
北京大學
PEKING UNIVERSITY

# Superradiant Instabilities of Bosons around ECOs

Kavli Institute for Astronomy and Astrophysics

**Speaker: Lijing Shao（邵立晶）**

ShanghaiTech University (2024)

# Disclaimer

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- **However**, I view modified gravity at classical and quantum levels goes hand by hand, not just at an **EFT** sense $\Leftarrow$ I could certainly be wrong at this point

# Collaborators

- **Richard Brito** (CENTRA, Instituto Superior Técnico, Lisbon Portugal)
- **Zhan-Feng Mai** (Postdoc at PKU $\rightarrow$ Guangxi University)
- **Lihang Zhou** (Undergraduate Student at PKU $\rightarrow$ Caltech)



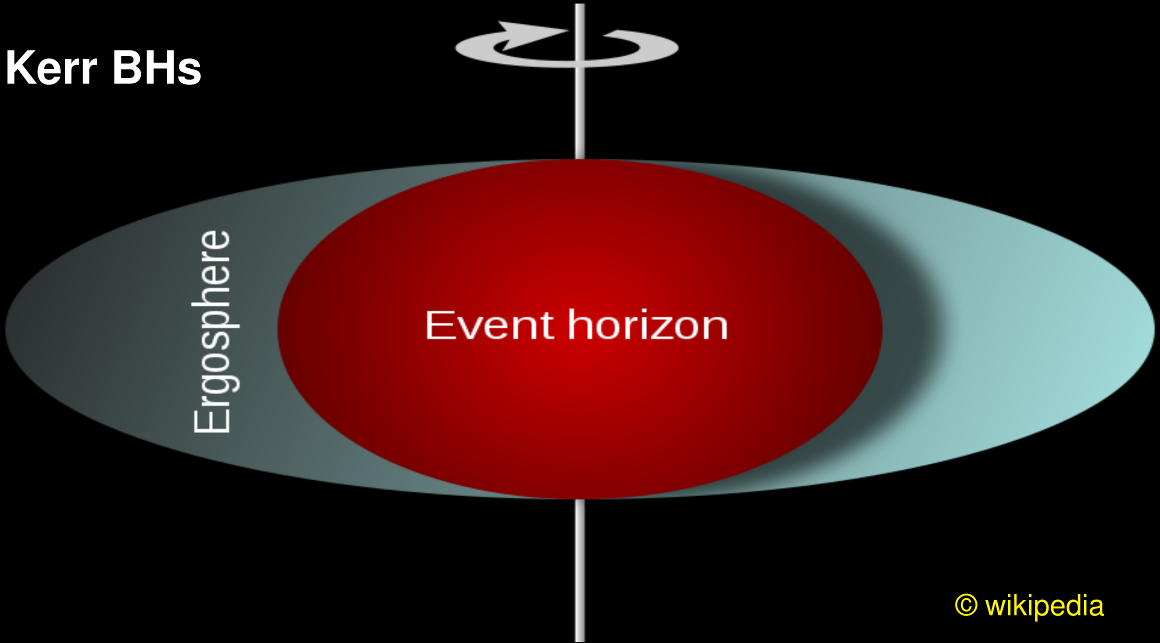
# Outline

- **Penrose process and BH superradiance**
- **Gravitational atoms**
- **Exotic compact objects (ECOs): alternatives to BHs**
- **Gravitational atoms with modified BH horizon**

**Based on:** Phys. Rev. D 108 (2023) 103025 [arXiv:2308.03091]

# 1. Penrose Process and BH Superradiance

# Kerr BHs

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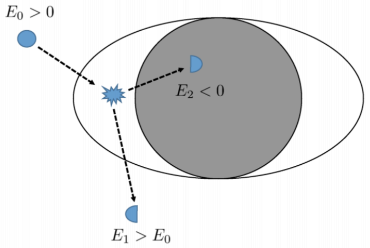
# Penrose Process

- A particle of rest mass $\mu_0$ at rest at infinity, decaying into two identical particles

$$\mathcal{E}^{(1)} + \mathcal{E}^{(2)} = \mathcal{E}^{(0)} = \mu_0 \,, \qquad \mathcal{L}^{(1)} + \mathcal{L}^{(2)} = \mathcal{L}^{(0)}$$




Brito et al. 2020 [arXiv:1501.06570]

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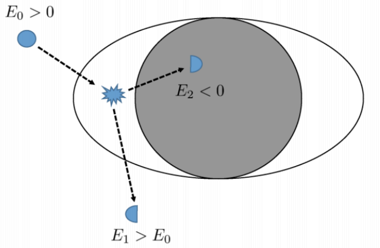
- If the decay takes place in the ergoregion, one of the particles could have a negative energy, in the view of an observer at infinity

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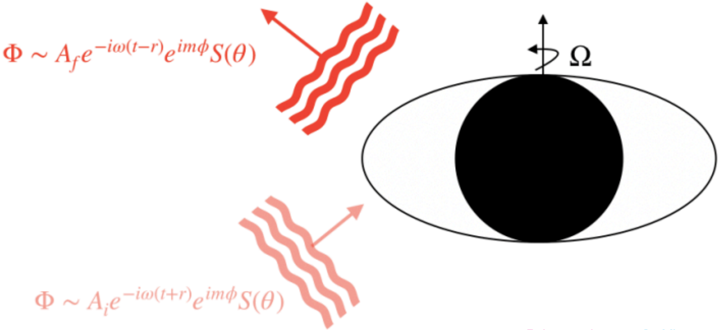
# Penrose Process: from particles to waves

- **BH superradiance**: the scattered wave gets amplified, $|A_f| > |A_i|$, when

$$\omega < m\Omega_{\rm H}$$

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# BH Superradiance

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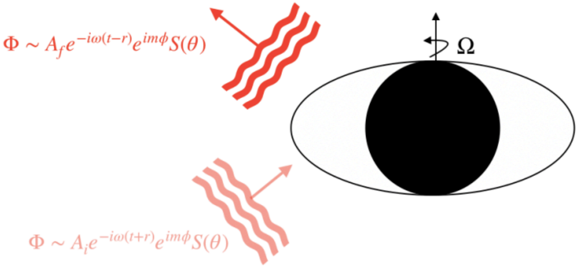
- Multiple ways to derive the SR condition

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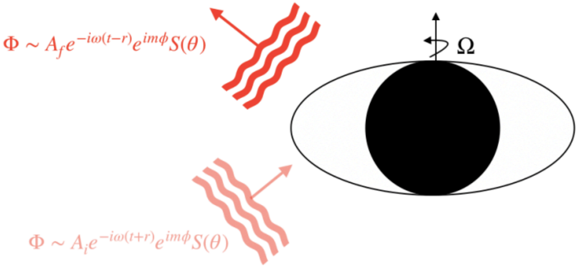


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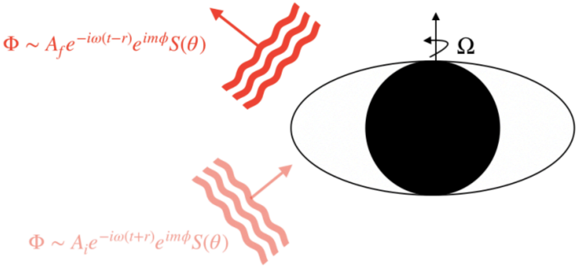


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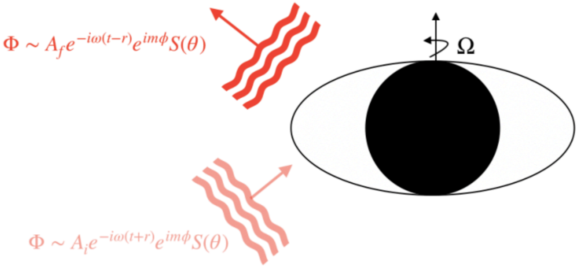
  3. Using the BH area theorem

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# BH Superradiance: BH area theorem

- The area of BH

$$A = 4\pi \left( r_+^2 + a^2 \right)$$

with

$$r_+ = M + \sqrt{M^2 - a^2 - Q^2}\,, \qquad \boldsymbol{a} = \boldsymbol{L}/M$$

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- Perform differentiation, we get **the first law of BH thermodynamics**

$$\mathrm{d}A = \frac{4A}{r_+ - r_-} \left( \mathrm{d}M - \boldsymbol{\Omega}_\mathrm{H} \cdot \mathrm{d}\boldsymbol{L} - \Phi \mathrm{d}Q \right)$$

with

$$\boldsymbol{\Omega}_\mathrm{H} = \frac{\boldsymbol{a}}{r_+^2 + a^2}, \qquad \Phi = \frac{Q r_+}{r_+^2 + a^2}$$

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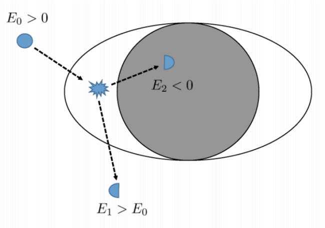
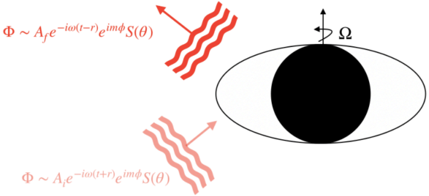
- The **BH area theorem** requires $\delta A \geq 0$, which gives

$$\omega < m\Omega_{\rm H} \quad \Rightarrow \quad \delta M \leq 0$$

# Penrose Process & BH Superradiance: subtleties

- Although similar, the **Penrose process** and the **BH superradiance** are **different** phenomena

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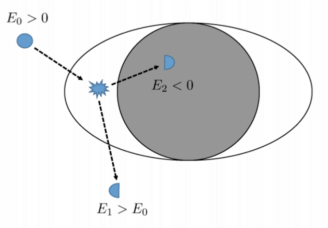


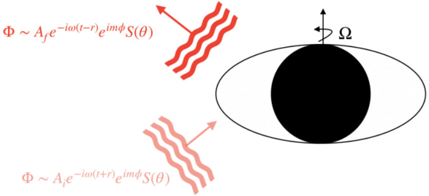


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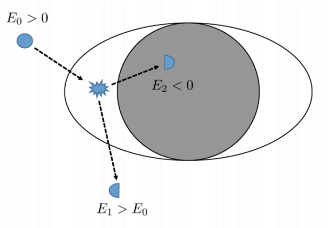
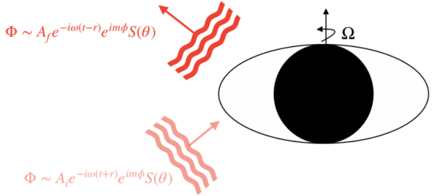
- **Penrose process** only requires the existence of an *ergoregion*, whereas the **BH superradiance** also requires some forms of *dissipation* (e.g., an event horizon)
- They are all related to the negative energy states in the ergoregion



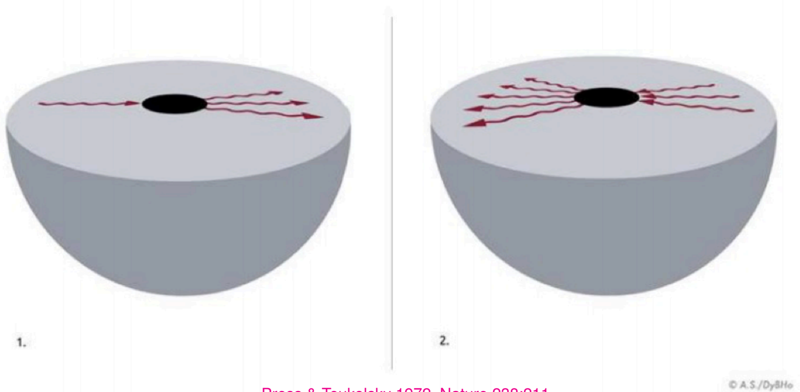




Brito et al. 2020 [arXiv:1501.06570]

# 2. Gravitational Atoms

# Black-hole Bomb

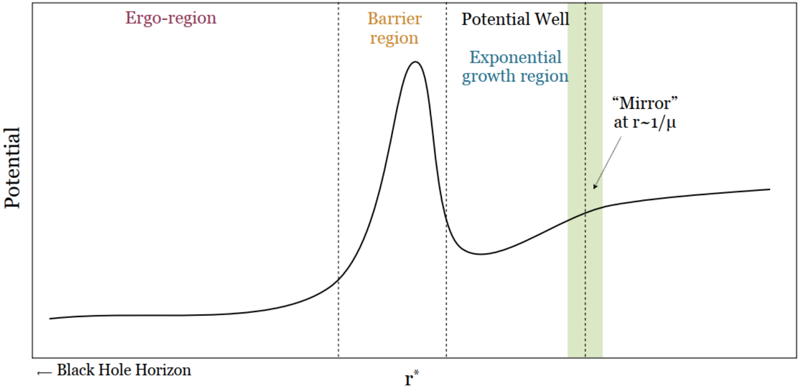
- **Superradiance + Confinement** ⇒ **Superradiant Instability**: reflected waves being amplified over and over again




Press & Teukolsky 1972, Nature 238:211

# Gravitational Atoms

- A natural **mirror**: mass $\mu$ of fields $\Rightarrow$ exponentially growing **gravitational atoms**




Arvanitaki & Dubovsky 2011 [arXiv:1004.3558]

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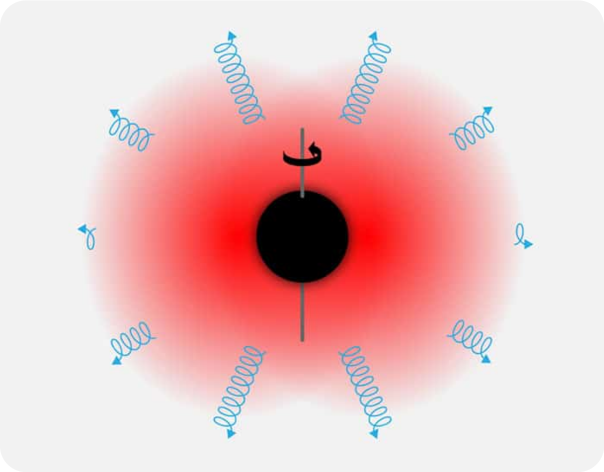

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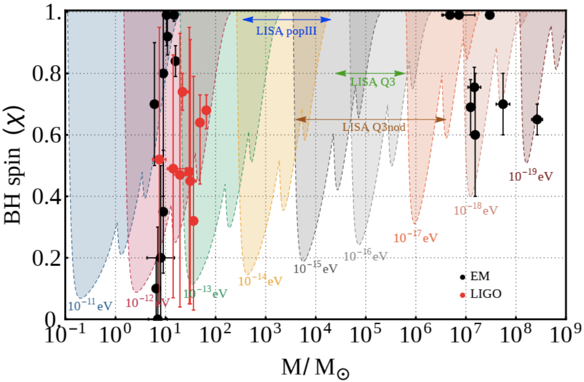
# Gravitational Atoms: consequences

- **Constraints on the mass of fundamental ultralight bosons**: measurements of $(a, M)$ of astrophysical BHs $\Rightarrow$ constraints on $\mu$




Cardoso et al. 2008 [arXiv:0709.0532]

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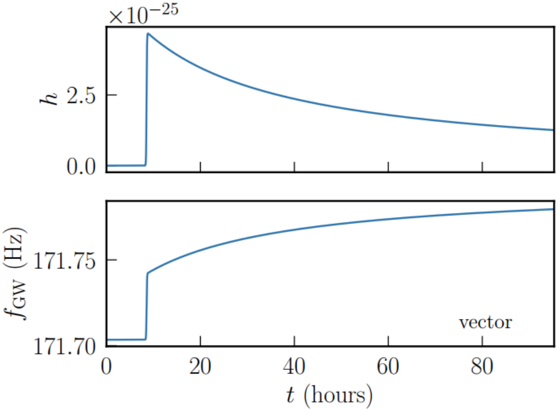
**Example:**

$$M = 62\, M_\odot$$

$$a = 0.67$$

$$\mu = 3.6 \times 10^{-13}\,\mathrm{eV}$$



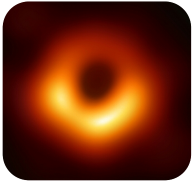

Siemonsen et al. 2023 [arXiv:2211.03845]

# 3. Exotic Compact Objects

## as alternatives to BHs

# Black Holes

- Dark, massive, and compact objects are known to exist in our universe (e.g. M87*)



Akiyama et al. 2019 [arXiv:1906.11238]

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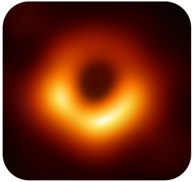

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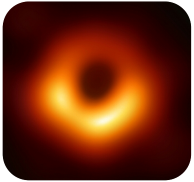

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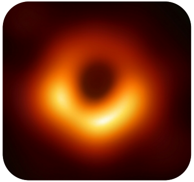

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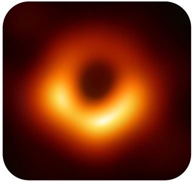

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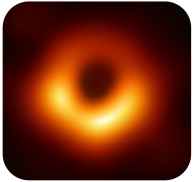
3. Astrophysical formation process is well understood
4. There are phenomena that can only be explained via **massive compact objects**



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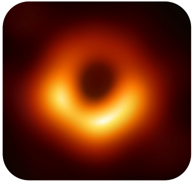
- **BHs** are so fundamental and important for gravity theories, and they should be carefully questioned and tested!



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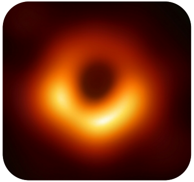

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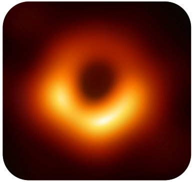

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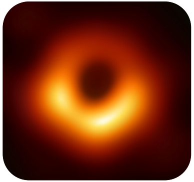

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# Alternatives to BHs

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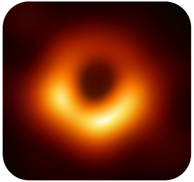
- **Motivation for considering alternatives to BH**
  1. **Pathological interior spacetime**: singularities and closed timelike curves
  2. **Information paradox**: a tremendously large entropy
  3. By considering and excluding alternatives, one gets a stronger paradigm

     $\Leftarrow$ in the same spirit of testing GR



Akiyama et al. 2019 [arXiv:1906.11238]

# Exotic Compact Objects

**A simply modified horizon**

$$r_0 = r_{\mathrm{H}}(1 + \epsilon)$$

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$$r_0 = r_{\mathrm{H}}(1 + \epsilon)$$

- **Buchdahl's theorem** (isotropic, perfect fluid, static, classical GR...): $\epsilon > 1/8$

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# Exotic Compact Objects

**A simply modified horizon**

$$r_0 = r_{\mathrm{H}}(1+\epsilon)$$

- **Buchdahl's theorem** (isotropic, perfect fluid, static, classical GR...): $\epsilon > 1/8$
- Studies on massless perturbations
  - long-lived modes
  - "echo" signals
  - ergoregion instability

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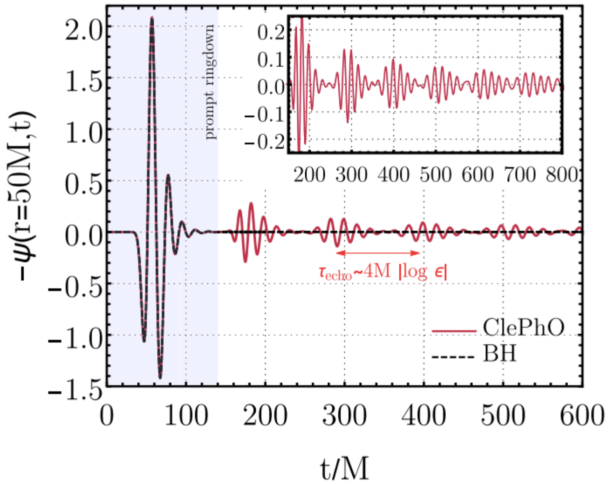
# ECOs: gravitational-wave echoes

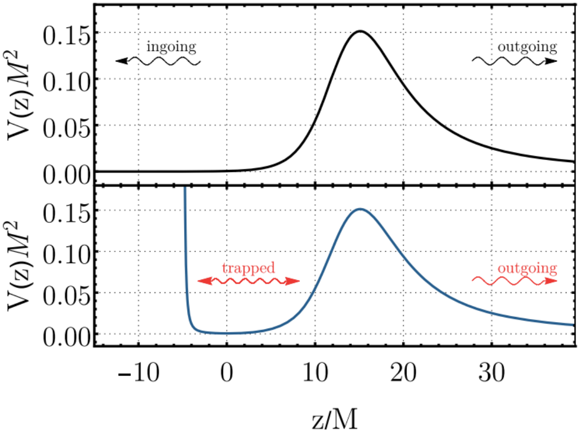



$\epsilon = 10^{-11}$

Cardoso & Pani 2019 [arXiv:1904.05363]

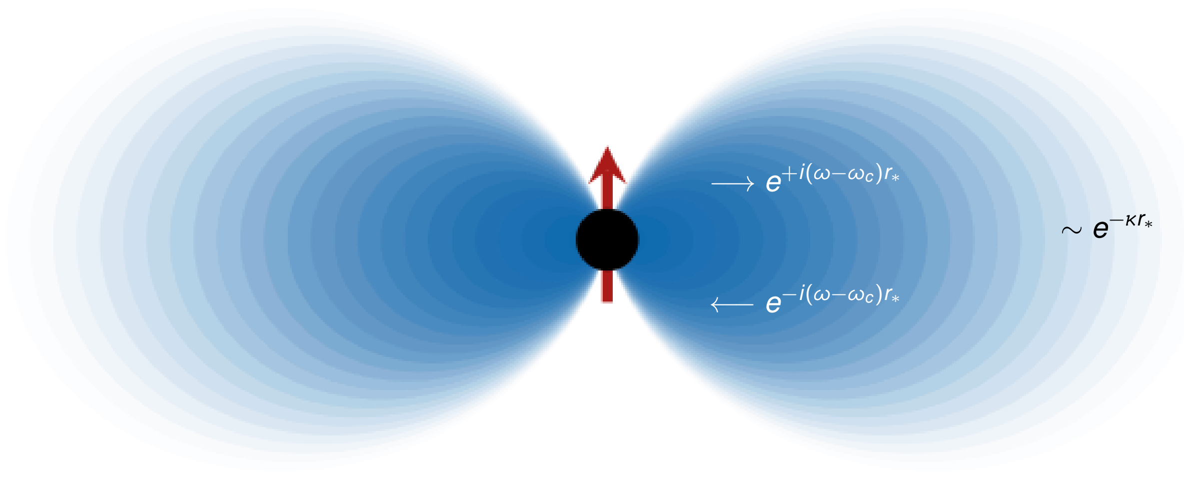
# ECOs: massive fields

- What about massive perturbations?




Cardoso & Pani 2019 [arXiv:1904.05363]; see also, Guo et al. 2022 [arXiv:2109.03376]

# 4. Grav. Atoms with Modified BH Horizon



$\longrightarrow e^{+i(\omega-\omega_c)r_*}$
$\sim e^{-\kappa r_*}$
$\longleftarrow e^{-i(\omega-\omega_c)r_*}$

# The Eigenvalue Problem

- Consider a massive scalar field in a Kerr spacetime

$$\left(\nabla_\nu \nabla^\nu - \mu^2\right) \Psi = 0$$

Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# The Eigenvalue Problem

- Consider a massive scalar field in a Kerr spacetime

$$\left(\nabla_\nu \nabla^\nu - \mu^2\right) \Psi = 0$$

- Using the Änsatz $\boxed{\Psi(t, r, \theta, \varphi) = e^{-i\omega t} e^{im\varphi} S(\theta) R(r)}$ the **angular part** reads

$$\frac{1}{\sin\theta} \frac{\mathrm{d}}{\mathrm{d}\theta} \left(\sin\theta \frac{\mathrm{d}S_{\ell m}}{\mathrm{d}\theta}\right) + \left[a^2 \left(\omega^2 - \mu^2\right) \cos^2\theta - \frac{m^2}{\sin^2\theta} + \Lambda_{\ell m}\right] S_{\ell m}(\theta) = 0$$

where $S_{\ell m}(\theta)$ is the spin-weighted spheroidal harmonics

Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# The Eigenvalue Problem

- The **radial part** reads

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta \frac{\mathrm{d}R_{\ell m}}{\mathrm{d}r}\right) + \left[\frac{\omega^2\left(r^2+a^2\right)^2 - 4Mam\omega r + m^2a^2}{\Delta} - \left(\omega^2 a^2 + \mu^2 r^2 + \Lambda_{\ell m}\right)\right] R_{\ell m}(r) = 0$$

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- Therefore, a massive boson field in the Kerr spacetime has eigenvalue $\{\Lambda_{\ell m}, \omega\}$, where

$$\Lambda_{\ell m}\approx\ell(\ell+1)+\mathcal{O}\left[a^2\left(\mu^2-\omega^2\right)\right]$$

$$\omega=\omega_R+i\Gamma$$

Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# The Radial Equation: I. analytic method

- **Far region** where $r \gg M$

$$\frac{\mathrm{d}^2[rR(r)]}{\mathrm{d}r^2} + \left[\left(\omega^2 - \mu^2\right) + \frac{2M\mu^2}{r} - \frac{\ell(\ell+1)}{r^2}\right][rR(r)] = 0$$

Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# The Radial Equation: I. analytic method

- **Far region** where $r \gg M$

$$\frac{\mathrm{d}^2[rR(r)]}{\mathrm{d}r^2} + \left[\left(\omega^2 - \mu^2\right) + \frac{2M\mu^2}{r} - \frac{\ell(\ell+1)}{r^2}\right][rR(r)] = 0$$

- **Near region** where $r \sim r_+$

$$z(z+1)\frac{\mathrm{d}}{\mathrm{d}z}\left[z(z+1)\frac{\mathrm{d}R}{\mathrm{d}z}\right] + \left[P^2 - \ell(\ell+1)z(z+1)\right]R = 0$$

where we have defined

$$z = \frac{r - r_+}{r_+ - r_-}$$

Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# The Radial Equation: I. analytic method

- **Boundary conditions**

$$R \to 0, \qquad r \to \infty$$

$$R \propto e^{-i(\omega-\omega_c)\Delta r_*} + \mathcal{K} e^{+i(\omega-\omega_c)\Delta r_*}, \qquad \Delta r_* \to 0$$

where

$$\Delta r_* \equiv r_* - r_*(r_0)$$

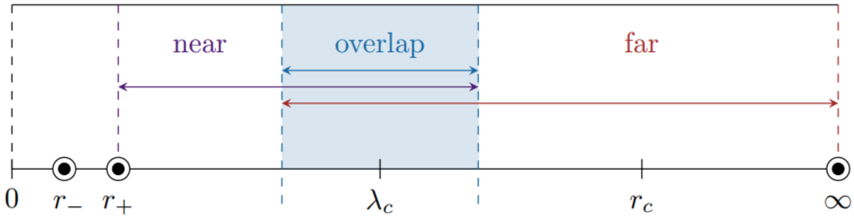
and $|\mathcal{K}|$ denotes the proportion of the incident wave reflected at $r_0$

Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# The Radial Equation: I. analytic method

- In our analytic method, **near** and **far** solutions are matched in the **overlap region**, to their leading order terms of $r^\ell$ and $r^{-\ell-1}$

$$R_{\rm near} \sim \alpha_1 r^\ell + \beta_1 r^{-\ell-1} + \cdots$$
$$R_{\rm far} \sim \alpha_2 r^\ell + \beta_2 r^{-\ell-1} + \cdots$$




Baumann et al. 2019 [arXiv:1908.10370]; Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# The Radial Equation: I. analytic method

- Our key result from the analytic method

$$\frac{M\omega_I}{g(\mathcal{K})} = \alpha^{4\ell+5}\left(\frac{ma}{2M} - \omega_R r_+\right)\left(1 - \frac{a^2}{M^2}\right)^{\ell} \frac{2^{4\ell+2}(2\ell+n+1)!}{(\ell+n+1)^{2\ell+4}n!}\left[\frac{\ell!}{(2\ell+1)!(2\ell)!}\right]^2 \prod_{j=1}^{\ell}\left(j^2+4P^2\right)$$

where the right-hand side is the **known result** for Kerr BH spacetime (Detweiler 1980), and

$$g(\mathcal{K}) = \frac{1-|\mathcal{K}|^2}{1+|\mathcal{K}|^2 + 2\mathrm{Re}\left(A^2 z_0^{2iP}\mathcal{K}\right)/|A|^2}$$

Here, $P$ and $A$ depend on $a$ and $\mu$

Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# The Radial Equation: I. analytic method

$$g(\mathcal{K}) = \frac{1 - |\mathcal{K}|^2}{1 + |\mathcal{K}|^2 + 2\mathrm{Re}\left(A^2 z_0^{2iP}\mathcal{K}\right)/|A|^2}$$

- Range of denominator: $\left[(1-|\mathcal{K}|)^2, (1+|\mathcal{K}|)^2\right]$

$$|\mathcal{K}| = 0 \quad \Rightarrow \quad g(\mathcal{K}) = 1 \qquad \text{(Kerr BHs)}$$

$$0 < |\mathcal{K}| < 1 \quad \Rightarrow \quad \frac{1-|\mathcal{K}|}{1+|\mathcal{K}|} \leq g(\mathcal{K}) \leq \frac{1+|\mathcal{K}|}{1-|\mathcal{K}|}$$

$$|\mathcal{K}| = 1 \quad \Rightarrow \quad g(\mathcal{K}) = 0 \qquad \text{(effectively)}$$

Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

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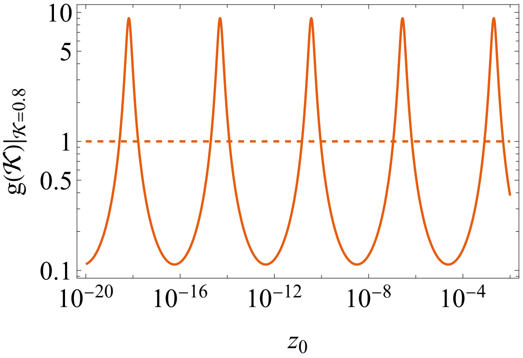


Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

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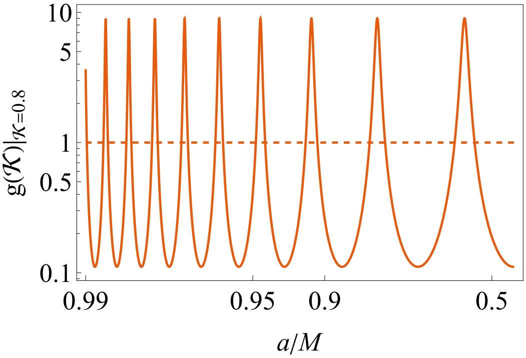


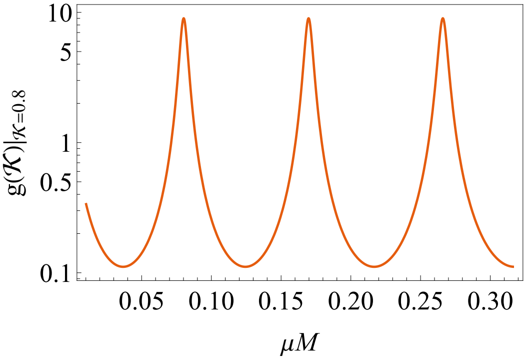


Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

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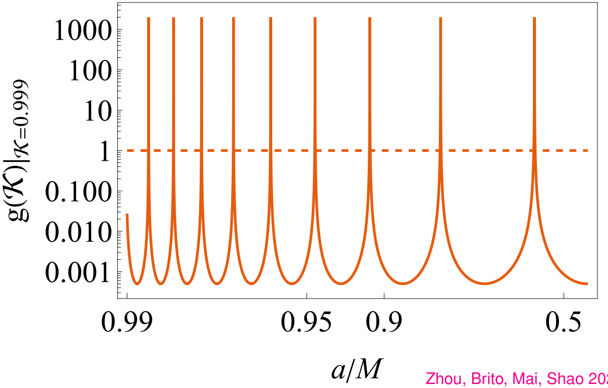



when $|\mathcal{K}| \sim 1$

Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# The Radial Equation: II. semi-analytic method

- We can match not only the leading terms $r^{\ell}$ and $r^{-\ell-1}$ of the near and far solutions, but also their full expressions, using special functions

$$W(r) \equiv \frac{\mathrm{d}R_{\mathrm{near}}}{\mathrm{d}r} R_{\mathrm{far}} - R_{\mathrm{near}} \frac{\mathrm{d}R_{\mathrm{far}}}{\mathrm{d}r}$$

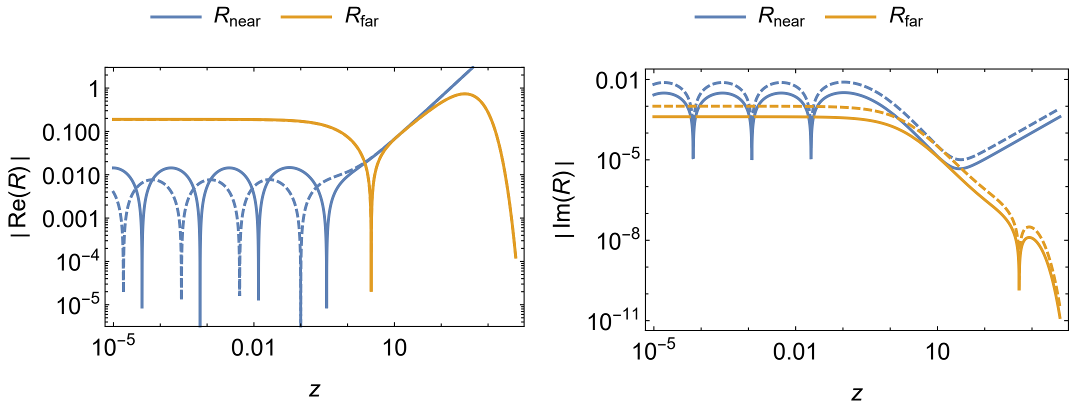
- We choose $r_{\mathrm{match}}$ with the idea of "equal errors" Arvanitaki & Dubovsky 2011 [arXiv:1004.3558]

$$W\left(r_{\mathrm{match}}\right) = 0$$

Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# The Radial Equation: II. semi-analytic method

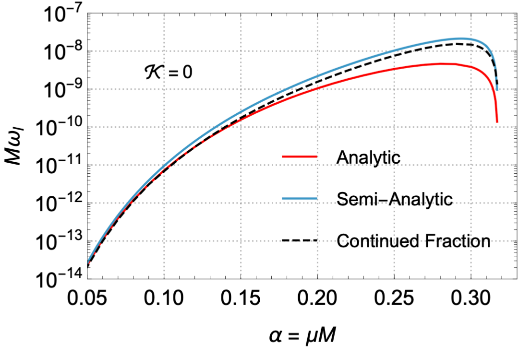
- Solid lines $\mathcal{K} = 0.8$ versus Dashed lines $\mathcal{K} = 0$ (BH)

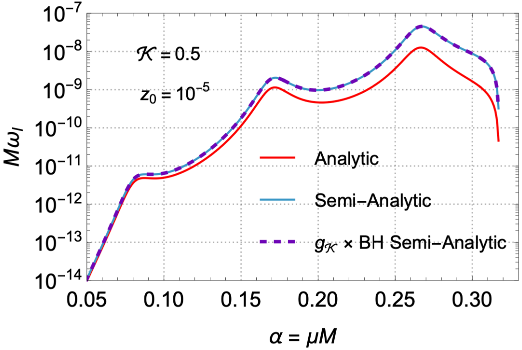



Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# The Radial Equation: III. comparison of methods

- **Analytic method**: explicit expression & physical meaning
- **Semi-analytic method**: better accuracy




Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

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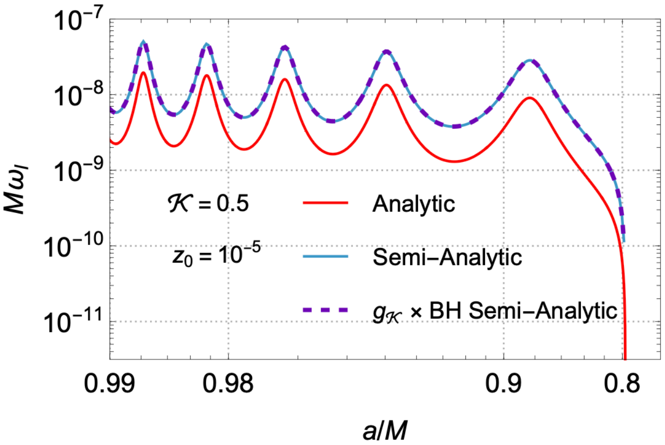
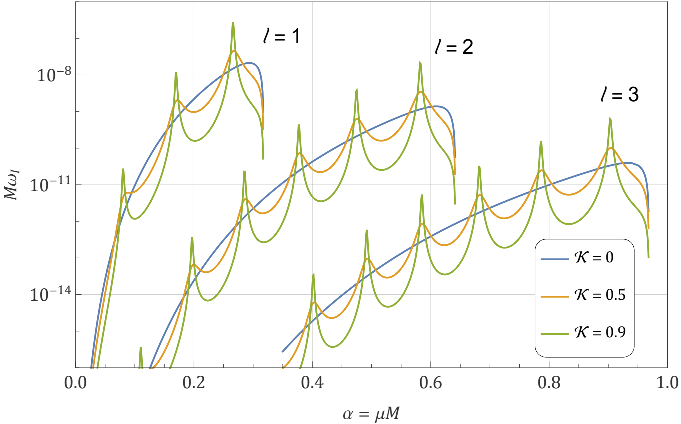



Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# Growth Rate

- Semi-analytic method




Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

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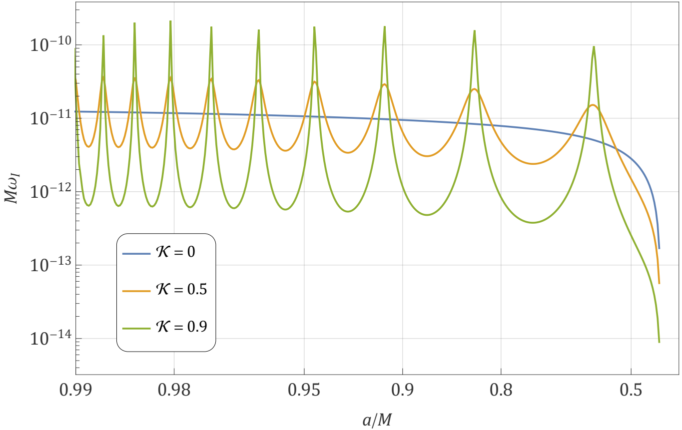


Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# Physical Interpretation of $g(\mathcal{K})$

- Energy-momentum tensor is $T^{\mu\nu} = \partial^{(\mu}\Psi\partial^{\nu)}\Psi^* - \frac{1}{2}g^{\mu\nu}\left(\partial^\rho\Psi\partial_\rho\Psi^* + \mu^2\Psi^*\Psi\right)$

Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

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- Using Gauss's theorem

$$\frac{\partial}{\partial\tilde{t}}\int_{\mathrm{3D}} T_0^0\rho^2\sin\theta \mathrm{d}r\,\mathrm{d}\theta\mathrm{d}\tilde{\varphi} = \int_{\mathrm{2D}} T_0^1\rho^2\sin\theta\mathrm{d}\theta\mathrm{d}\tilde{\varphi}$$

Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

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- Using the above **energy balance** at the boundary $r = r_0$ with the radial function $R_{\ell m}(r) = C_{\ell m}\left[(z/z_0)^{iP} + \mathcal{K}\,(z/z_0)^{-iP}\right]$ we get

$$2\omega_I = \frac{\omega_R\,(2Mr_+\omega_R - ma)}{\int_{3\mathrm{D}} T^0_0\big|_{\tilde{t}=0}\rho^2\sin\theta\mathrm{d}r\mathrm{d}\theta\mathrm{d}\tilde{\varphi}}|C_{\ell m}|^2\left(1 - |\mathcal{K}|^2\right)$$

Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# Physical Interpretation of $g(\mathcal{K})$

- Blue factor $|C_{\ell m}|^2$ gives the “relative amplitude” of the wave function at $r_0$

Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

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Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

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$$g(\mathcal{K}) = \frac{1 - |\mathcal{K}|^2}{1 + |\mathcal{K}|^2 + 2\mathrm{Re}\left(A^2 z_0^{2iP} \mathcal{K}\right) / |A|^2}$$

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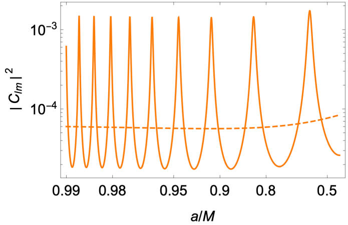


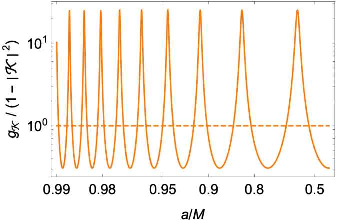


Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# Influence of Reflection on Cloud's Evolution

- Cloud growth is bumpy

$$\frac{1-|\mathcal{K}|}{1+|\mathcal{K}|}\omega_l^{\mathrm{BH}} < \omega_l^{\mathrm{ECO}} < \frac{1+|\mathcal{K}|}{1-|\mathcal{K}|}\omega_l^{\mathrm{BH}}$$

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Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

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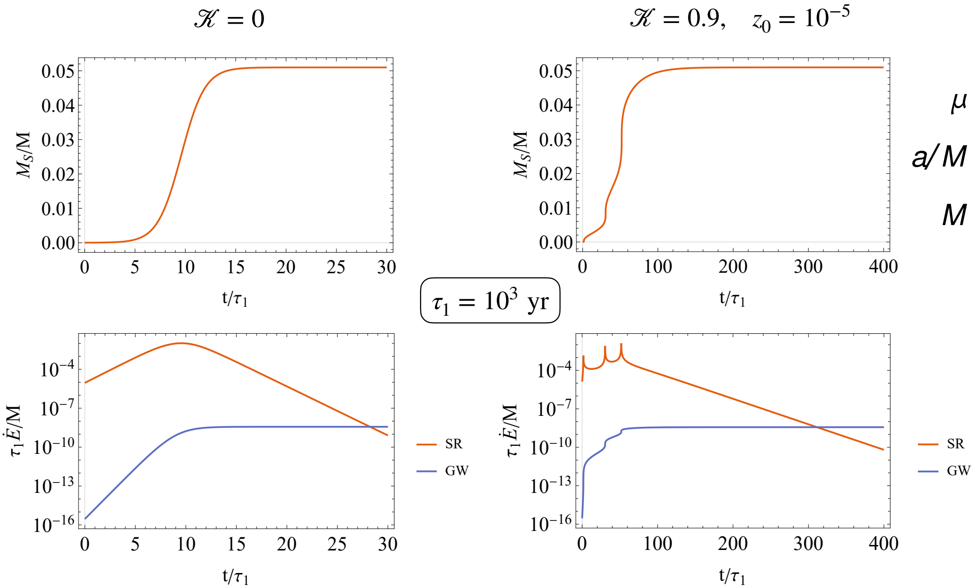
- The maximal mass does not change, but the timescale is changed
- Even with a boundary reflection, the GW dissipation is always marginal before the saturation of superradiance

Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]




$\mu = 10^{-15}\,\mathrm{eV}$

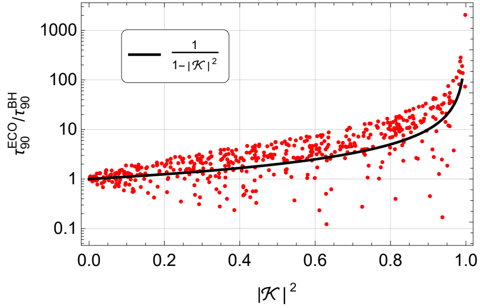
$a/M = 0.9$

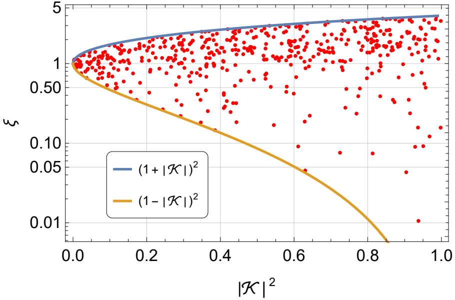
$M = 10^4\,M_\odot$

Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# Influence of Reflection on Cloud's Evolution

$$\omega_l^{\rm ECO} = \omega_l^{\rm BH} \frac{1 - |\mathcal{K}|^2}{1 + |\mathcal{K}|^2 + 2{\rm Re}\left(A^2 z_0^{2iP} \mathcal{K}\right)/|\mathcal{K}|^2} \quad \Rightarrow \quad \frac{\tau_{90}^{\rm ECO}}{\tau_{90}^{\rm BH}} = \xi \frac{1}{1 - |\mathcal{K}|^2}$$

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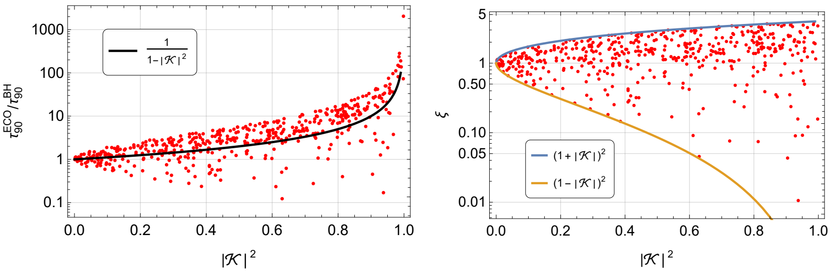
# Influence of Reflection on Cloud's Evolution

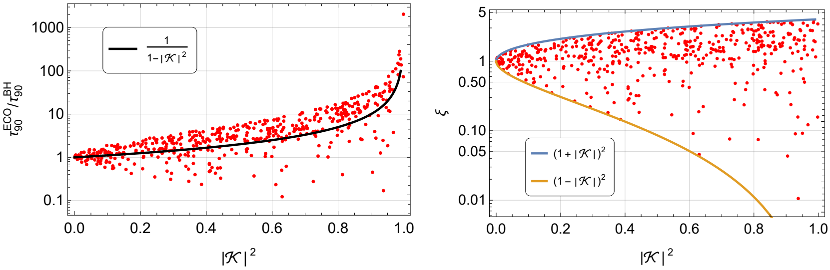
- In most cases, the growth timescale is prolonged by a boundary reflection




Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# Influence of Reflection on Cloud's Evolution

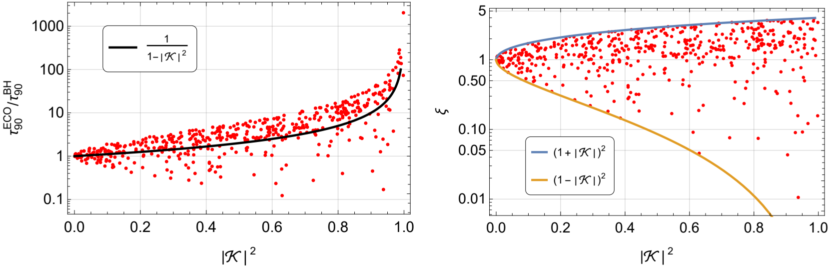
- In most cases, the growth timescale is prolonged by a boundary reflection
- For $|\mathcal{K}|^2 < 0.6|$, the change is within an order of magnitude




Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# Influence of Reflection on Cloud's Evolution

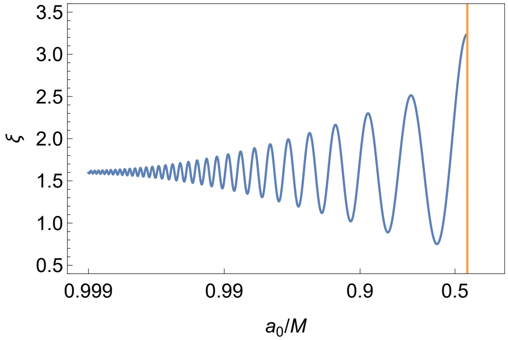
- In most cases, the growth timescale is prolonged by a boundary reflection
- For $|\mathcal{K}|^2 < 0.6|$, the change is within an order of magnitude
- Most samples have $0.1 < \xi < 1$ including samples with large $|\mathcal{K}|$, indicating that $\tau_{90}^{\rm BH} / \left(1 - |\mathcal{K}|^2\right)$ is a good estimation for the timescale of ECOs

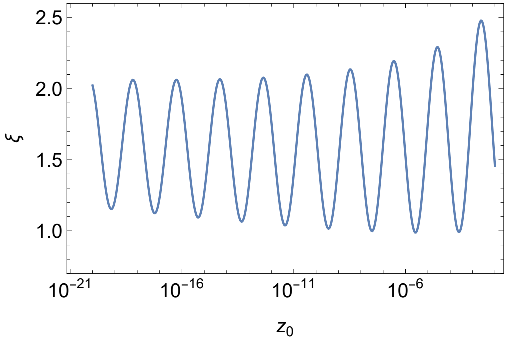



Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# Influence of Reflection on Cloud's Evolution

- Outliers: $\xi$ could be far away from 1 only when $a_0$ is near the criticality of superradiance

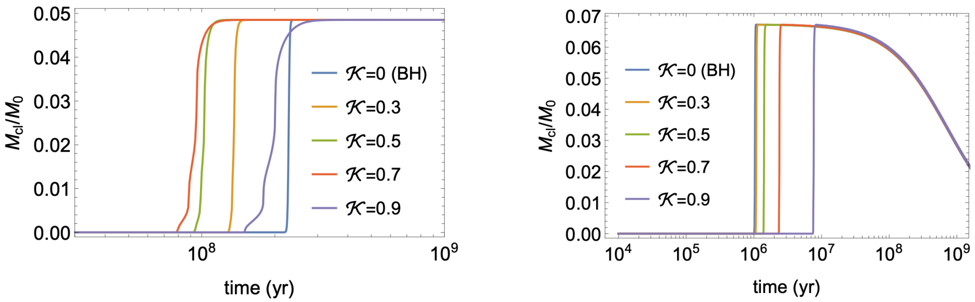



Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# Influence of Reflection on Cloud's Evolution

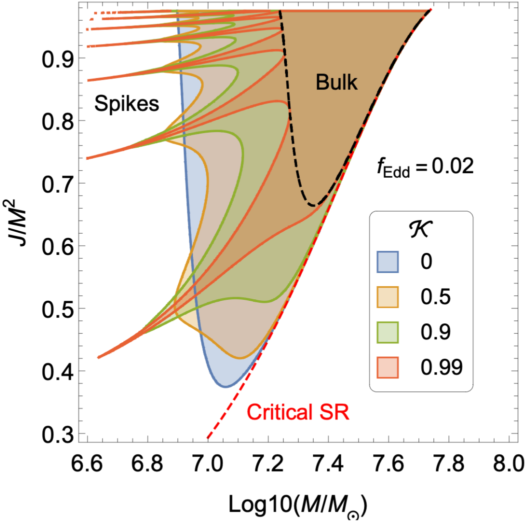
$\mu = 1 \times 10^{-18}\,\mathrm{eV}$ $\mu = 2 \times 10^{-18}\,\mathrm{eV}$




Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# Exclusion Regions

- For a $10^6\,M_\odot$ ECO, it takes about 205 e-folds to grow a $0.1\,M_\odot$ cloud, when $\mu \sim 10^{-18}\,\mathrm{eV}$

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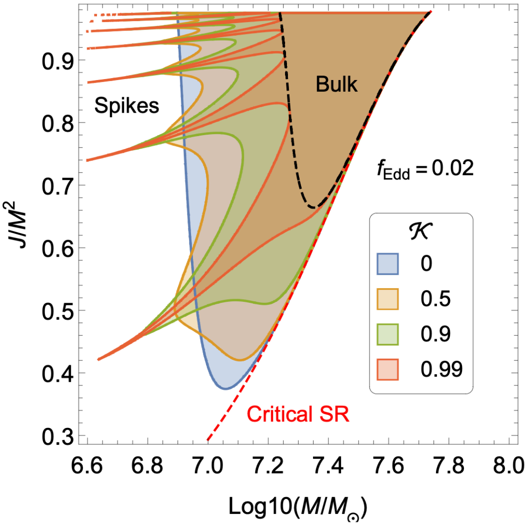
- For a $10^6\, M_\odot$ ECO, it takes about 205 e-folds to grow a $0.1\, M_\odot$ cloud, when $\mu \sim 10^{-18}\,\mathrm{eV}$
- We define **fast-superradiance regime**

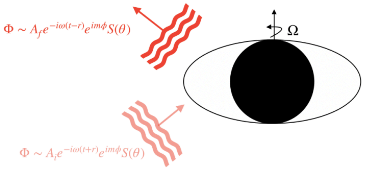
$$205 \ln\left(\frac{M}{10^6 M_\odot}\frac{10^{-18}\mathrm{eV}}{\mu}\right)\tau_{\mathrm{SR}} < \tau_{\mathrm{Acc}}$$

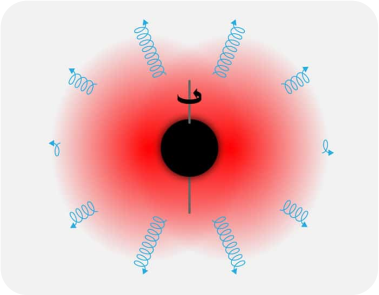



Zhou, Brito, Mai, Shao 2023 [arXiv:2308.03091]

# Summary

- In the background of ECOs, which has **a reflective boundary** as alternatives to the event horizon, we calculated the **growth rate** of **gravitational atoms**



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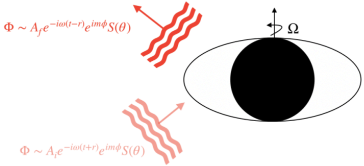
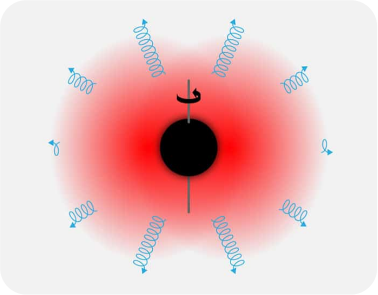



- In the background of ECOs, which has **a reflective boundary** as alternatives to the event horizon, we calculated the **growth rate** of **gravitational atoms**
- When $\alpha \ll 1$ holds, $\omega_l^{\rm ECO}$ can be connected to its BH counterpart with a factor $g(\mathcal{K})$



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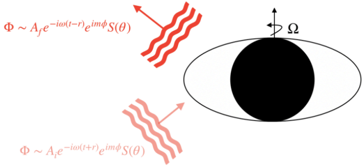
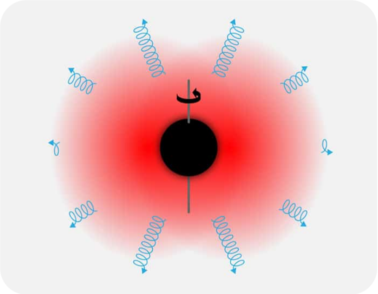
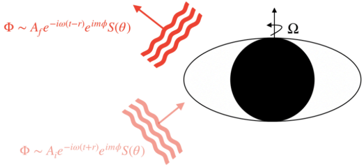


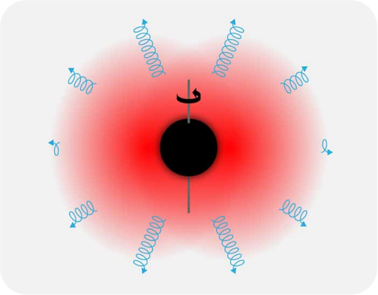

- In the background of ECOs, which has **a reflective boundary** as alternatives to the event horizon, we calculated the **growth rate** of **gravitational atoms**
- When $\alpha \ll 1$ holds, $\omega_I^{\mathrm{ECO}}$ can be connected to its BH counterpart with a factor $g(\mathcal{K})$
- For larger $\alpha$, although the analytic method doesn't hold, the feature of $g(\mathcal{K})$ is maintained in the growth rate calculated with the **semi-analytic method**



# Summary

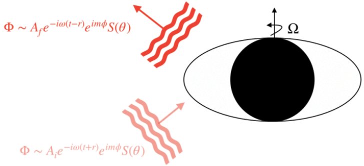
- The physical interpretation of $g(\mathcal{K})$ is made by considering **energy balance** at the boundary

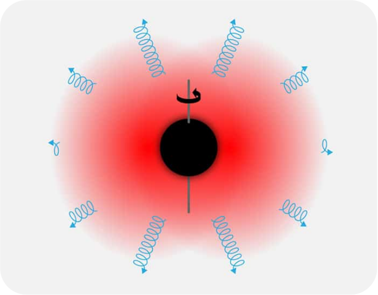


# Summary

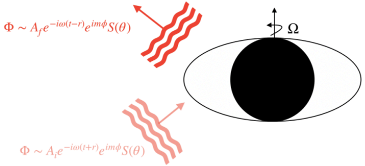
- The physical interpretation of $g(\mathcal{K})$ is made by considering **energy balance** at the boundary
- The main influence of the boundary reflection is the **prolonged growth timescale**

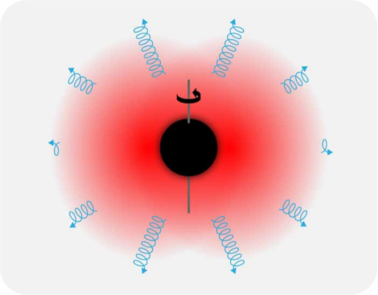


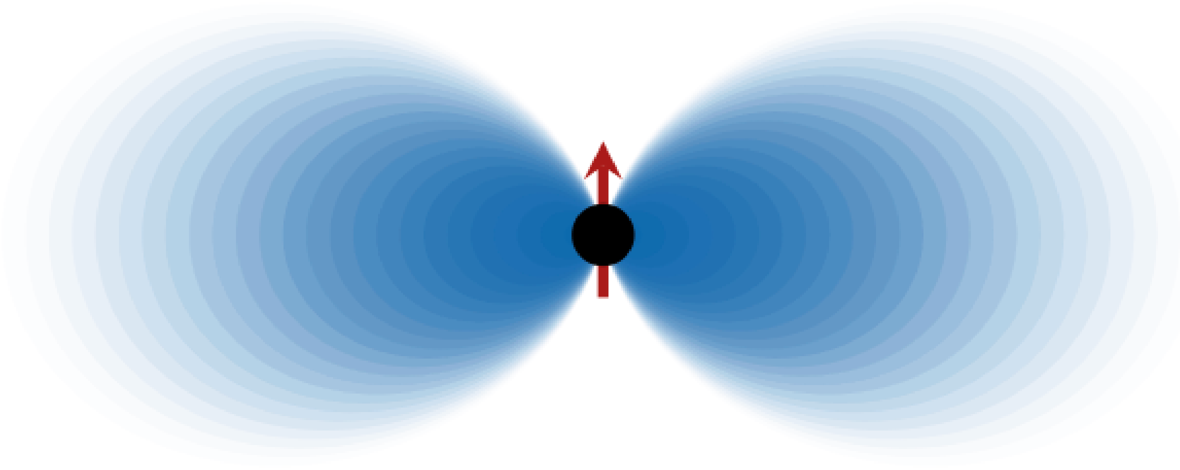



# Summary

- The physical interpretation of $g(\mathcal{K})$ is made by considering **energy balance** at the boundary
- The main influence of the boundary reflection is the **prolonged growth timescale**
- The timescale on average scales as $1/\left(1-|\mathcal{K}|^2\right)$ and the correction is always within an order of magnitude as long as $|\mathcal{K}| < 0.8$





**Thank you very much for your attention!**