SCALE-DEPENDENT GRAVITY AND PLANCK STARS

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(Collaboration with G.Lambiase)
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Scale-dependent (Asymptotically safe) gravity

- In many QG approaches, the **basic parameters** that enter the action, such as Newton's constant, electromagnetic coupling, cosmological constant, etc, **become scale dependent quantities**.
- **Scale dependence** at the level of the effective action is a **generic feature** of ordinary quantum field theories.

1. A typical Scale dependent effective gravitational action

$$\Gamma_k[\mathfrak{g}] = \frac{1}{16\pi G(k)} \int \mathrm{d}^4x \sqrt{\mathfrak{g}} \Big(-R(\mathfrak{g}) + 2\bar{\lambda}(k) \Big).$$

$\mathfrak{g}$ = metric, k = energy/momentum scale, G(k) and λ(k) are the running Newton's and cosmological constants. Cosmological constant $\lambda(k) \sim 0$ in local problems. Dimensionless Newton's constant $g(k) = k^2 G(k)/\hbar$

The evolution of scale-dependent quantities is governed by a differential equation, from which we derive an evolution equation for g(k)

$$\frac{\mathrm{d}g(t)}{\mathrm{d}t} \equiv \beta(g(t)) = \left[2 + \frac{B_1 g(t)}{1 - B_2 g(t)}\right] g(t)$$

where t=log(k)

Integrating for g(k), and using $g(k) = k^2 G(k)/\hbar$ we get the dimensionful Newton's coupling

$$G(k) = \frac{G_0}{1 + \tilde{\omega} G_0 k^2/\hbar}$$

Deviations at high energy: $G(k) \longrightarrow 0$. Classical space-time recovered for $k \longrightarrow 0$.

The ***SIGN*** of $\boldsymbol{\tilde{\omega}}$ is ***crucial*** for the physics of the theory.

Asymptotically Safe Gravity (ASG) assumes $\tilde{\omega} > 0$. So that gravity can be "asymptotically safe" from divergences.

If instead $\tilde{\omega} < 0$ ⟹ $G(k) \rightarrow \infty$ for $k \sim k_{Planck}$

When $\tilde{\omega} > 0$, it is a proper ASG theory. When $\tilde{\omega} < 0$, it is a general SDG theory!

2. Link between the energy scale k and the radial coordinate r, namely $k = k(r)$:

$$k(r) \equiv \hbar \left(\frac{r + \gamma G_0 M}{r^3} \right)^{1/2}$$

The energy scale k(r) is a modified proper distance, $k(r) \sim 1/d(r)$. $\gamma = 9/2$ by GR argument (infrared cutoff)

3. Finally the running Newton coupling constant is

$$G(r) = \frac{G_0 r^3}{r^3 + \tilde{\omega} G_0 \hbar (r + \gamma G_0 M)}$$

$G_0 \hbar = \ell_p^2$ = quantum effects!

$\tilde{\omega}, \gamma$ numerical constants

In the SD (AS) gravity approach, in order to get scale-dependent solutions (Newtonian or general relativistic), we replace everywhere the Newton constant G_0 with the running constant G(r).

For a spherically symmetric Lorentzian metric

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2\, d\Omega^2$$

the lapse function of the SD/AS-Schwazschild metric reads

$$f(r) = 1 - \frac{2MG(r)}{r} = 1 - \frac{2G_0 M r^2}{r^3 + \tilde{\omega} G_0 \hbar (r + \gamma G_0 M)}$$

Important limits: large r, low energy k

$$f(r \to \infty) \simeq 1 - \frac{2G_0 M}{r}$$

Small r, high energy k

$\gamma > 0$ $$f(r \to 0) \simeq 1 - \frac{2r^2}{\tilde{\omega}\gamma G_0 \hbar}$$

$\gamma = 0$ $$f(r \to 0) \simeq 1 - \frac{2Mr}{\tilde{\omega}\hbar}$$

$\tilde{\omega}\gamma > 0$ DeSitter; $\tilde{\omega}\gamma < 0$ Anti-DeSitter.

Conic singularity

SDG modified Newtonian potential

A SDG modified Newtonian potential can be obtained from the standard Newton formula by simply replacing the experimentally observed Newton constant G with the running coupling G(r)

$$V^{SDG}(r) = -\frac{G(r)Mm}{r} = -\frac{G\,M\,m\,r^2}{r^3 + \tilde{\omega}\,G\,\hbar\,(r + \gamma GM)}\,,$$

By expanding for *large r* we get

$$V^{SDG}(r) = -\frac{GMm}{r}\left[1 - \frac{\tilde{\omega}G\hbar}{r^2} - \frac{\gamma\tilde{\omega}G^2\hbar M}{r^3} + \mathcal{O}\left(\frac{G^2\hbar^2}{r^4}\right)\right]$$

- The corrections predicted by the SDG (ASG) approach are of genuine quantum nature (presence of ħ).
- To fix the value of $\tilde{\omega}$, people in ASG usually compared this potential with the Quantum corrected Newtonian Potential obtained via GR-EFT by Donoghue, Khriplovich, Shapiro, etc.
- Even if, recently (~2020 and later), there are doubts on the comparability between ASG/GR-EFT approaches (different classes of Feynman diagrams).

Possible $\tilde{\varpi} < 0$ values from quantum corrections to the Newtonian potential

Computed by John Donoghue (PRD 1994, PRD 2003). GR is seen as a low energy Effective Field Theory. At ordinary energies GR gravity is a well-behaved QFT. The quantum corrections at low energy, and dominant effects at large distances can be isolated. **Potential energy between two heavy objects close to rest is (to the lowest order)**

$$V^{QGR}(r) = -\frac{GMm}{r}\left[1 + \frac{41}{10\pi}\frac{G\hbar}{r^2} + \ldots\right]$$

The last term is a true long distance quantum effect linear in ħ.
(We neglect post Newtonian classical correction terms). By comparing with

$$V^{SDG}(r) = -\frac{GMm}{r}\left[1 - \frac{\tilde{\omega} G\hbar}{r^2} - \frac{\gamma\tilde{\omega} G^2\hbar M}{r^3} + \mathcal{O}\left(\frac{G^2\hbar^2}{r^4}\right)\right]$$

we get at the first order in ħ

$$\tilde{\omega} = -\frac{41}{10\pi}$$

Ilya Shapiro:

$\tilde{\varpi}$ = - 17 / 20 π

Main message:

$\tilde{\varpi}$ can be Negative!

The γ parameter is not fixed by previous considerations.

$\gamma = 9/2$ by Gen.Rel. arguments (Bonanno-Reuter 2000): correct identification of the infrared cutoff

$\gamma = 0$ by GUP considerations

Anyway $\gamma \geq 0$

The ***SIGN*** of $\tilde{\omega}$ is ***crucial*** for the physics of SDG/ASG-black hole.

Since $$G(k) = \frac{G_0}{1 + \tilde{\omega} G_0 k^2/\hbar}$$ gravity is "asymptotically safe" from divergences only when $\tilde{\omega} > 0$.

$\tilde{\omega} > 0$ → singularity-free BH metric, with DeSitter or Anti DeSitter core

Instead when $\tilde{\omega} < 0$ ⟹ $G(k) \to \infty$ for $k \sim k_{Planck}$ (SDG !)

- GR as Effective QFT [Donoghue, Krilipovich, Froeb, Shapiro] (2003, 2005, 2022): all results point to a ***negative value of $\tilde{\omega}$***.

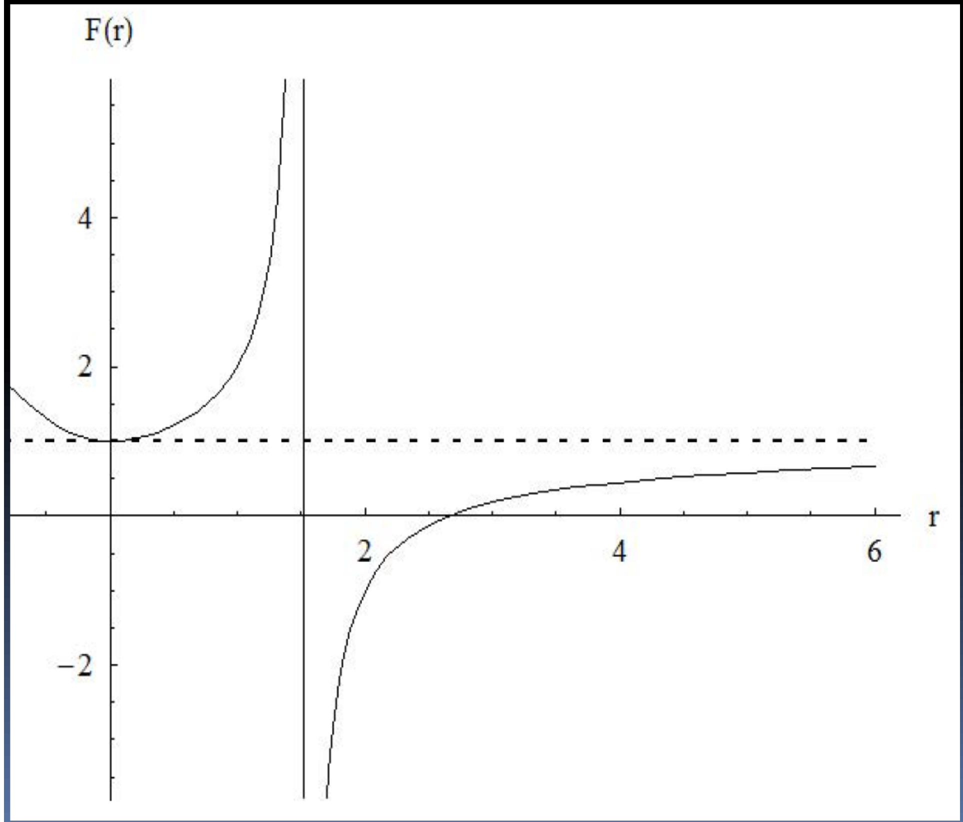
$\tilde{\omega} < 0$: deep consequences on the structure of the metric of the black hole.

A NEW SDG-SCHWARZSCHILD METRIC

Under the condition $\tilde{\omega} < 0 \;\Rightarrow\; \tilde{\omega} = -|\tilde{\omega}| \,; \qquad \gamma > 0 \,.$ the lapse F(r) is

$$F(r) = 1 - \frac{2GMr^2}{r^3 - |\tilde{\omega}|G\hbar(r + \gamma GM)}$$

Since $\widetilde{\omega} < 0$ the denominator can develop zeros, and hence the metric can have singularities.



Lapse F(r) for any **r > 0, γ > 0, M > 0**.

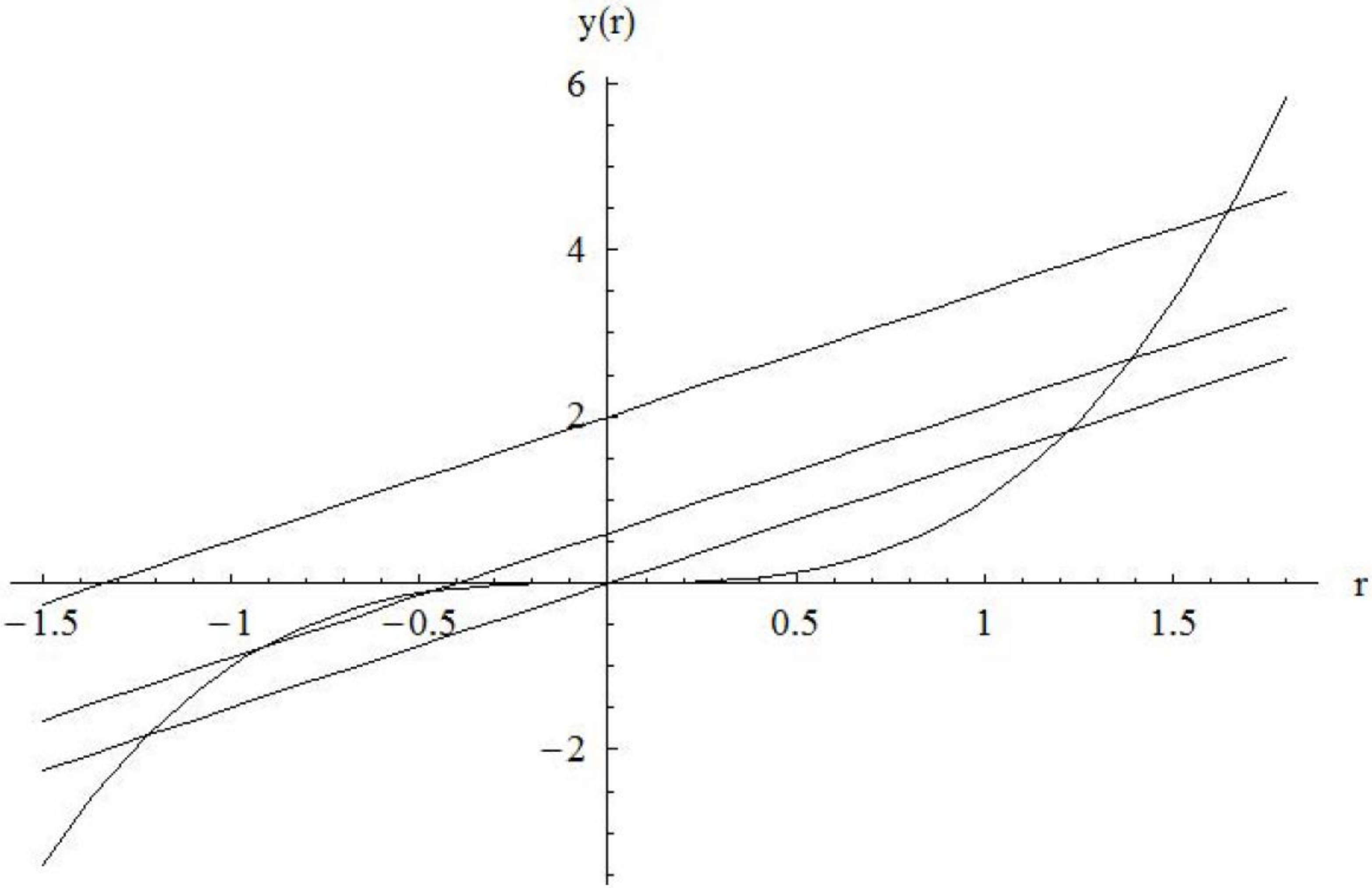
Essential singularity at $\boldsymbol{r = r_o > 0}$ (instead of being at $\mathrm{r} = 0$ as in Schwarzschild) (Kretschmann scalar diverges) .

Always one single positive zero $F(r_h) = 0$

SDG-BH Horizon at $\boldsymbol{r = r_h}$

Always $\boldsymbol{r_o < r_h}$ for any **M > o**.

No naked singularity in full accordance with the Cosmic Censorship Conjecture of Penrose.


y(r)
6
4
2
−2
r
−1.5
−1
−0.5
0.5
1
1.5

Horizon $r = r_h$, Singularity $r = r_o$

Behavior for large or small *M*

$$r_h \simeq 2GM + \frac{(2+\gamma)|\tilde{\omega}|\hbar}{4M}$$

for $M \to \infty$

(Schwarzschild recovered)

$$r_h \simeq \sqrt{|\tilde{\omega}|G\hbar} + \left(1 + \frac{\gamma}{2}\right) GM$$

for $M \to 0$

(essentially, $r_h \sim \ell_p$ which is reasonable, since any length below Planck length is physically meaningless)

$$r_0 \simeq \sqrt{|\tilde{\omega}|G\hbar} + \frac{\gamma}{2} GM$$

for $M \to 0$

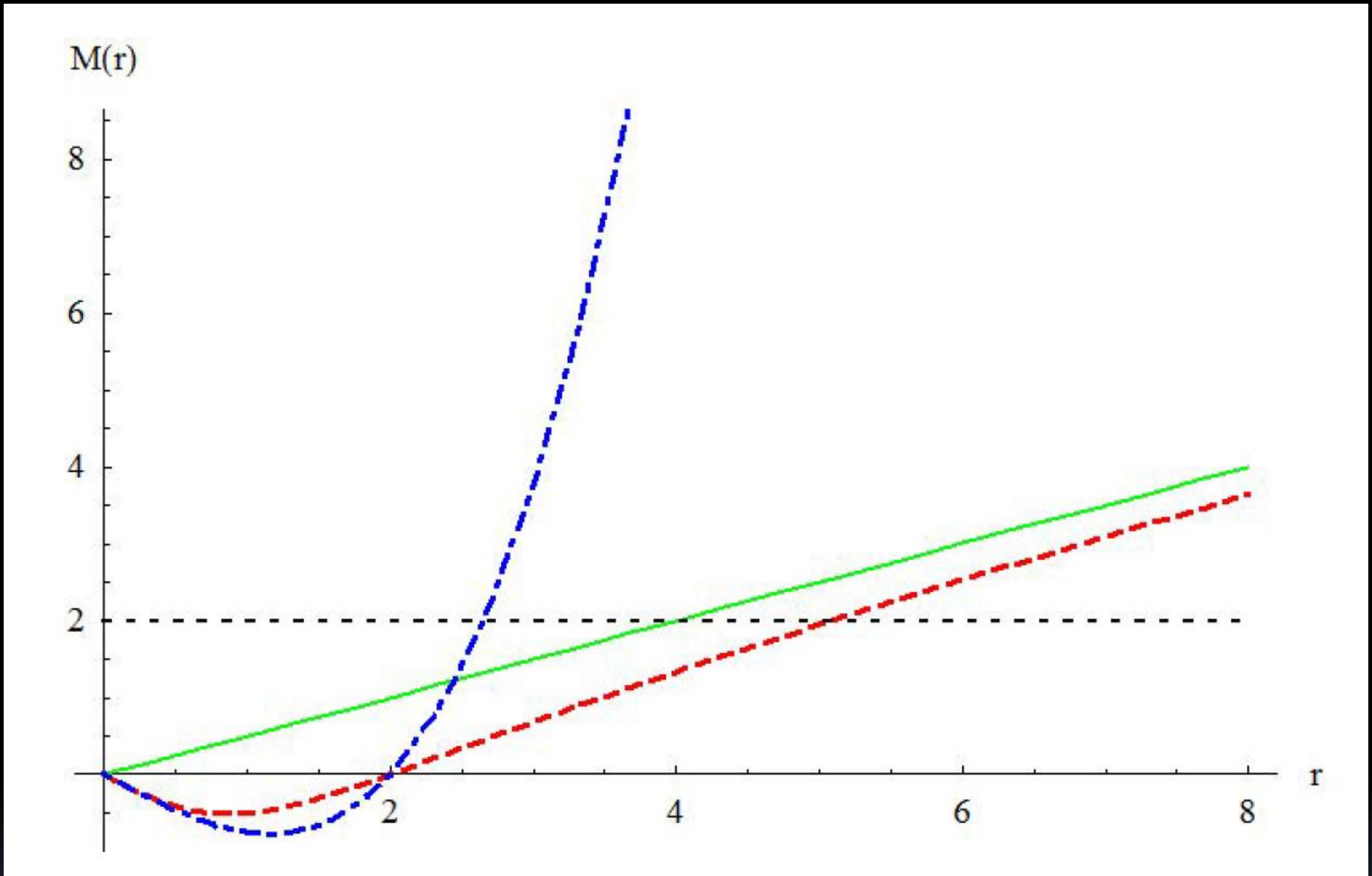
$$r_0 \simeq (\gamma|\tilde{\omega}|G^2\hbar M)^{1/3} + \frac{|\tilde{\omega}|G\hbar}{3(\gamma|\tilde{\omega}|G^2\hbar M)^{1/3}}$$

for $M \to \infty$

Note that, for both limits $M \to \infty$ or $M \to 0$, we always have $r_o < r_h$

So the ***singularity results always to be protected by the horizon***.

Horizon & Singularity Mass Functions



$$GM(r_h) = \left.\frac{r^3 - |\tilde{\omega}|\ell_P^2 r}{2r^2 + |\tilde{\omega}|\ell_P^2\gamma}\right|_{r=r_h}$$

RED LINE

$$GM(r_0) = \left.\frac{r^3 - |\tilde{\omega}|\ell_P^2 r}{|\tilde{\omega}|\ell_P^2\gamma}\right|_{r=r_0}$$

BLUE LINE

A METRIC FOR THE PLANCK STARS

The central hard singularity is located at

$$r = r_o = \, [\, \gamma \, |\widetilde{\omega}| \, \ell_p^2 \, G \, M \,]^{1/3} \; > \; 0$$

(Schwarzschild: $r_o = 0$). Finite size of QUANTUM ORIGIN: $r_o \to 0$ for $\hbar \to 0$

Consider the whole collapsing mass M as concentrated into the central hard sphere of radius r_o. Its (non covariant) volume, and hence the density of this matter (as seen by an observer at infinity), results to be finite:

$$\varrho = \frac{M}{V_{core}} = \frac{3}{2\pi} \frac{m_p}{\gamma|\tilde{\omega}|\ell_P^3} \simeq \frac{m_p}{2\gamma|\tilde{\omega}|\ell_P^3} = \frac{\varrho_{Planck}}{2\gamma|\tilde{\omega}|} \;\Rightarrow\; \varrho_{Planck} = \frac{m_p}{\ell_P^3}$$

This BH has a central hard kernel of finite size,

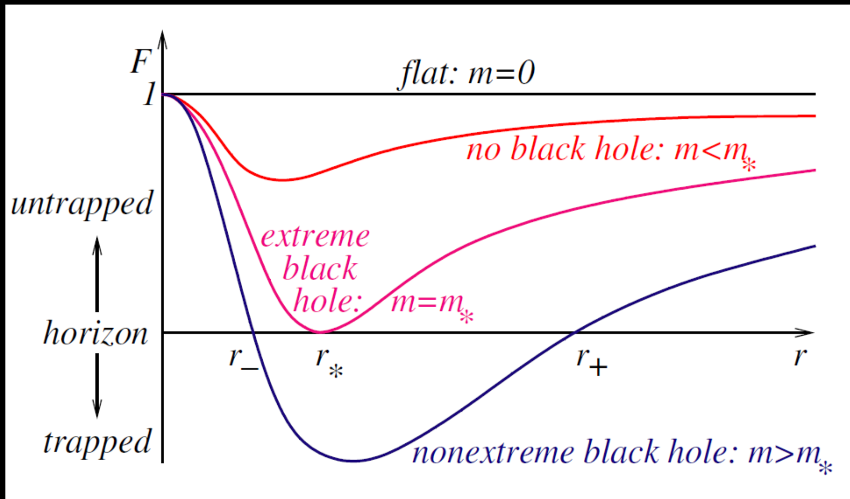
with density ~ Planck density.

These are exactly the characteristics of Planck stars, proposed by Rovelli in 2014, on the ground of general qualitative considerations.

The finite size of the central core, being of pure quantum origin, is presumably due to the action of the Heisenberg uncertainty principle, which prevents matter to be arbitrarily concentrated into a geometrical point of size zero.

The central kernel can in principle keep trace of the information swallowed by the black hole: a possible way out of the information paradox? (Of course $\gamma > 0$ strictly.)

COMPARISON WITH (MODIFIED) HAYWARD METRIC



Original "educated guess" to describe a Planck star: the Hayward metric

$$F(r) = 1 - \frac{2GMr^2}{r^3 + 2GML^2}$$

No particular compelling argument apart that Hayward metric is an example of **singularity-free** metric

This metric presents two problems:

- No $\mathbf{1/r^3}$ term in the large *r* expansion, to match the quantum corrections to Schwarzschild metric
- Horizons for large M: $r_- \simeq L + \frac{L^2}{4GM}$ $r_+ \simeq 2GM - \frac{L^2}{2GM}$

Even assuming $\boldsymbol{r_-}$ as the size of the hard kernel of a Planck star, certainly such hard kernel does not increase in size with an increasing M, as instead $\boldsymbol{r_o}$ does.

These odds have been cured by hand but **at the price** of a much more complicate lapse

$$ds^2 = -H(r)F(r)dt^2 + \frac{1}{F(r)}dr^2 + r^2 d\Omega^2, \quad \text{with} \quad H(r) = 1 - \frac{\beta GM\alpha}{\alpha r^3 + \beta GM}$$

CONCLUSIONS

- The SDG(ASG) approach suggests a general metric describing quantum effects (containing ħ)
- The matching of a SDG-metric with Donoghue quantum corrections to Schwarzschild metric suggests $\tilde{\omega} < 0$
- This $\tilde{\omega} < 0$ implies a new geometry for SDG black hole metric.
- Prediction of a **finite-size singularity** at the **core** of the black hole.
- The size of this "black core" describes exactly the **Planck stars**. Introduced years ago via semi-qualitative arguments, while in our context they appear as a natural **consequence** of a SDG metric with $\tilde{\omega} < 0$
- Potentially rich phenomenology:

- Black hole evaporation, with the final explosion occurring at scales much bigger than the Planck scale.

- Naturally associated with short gamma-ray bursts (SGRBs) (several bursts per day from region around us ~100 l.y.).

- Also with fast radio bursts, of extragalactic origin, with wavelengths close to the expected size of the exploding hole.