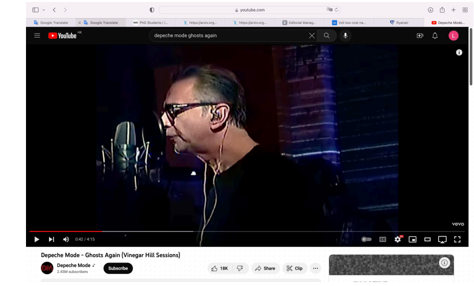
Propagator

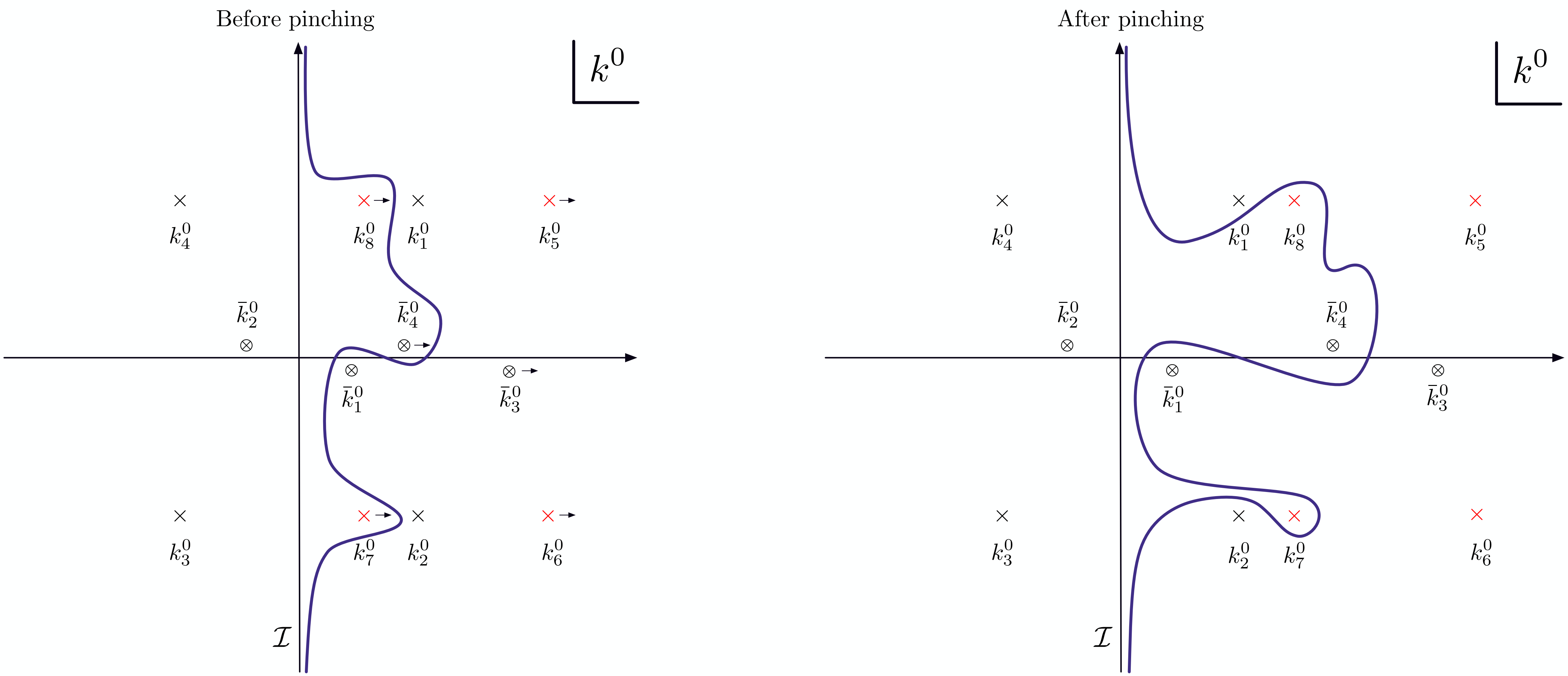
$$G(k)=i\Delta_{\rm F}(k)=\frac{iM^4}{(k^2-m^2+i\epsilon)\left[(k^2)^2+M^4\right]},$$

Perturbative Unitarity

Quantum field theory with ghost pairs

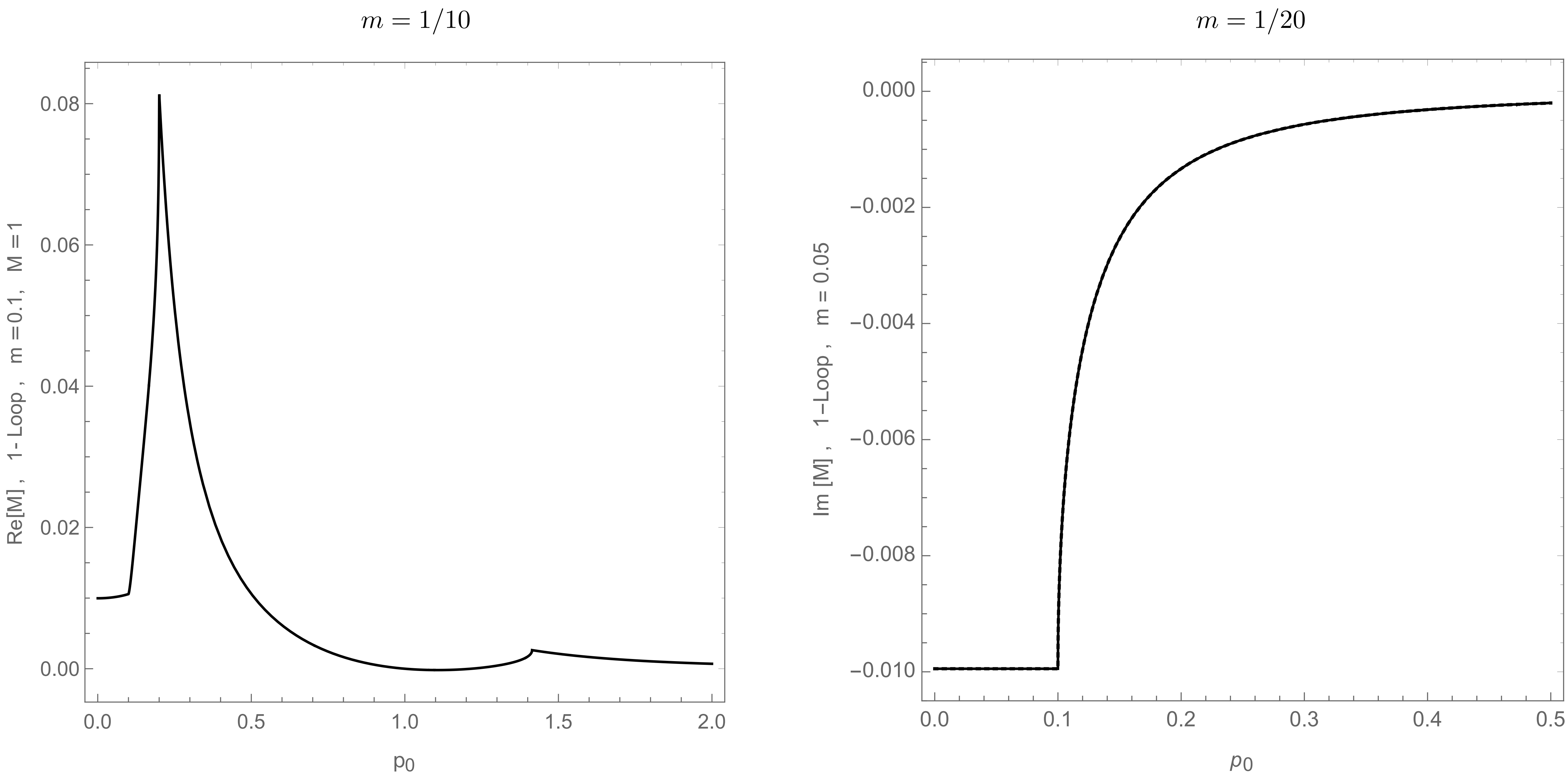
Jiangfan Liu,[a] Leonardo Modesto[a] and Gianluca Calcagni[b,1]





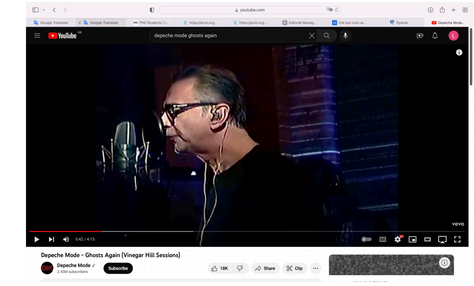
$$\mathcal{M} = -\frac{i\lambda^2}{2}\left[\frac{M^4}{M^4+m^4}\right]^2 \int \frac{d^4k}{(2\pi)^4} \frac{M^4}{(k^2-m^2+i\epsilon)\left[(k^2)^2+M^4\right]} \times \frac{M^4}{[(p-k)^2-m^2+i\epsilon]\{[(p-k)^2]^2+M^4\}},$$

$$\mathcal{M} = \mathcal{M}_{\rm I} + \mathcal{M}_{\rm ResR} + \mathcal{M}_{\rm ResC}\,. \qquad {\rm Im}\mathcal{M}_{\rm ResC} = 0\,.$$



Cusps at : $p_0 = 2m$ and $p_0 = \sqrt{2}M$.

Real Ghosts



Self-energy with fakeons.

(D. Anselmi, M. Piva).

$$\mathrm{i}\,\mathcal{M} = \frac{\lambda^2}{2}\int \frac{\mathrm{d}^D k}{(2\pi)^D}\frac{1}{-k^2-m^2+\mathrm{i}\epsilon}\,\frac{1}{-(k-p)^2-m^2+\mathrm{i}\epsilon}$$

$$= -\frac{\lambda^2}{2}\int \frac{\mathrm{d}^{D-1}\boldsymbol{k}}{(2\pi)^{D-1}}\frac{1}{2\omega_k}\frac{1}{2\omega_{k-p}}\left(\frac{\mathrm{i}}{-p^0-\omega_k-\omega_{k-p}+\mathrm{i}\epsilon}+\frac{\mathrm{i}}{p^0-\omega_k-\omega_{k-p}+\mathrm{i}\epsilon}\right)$$

$$= -\frac{\lambda^2}{2}\int \frac{\mathrm{d}^{D-1}\boldsymbol{k}}{(2\pi)^{D-1}}\frac{1}{2\omega_k}\frac{1}{2\omega_{k-p}}\left[\mathrm{PV}\frac{\mathrm{i}}{-p^0-\omega_k-\omega_{k-p}}+\mathrm{PV}\frac{\mathrm{i}}{p^0-\omega_k-\omega_k}\right.$$

$$\left.+\pi\delta(-p^0-\omega_k-\omega_{k-p})+\pi\delta(p^0-\omega_k-\omega_{k-p})\right],$$

AP prescription: drop any delta with the frequency of a fakeon particle:

$$\mathrm{i}\,\mathcal{M} = -\frac{\lambda^2}{2}\int \frac{\mathrm{d}^{D-1}\boldsymbol{k}}{(2\pi)^{D-1}}\frac{1}{2\omega_k}\frac{1}{2\omega_{k-p}}\left(\mathrm{PV}\frac{\mathrm{i}}{-p^0-\omega_k-\omega_{k-p}}+\mathrm{PV}\frac{\mathrm{i}}{p^0-\omega_k-\omega_{k-p}}\right).$$

$$\mathcal{M} = -\frac{\lambda^2}{32\pi^2}\ln\frac{-p^2}{\tilde{\Lambda}_{\mathrm{UV}}^2} = -\frac{\lambda^2}{32\pi^2}\ln\frac{|p^2|}{\tilde{\Lambda}_{\mathrm{UV}}^2}.$$

... in short:

Feynman Prescription

$$\mathcal{M}_{\text{Feynman}} \sim -\ln(p^2 - \mathrm{i}\epsilon).$$

Anselmi-Piva Prescription

$$\mathcal{M} = \frac{\mathcal{M}_{\text{Feynman}} + \mathcal{M}^*_{\text{Feynman}}}{2}$$

$$\sim -\frac{1}{2}\left[\ln(p^2 - \mathrm{i}\epsilon) + \ln(p^2 + \mathrm{i}\epsilon)\right]$$

$$= -\frac{1}{2}\ln[(p^2)^2 + \epsilon^2] \xrightarrow{\epsilon \to 0} -\ln|p^2|.$$

Any number of degenerate Phantom-Pairs

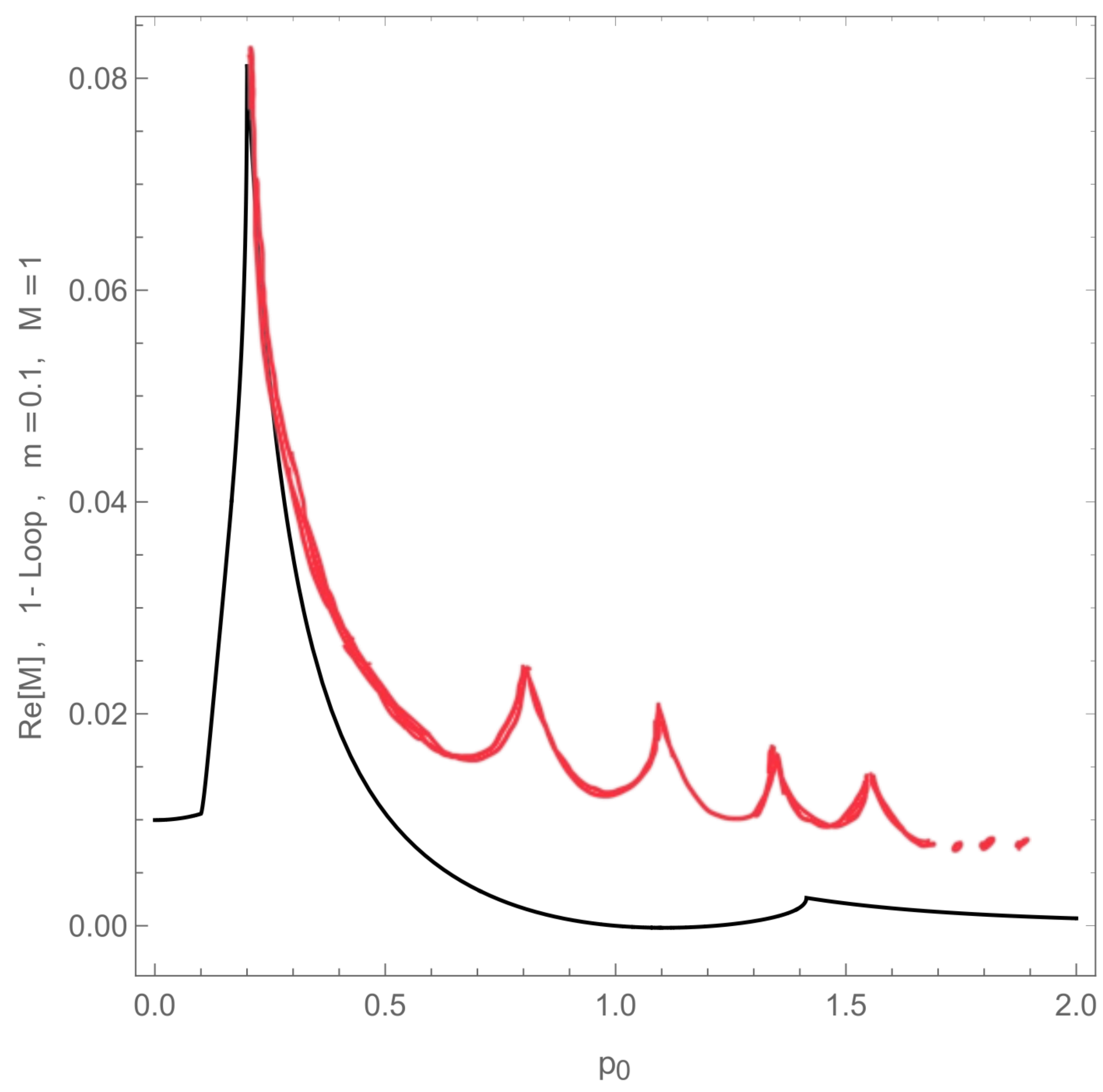
(Jiangfan Liu, L.M., Yan Huyi, F. Briscese, G. Calcagni).

$$S_{\rm cg} \equiv \int d^4x L_{\rm cg} = \int d^4x \left\{ -\frac{1}{2}\phi\left(\Box + m^2\right)\left[\prod_i^N \left(\frac{\Box^2 + M_{2,i}^4}{M_{1,i}^4}\right)^{N_i}\right]\phi + V(\phi)\right\}.$$

$$G(k) = i\Delta_{\rm F}(k) = \frac{i}{k^2 - m^2 + i\epsilon}\prod_{i=1}^N \frac{M_i^4}{(k^2)^2 + M_i^4}\,, \qquad G(k) = \frac{i}{k^2 - m^2 + i\epsilon}\left(\frac{M^4}{(k^2)^2 + M^4}\right)^N,$$

UNITARY

ghost pairs give complex conjugated contributions to any loop-perturbative order.


Re[M] , 1-Loop , m = 0.1, M = 1
0.08
0.06
0.04
0.02
0.00
0.0
0.5
1.0
1.5
2.0
p0

Super-renormalizability

The Simplest Theory

$$\mathcal{L}_4 = \lambda + \frac{2}{\kappa_4^2} R + \frac{2}{\tilde{\kappa}_4^2} G_{\mu\nu} \frac{e^{H(-\Box_\Lambda)} - 1}{\Box} R^{\mu\nu}$$

$$= \lambda + \frac{2}{\kappa_4^2} R + \frac{2}{\tilde{\kappa}_4^2} G_{\mu\nu} \left(\frac{e^{H(-\Box_\Lambda)} - e^{\gamma_E} p(-\Box_\Lambda) - 1}{\Box} + \frac{e^{\gamma_E} p(-\Box_\Lambda)}{\Box} \right) R^{\mu\nu}$$

Local HD limit

Super-renormalizability in a nutshell

Asorey, Lopez, Shapiro,M, Rachwal.

Action: $\int d^D x \sqrt{-g}\, \omega_\gamma \mathcal{R}\, \Box^{\gamma + \frac{D-4}{2}}\, \mathcal{R}\,, \quad \omega_\gamma = \frac{\tilde{\omega}_\gamma}{\Lambda^{2\gamma}}\,, \quad \gamma \in \mathbb{N}\,.$

Propagator: $G(p) \sim \frac{1}{\omega_\gamma\, p^{2\gamma+D}}\,,$ **Vertex:** $\mathcal{V}(p) \sim \omega_\gamma\, p^{2\gamma+D}\,.$

$$
\begin{aligned}
\mathcal{A}^{(L)} &= \delta^D(K) \int (d^D p)^L\, G(p)^I\, \mathcal{V}(p)^V \\
&= \delta^D(K) \int (d^D p)^L \left(\frac{1}{\omega_\gamma\, p^{2\gamma+D}}\right)^I \left(\omega_\gamma\, p^{2\gamma+D}\right)^V \\
&= \delta^D(K)\, \Lambda^{2\gamma(L-1)} \int (d^D p)^L \left(\frac{1}{p^{2\gamma+D}}\right)^I \left(p^{2\gamma+D}\right)^V \\
&= \delta^D(K)\, \Lambda^{2\gamma(L-1)} \int (d^D p)^L \left(\frac{1}{p^{2\gamma+D}}\right)^{L-1} \\
&= \delta^D(K)\, \Lambda^{2\gamma(L-1)}\, (\Lambda_{\mathrm{cut-off}})^{\omega(G)}\,, \quad \boxed{\omega(G) \equiv D - 2\gamma(L-1)}\,.
\end{aligned}
$$

For $\gamma > \frac{D}{2}$ only one loop divergences.

Counterterms

$$\delta^D(K)\,\Lambda^{2\gamma(L-1)}\ (\Lambda_{\text{cut-off}})^{D-2\gamma(L-1)}$$

$$\Downarrow$$

$$\boxed{\frac{1}{\epsilon}\int d^Dx\sqrt{-g}\,\mathcal{R}^{\frac{D}{2}}\,.}$$

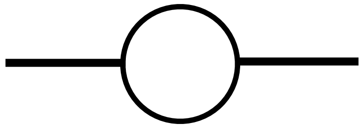
In general (in DIMREG) :

$$\mathcal{I}_{k,n} \equiv \int d^Dp\,\frac{(p^2)^k}{(p^2+C)^n} \quad\Longrightarrow\quad \frac{1}{\epsilon}\mathcal{R}^{\frac{D}{2}}\,, \quad \frac{1}{\epsilon}\mathcal{R}^{\frac{D}{2}-1}\,, \quad \ldots\,, \frac{1}{\epsilon}\mathcal{R}\,, \quad \frac{1}{\epsilon}\mathbf{1}\,,$$

$k=n$ $\quad k=n-1$ $\quad k=n-2$ $\quad k=n-D$

Counterterms in $D = 4$

$$\frac{1}{\epsilon}\sqrt{|g|}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\,, \quad \frac{1}{\epsilon}\sqrt{|g|}\Box R\,, \quad \frac{1}{\epsilon}\sqrt{|g|}R^2\,, \quad \frac{1}{\epsilon}\sqrt{|g|}R^2_{\mu\nu}\,, \quad \frac{1}{\epsilon}\sqrt{|g|}R\,, \quad \frac{1}{\epsilon}\sqrt{|g|}\,,$$

The theory in D = 4 at 

$$
\begin{aligned}
\mathcal{L}_4 &= \lambda + \frac{2}{\kappa_4^2} R + \frac{2}{\tilde{\kappa}_4^2} G_{\mu\nu} \frac{e^{H(-\Box_\Lambda)} - 1}{\Box} R^{\mu\nu} + a R_{\mu\nu} R^{\mu\nu} + b R^2 \\
&= \lambda + \frac{2}{\kappa_4^2} R + a R_{\mu\nu} R^{\mu\nu} + b R^2 + \frac{2}{\tilde{\kappa}_4^2} G_{\mu\nu} \left(\frac{e^{H(-\Box_\Lambda)} - e^{\gamma_E} p(-\Box_\Lambda) - 1}{\Box} + \frac{e^{\gamma_E} p(-\Box_\Lambda)}{\Box} \right) R^{\mu\nu} \\
&\to \lambda + \frac{2}{\kappa_4^2} R + a R_{\mu\nu} R^{\mu\nu} + b R^2 + \frac{2}{\tilde{\kappa}_4^2} G_{\mu\nu} \left(\frac{e^{-p(-\Box_\Lambda)} + \cdots - 1}{\Box} + \frac{e^{\gamma_E} p(-\Box_\Lambda)}{\Box} \right) R^{\mu\nu} \,.
\end{aligned}
$$

$$
\text{Couplings}: \quad \alpha_i(t) \in \{\kappa_4(t),\, \lambda(t),\, a(t),\, b(t)\} \,, \quad \kappa_4(0) = \tilde{\kappa}_4 \,, \quad a(0) = b(0) = 0 \,,
$$

$$
\begin{aligned}
\mathcal{L}_{4\,\mathrm{Ren}} &= \mathcal{L}_4 + \mathcal{L}_{\mathrm{ct}} \\
&= \mathcal{L}_4 + (Z_\lambda - 1)\lambda + (Z_\kappa - 1) \frac{2}{\kappa_4^2} R + (Z_a - 1) a R_{\mu\nu} R^{\mu\nu} + (Z_b - 1) b R^2 \,.
\end{aligned}
$$

$$
(Z_{\alpha_i} - 1)\alpha_i = \frac{1}{\epsilon} \beta_i \quad \Longrightarrow \quad Z_{\alpha_i} = 1 + \frac{1}{\epsilon} \beta_i \frac{1}{\alpha_i} \,.
$$

$$
\text{Couplings}: \quad \alpha_i(t) \in \{\kappa_4(t),\, \lambda(t),\, a(t),\, b(t)\} \,, \quad \kappa_4(0) = \tilde{\kappa}_4 \,, \quad a(0) = b(0) = 0 \,.
$$

$$
\boxed{\alpha_i(t) = \alpha(0) + \beta_i t \,, \quad \beta_i = \text{const.}}
$$

For example: $\kappa_4^2(t) = \dfrac{\kappa_4^2(0)}{1 + \kappa_4^2(0)\, \beta_\kappa\, t} \,.$

Yang-Mills & Gravity

Asymptotic Freedom

$$\mathcal{L}_{\rm YM} = \frac{1}{2g^2(t)} {\rm Tr} F^2 \sim dA\, dA + g(t) A^2 dA + g^2(t) A^4 \,,$$

$$g(t)^{-2} = g_o^{-2} + \beta_g t \,,$$

$$\mathcal{L}_{\rm G} = \frac{2}{\kappa_D^2(t)} \left(R + G_{\mu\nu} \frac{e^{H(-\Box_\Lambda)} - 1}{\Box} R^{\mu\nu} \right) \sim \partial h \, e^{H(-\Box_\Lambda)} \, \partial h + \kappa_D \, h \partial h \, \partial h + O(\kappa_D^2) \,,$$

$$\kappa_D^2(t) = \kappa_{Do}^{-2} + \beta_{\kappa_D} \, t \quad \Longrightarrow \quad \kappa_D^2(t) = \frac{\kappa_{Do}^2}{1 + \kappa_{Do}^2 \, \beta_{\kappa_D} \, t} \,.$$

All the beta-functions do not depend on the gauge fixing condition.

I. Shapiro.

Asymptotic Freedom solves the Cosmological Trans-Planckian Problem

(F. Briscese, LM).

Beyond Renormalizability

$$\int < \infty$$

$$\beta_{\alpha_i} \equiv 0$$

Could we make quantum gravity finite?

$$\beta_{\alpha_i} \equiv 0$$

- Multidimensional Gravity .
- Terminating Curvature Potential $\mathcal{V} \sim O(R^3)$.

i. Multidimensional Gravity

Finite Quantum Gravity in Odd dimension

$$\beta_i = 0$$

Prototype 1-loop integral

$$\mathcal{I}_{k,n} = \int d^D p \frac{(p^2)^k}{(p^2 + C)^n} = i \frac{C^{\frac{D}{2}-(n-k)}}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma(n-k-D/2)\Gamma(k+D/2)}{\Gamma(D/2)\Gamma(n)},$$

$$\mathcal{I}_{\rm null} = \int d^D p \frac{1}{p^{2N}} \equiv 0 \;\; \text{for} \;\; N < D/2\,.$$

For $\quad k, n \in \mathbb{N} \quad$ and $\quad D$ odd : $\quad \Gamma\left(n - k - \frac{D}{2}\right) < \infty.$

$$\mathcal{L}_{\rm odd} = \frac{2}{\kappa_{\rm odd}^2}\left(R + G_{\mu\nu}\frac{e^{H(\Box)} - 1}{\Box} R^{\mu\nu}\right)\,.$$

$ii.$ Theory in $D = 4$

$V(R) \sim O(R^3)$

L.M. , Leslaw Rachwal

$$\mathcal{L}_{\rm g} = 2\kappa_D^{-2}\sqrt{-g}\left(R + G_{\mu\nu}\,\frac{e^{H(-\Box_\Lambda)}-1}{\Box}R^{\mu\nu} + s_1 R^2\,\Box^{\gamma-2}R^2 + s_2 R_{\mu\nu}R^{\mu\nu}\,\Box^{\gamma-2}R_{\rho\sigma}R^{\rho\sigma}\right).$$

$$\Longrightarrow \quad 2\kappa_4^{-2}\sqrt{|g|}\left[\omega_1\,R\,\Box^\gamma\,R + \omega_2\,R_{\mu\nu}\,\Box^\gamma\,R^{\mu\nu} + s_1 R^2\,\Box^{\gamma-2}\,R^2 + s_2 R_{\mu\nu}R^{\mu\nu}\,\Box^{\gamma-2}\,R_{\rho\sigma}R^{\rho\sigma}\right],$$

$$\omega_2 = -2\,\omega_1 = e^{\gamma_E/2}/\Lambda^{2\gamma+2}.$$

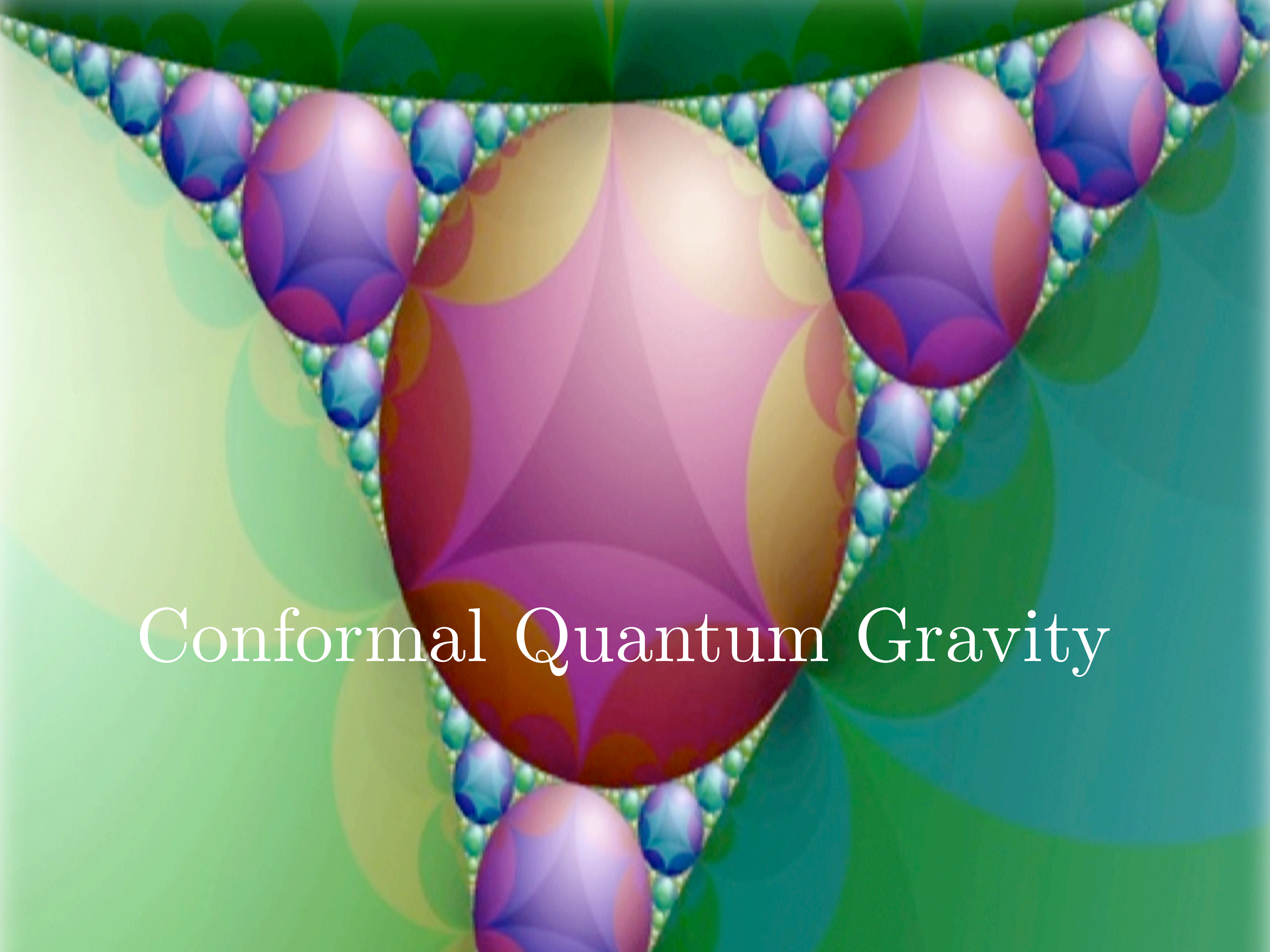
$$\Gamma^{(1)}_{\rm div} = \frac{1}{\epsilon}\left(\beta_{R^2_{\mu\nu}}\,R_{\mu\nu}R^{\mu\nu} + \beta_{R^2}\,R^2\right),$$

$$\beta_{R^2} := a_1 s_1 + a_2 s_2 + c_1 = 0$$
$$\beta_{R^2_{\mu\nu}} := b_2 s_2 + c_2 = 0$$

$\Rightarrow$

$$s_1 = \frac{-(3\omega_1+\omega_2)(40c_1\omega_1 + 10c_2\omega_1 + 14c_1\omega_2 - c_2\omega_2)}{48(20\omega_1+7\omega_2)},$$
$$s_2 = \frac{-3c_2\omega_2(3\omega_1+\omega_2)}{2(20\omega_1+7\omega_2)}.$$

Conformal Quantum Gravity



Nonlocal Conformal Gravity

L.M., L. Rachwal

$$\mathcal{L}_g = 2\kappa_D^{-2}\sqrt{-g}\Big(R + R\gamma_0(\Box)R + R_{\mu\nu}\gamma_2(\Box)R^{\mu\nu} + R_{\mu\nu\rho\sigma}\gamma_4(\Box)R^{\mu\nu\rho\sigma} + \mathcal{V}(\mathcal{R})\Big),$$

$$g_{\mu\nu} = (\phi^2\,\kappa_D^2)^{\frac{2}{D-2}}\,\hat{g}_{\mu\nu}\,,$$

$$\hat{g}_{\mu\nu} \to \Omega^2(x)\,\hat{g}_{\mu\nu}\,, \quad \phi \to \Omega^{\frac{2-D}{2}}(x)\,\phi\,.$$

$$\mathcal{L}_{\rm g} = -2\sqrt{\hat{g}}\left[\phi^2 R(\hat{g}) + \frac{4(D-1)}{D-2}\hat{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right] - \frac{2}{\kappa_D^2}\sqrt{g}\,[\mathbf{R}(g)\,\gamma_0(\Box)\mathbf{R}(g) + \mathbf{Ric}(g)\,\gamma_2(\Box)\mathbf{Ric}(g) + \mathbf{V}(g)\,]\Big|_{\phi\hat{g}}\,.$$

Finiteness $\Longrightarrow$ Quantum Conformal Invariance

No Weyl Conformal Anomaly

Counterterms In DIMREG :

$$\Gamma^{(1)} = S_{\rm cl} + \frac{1}{\varepsilon}\sum_i \beta_i \int \mathrm{d}^{4-2\varepsilon}x\, \mathcal{O}_i(g) + \Gamma_{\rm finite}\,, \qquad \frac{1}{\varepsilon} = \frac{2}{4-D} \equiv \ln\left(\frac{\Lambda_{\rm UV}}{\mu}\right)^2,$$

$$\mathcal{O}_i(g) \in \left\{\sqrt{|g|}\Box R\,,\ \sqrt{|g|}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma},\ \sqrt{|g|}R^2\,,\ \sqrt{|g|}R_{\mu\nu}R^{\mu\nu},\ \sqrt{|g|}R\,,\ \sqrt{|g|}\right\}\Big|_{g=g(\phi,\hat{g}),\, D=4-2\varepsilon},$$

$$g_{\mu\nu} = g_{\mu\nu}(\phi, \hat{g}_{\mu\nu}) = \phi^{\frac{4}{D-2}}\hat{g}_{\mu\nu}\,,$$

$$\left(\sqrt{|g|}\right)_{D=4-2\varepsilon} \simeq \phi^{4+2\varepsilon}\sqrt{|\hat{g}|} = \phi^{2\varepsilon}\underbrace{\phi^4\sqrt{|\hat{g}|}}_{\text{conf. inv.}} = \phi^{2\varepsilon}\sqrt{|g|}\,.$$

$$\frac{1}{\varepsilon}\beta_\Lambda\phi^{2\varepsilon}\sqrt{|g|} = \frac{1}{\varepsilon}\beta_\Lambda[1+2\varepsilon\ln\phi + O(\varepsilon^2)]\sqrt{|g|} = \frac{1}{\varepsilon}\beta_\Lambda\sqrt{|g|} + 2\beta_\Lambda\ln\phi\,\sqrt{|g|} + O(\varepsilon)\,.$$

$$\begin{aligned}\lim_{\varepsilon\to 0}\frac{\beta_i}{\varepsilon}\phi^{2\varepsilon}\mathcal{O}_i(\phi^2\hat{g};\varepsilon) &= \lim_{\varepsilon\to 0}\frac{\beta_i}{\varepsilon}(1+2\varepsilon\ln\phi)\,\mathcal{O}_i(\phi^2\hat{g};\varepsilon)\\ &= \lim_{\varepsilon\to 0}\frac{\beta_i}{\varepsilon}\,\mathcal{O}_i(\phi^2\hat{g}) + \beta_i\tilde{\mathcal{O}}_i(\phi^2\hat{g}) + 2\beta_i\ln\phi\,\mathcal{O}_i(\phi^2\hat{g}),\end{aligned}$$

where $\tilde{\mathcal{O}}_i(\phi^2\hat{g})$ is the finite contribution to $\lim_{\varepsilon\to 0}\mathcal{O}_i(\phi^2\hat{g};\varepsilon)/\varepsilon$.

Spontaneous symmetry breaking: vacuum exact solution + gauge fixing .

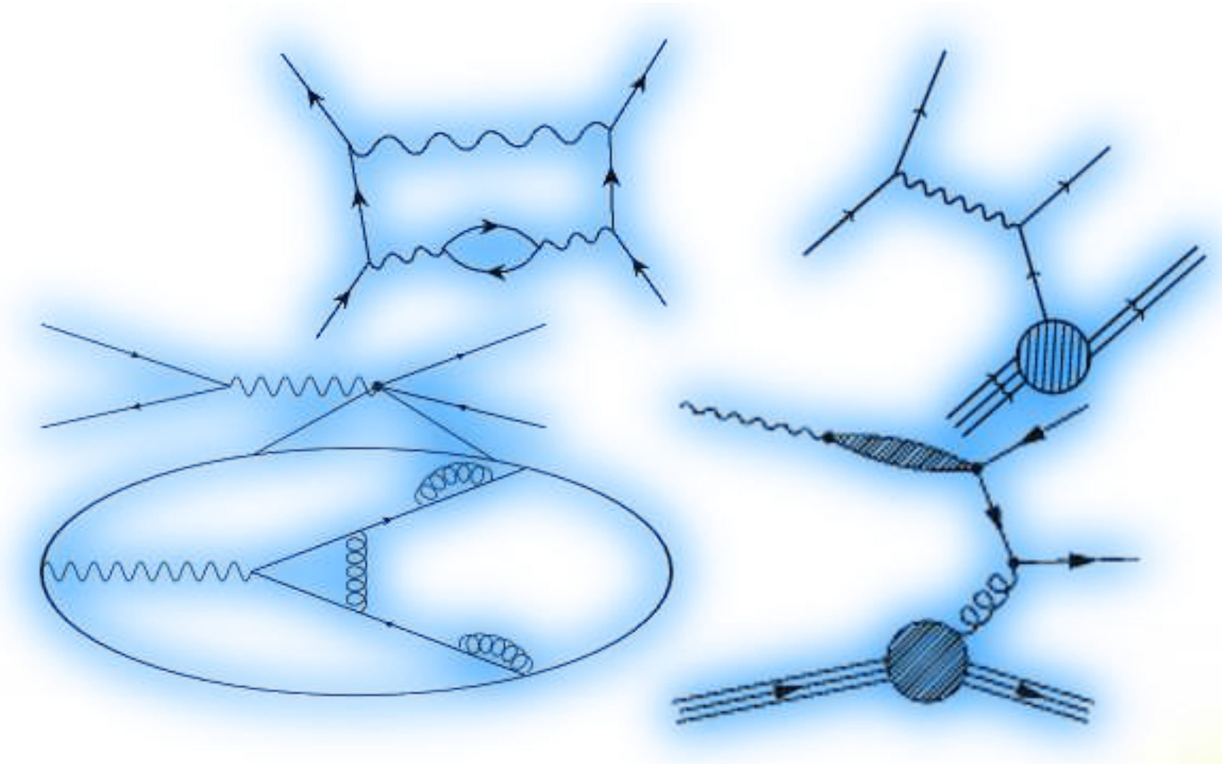
$$\phi = v + \varphi \,, \quad v = \kappa_D^{-1} \,,$$

$$\text{unitary gauge}: \quad \varphi = 0 \,,$$

$$\mathcal{L}_{\rm g} = 2\sqrt{\hat{g}}\left[\phi^2 R(\hat{g}) + \frac{4(D-1)}{D-2}\hat{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right]\Bigg|_{\phi=\kappa_D^{-1}} = \frac{2}{\kappa_D^2}\sqrt{\hat{g}}R(\hat{g}) \,.$$

Conformal invariance spontaneously broken $\Longrightarrow$ Observables are only Diff. Invariant.

Scattering Amplitudes



Scattering Amplitudes

P. Donà, S. Giaccary, L.M., L. Rachwal, Yiwei-Zhu.

$$\mathcal{L}_{\rm g} = -2\kappa_D^{-2}\sqrt{g}\left[R + R\,\gamma_0(\Box)R + R_{\mu\nu}\gamma_2(\Box)R^{\mu\nu} + R_{\mu\nu\rho\sigma}\,\gamma_4(\Box)R^{\mu\nu\rho\sigma}\right].$$

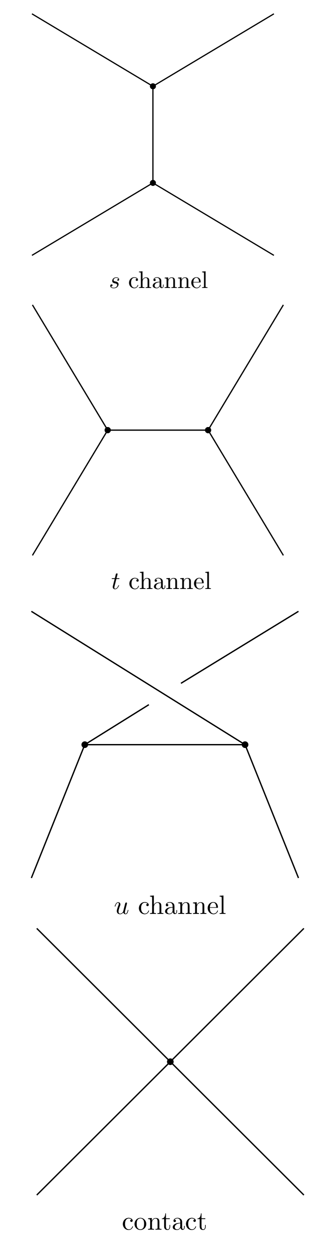
$$\gamma_4(\Box) = 0,$$

$$A_s(++,++) = -\frac{9}{8}\frac{t(s+t)}{s} + \frac{9}{32}\gamma_2(s)\left(s^2 + (s+2t)^2\right) + \frac{9}{8}s^2\gamma_0(s)\,,$$

$$\mathcal{A}_t(++,++) = -\frac{1}{8}\frac{\left(s^3 - 5s^2t - st^2 + t^3\right)(s+t)^2}{s^3t} + \frac{1}{16}\gamma_2(t)\frac{\left(2s^4 - 10s^3t - s^2t^2 + 4st^3 + t^4\right)(s+t)^2}{s^4} + \frac{1}{8}\gamma_0(t)\frac{t^2(s+t)^4}{s^4}\,,$$

$$\mathcal{A}_u(++,++) = -\frac{1}{8}\frac{\left(s^3 - 5s^2u - su^2 + u^3\right)(s+u)^2}{s^3u} + \frac{1}{16}\gamma_2(u)\frac{\left(2s^4 - 10s^3u - s^2u^2 + 4su^3 + u^4\right)(s+u)^2}{s^4} + \frac{1}{8}\gamma_0(u)\frac{u^2(s+u)^4}{s^4}\,,$$

$$\mathcal{A}_{\rm contact}(++,++) = -\frac{1}{4}\frac{s^4 + s^3t - 2st^3 - t^4}{s^3} + -\frac{9}{32}\gamma_2(s)\left(s^2 + (s+2t)^2\right) - \frac{9}{8}s^2\gamma_0(s) - \frac{1}{16}\gamma_2(t)\frac{\left(2s^4 - 10s^3t - s^2t^2 + 4st^3 + t^4\right)(s+t)^2}{s^4} - \frac{1}{8}\gamma_0(t)\frac{t^2(s+t)^4}{s^4} - \frac{1}{16}\gamma_2(u)\frac{\left(2s^4 - 10s^3u - s^2u^2 + 4su^3 + u^4\right)(s+u)^2}{s^4} - \frac{1}{8}\gamma_0(u)\frac{u^2(s+u)^4}{s^4}\,.$$



$$\mathcal{A}(++,++) = \mathcal{A}_s(++,++) + \mathcal{A}_t(++,++) + \mathcal{A}_u(++,++) + \mathcal{A}_{\rm contact}(++,++) = \mathcal{A}(++,++)_{\rm EH}$$

Stelle, Weyl gravity, etc.

Scattering Amplitudes in Higher-derivative Gravities

Stelle Quadratic Gravity: $\boxed{\kappa_4^{-2} R + \alpha R^2 + \beta R_{\mu\nu}^2} \quad \rightarrow \quad \text{same of } \boxed{\kappa_4^{-2} R}$ (ext. gravitons) .

Weyl Gravity $\boxed{\alpha C_{\mu\nu\rho\sigma}^2} \quad \rightarrow \quad \text{same of } \boxed{0}$ (ext. gravitons) .

Nonlocal Gravity: $\boxed{R + R\gamma_0(\Box)R + R_{\mu\nu}\gamma_0(\Box)R^{\mu\nu}} \quad \rightarrow \quad \text{same of } \boxed{\kappa_4^{-2} R}$.

Anselmi's Field Redefinition Theorem

Theorem 9.1. *Consider two generic weakly nonlocal action functionals* $S'[g]$ *and* $S[g']$*, respectively defined in terms of the metric fields* g *and* g'*, such that*

$$S'[g] = S[g] + E_i(g)\, F_{ij}(g)\, E_j(g)\,, \tag{9.59}$$

where $E_i = \delta S/\delta g_i$ *is the equation of motion of the theory with action* $S[g]$ *and* F_{ij} *can contain derivative operators acting on the left and on the right. Let* Δ_{ij} *be a (possibly nonlocal) symmetric operator acting linearly on the equation of motion* E_j*, with indices* i *and* j *in the field space. Then, there exists a field redefinition*

$$g'_i = g_i + \Delta_{ij} E_j\,, \qquad \Delta_{ij} = \Delta_{ji}\,, \tag{9.60}$$

such that, perturbatively in F *and to all orders in powers of* F*, we have the equivalence*

$$S'[g] = S[g']\,. \tag{9.61}$$

From Perturbative Solutions to Scattering Amplitudes

(Galcagni, LM)

$$\mathcal{A}_n^{i_1\dots i_n}(p_1,\dots,p_n) = i \lim_{p_n^2\to -m_n^2} p_n^2\, \frac{\delta^{n-1}\tilde{\Phi}^{(n-2)}_{i_n}(-p_n)}{\delta\tilde{\Phi}^{(0)}_{i_1}(p_1)\cdots\delta\tilde{\Phi}^{(0)}_{i_{n-1}}(p_{n-1})}\,.$$

Tree-Level Scattering Amplitudes

$$\mathcal{M}_{\rm tree} \propto s\,.$$

Quantum Scattering Amplitudes

(work in progress)

Softness : $\mathcal{M}_Q \propto s\, e^{-e^{s^n}}\,.$

Causality

Tree-level causality

Same amplitudes of Einstein's theory,

↓

same tree-level causality properties of Einstein's theory.

(see Carone, and Gristein, O'Connel, Wise.)

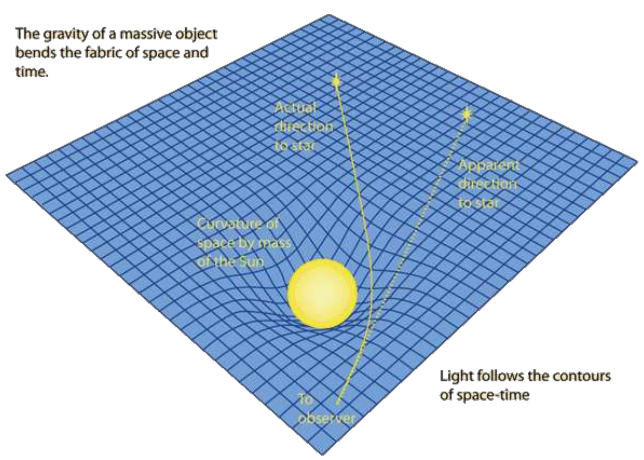
Shapiro's time delay

Camanho, Edelstein, Maldacena, Zhiboedov

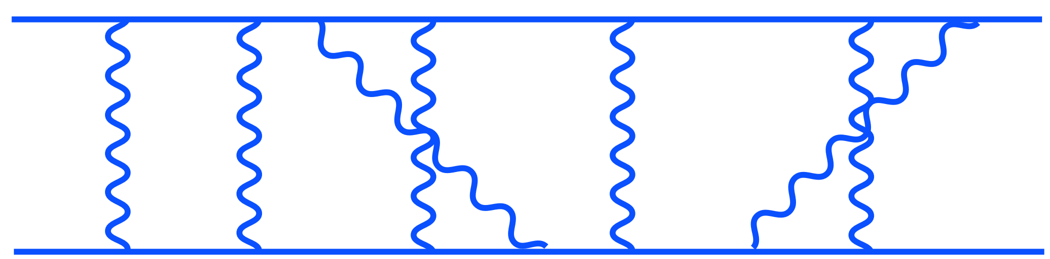
Scattering amplitude in the EIKONAL limit:

sum of diagrams in the Regge limit: large s , $t \ll s$,

$$\ell_{\rm P} \ll b \ll \ell_\Lambda \,.$$



Eikonal Approximation



$$iA_{\rm eik} = 2s \int d^{D-2}\vec{b}\, e^{-i\vec{q}\cdot\vec{b}} \left[e^{i\delta(b,s)} - 1 \right] ,$$

$$\delta(b,s) = \frac{1}{2s} \int \frac{d^{D-2}\vec{q}}{(2\pi)^{D-2}} e^{i\vec{q}\cdot\vec{b}} A_{\rm tree}(s, -\vec{q}^{\,2}) \,,$$

$$\Delta t = 2\partial_E \delta(E,b) \,.$$

Causality Violation in Gauss-Bonnet Gravity

$$\mathcal{L}_{\mathrm{EH-GB}} = \frac{2}{\kappa_D^2}\left[R + \lambda_{\mathrm{GB}}(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2)\right].$$

$$A_t = A_{t\mathrm{EH}} + A_{t\mathrm{GB}} \approx -\frac{8\pi G s^2}{t}(\epsilon_1\cdot\epsilon_3)(\epsilon_2\cdot\epsilon_4) + \frac{4\kappa_D^2\lambda_{\mathrm{GB}}s^2}{t}\left(k_2^\mu k_4^\nu \epsilon^\rho_{2\nu}\epsilon_{4\rho\mu}\epsilon_1\cdot\epsilon_3 + k_1^\mu k_3^\nu \epsilon^\rho_{1\nu}\epsilon_{3\rho\mu}\epsilon_2\cdot\epsilon_4\right),$$

$$\delta_{\mathrm{GB}}(b,s) = -4\lambda_{\mathrm{GB}}\frac{\Gamma\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}}\frac{Gs}{b^{D-2}}\left[2(\epsilon_1\cdot\epsilon_1)(\epsilon_2\cdot\epsilon_2) - (D-2)(\epsilon_2\cdot\epsilon_2)(n\cdot\epsilon_1)^2 - (D-2)(\epsilon_1\cdot\epsilon_1)(n\cdot\epsilon_2)^2\right],$$

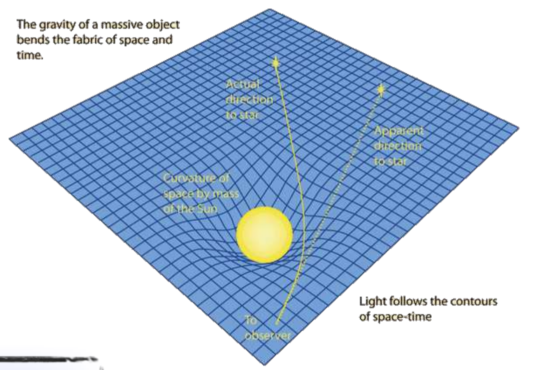
$$\vec{n} = \frac{\vec{b}}{b},$$

$$\Delta t_{\mathrm{g-GB}} = \frac{\Gamma\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}}\frac{16EG}{b^{D-4}}(\epsilon_1\cdot\epsilon_1)(\epsilon_2\cdot\epsilon_2)\left[1 + \frac{4\lambda_{\mathrm{GB}}(D-2)(D-4)}{b^2}\left(\frac{(n\cdot\epsilon_1)^2}{\epsilon_1\cdot\epsilon_1} + \frac{(n\cdot\epsilon_2)^2}{\epsilon_2\cdot\epsilon_2} - \frac{2}{D-2}\right)\right].$$

$\Delta t_{\mathrm{g-GB}} < 0$ for $b^2 < \lambda_{\mathrm{GB}}$.

Causality in Nonlocal Gravity

S. Giaccari, L.M.



$$A_{t\,\mathrm{NG}} = A_{t\,\mathrm{EH}} = -\frac{8\pi G s^2}{t}\,.$$

$$\delta_{\mathrm{g}}(b,s) = \frac{\Gamma\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}}\frac{Gs}{b^{D-4}}\,(\epsilon_1\cdot\epsilon_3)(\epsilon_2\cdot\epsilon_4)\,,$$

$$\Delta t_{\mathrm{g}} = \frac{\Gamma\left(\frac{D-4}{2}\right)}{\pi^{\frac{D-4}{2}}}\frac{16EG}{b^{D-4}}\,(\epsilon_1\cdot\epsilon_3)(\epsilon_2\cdot\epsilon_4)\,.$$

No time advanced $\Longrightarrow$ No causality violation .

Nonlocal Gravity-Matter Theory

&/or

Finite Standard Model of Particle Physics

$$S[\Phi_i] = \int \mathrm{d}^D x \sqrt{-g}\,(\mathcal{L}_\ell + E_i\, F_{ij}\, E_j)\ ,$$

$$S_\ell = \int \mathrm{d}^D x \sqrt{-g}\,\mathcal{L}_\ell\,, \quad \mathcal{L}_\ell = \frac{2}{\kappa^2} R + \mathcal{L}_m\ ,$$

$$E_i(x) = \frac{\delta S_\ell}{\delta \Phi_i(x)}\ ,$$

$$\Delta_{ki}(y,x) \equiv \frac{\delta E_i(x)}{\delta \Phi_k(y)} = \frac{\delta^2 S_\ell}{\delta \Phi_k(y) \delta \Phi_i(x)} = \Delta_{ki}(x)\, \frac{\delta^D(x-y)}{\sqrt{-g(y)}}\ ,$$

$$2\Delta_{ik} F(\Delta)_{kj} \equiv \left(e^{H(\Delta_\Lambda)} - 1\right)_{ij}\ .$$

EoM: $$\mathcal{E}_k = E_k + 2\left(\frac{\delta E_i}{\delta \Phi_k}\right) F_{ij} E_j + O(E^2) = 0\,,$$

$$\mathcal{E}_k = E_k + 2\Delta_{ki} F_{ij} E_j + O(E^2) = 0\,,$$

$$\mathcal{E}_k = \left(e^{H(\Delta_\Lambda)}\right)_{kj} E_j + O(E^2) = 0\,,$$

or :

$$\tilde{\mathcal{E}}_i \equiv E_i + \left(e^{-H(\Delta_\Lambda)}\right)_{ik} \left[O(E^2)\right]_k = 0\,.$$

Linear and Nonlinear stability

Perturbative expansion:

$$\Phi_i = \sum_{n=0}^{\infty} \epsilon^n \Phi_i^{(n)} \,,$$

$$E_k(\Phi_i) = \sum_{n=0}^{\infty} \epsilon^n E_k^{(n)} \,, \qquad \mathcal{E}_k(\Phi_i) = \sum_{n=0}^{\infty} \epsilon^n \mathcal{E}_k^{(n)} \,.$$

Background solution:

$$E_k^{(0)}(\Phi_i^{(0)}) = 0 \,,$$

$$\mathcal{E}_k^{(n)}(\Phi_i^{(n)}) = 0 \quad \Longrightarrow \quad E_k^{(n)}(\Phi_i^{(n)}) = 0 \;\; \text{for} \;\; n > 0 \,.$$

$$\mathcal{E} = \mathbf{e}^{\mathbf{H}(\mathbf{\Delta_\Lambda})}\,\mathbf{E} + O(\mathbf{E^2}) = 0\,.$$

We expand around a metric consistent with $\mathbf{E}^{(0)} = 0$

At the zero order in ϵ, i.e. ϵ^0: $\quad \mathbf{e}^{\mathbf{H}^{(0)}(\mathbf{\Delta_\Lambda})}\,\mathbf{E}^{(0)} + O(\mathbf{E}^{(0)\mathbf{2}}) = 0\,,$

which is satisfied because by hypothesis $\mathbf{E}^{(0)} = 0$.

At the first order ϵ^1 : $\quad \mathbf{e}^{\mathbf{H}^{(1)}(\mathbf{\Delta_\Lambda})}\,\mathbf{E}^{(0)} + \mathbf{e}^{\mathbf{H}^{(0)}(\mathbf{\Delta_\Lambda})}\,\mathbf{E}^{(1)} + O(\mathbf{E}^{(0)}\mathbf{E}^{(1)}) = 0$

$$\Longrightarrow \quad \mathbf{E}^{(1)} = 0\,.$$

where we used $\mathbf{E}^{(0)} = 0$.

At the second order ϵ^2 :

$$\mathbf{e}^{\mathbf{H}^{(2)}(\mathbf{\Delta_\Lambda})}\,\mathbf{E}^{(0)} + \mathbf{e}^{\mathbf{H}^{(1)}(\mathbf{\Delta_\Lambda})}\,\mathbf{E}^{(1)} + \mathbf{e}^{\mathbf{H}^{(0)}(\mathbf{\Delta_\Lambda})}\,\mathbf{E}^{(2)} + O(\mathbf{E}^{(1)}\mathbf{E}^{(1)}) + O(\mathbf{E}^{(2)}\mathbf{E}^{(0)}) = 0$$

$$\Longrightarrow \quad \mathbf{E}^{(2)} = 0\,,$$

where we used $\mathbf{E}^{(0)} = 0$ and $\mathbf{E}^{(1)} = 0$.

Finally, at the order ϵ^n,

$$\mathbf{e}^{\mathbf{H}^{(\mathbf{n})}(\mathbf{\Delta_\Lambda})}\,\mathbf{E}^{(0)} + \mathbf{e}^{\mathbf{H}^{(\mathbf{n-1})}(\mathbf{\Delta_\Lambda})}\,\mathbf{E}^{(1)} + \mathbf{e}^{\mathbf{H}^{(\mathbf{n-2})}(\mathbf{\Delta_\Lambda})}\,\mathbf{E}^{(2)} + \cdots + \mathbf{e}^{\mathbf{H}^{(0)}(\mathbf{\Delta_\Lambda})}\,\mathbf{E}^{(\mathbf{n})} + \\ + O(\mathbf{E}^{(\mathbf{n})}\mathbf{E}^{(0)}) + O(\mathbf{E}^{(\mathbf{n-1})}\mathbf{E}^{(1)}) + \cdots + O(\mathbf{E}^{(1)}\mathbf{E}^{(\mathbf{n-1})}) + O(\mathbf{E}^{(0)}\mathbf{E}^{(\mathbf{n})}) = 0$$

$$\Longrightarrow \quad \mathbf{E}^{(\mathbf{n})} = 0\,,$$

where we used : $\mathbf{E}^{(0)} = 0,\ \mathbf{E}^{(1)} = 0, \ldots,\ \mathbf{E}^{(\mathbf{n-1})} = 0\,.$

Therefore, $\boxed{\mathcal{E}^{(\mathbf{n})} = 0 \quad \Longrightarrow \quad \mathbf{E}^{(\mathbf{n})} = 0}\,.$

The Theory

$$S[\Phi_i] = \int d^D x \sqrt{-g}\,(\mathcal{L}_\ell + E_i\, F_{ij}\, E_j)\ ,$$

$$S_\ell = \int d^D x \sqrt{-g}\,\mathcal{L}_\ell\,, \quad \mathcal{L}_\ell = \frac{2}{\kappa^2} R + \mathcal{L}_m\,,$$

$$E_i(x) = \frac{\delta S_\ell}{\delta \Phi_i(x)}\ ,$$

$$\Delta_{ki}(y,x) \equiv \frac{\delta E_i(x)}{\delta \Phi_k(y)} = \frac{\delta^2 S_\ell}{\delta \Phi_k(y) \delta \Phi_i(x)} = \Delta_{ki}(x)\, \frac{\delta^D(x-y)}{\sqrt{-g(y)}}\ ,$$

$$2\Delta_{ik} F(\Delta)_{kj} \equiv \left(e^{H(\Delta_\Lambda)} - 1\right)_{ij}\ .$$

EoM : $e^{H(\Delta_\Lambda)} E_i + O(E^2) = 0\,.$

Properties:

(i) same solutions of Einstein's gravity,

(ii) same tree-level scattering amplitudes of Einstein's theory,

(iii) same stability of Einstein's theory,

(iv) super-renormalizable or finite and unitary at quantum level,

(iv) causality.

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Renormalizability of nonlocal quantum gravity coupled to matter

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Statement 1. *In order to have only a finite number of superficially divergent diagrams, in all operators having the higher number of derivatives (the same number as in the kinetic operator), any matter field must carry at least one derivative.*

Statement 2 (**too strong**). *In order to have only a finite number of superficially divergent diagrams, all operators must contain a finite number of matter fields. In particular, the potential for the scalar field has to be polynomial, it cannot be an analytic nonpolynomial function (like the Starobinsky potential) because it will produce an infinite number of counter-terms.*

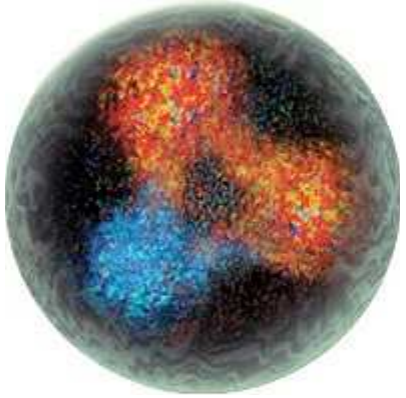
Statement 3. *Once (i) the requirement* **1** *is secured, (ii) the number of derivatives in the vertices is less than in the propagator, and (iii) the theory has divergences only at one loop, then the number of divergent one-loop diagrams is finite, regardless of the type of potential.*

Mimetic killers

$$p(\hat{\Delta}_{\Lambda_*}) = \hat{\Delta}\left[\tilde{a}_{n+1}\hat{\Delta}^n + \tilde{a}_n\hat{\Delta}^{n-1} + \cdots + \tilde{a}_1 + \left(\sum_r c_r\mathcal{O}_r\right)\Box^{n-2}\right], \qquad n \geqslant 2$$

Implications and Applications

Fundamental Confinement



Action:

$$S_{\rm HOP} = \int d^4x \left[-\frac{1}{2}\phi\,(\Box + m^2)\left(\frac{\Box^2 + \mathcal{E}^4}{M^4}\right)^N \phi + V(\phi) \right], \quad \mathcal{E} \to 0\,.$$

EoM:

$$(\Box + m^2)\left(\frac{\Box^2}{M^4}\right)^N \phi = J\,.$$

Potential: $V(r) \propto r^{4N-3}$.

Ghost-Pairs in Nonlocal Theories

Action:

$$S_{\rm NL} = \int d^4x \left\{ -\frac{1}{2}\phi\,(\Box + m^2) e^{-H\left[\left(\frac{M^4}{(k^2)^2+\mathcal{E}^4}\right)^N\right]}\phi + V(\phi) \right\}, \quad \mathcal{E} \to 0,$$

$$e^{H(z)} \equiv \sum_{r=0}^{+\infty} c_r z^r\,, \quad \text{where} \quad z = \frac{M_1^4}{k^4+\mathcal{E}^4}.$$

$$H(z) = \gamma_E + \Gamma(0, p(z)) + \log p(z)\,,$$

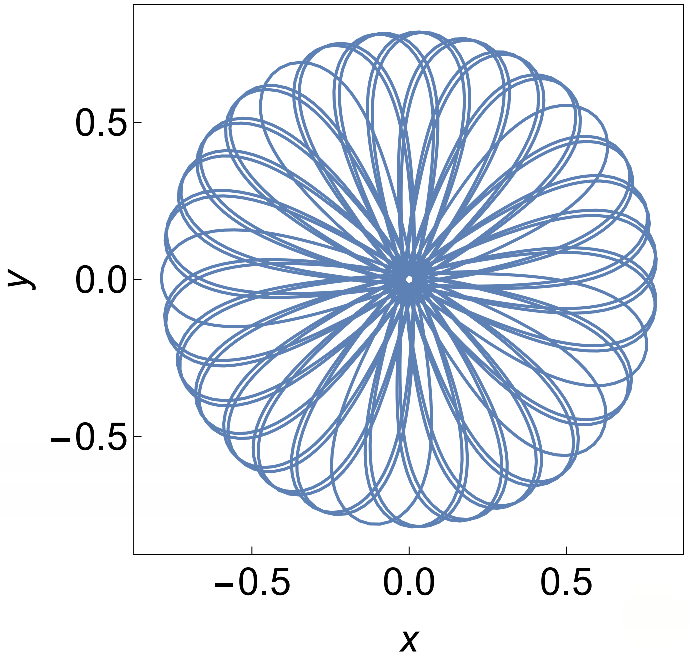
$$\lim_{z\to 0\ (k^2\to+\infty)} H(z) = 0\,, \quad \text{while} \quad \lim_{z\to+\infty\ (k^2\to 0)} \lim_{\mathcal{E}\to 0} H = \left(\frac{M_1^4}{k^4}\right)^N.$$

$$G(k) = \frac{i}{k^2 - m^2 + i\epsilon}\, e^{H\left[\left(\frac{M_1^4}{(k^2)^2+\mathcal{E}^4}\right)^N\right]} \underset{k^2\to 0,\ \mathcal{E}\to 0}{\longrightarrow} \frac{i}{k^2-m^2+i\epsilon}\left(\frac{M^4}{k^4}\right)^N,$$

$$V(r) \ \to \ r^{4N-3}\,.$$

Perturbative Bound States

(Zhongyou Mo, Tibério de Paula Netto, Nicolò Burzillà, LM)



In the Regge's limit $s \gg t\,, \quad A_t(+,+;+,+) \approx -8\pi G \frac{s^2}{t}\,\frac{1}{Q(t)}$

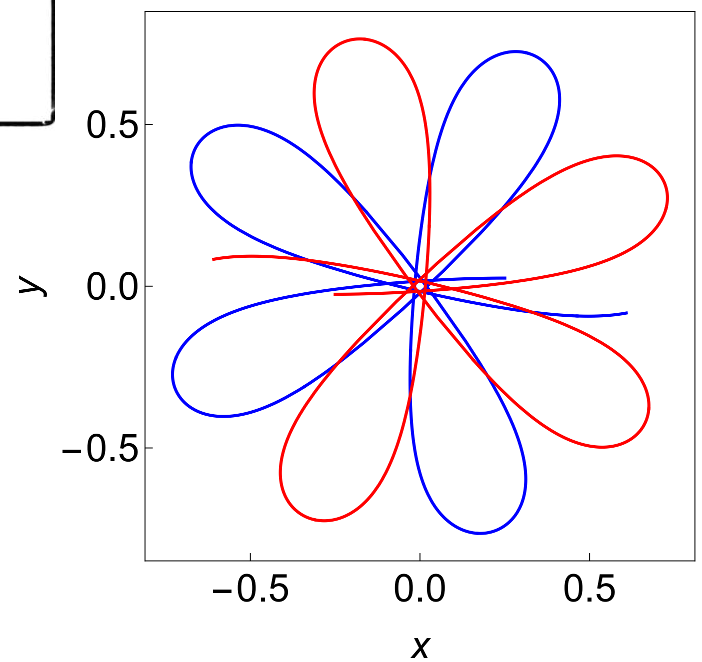
$$\Downarrow$$

$$V(|\vec{r}_1-\vec{r}_2|) = -\frac{1}{4E_1E_2}\int \frac{d^3q}{(2\pi)^3}\, e^{i\vec{q}\cdot(\vec{r}_1-\vec{r}_2)} A_t(-\vec{q}^{\,2})\,,$$

EOM for massless particles :

$$\vec{v} \equiv \dot{\vec{r}},$$
$$\dot{\vec{v}} = \frac{1}{\omega}[\vec{F} - (\vec{v}\cdot\vec{F})\vec{v}]\,,$$
$$\vec{v}^{\,2} = 1\,,$$
$$\vec{r} \equiv \frac{\vec{r}_1 - \vec{r}_2}{2}\,.$$

$\Longrightarrow$ Stringballs, Graviballs in NLG.



Nonlocal Quantum Black Holes

(Approximate Solution)

Regular Black Holes

(Approximate Solution)

$$Q(\Box)G_{\mu\nu} + O(\mathrm{Riem}^3) = 8\pi G_N\left(\tau_{\mu\nu} + T_{\mu\nu}\right), \quad \tau_{\mu\nu} = O(\mathrm{Riem}^2)\,.$$

Spallucci-Nicolini Toy Model $Q(\Box) = e^{-\frac{\Box}{\Lambda^2}}$,

$$ds^2 = -\left(1 - \frac{2m(r)}{r}\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2m(r)}{r}\right)} + r^2 d\Omega^{(2)}\,,$$

$$m(r) = M\left[1 - \frac{\Gamma\left(3/2; r^2\Lambda^2/4\right)}{\Gamma(3/2)}\right].$$

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I. Diminikova.

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Breno L. Giacchini,
Tibério de Paula Netto.

A. Mazumdar, L. Buoninfante.
A. Koshelev, A. Tokareva.

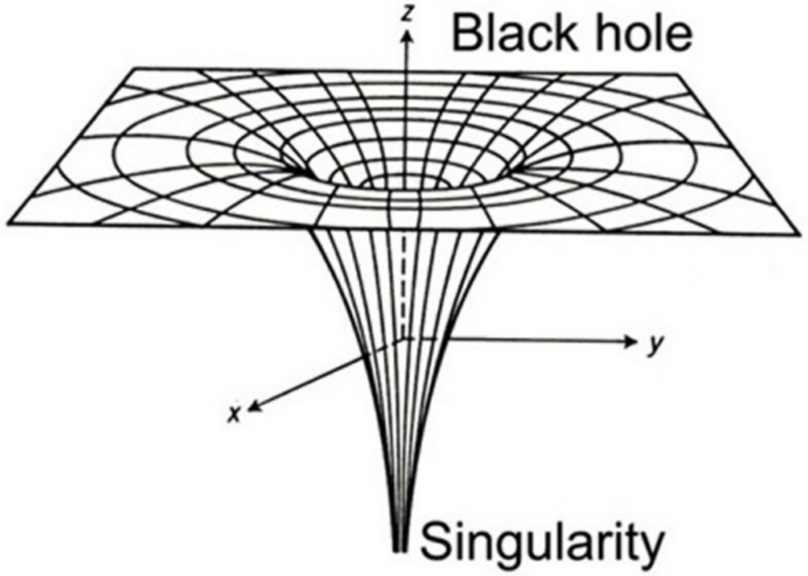
However, ...

ALL the same SOLUTIONS of Einstein's theory

$$\left(e^{H(\Delta_\Lambda)}\right)_{kj} E_j + O(E^2) = 0\,, \qquad E_k = 0 \quad \Longrightarrow \quad \mathcal{E}_k = 0\,.$$

$$\Downarrow$$

ALL the same SINGULARITIES of Einstein's theory



!

Singularity-free Black Holes

Nonlocal Conformal Gravity

L.M., L. Rachwal

$$\mathcal{L}_g = 2\kappa_D^{-2}\sqrt{-g}\Big(R + R\gamma_0(\Box)R + R_{\mu\nu}\gamma_2(\Box)R^{\mu\nu} + R_{\mu\nu\rho\sigma}\gamma_4(\Box)R^{\mu\nu\rho\sigma} + \mathcal{V}(\mathcal{R})\Big),$$

$$g_{\mu\nu} = (\phi^2\,\kappa_D^2)^{\frac{2}{D-2}}\,\hat{g}_{\mu\nu}\,,$$

$$\hat{g}_{\mu\nu} \to \Omega^2(x)\,\hat{g}_{\mu\nu}\,, \quad \phi \to \Omega^{\frac{2-D}{2}}(x)\,\phi\,.$$

$$\mathcal{L}_{\rm g} = -2\sqrt{\hat{g}}\left[\phi^2 R(\hat{g}) + \frac{4(D-1)}{D-2}\hat{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right] - \frac{2}{\kappa_D^2}\sqrt{g}\,[\mathbf{R}(g)\,\gamma_0(\Box)\mathbf{R}(g) + \mathbf{Ric}(g)\,\gamma_2(\Box)\mathbf{Ric}(g) + \mathbf{V}(g)\,]\Big|_{\phi\hat{g}}\,.$$

Spacetime Singularities and Conformal Invariance

Narlikar, Kembhavi (1977).

If $(\hat{g}_{\mu\nu}, \phi)$ is a solution $\Longrightarrow$ $(\hat{g}^*_{\mu\nu}, \phi^*)$ is a solution, where $\hat{g}^*_{\mu\nu} = \Omega^2\,\hat{g}_{\mu\nu}\,,\quad \phi^* = \Omega^{\frac{2-D}{2}}\,\phi\,.$

FRW :

$$ds^{*2} = S(t)ds^2\,,\quad ds^2 = a^2(t)(-dt^2 + d\vec{x}^2)\,;$$
$$S(t)^{-1} = a^2(t)\,.$$

$$\Longrightarrow \quad ds^{*2} = -dt^2 + d\vec{x}^2\,.$$

Singularity-free Back Holes in Conformal Gravity

Bambi, M., Rachwal

Narlikar, Kembhavi (1977).

If $(\hat{g}_{\mu\nu}, \phi)$ is a solution $\Longrightarrow$ $(\hat{g}^*_{\mu\nu}, \phi^*)$ is a solution, where $\hat{g}^*_{\mu\nu} = \Omega^2 \, \hat{g}_{\mu\nu}$, $\phi^* = \Omega^{\frac{2-D}{2}} \, \phi$.

$$ds^2 = \hat{g}^*_{\mu\nu} dx^\mu dx^\nu = -S(r) \left(1 - \frac{2M}{r}\right) dt^2 + \frac{S(r)\, dr^2}{1 - \frac{2M}{r}} + S(r) r^2 d\Omega^2 \,,$$

$$S(r) = 1 + \frac{L^4}{r^4} \,, \qquad \phi^* = S(r)^{-1/2} \kappa_4^{-1} \,.$$

$r = 0$ Singularity in Conformal Gravity $\equiv$ $r = 2m$ Singularity in GR.

Schwarzschild Singularity = one element of the gauge orbit

$$ds^2 = \hat{g}^*_{\mu\nu} dx^\mu dx^\nu = -S(r)\left(1 - \frac{2M}{r}\right) dt^2 + \frac{S(r)\, dr^2}{1 - \frac{2M}{r}} + S(r) r^2 d\Omega^2 \,,$$

$$S(r) = 1 + \frac{L^4}{r^4} \,, \qquad \phi^* = S(r)^{-1/2} \kappa_4^{-1} \,.$$

The Kretschmann invariant : $\hat{\mathbf{K}} = \hat{\mathbf{R}}\mathbf{iem}^2$,

$$\hat{\mathbf{K}} = \frac{1}{(L^4 + r^4)^6} [16 r^2 (L^{16}(39m^2 - 20mr + 3r^2) + 2L^{12} r^4 (66m^2 - 32mr + 3r^2)$$
$$+ L^8 r^8 (342m^2 - 284mr + 63r^2) + 12 L^4 m^2 r^{12} + 3m^2 r^{16})].$$

The Ricci scalar :

$$\hat{R} = -\frac{12 L^4 r \left(L^4 (r - 4m) + r^4 (3r - 8m)\right)}{(L^4 + r^4)^3}$$

Conformal Invariant Observable

Hawking Temperature : $\quad T_H = \dfrac{1}{8\pi m} \quad \forall\ S(r) \,.$

Geodesic Completion

Massive particle probe: $S_{\rm m} = -m \int \sqrt{-\hat{g}_{\mu\nu} dx^\mu dx^\nu} = -m \int \sqrt{-\hat{g}_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}\, d\lambda\,,$

Proper time gauge : $\frac{d\hat{s}^2}{d\tau^2} = -1\,.$

Conformally coupled probe: $S_{\rm m} = -\int \sqrt{-f^2\phi^2 \hat{g}_{\mu\nu} dx^\mu dx^\nu} = -\int \sqrt{-f^2\phi^2 \hat{g}_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}\, d\lambda\,,$

Proper time gauge : $\frac{d\hat{s}^2}{d\tau^2} = -1\,,$

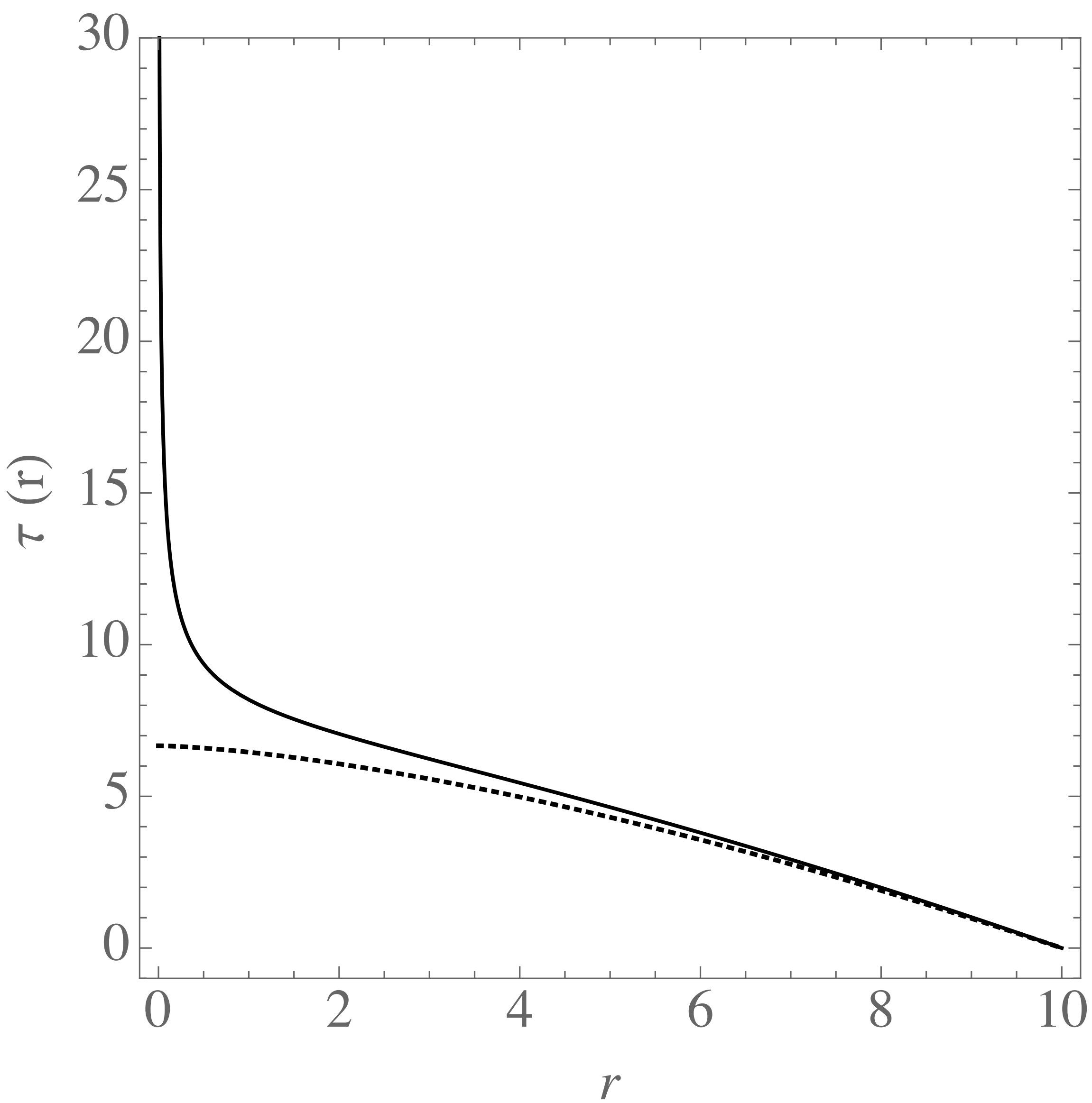
solution : $$\tau = \frac{4M^2 - 3L^2}{3M} - \frac{\left(r^2 - 3L^2\right)\sqrt{\frac{2r}{M}}}{3r}\,.$$

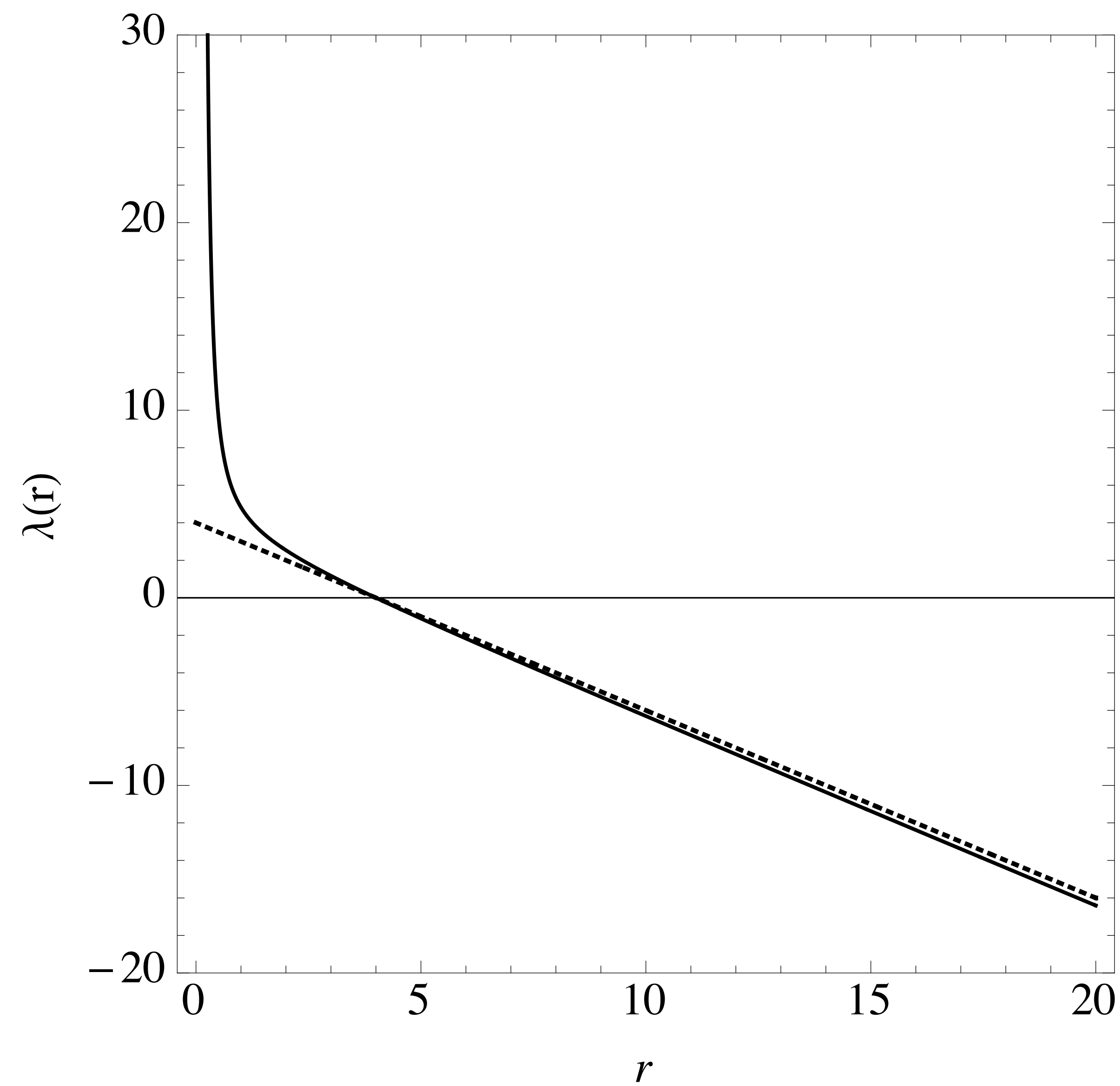
Light: $d\hat{s}^2 = 0\,,$

Killing vectors: $\xi^\alpha = (1,0,0,0)\,, \quad \eta^\alpha = (0,0,0,1)\,,$

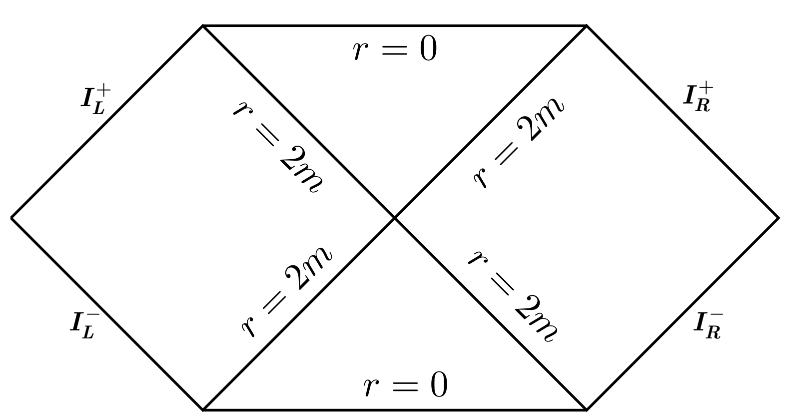
$$e = -\xi \cdot u = -\xi^\alpha u^\beta \hat{g}_{\alpha\beta} = -\hat{g}_{tt} u^t = S(r)\left(1 - \frac{2M}{r}\right)\frac{dt}{d\lambda} = S(r)\left(1 - \frac{2M}{r}\right)\dot{t}\,,$$

solution : $$\lambda(r) = \frac{1}{e}\left[\frac{L^4}{3r^3} - \frac{L^4}{3{r_0}^3} + \frac{2L^2}{r} - \frac{2L^2}{r_0} - r + r_0\right].$$





Spacetime Causal Structure



Kerr Singularity

Bambi, M.,Rachwal, Wang.

$$ds^{*2} \equiv \hat{g}^{*}_{\mu\nu} dx^\mu dx^\nu = S(r,\theta) \hat{g}^{\rm Kerr}_{\mu\nu} dx^\mu dx^\nu \,,$$

$$\phi^* = S(r,\theta)^{-1/2} \kappa_4^{-1} \,,$$

$$S(r) = \left(1 + \frac{L^2}{r^2 + a^2 \cos^2\theta} \right)^4 .$$

$$\mathbf{K} = \frac{1}{(L^2 + r^2 + a^2\cos^2\theta)^{12}} \times \text{polynomial}(r, \theta, m, a, L) \,.$$

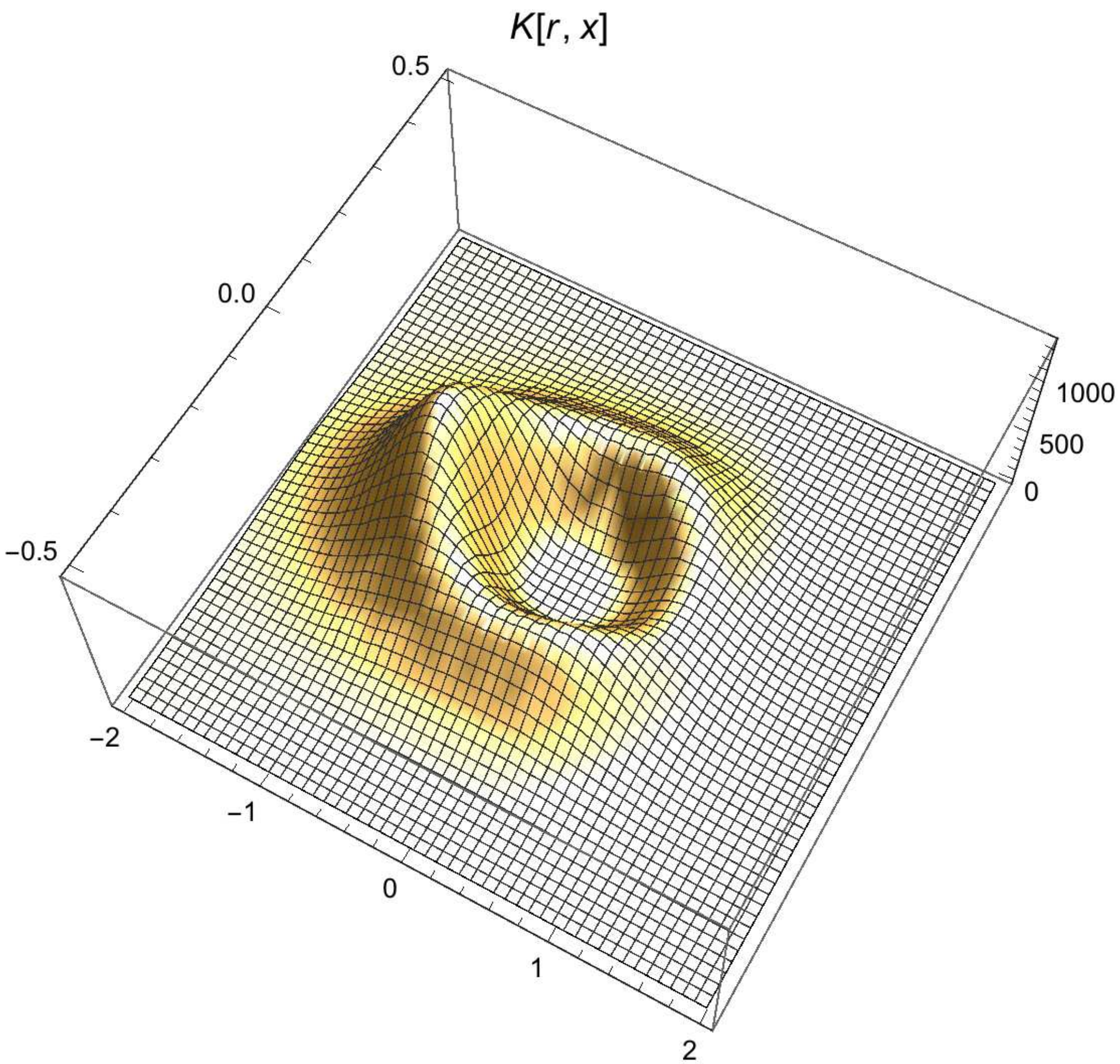
$$L \propto m \quad \text{(black hole mass).}$$

C. Bambi and collaborators: Upper bound on L:

$$L < 0.6m.$$

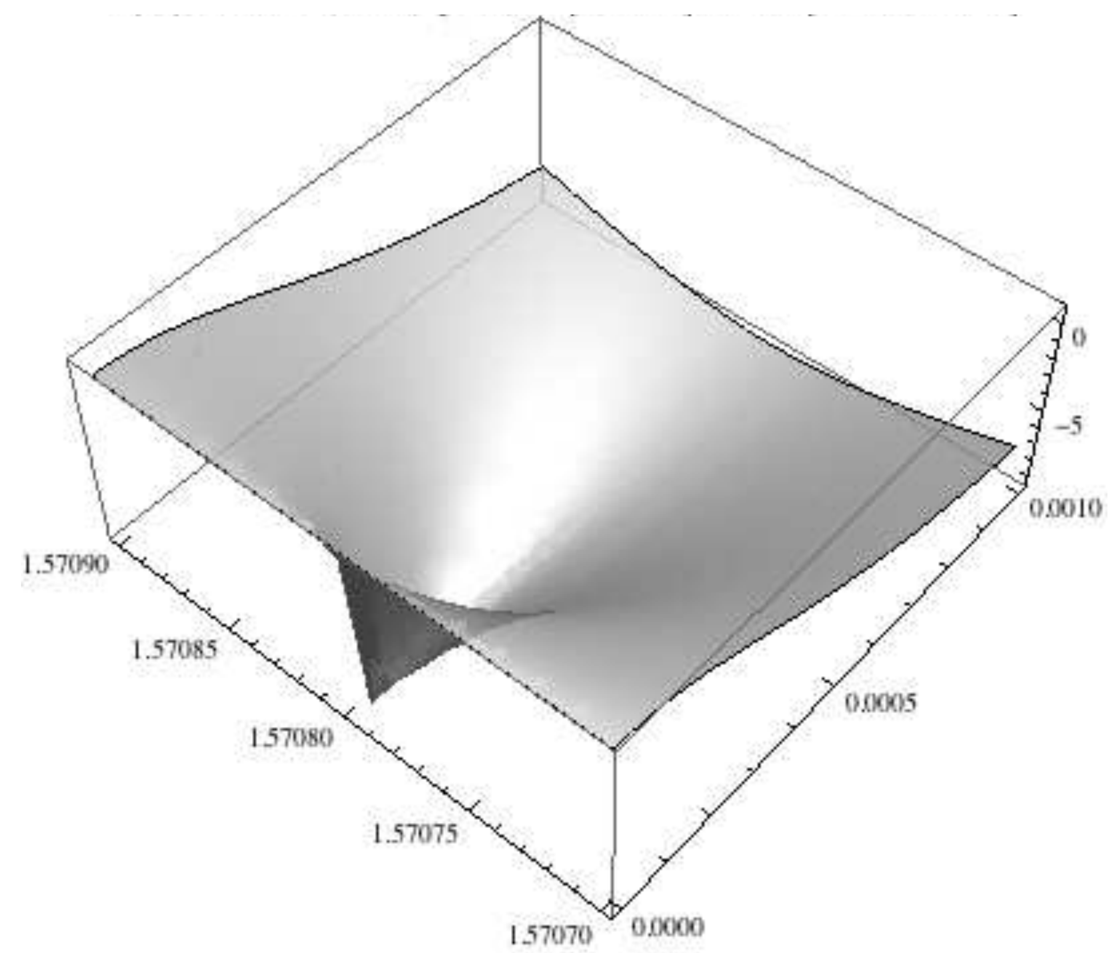
Kretschmann Scalar

Conformal Gravity



Analytic

Other ...



$$\lim_{r\to 0}\left(\lim_{\vartheta\to\pi/2} K(r,\vartheta)\right) = \frac{8M^2}{3\pi\theta^3},$$

$$\lim_{\vartheta\to\pi/2}\left(\lim_{r\to 0} K(r,\vartheta)\right) = 0.$$

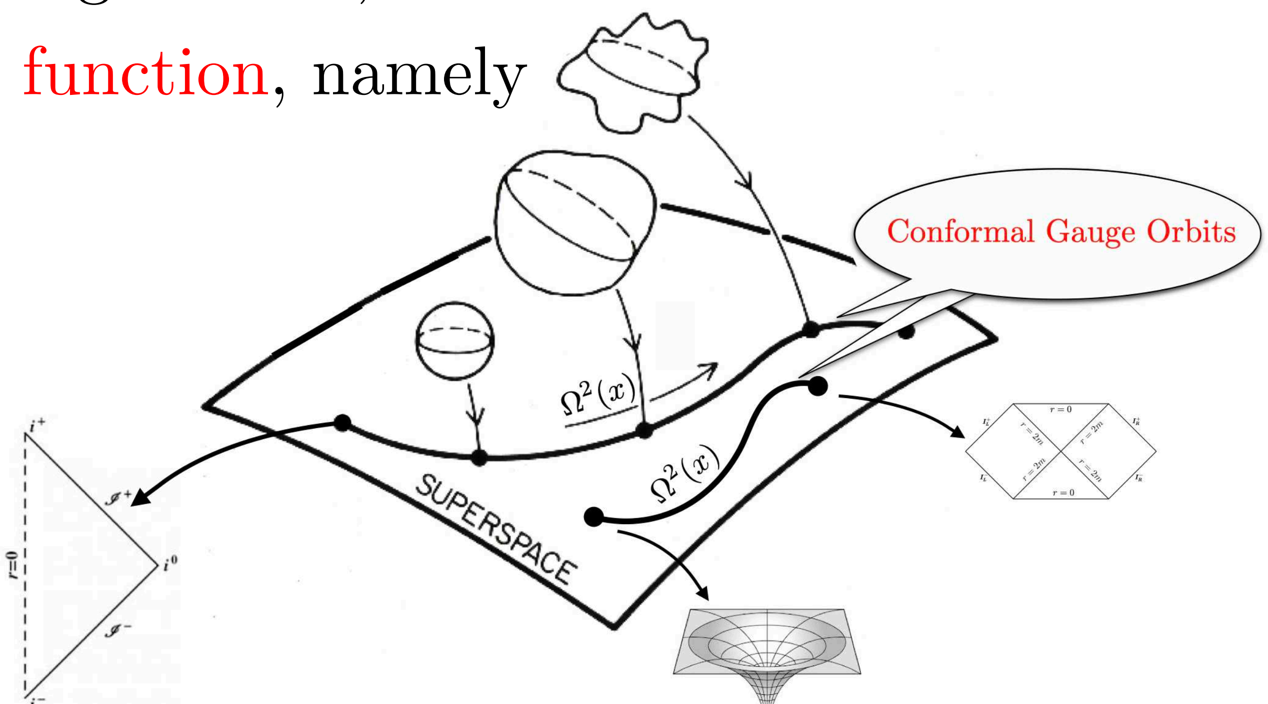
Non-Analytic

Towards a General Theorem

$$(\text{SINGULAR SPACETIME}) = \Omega^2(x) \times (\text{REGULAR SPACETIME}).$$

The Big Bang singularity, the Black Holes' singularities, and the Kasner singularity are all characterized by only one function, namely

$$g_{\mu\nu} = \Omega^2 \times \underbrace{\Omega^{-2} \times g_{\mu\nu}}_{\text{complete spacetime}} \, .$$

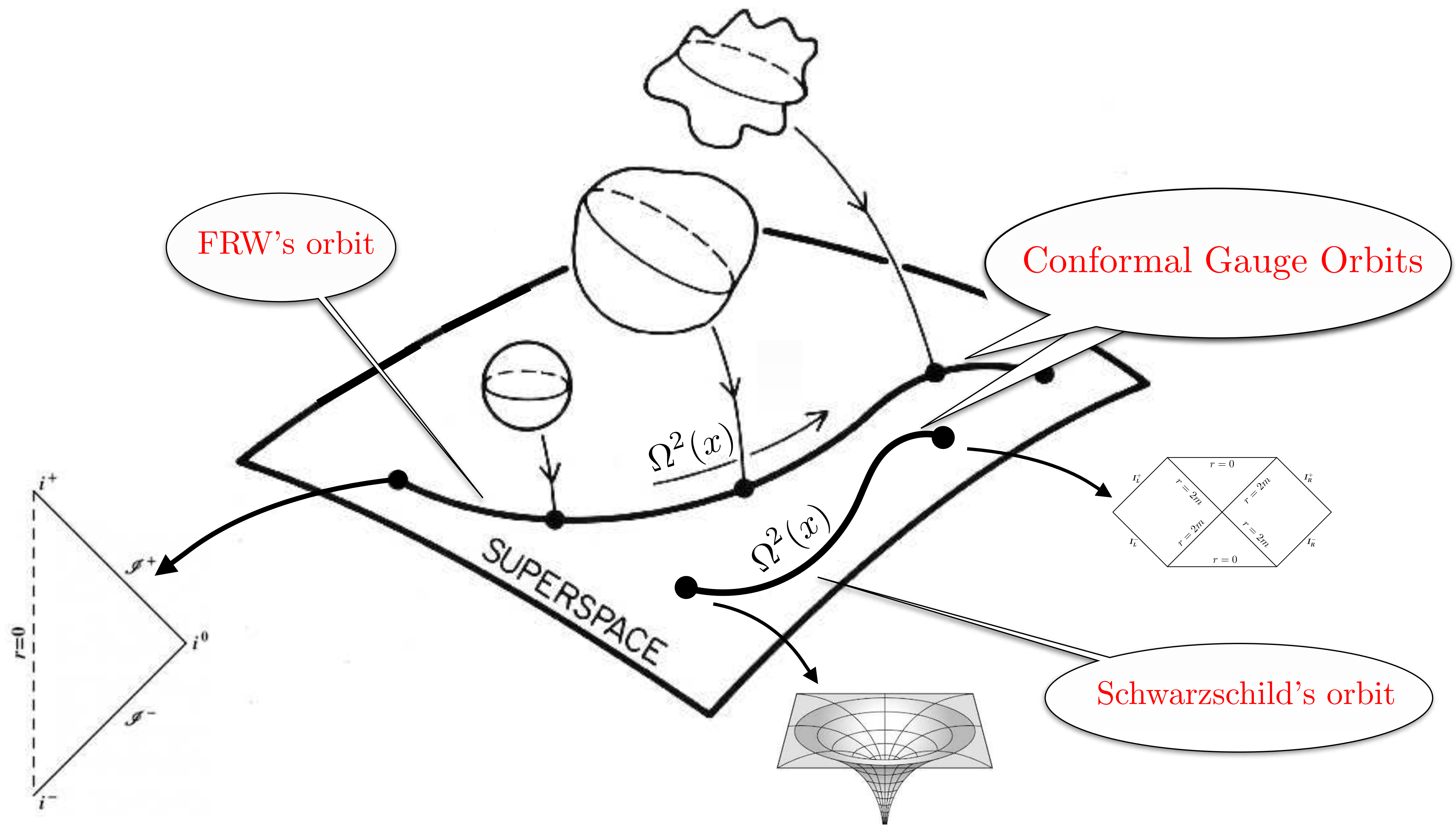


However, the overall function Ω^2 is actually not relevant at all to describe conformal geometry.

Therefore, it has no meaning to speak of black holes' or cosmological singularities in conformal gravity.

Metric Superspace

If $(\hat{g}_{\mu\nu}, \phi)$ is a solution $\Longrightarrow$ $(\hat{g}^*_{\mu\nu}, \phi^*)$ is a solution, where $\hat{g}^*_{\mu\nu} = \Omega^2\, \hat{g}_{\mu\nu}\,, \quad \phi^* = \Omega^{\frac{2-D}{2}}\, \phi\,.$



General Theorem

"Non-singularity theorem in conformal gravity".

Zhou Tian, LM.

Proven for massless and conformally coupled particles (timelike singularities).

So far so good, but ...

What is the contribution of QG to the singularity resolution?

Answer :

Preserving Conformal Invariance!

Indeed,

Einstein's gravity is classically conformal invariant, but not at quantum level,

only a finite quantum gravity is Weyl anomaly-free.

Galactic Rotation Curves

Galactic Rotation Curves

Li Qinang, Zhou Tian, LM.

In x coordinate :

$$d\hat{s}^{*2} = Q^2(x)\left[-\left(1-\frac{2GM}{x}\right)dt^2 + \frac{dx^2}{1-\frac{2GM}{x}} + x^2(d\theta^2+\sin^2\theta d\varphi^2)\right],$$

$$Q(x) = \frac{1}{1-\frac{\gamma^*}{2}x}, \phi^* = Q(x)^{-1}\,\kappa_4^{-1}.$$

Singular in : $x = \frac{2}{\gamma^*}$.

In r coordinate :

$$d\hat{s}^{*2} = -Q^2(r)\left(1-\frac{2GMQ(r)}{r}\right)dt^2 + \frac{dr^2}{Q^2(r)\left(1-\frac{2GMQ(r)}{r}\right)} + r^2(d\theta^2+\sin^2\theta d\varphi^2),$$

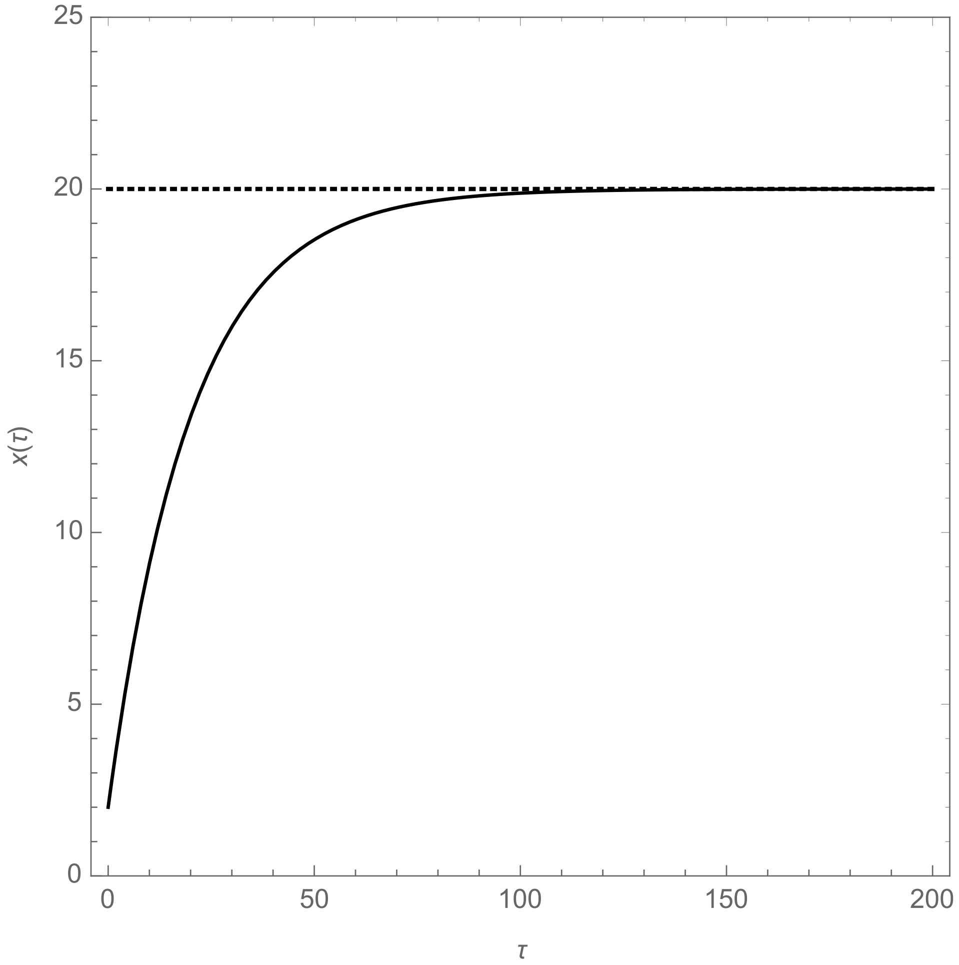
$$Q(r) = 1+\frac{\gamma^*}{2}r \qquad \left(\text{notice that } x = \frac{r}{Q(r)}\right),$$

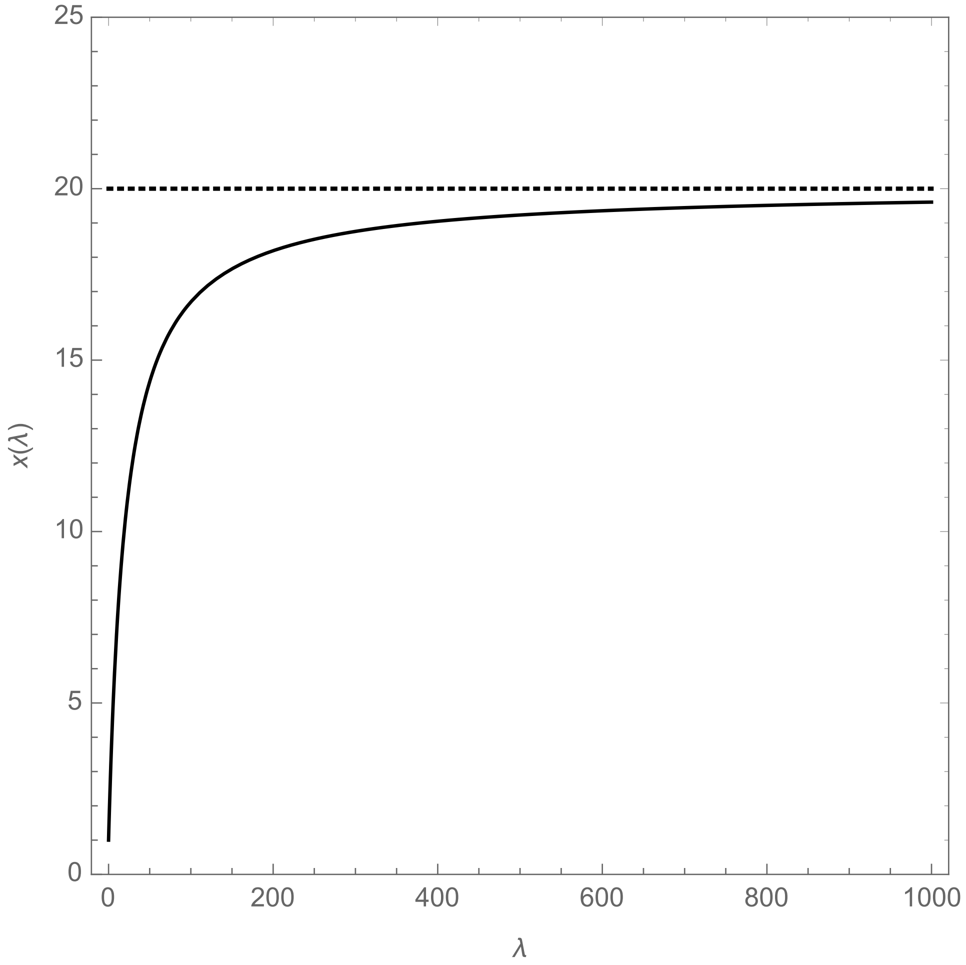
$$\phi^*(r) = Q(r)^{-1}\,\kappa_4^{-1}.$$

Geodesic Completion

$$x(\tau) = \frac{2}{\gamma^*}\left[1 - e^{-\frac{\gamma\tau}{2}}\left(1 - \frac{\gamma^*}{2}x_0\right)\right].$$

$$x(\lambda) = \frac{4\lambda - 2\gamma^*\lambda x_0 + 4x_0}{2\gamma^*\lambda - \gamma^{*2}\lambda - x_0 + 4}$$





Velocity

$$v^2(x) = \frac{g'_{tt}}{g'_{\varphi\varphi}}\frac{g_{\varphi\varphi}}{g_{tt}} = \frac{GM}{x-2GM} \approx \frac{GM}{x} \quad \to \quad \frac{GM\gamma^*}{2}\,.$$

$$v^2(\ell_r) = \frac{GM\gamma^*}{4}\left[1+\coth\left(\frac{\ell_r\gamma^*}{4}\right)\right] \quad \Longrightarrow \quad \boxed{v^2(r) \approx \frac{GM}{\ell_r} + \frac{GM\gamma^*}{2}}.$$

Effective Gravitational Potential

$$\Phi(\ell_r) \approx -\frac{GM}{\ell_r} + \frac{GM\gamma^*}{2}\log(\ell_r) + \text{const.}\,.$$

UNIQUENESS OF THE SOLUTION

In the first part of this paper the rescaling of the metric $Q(x)$ was chosen compatibly with the relation $g_{00} = -1/g_{11}$, as evident in the coordinate r. In this section we would like to provide three fundamental reasons to support such choice. (i) The first one is related to the null energy condition, which asserts that $p + \rho \geqslant 0$ [32]. Indeed, in order to preserve the null energy condition we must impose $g_{00} = -1/g_{11}$.

(ii) The second one is related to the acceleration of the light in the Newtonian regime. Indeed, if the velocity of light has to remain constant in empty space surrounding a point-like mass, then photons should experience zero acceleration [33]. Using the last result in the previous subsection, namely $|\dot{r}| = e$ we get $\ddot{r} = 0$, which is true only if the relation $g_{00} = -1/g_{11}$ for the components of the metric tensor is satisfied. Let us expand on this point. For a general spherically symmetric metric,

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\Omega^2, \tag{55}$$

making use again of (45), namely

$$e = A(r)\dot{t}\,, \tag{56}$$

and $ds^2 = 0$, the radial geodesic equation reads

$$-A(r)\dot{t}^2 + B(r)\dot{r}^2 = 0 \implies -\frac{e^2}{A(r)} + B(r)\dot{r}^2 = 0 \implies \dot{r}^2 = \frac{e^2}{A(r)B(r)} \implies 2\,\dot{r}\,\ddot{r} = \left(\frac{e^2}{A(r)B(r)}\right)'\dot{r}\,, \tag{57}$$

where $'$ means derivative respect to r. Finally,

$$\ddot{r} = \left(\frac{e^2}{2A(r)B(r)}\right)'. \tag{58}$$

Therefore, in order to do not experience acceleration in the radial coordinate we must have: $A(r)B(r) = \text{const.}$. Notice that here the radial coordinate is not the physical radial distance because the spacetime is not asymptotically flat. However, according to the Taylor expansion of (88) in the Newtonian intermedium regime $\ell_r \approx r$ and the acceleration above vanishes.

(iii) Last but not least we should consider the impact of the large distance modification of the Schwarzschild metric on the homogeneity and isotropy of the Universe.

Let us start considering the following coordinate transformation from the radial coordinate r to ρ,

$$\rho = \frac{4r}{2(1 + \alpha r + \beta r^2)^{1/2} + 2 + \alpha r}\,, \tag{59}$$

$$\tau = \int dt R(t)\,, \tag{60}$$

in the following general not asymptotically flat metric,

$$d\hat{s}^{*2} = -\left(1 + \alpha r + \beta r^2\right) dt^2 + \frac{dr^2}{(1 + \alpha r + \beta r^2)} + r^2 d\Omega^{(2)}\,. \tag{61}$$

The above metric (61) in the new coordinates reads:

$$d\hat{s}^{*2} = \frac{1}{R^2(\tau)}\left[\frac{1 - \frac{\alpha^2\rho^2}{16} + \frac{\beta\rho^2}{4}}{\left(1 - \frac{\alpha\rho}{4}\right)^2 - \frac{\beta\rho^2}{4}}\right]^2 \left\{-d\tau^2 + \frac{R(\tau)^2}{\left[1 - \left(\frac{\alpha^2}{16} - \frac{\beta}{4}\right)\rho^2\right]^2}\left(d\rho^2 + \rho^2 d\Omega^{(2)}\right)\right\}, \tag{62}$$

where $R(\tau) := R(t(\tau))$.

Now, in a geometry which is both homogeneous and isotropic about all points, any observer can serve as the origin of the radial coordinate ρ; thus in his own local rest frame each observer is able to make the above general coordinate transformation using his own particular ρ. Moreover, in conformal gravity we can make an overall rescaling of the metric to finally end up with a comoving Robertson-Walker (RW) spacetime written in spatially isotropic coordinates with spatial curvature $K = \beta - \alpha^2/4$,

$$d\hat{s}^{*2} = F(\tau,\rho)\left[-d\tau^2 + \frac{R(\tau)^2}{(1 + K\rho^2/4)^2}\left(d\rho^2 + \rho^2 d\Omega^{(2)}\right)\right]. \tag{63}$$

For the case of the metric (10), taking $r \gg 2GM$ and $GM\gamma^* \ll 1$,

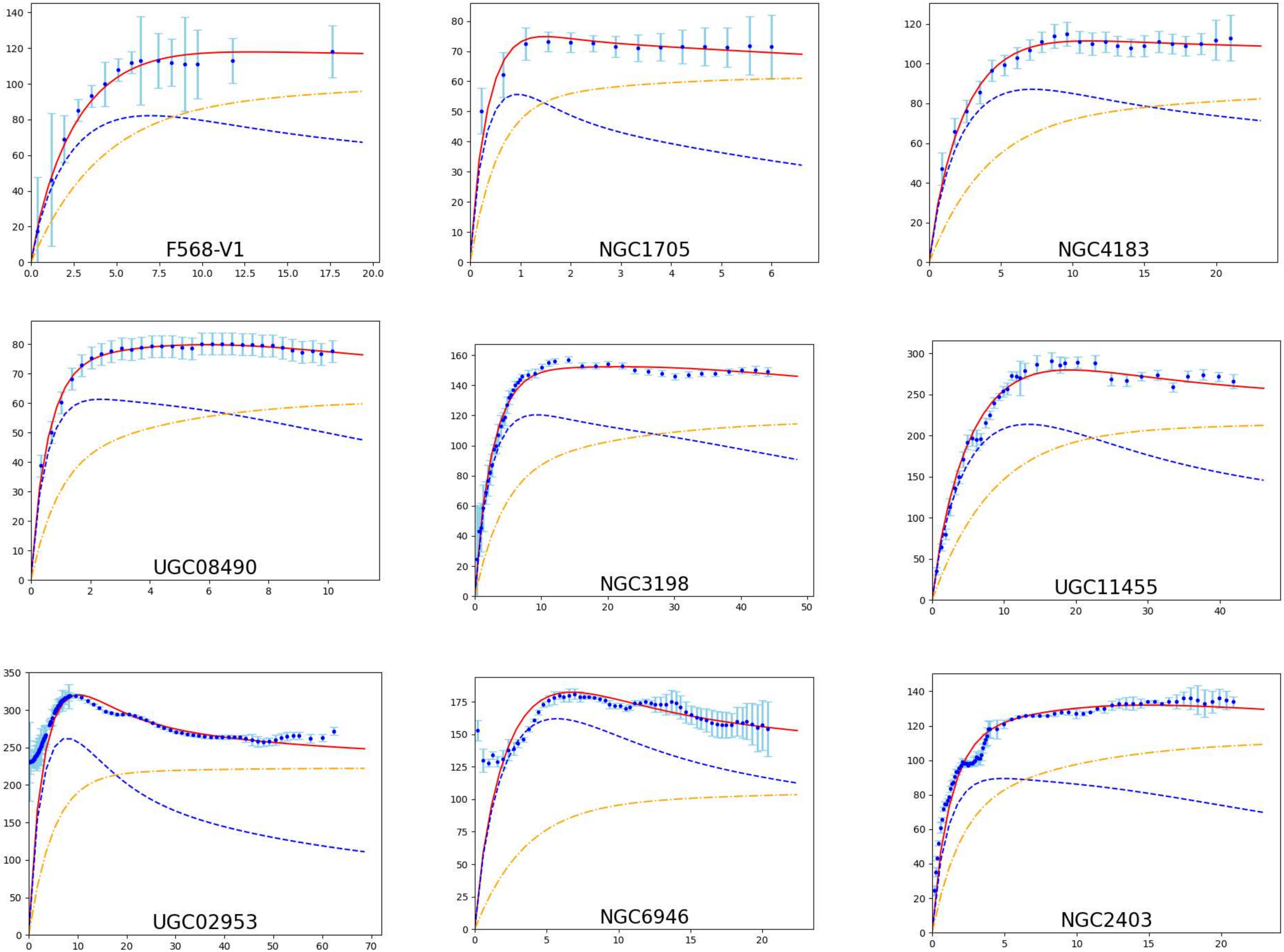
$$d\hat{s}^{*2} \approx -\left(1 + \gamma^* r + \frac{\gamma^{*2}}{4}r^2\right) dt^2 + \frac{dr^2}{\left(1 + \gamma^* r + \frac{\gamma^{*2}}{4}r^2\right)} + r^2 d\Omega^{(2)}\,. \tag{64}$$

Therefore, we can identify the constants $\alpha = \gamma^*$ and $\beta = \gamma^{*2}/4$, and in the new coordinates (τ, ρ) the metric (64) takes the following RW form,

$$d\hat{s}^{*2} = \frac{1}{R^2(\tau)}\frac{1}{\left(1 - \frac{\gamma^*}{2}\rho\right)^2}\left[-d\tau^2 + R(\tau)^2\left(d\rho^2 + \rho^2 d\Omega^{(2)}\right)\right], \tag{65}$$

which coincides with the metric (7) for $x \gg 2GM$ upon reintroducing the time coordinate t defined in (60).

Therefore, the metric proposed in this paper is the *only one* that does not affect the homogeneity of the Universe at large scales. Finally, we notice that the metric (64) is asymptotically (for large r) Anti-de Sitter, whose stability is guarantee from the fact that it comes from a rescaling of the Schwarzschild metric, which is known to be stable.


F568-V1
NGC1705
NGC4183
UGC08490
NGC3198
UGC11455
UGC02953
NGC6946
NGC2403

Conclusions

- Local Gravitational Theories.
  - Unitarity: complex pairs, fakions.
- Super-renormalizable Gravitational Theories.
  - Unitarity (no ghosts).
  - Super-renormalizability.
- Finite Quantum Gravity :
  - Finiteness in Odd Dimension.
  - Finiteness in Even Dimension.
  - Conformal invariant quantum gravity.
- Scattering amplitudes and Causality.
- Nonlocal Unification of the Fundamental Interactions.
- Unitarity: Fundamental confinement.
- Exact solutions and spacetime singularities.
- Planck-Balls.
- Nonlocal Quantum Black Holes.
- Nonsingular Spacetimes in Conformal Gravity.
- Galactic Rotation Curves in Conformal gravity.
- Finite Entanglement Entropy.
- Information loss problem and Spacetime Singularity.
- The Early Universe in Quantum Gravity.

Achievements

1. Perturbative Unitarity
2. Super-renormalizability ,
3. Finiteness ,
4. Non-perturbative Unitarity (work in progress) ,
5. n-points scattering amplitudes and causality,
6. One loop exact beta functions,
7. Universal regular Newtonian potential ,
8. Quantum Conformal Invariance ,
9. Singularity free spacetimes (black holes, FLRW, etc.) ,
10. Finite Entanglement(-conical) Entropy ,
11. Well define theory in (A)dS ,
12. Starobinsky Inflation ,
13. Finite nonlocal gauge theory ,
14. Towards a finite Standard Model .
15. Quantum Cosmology.

Quantum Conformal Cosmology

(G. Calcagni, L.M.)

No Inflation