Quadratic Gravity and Quantum Cosmology

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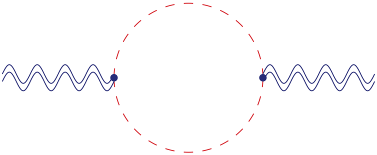
Quadratic-in-curvature terms and their relevance

$$S_{\rm quad} = \int d^4x\sqrt{-g}\,\mathscr{L}_{\rm quad}, \qquad \mathscr{L}_{\rm quad} = c_2 R^2 + c_2' R_{\mu\nu}R^{\mu\nu}$$

Such terms are unavoidably generated by matter loops, such as



At one loop

$$\begin{aligned}(4\pi)^2\frac{dc_2}{d\ln\bar\mu} &= \frac{N_V}{15}+\frac{N_f}{60}+\frac{N_s}{180}-\frac{(\delta_{ab}+6\xi_{ab})(\delta_{ab}+6\xi_{ab})}{72}\\ (4\pi)^2\frac{dc_2'}{d\ln\bar\mu} &= -\frac{N_V}{5}-\frac{N_f}{20}-\frac{N_s}{60}\end{aligned}$$

N_V, N_f, N_s are the numbers of vectors, Weyl fermions and real scalars ϕ_a with non-minimal couplings ξ_{ab} (that is $\mathscr{L} \supset -\xi_{ab}\phi_a\phi_b R/2$)

Quadratic gravity

The quadratic terms make gravity renormalizable *[Stelle (1977)]*

Intuitive reason: in the UV the theory is the most general dimensionless one

The general quadratic gravity (QG) Lagrangian:

$$\mathscr{L} = \frac{R^2}{6f_0^2} - \frac{W^2}{2f_2^2} + \mathscr{L}_{\rm EH} + \mathscr{L}_m$$

where

- $W^2 \equiv W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma}$
- $\mathscr{L}_{\rm EH}$ is the Einstein-Hilbert piece plus a cosmological constant
- $\mathscr{L}_m$ is the matter piece

Another motivation:

The Higgs mass M_h can be naturally much smaller than the Planck mass $\bar{M}_{\mathrm{Pl}}$

't Hooft definition: a physical quantity is naturally small when setting it to zero leads to enhanced symmetry.

This ensures that quantum corrections respect the smallness of the physical quantity

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The theory is renormalizable

$\Longrightarrow$ one can consistently compute the quantum corrections δM_h to the Higgs mass:

$$\delta M_h^2 \sim \frac{\bar{M}_{\rm Pl}^2 f_i^4}{(4\pi)^2}, \qquad \delta M_h^2 \lesssim M_h^2 \ \to \ f_2 \lesssim \sqrt{\frac{4\pi M_h}{\bar{M}_{\rm Pl}}} \sim 10^{-8}$$

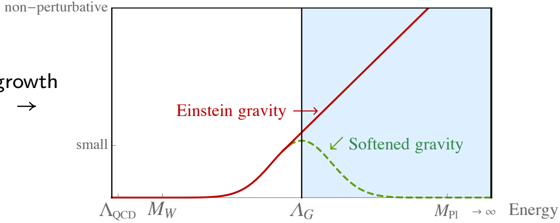
[Salvio, Strumia (2014)]

Quadratic gravity is a realization of “softened gravity”

(Einstein) gravitational interactions increase with energy

Idea (*softened gravity*):

consider theories where the growth of the gravitational coupling $\rightarrow$ *stops at some* $\Lambda_G \ll M_{\rm Pl}$.



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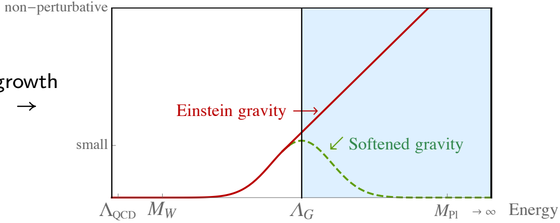

The gravitational contribution to the Higgs mass is

$$\delta M_h^2 \sim \frac{G_N \Lambda_G^4}{(4\pi)^2}$$

Requiring $\delta M_h \lesssim M_h \rightarrow \Lambda_G \lesssim 10^{11}$ GeV *[Giudice, Isidori, Salvio, Strumia (2014)]*

(the Higgs field acquires an approximate shift symmetry that protects M_h)

The Ostrogradsky theorem

<u>Classical</u> Lagrangians that depend on the second derivatives have Hamiltonians unbounded from below *[Ostrogradsky (1848)]*

Indeed, looking at the degrees of freedom (expanding around the flat spacetime):

(i) massless graviton

(ii) scalar ω with squared mass $M_0^2 \sim \frac{1}{2} f_0^2 \bar{M}_{\rm Pl}^2$

(iii) massive spin-2 field with an abnormal-sign kinetic term (ghost) and squared mass $M_2^2 = \frac{1}{2} f_2^2 \bar{M}_{\rm Pl}^2$ (a manifestation of the Ostrogradsky theorem)

This <u>abnormal</u> <u>graviton</u> is associated with $\frac{W^2}{2f_2^2}$.

By linearizing the theory one finds the spin-2 classical Hamiltonians *[Salvio (2017)]*

$$H_{\rm graviton} = \sum_{\lambda=\pm 2} \int d^3q \left[P_\lambda^2 + q^2 Q_\lambda^2\right]$$

$$H_{\rm ghost} = -\sum_{\lambda=\pm 2,\pm 1,0} \int d^3q \left[\tilde{P}_\lambda^2 + (q^2 + M_2^2)\tilde{Q}_\lambda^2\right]$$

Proceeding perturbatively

Let us split the metric $g_{\mu\nu}$ as follows:

$$g_{\mu\nu} = g^{\mathrm{cl}}_{\mu\nu} + \hat{h}_{\mu\nu}$$

- $g^{\mathrm{cl}}_{\mu\nu}$ is a classical background that solves the classical EOMs
- $\hat{h}_{\mu\nu}$ is a *quantum* fluctuation

Classical theory

Can we avoid the possible Ostrogradsky instabilities?

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- The weak coupling case $f_2 \ll 1$ (compatible with Higgs naturalness) must have an energy thresholds much larger than M_2:
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This argument can be made precise in classical quadratic gravity. The whole cosmology can only involve energies below this threshold and avoid runaways

→ "metastability in quadratic gravity"

[Salvio (2019)] see also *[dos Reis, Chapiro, Shapiro (2019); Gross, Strumia, Teresi, Zerilli (2020); Held, Lim (2021); Held, Lim (2023)]*

This can be shown with a two-derivative formulation

One separates the two-derivative d.o.f.: ω, ordinary and abnormal gravitons

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First perform the field redefinition $\quad g_{\mu\nu} \to \frac{\bar{M}_{\rm Pl}^2}{f} g_{\mu\nu}, \qquad f \equiv \bar{M}_{\rm Pl}^2 - \frac{2R}{3f_0^2} > 0,$

(where the Ricci scalar above is computed in the Jordan frame metric) that gives

$$S = \int d^4x\sqrt{-g}\left(-\frac{W^2}{2f_2^2} - \frac{\bar{M}_{\rm Pl}^2}{2}R + \mathscr{L}_m^E\right) \qquad \text{“Einstein frame action”}$$

The Einstein-frame matter Lagrangian, $\mathscr{L}_m^E$, also contains an effective scalar ω (a.k.a. the scalaron), which corresponds to the R^2 term in the Jordan frame:

$$\mathscr{L}_\omega^E = \frac{(\partial\omega)^2}{2} - U(\omega), \qquad U(\omega) = \frac{3f_0^2\bar{M}_{\rm Pl}^4}{8}\left(1 - e^{-2\omega/\sqrt{6}\bar{M}_{\rm Pl}}\right)^2$$

To make the abnormal graviton explicit consider an auxiliary field $\gamma_{\mu\nu}$:

$$S = \int d^4x\sqrt{-g}\left\{\frac{M_2^2\bar{M}_{\rm Pl}^2}{8}\left[\gamma_{\mu\nu}\gamma^{\mu\nu} - (\gamma_{\mu\nu}g^{\mu\nu})^2\right] - \frac{\bar{M}_{\rm Pl}^2}{2}G_{\mu\nu}\gamma^{\mu\nu} - \frac{\bar{M}_{\rm Pl}^2}{2}R + \mathscr{L}_m^E\right\}$$

where $G_{\mu\nu}$ is the Einstein tensor.

One has a mixing between $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$ and $\gamma_{\mu\nu}$. The tensors $\bar{h}_{\mu\nu} = h_{\mu\nu} + \gamma_{\mu\nu}$ and $\gamma_{\mu\nu}$ represent the ordinary and abnormal gravitons

Interactions of the abnormal graviton and energy thresholds

The two-derivative formulation is good to understand the abnormal graviton interactions. First, one can easily see that they are suppressed by f_2

Next, $\frac{M_2^2 \bar{M}_{\rm Pl}^2}{8}\left[\gamma_{\mu\nu}\gamma^{\mu\nu} - (\gamma_{\mu\nu}g^{\mu\nu})^2\right]$ leads to mass and interaction terms of the <u>schematic</u> form

$$\frac{M_2^2}{2}\left(\phi_2^2 + \frac{\phi_2^3}{\bar{M}_{\rm Pl}} + \frac{\phi_2^4}{\bar{M}_{\rm Pl}^2} + \dots\right),$$

(ϕ_2 represents the canonically normalized spin-2 fields)

The mass term has the same order of magnitude of the interactions for $\phi_2 \sim \bar{M}_{\rm Pl}$, which gives $M_2^2\phi_2^2/2 = M_2^4/f_2^2 \equiv E_2^4$, where

$$E_2 \equiv \frac{M_2}{\sqrt{f_2}} = \sqrt{\frac{f_2}{2}}\bar{M}_{\rm Pl}$$

For energies $\boxed{E \ll E_2}$ *the Ostrogradsky instabilities are avoided*

This bound applies to the boundary conditions (BCs) of <u>derivatives of the spin-2 fields</u>.

Analogously, one can show that the energy E in the <u>matter sector</u> must satisfy

$$\boxed{E \ll E_m} \qquad E_m \equiv \sqrt[4]{f_2}\,\bar{M}_{\rm Pl} \qquad \text{(matter sector)}$$

(one has to impose it on the BCs)

Relations with chaotic inflation *[Linde (1983)]*

For a natural Higgs mass ($f_2 \sim 10^{-8}$, $M_2 \sim 10^{10}$ GeV)

$$E_2 \sim 10^{-4}\bar{M}_{\rm Pl}, \qquad E_m \sim 10^{-2}\bar{M}_{\rm Pl}$$

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But we live in one of those patches where the energy scales of inhomogeneities (E_i) and anisotropies (E_a) were small enough:

$$E_i \ll |U'(\phi)/\phi|^{1/2}, \qquad E_i, E_a \ll H$$

these conditions justify the use of homogeneous and isotropic solutions to describe the classical part of inflation (Linde's idea)

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The chaotic theory automatically ensures that the conditions to avoid runaway solutions are satisfied. The fatal runaways above the energy thresholds give an (anthropic) rationale for a homogeneous and isotropic universe
(verified for Starobinsky inflation, Higgs inflation, natural inflation and other models)

Let us go back to the the following metric splitting

$$g_{\mu\nu} = g^{\mathrm{cl}}_{\mu\nu} + \hat{h}_{\mu\nu}$$

- $g^{\mathrm{cl}}_{\mu\nu}$ is a classical background that solves the classical EOMs.
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Quantization and probabilities

Renormalizability implies that the *quantum* Hamiltonian governing $\hat{h}_{\mu\nu}$ is bounded from below *[Stelle (1977)]*

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However, the space of states must be endowed with an indefinite metric

How can we define probabilities consistently?

A derivation of probabilities

- Define observable any operator A with complete eigenstates $\{|a\rangle\}$ *[Salvio (2019)]*: for any state $|\psi\rangle$ there is a decomposition

$$|\psi\rangle = \sum_a c_a |a\rangle$$

  One can show that the basic canonical operators q, p have complete eigenstates
- Interpret $|a\rangle$ as the state where A assumes certainly the value a (deterministic Born rule)

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Experimentalists prepare a large number N of times the same state, so consider

$$|\Psi_N\rangle \equiv \nu^N \underbrace{|\psi\rangle ... |\psi\rangle}_{N \text{ times}} = \sum_{a_1 \dots a_N} \nu^N c_{a_1} ... c_{a_N} |a_1\rangle ... |a_N\rangle, \qquad \nu \equiv \frac{1}{\sqrt{\sum_b |c_b|^2}}$$

Define a frequency operator F_a which counts the number N_a of times there is the value a in the state $|a_1\rangle ... |a_N\rangle$:

$$F_a |a_1\rangle ... |a_N\rangle \equiv \frac{N_a}{N} |a_1\rangle ... |a_N\rangle$$

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One can show that $$\lim_{N\to\infty} F_a |\Psi_N\rangle = B_a |\Psi_N\rangle, \qquad B_a \equiv \frac{|c_a|^2}{\sum_b |c_b|^2}$$

(all coefficients in the basis $|a_1\rangle ... |a_N\rangle$ converge to the same quantities)

The probabilities are positive and sum up to one at any time (the theory is unitary)

The emergent norms to compute probabilities

$\{|a\rangle\}$ is complete so we can *define* a "norm" operator P_A:

$$\langle a'|P_A|a\rangle \equiv \delta_{aa'}$$

where for any pair of states $|\psi_1\rangle$, $|\psi_2\rangle$, we denote the indefinite metric with $\langle\psi_2|\psi_1\rangle$. The definition above provides a positive-definite inner product:

$$\langle\psi_2|\psi_1\rangle_A \equiv \langle\psi_2|P_A|\psi_1\rangle = \sum_a c^*_{a2} c_{a1}$$

(it is non negative for $|\psi_1\rangle = |\psi_2\rangle$)

$$B_a \equiv \frac{|c_a^2|}{\sum_b |c_b^2|} = \frac{|\langle a|\psi\rangle_A|^2}{\langle\psi|\psi\rangle_A}$$

We recover the full probabilistic Born rule, but expressed in terms of the *positive norm* not in terms of the indefinite one

Dirac-Pauli (DP) quantization of canonical variables

[Dirac (1941); Pauli (1943); Salvio, Strumia (2015)]

A is normal with respect to the A-norm $\Rightarrow A = A_h + A_a$, where A_h (A_a) is an (anti)Hermitian operator with respect to the A-norm and $[A_h, A_a] = 0$.

So we restrict to

$$A|a\rangle = \lambda_a|a\rangle, \qquad \lambda_a = \alpha_a \quad \text{or} \quad \lambda_a = i\alpha_a \quad (\text{with } \alpha_a \text{ real})$$

In quadratic gravity there are also observables that realize the second possibility. This is the only option which allows a Hamiltonian of the form

$$\hat{H} = -\frac{1}{2}\left(P^2 + \omega^2 Q^2\right)$$

to have a spectrum bounded from below *and normalizable eigenfunctions*

A non-perturbative and background-independent formulation of quadratic gravity

Key idea: construction of a consistent Euclidean path integral (PI)

To obtain the PI we need the quantum Hamiltonian. Take first the Gauss coordinates,

$$ds^2 = g_{ij}(x)dx^i dx^j - dt^2,$$

In Ostrogradsky's canonical method one can choose g_{ij} and $K_{ij} \equiv -\frac{1}{2}\dot{g}_{ij}$ as coordinates and the following fields as corresponding conjugate momenta

$$\pi^{ij} \equiv \frac{\partial \mathscr{L}}{\partial \dot{g}_{ij}} - \frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \ddot{g}_{ij}}, \quad P^{ij} \equiv \frac{\partial \mathscr{L}}{\partial \dot{K}_{ij}} - \frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \ddot{K}_{ij}}$$

The classical Hamiltonian density is then given in terms of the Lagrangian density $\mathscr{L}$:

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Idea! *DP quantize* K_{ij}. With appropriate ordering one obtains

$$\boxed{\langle q_{f\eta}, \tau_f | q_i, \tau_i \rangle^J = \int_{q(\tau_i)=q_i}^{q(\tau_f)=q_f} C\delta g \, \exp\left(-S_E/\hbar + \int_{\tau_i}^{\tau_f} d\tau \int d^3x \, J^{ij} g_{ij}\right).}$$

$$S_E = \int_{\tau_i}^{\tau_f} d\tau \int d^3x \sqrt{g}\left(\frac{\alpha}{2}W^2 + \beta R^2 + \gamma R + \lambda\right), \qquad C = \prod_{x'_E}\left[\left(\frac{\Delta V_3\sqrt{g}}{\pi\hbar\Delta\tau^3}\right)^3 \frac{2\sqrt{3\beta\alpha^5}}{g^2}\right]$$

Transition to a generic coordinate system (generic gauge)

In a generic gauge

$$\langle q_{f\eta}, \tau_f | q_i, \tau_i \rangle^J = \int_{q(\tau_i)=q_i}^{q(\tau_f)=q_f} \mathcal{D}g \left| \det \frac{\partial f}{\partial \xi} \right| \delta(f) \exp\left(-S_E/\hbar + \int_{\tau_i}^{\tau_f} d\tau \int d^3x J^{\mu\nu} g_{\mu\nu} \right)$$

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The convergence of the Euclidean PI (EPI) requires the following conditions

$$\alpha > 0, \quad \beta > 0, \quad \lambda > \frac{\gamma^2}{4\beta}$$

in a generic gauge. These are also sufficient conditions in a spacetime discretization.

This is opposed to the case of Einstein gravity, where there are no values of the parameters for which the EPI could converge as the Euclidean action is always unbounded from below (the conformal-factor problem).

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Recall α, β, γ and λ are bare parameters: they generically diverge as a function of the cutoff to ensure the convergence of the physical quantities. So, it is not possible to infer that the same conditions hold for renormalized quantities.

Path integral for Green's functions

One finds the following EPI for the generating functional of Green's function

$$Z(J) = \frac{1}{\text{“}J \to 0\text{”}} \int \mathcal{D}g \left(\det \frac{\delta f}{\delta \xi}\right) \delta(f) \exp\left(-S_E/\hbar + \int d^4x_E\, J^{\mu\nu} g_{\mu\nu}/\hbar\right)$$

while in Lorentzian signature

$$\mathcal{Z}(J) = \frac{1}{\text{“}J \to 0\text{”}} \int \mathcal{D}g \left(\det \frac{\delta f}{\delta \xi}\right) \delta(f) \exp\left(iS/\hbar + i \int d^4x\, J^{\mu\nu} g_{\mu\nu}/\hbar\right)$$

Path integral for Green's functions

One finds the following EPI for the generating functional of Green's function

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Previous perturbative calculations performed expanding around the Minkowski metric $\eta_{\mu\nu}$ (e.g. those of *[Stelle (1977)]*) are recovered when applicable

The quadratic gravity effective action

The expectation value of $g_{\mu\nu}$ in the presence of a physical energy-momentum tensor, $J^{\mu\nu}$ is

$$\langle g_{\mu\nu} \rangle_J \equiv \frac{1}{\text{“}J \to 0\text{”}} \frac{\int \mathcal{D}g \left(\det \frac{\delta f}{\delta \xi}\right) \delta(f)\, g_{\mu\nu} \, \exp\left(iS/\hbar + i \int d^4x\, J^{\mu\nu} g_{\mu\nu}/\hbar\right)}{\int \mathcal{D}g \left(\det \frac{\delta f}{\delta \xi}\right) \delta(f) \exp\left(iS/\hbar + i \int d^4x\, J^{\mu\nu} g_{\mu\nu}/\hbar\right)}$$

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The quadratic gravity effective action Γ can then be defined as a functional of $\langle g_{\mu\nu}\rangle_J$:

$$\Gamma(\langle g_{\mu\nu}\rangle_J) \equiv W(J) - \int d^4x\, J^{\mu\nu}\langle g_{\mu\nu}\rangle_J, \qquad \mathcal{Z}(J) \equiv \exp(iW(J)/\hbar)$$

The classical limit

In the classical limit, $\hbar \to 0$, the path integral is dominated by a metric satisfying

$$\frac{\delta S}{\delta g_{\mu\nu}} + J^{\mu\nu} = 0$$

Let us call such a metric $g^J_{\mu\nu}$ to highlight its dependence on $J_{\mu\nu}$. For $\hbar \to 0$

$$\Gamma(\langle g_{\mu\nu}\rangle_J) = S|^{\text{ren}}_{g^J} - S|^{\text{ren}}_{g^0}$$

where $S|^{\text{ren}}_{g^J}$ is the starting action with renormalized coefficients and computed in the corresponding $g^J_{\mu\nu}$, namely the solution of

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This can happen even if the spectrum of the quantum Hamiltonian H is bounded from below thanks to the presence of DP variables, which allow to break the equality between $\bar{H} \equiv \frac{\langle p|H|q\rangle}{\langle p|q\rangle}$ and the classical Hamiltonian H_c.

Nevertheless, the metric remains real because the expectation value of a canonical variable is always real *[Salvio (2020)]*

Non-perturbative quantum cosmology

- In quantum cosmology one describes the universe through a quantum-mechanical wave function *[Hartle, Hawking (1983)]* (HH), but HH based this on Einstein's gravity, with the conformal-factor problem. Quadratic gravity in the quantization specified does not suffer from this problem, so it can rigorously implement the HH idea.

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- In our fully quantum construction the configuration that leads to the smallest action corresponds to peaks in the wave function of the ground state and are homogeneous and isotropic, while inhomogeneities and anisotropies lead to a larger action and are associated with excited states (relation with the finite-action principle of *[Lehners, Stelle (2019); Lehners, Stelle (2023)]*)

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- The homogeneity and isotropy of the initial conditions for inflation are explained rather than postulated. It is only in the nearly homogeneous and isotropic patches that inflation can occur and observations can eventually be made.

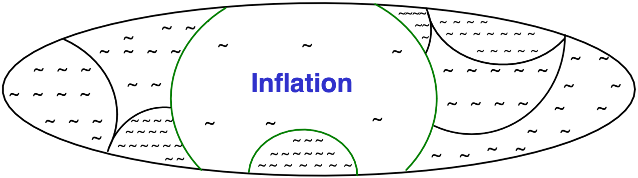


Figure: *Cartoon for the primordial universe. The density of wiggle represents the amount of inhomogeneity and anisotropy.*

Inflationary observables

Observational consequences: $M_2 > H$

No differences compared to GR

Observational consequences: $M_2 < H$

The modifications:

- The tensor-to-scalar ratio r gets suppressed

$$r = \frac{r_E}{1 + \frac{2H^2}{M_2^2}}, \qquad r_E = \text{tensor-to-scalar ratio in Einstein gravity}$$

  models that are excluded for a large r_E (e.g. quadratic inflation) become viable
- There is an isocurvature mode (which fullfils the observational bounds) corresponding to the scalar component of the abnormal graviton

The isocurvature power spectrum P_B is the same as the tensor power spectrum in Einstein's gravity, except that it is smaller by a factor of $3/16 \approx 1/5$:

$$P_B = \frac{3}{2\bar{M}_{\rm Pl}^2}\left(\frac{H}{2\pi}\right)^2$$

and the correlation $P_{\mathcal{R}B}$ is highly suppressed

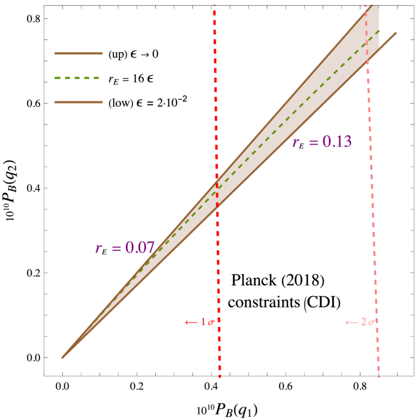
[Ivanov, Tokareva (2016)], *[Salvio (2017)]*

Ghost-isocurvature power spectrum ($M_2 < H$)

$q_1 = 0.002\ \mathrm{Mpc}^{-1}$ and $q_2 = 0.1\ \mathrm{Mpc}^{-1}$.

The strongest constraints from *Planck (2018)* have been taken

[Salvio (2017), (2019)]



Some specific inflationary scenarios

- **Scalaron inflation**. We have an R^2 term so we can implement scalaron inflation *[Starobinsky (1980)]*.

- **Natural inflation**. The inflaton is a pseudo-Goldstone boson *[Freese, Frieman, Olinto (1990)]*. In this case one can elegantly obtain a UV completion within an asymptotically free QCD-like theory *[Salvio (2019), (2021)]*.

$$V_N(\phi) = \Lambda^4 \left(1 + \cos\left(\frac{\phi}{f}\right)\right)$$

- **Non-minimal coupling inflation** The inflaton is a fundamental scalar with a non-minimal coupling to the Ricci scalar. Examples are Higgs inflation *[Shaposhnikov, Bezrukov (2008)]* and hilltop inflation *[Boubekeur, Lyth (2005); Ballesteros, Tamarit (2015)]*.

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The conditions to avoid the Ostrogradsky instability are satisfied in all these cases

Natural-scalaron inflation in quadratic gravity

We can go to the pure-natural and to the pure-scalaron inflation by taking respectively a small and large value of

$$\rho \equiv \frac{\Lambda^2}{\sqrt{6} f_0 \bar{M}_{\rm Pl}^2}$$

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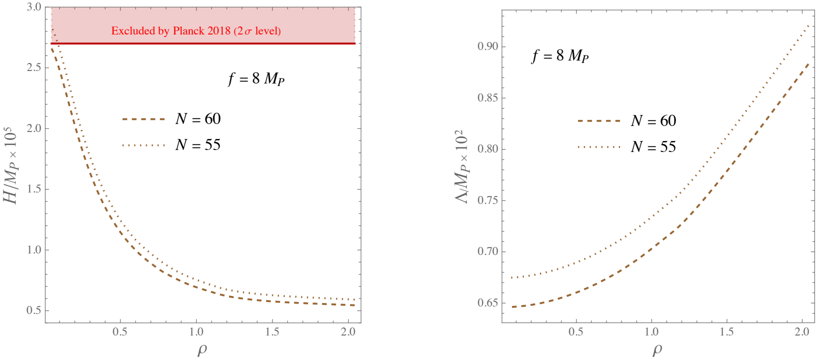
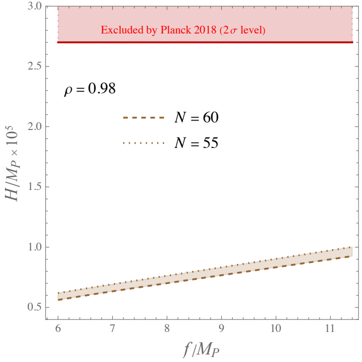
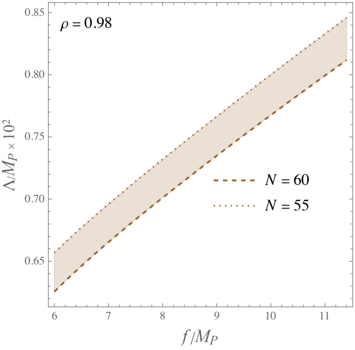


Λ is obtained by requiring P_R to match the observed value

Natural-scalaron inflation in quadratic gravity

Another important parameter is $f/\bar{M}_{\rm Pl}$





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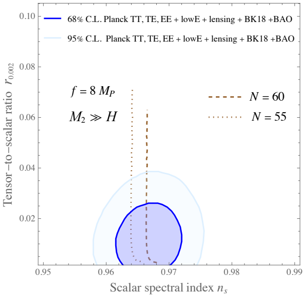

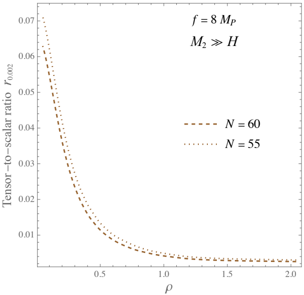

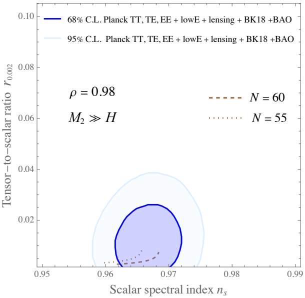

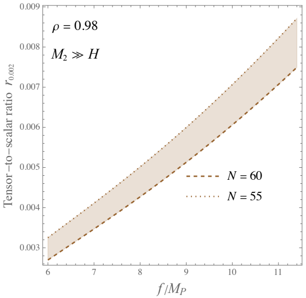

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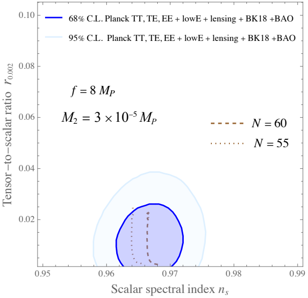

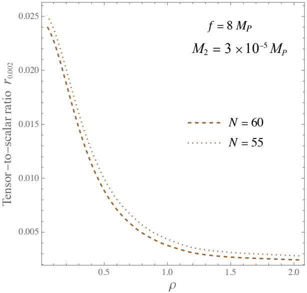

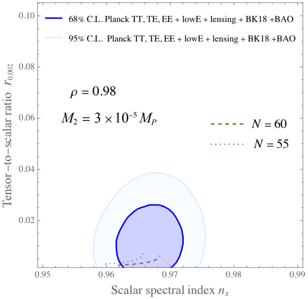

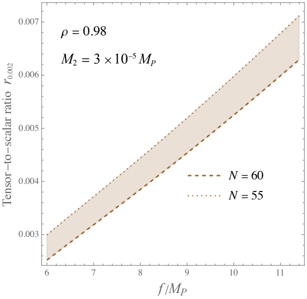

What about the cosmological constant (CC)?

Unimodular gravity

In unimodular gravity (UG) one requires by definition that the spacetime volume is not dynamical (see *[Weinberg (1989)]* and refs. therein)

The classical limit is the same independently of the theory of gravity, on which the unimodular condition is imposed *[Percacci (2017)]*. But the CC is an integration constant of the *classical* field equations in unimodular gravity

→ the CC and vacuum energy are independent. Unimodular gravity addresses the old cosmological constant (CC) problem: it explains why such constant is not at least as large as the largest particle mass scale.

Unimodular gravity: a non-perturbative and background-independent formulation *[Salvio (2024)]*

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Can we distinguish unimodular and non-unimodular theories at least quantum mechanically? ← use the non-perturbative and background-independent quantum formulation of quadratic gravity

Insert the following constraining δ-function factor in the Euclidean path integrand

$$\prod_{x_E} \delta(\Delta\tau \Delta V_3 \sqrt{g(x_E)} - \Delta V_E)$$

while in the Lorentzian path integrand insert

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$$\prod_x \delta(\Delta t \Delta V_3 \sqrt{-g} - \Delta V) = \int \left(\prod_x \frac{dl(x)}{2\pi}\right) \times \exp\left(i \int d^4x \, l(x)(\sqrt{-g(x)} - \omega(x))\right)$$

Distinguishing the two theories at quantum level: unimodular inflation

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At *quantum* level choose the conformal Newtonian gauge

$$ds^2 = a(u)^2 \left\{ \left[(1 - 2\Psi(u,\vec{x}))\delta_{ij} + h_{ij}(u,\vec{x}) \right] dx^i dx^j + 2V_i(u,\vec{x}) du dx^i - (1 + 2\Phi(u,\vec{x})) du^2 \right\}$$

where perturbations satisfy $\partial_i V_i = 0, \qquad h_{ij} = h_{ji}, \qquad h_{ii} = 0, \qquad \partial_i h_{ij} = 0$

g is reduced to a c-number function in the unimodular theory.

At linear level in the perturbations $\qquad g = -a^8(u)(1 + 2\Phi - 6\Psi) \rightarrow \boxed{\Phi = 3\Psi}$

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Assuming that inflation is driven by a minimally coupled scalar field, one finds $\Psi \to 0$ in the superhorizon limit, $u \to 0^-$, when $a \to +\infty$

- The curvature perturbation $\mathcal{R}$ acquires the expression in Einstein gravity *[Salvio (2017)]* → The predictions for r and n_s are the same as in quadratic gravity without the unimodular constraint
- However, the extra isocurvature mode B present in quadratic gravity decouples in unimodular quadratic gravity. Since future CMB observations may detect the power spectrum of B, we can distinguish unimodular and non-unimodular quadratic gravity

A time-multiverse for the new CC problem

Unimodular gravity per se does not solves the new CC problem (why the CC has the observed value?) even in this realization

Let us consider the post-inflationary cosmology

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Note that here the value of the CC ($\rho_\Lambda \sim \rho_M$ today) is explained anthropically with a multiverse made by different eras in a single big bang; a type of multiverse mentioned before, see e.g. *[Banks (1985)]*.

Decays and scattering

Formulæ from the appropriate quantization of quadratic gravity

The DP quantization of quadratic gravity and the appropriate Born rule lead to the following formulæ for

Decays: $$P_{\text{decay}} = \frac{|\langle\sigma|P_{H_0}U(t)|g\rangle|^2}{\langle g|U(t)^\dagger P_{H_0}U(t)|g\rangle}$$

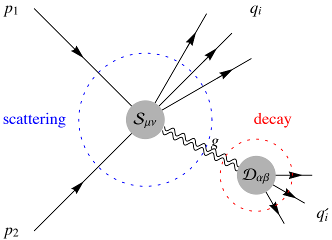
Scattering processes: $$P(\sigma \to \sigma') = \frac{\left|\langle\sigma'|P_{H_0}U(t)|\sigma\rangle\right|^2}{\langle\sigma|U(t)^\dagger P_{H_0}U(t)|\sigma\rangle} > 0$$

At tree level one recovers the standard QFT formulæ but at loop level the denominators may play a non-trivial role.

Gravitational scattering between asymptotic states and the UV

The radiative corrections to the high energy behavior in quadratic gravity should (like any other radiative corrections) take into account the denominators in the previous formulæ.

So let us focus on the tree-level approximation



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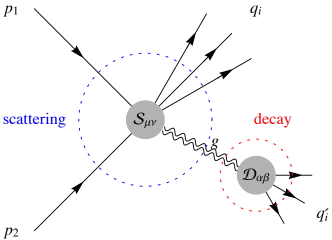

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One obtains a lot of information about the high energy behaviour of the theory already at tree-level, where a classicalization effect takes place due to IR effects in quadratic gravity *[Salvio, Strumia, Veermäe (2018)]*: super-Planckian scatterings get downgraded to Planckian by radiating hard gravitons and ghosts, which are weakly coupled and carry away the energy.

Conclusions

- The classical runaway solutions can be avoided even at energies exceeding the ghost mass M_2. The runaways occurring above an energy threshold ($\gg M_2$) can explain homogeneity and isotropy for $f_2 \sim 10^{-8}$
- Interpreting probabilities as emergent from repeated experiments leads us to a way of quantizing the theory where all probabilities are positive and they sum up to 1 (unitarity).
- A non-perturbative and background formulation was found through a consistent Euclidean PI, which can be applied to quantum cosmology
- For $f_2 \sim 10^{-8}$ (maximal value compatible with $M_h \ll \bar{M}_{\rm Pl}$ being natural) quadratic gravity leads to testable predictions for the inflationary observables. Also it can suppress the value of r and render viable many models that are ruled out in Einstein gravity.
- An unimodular version (with different predictions) addresses the old CC problem and a time-multiverse explains the observed dark energy

Thank you very much for your attention!

Extra slides

Ultracompact objects

There must be a linear description of some ultracompact objects

Since gravity is softened above Λ_G a "linearization mechanism" takes place:

light objects can be described by the linearized theory when their Schwarzschild radius $r_h = 2G_N M$ satisfies

$$\boxed{r_h \ll 1/\Lambda_G}$$

We can therefore, describe *analytically* ultracompact objects (UCOs): the mass-to-radius ratio is higher than for a Schwarzschild black hole (BH).

[Salvio, Veermäe (2019)]

For $r_h \gtrsim 1/\Lambda_G$ non-linear horizonless solutions mimicking BHs have been found numerically *[Holdom, Ren (2016)]*

The linearization mechanism

To easily understand this mechanism consider first a point mass

$$T^{\mu}_{\ \nu} = \mathrm{diag}(-M\delta(r), 0, 0, 0)$$

Note that $T^{\mu}_{\ \mu} \neq 0$ in this case and so $\Lambda_G = \max(M_0, M_2)$.
This source generates a Newtonian potential *[Stelle (1977)]*

$$V_N(r) = -\frac{r_h}{2r}\left(1 - \frac{4}{3}e^{-M_2 r} + \frac{1}{3}e^{-M_0 r}\right)$$

Noting

$$|V_N(r)| = \frac{r_h}{2r}\left|\frac{4}{3}(1 - e^{-M_2 r}) - \frac{1}{3}(1 - e^{-M_0 r})\right| \leq \frac{r_h}{6}(4M_2 + M_0),$$

we see that for $r_h \ll \min(1/M_0, 1/M_2)$ the potential $V_N(r)$ should be small and a horizon does not form even for a UCO. *[Salvio, Veermäe (2019)]*

Singular δ-function sources generate a singular response, just like in (even asymptotically free) Yang-Mills theories

The linearization mechanism

To easily understand this mechanism consider first a point mass

$$T^{\mu}_{\ \nu} = \mathrm{diag}(-M\delta(r), 0, 0, 0)$$

Note that $T^{\mu}_{\ \mu} \neq 0$ in this case and so $\Lambda_G = \max(M_0, M_2)$.
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However, the argument can be extended to general sources and spacetimes that are regular everywhere *[Salvio, Veermäe (2019)]*

$$ds^2 = -a(r)\, dt^2 + \frac{dr^2}{b(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$T^{\mu}_{\ \nu} = \mathrm{diag}(-\rho, P, P_\perp, P_\perp)$$

General analytic solution

Defining $\delta a \equiv a - 1$ and $\delta b \equiv b - 1$

$$\begin{aligned}
\delta a/2 &= V(r;0) + \frac{1}{3}V(r;M_0) - \frac{4}{3}V(r;M_2) + U(r;0) - \frac{1}{3}U(r;M_0) - \frac{2}{3}U(r;M_2) \\
\delta b/2 &= -rV'(r;0) + \frac{1}{3}rV'(r;M_0) + \frac{2}{3}rV'(r;M_2) - \frac{1}{3}rU'(r;M_0) + \frac{1}{3}rU'(r;M_2)
\end{aligned}$$

where

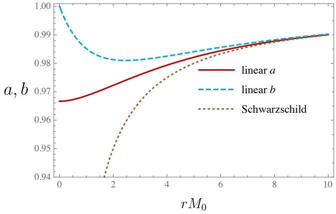
$$\begin{aligned}
V(r;m) &= -G_N \int d^3x \frac{e^{-m|\vec{x}-\vec{r}|}}{|\vec{x}-\vec{r}|}\rho(\vec{x}) \\
U(r;m) &= -G_N \int d^3x \frac{e^{-m|\vec{x}-\vec{r}|}}{|\vec{x}-\vec{r}|}\nabla_{\vec{x}} \cdot (\vec{x}P(\vec{x}))
\end{aligned}$$

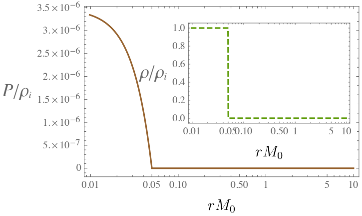
Example: a ball of constant density ρ_i, radius $\mathcal{R}$ and mass M

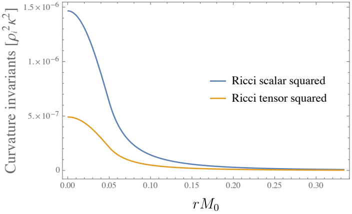
$$V_{\text{ball}}(r;m) = -G_N M \begin{cases} \frac{3}{\mathcal{R}^3 m^2}\left(1 - \frac{\sinh(rm)}{rm} e^{-\mathcal{R}m}(1+\mathcal{R}m)\right) & , \quad r < \mathcal{R} \\ \frac{3}{r} e^{-mr} \frac{\mathcal{R}m\cosh(\mathcal{R}m) - \sinh(\mathcal{R}m)}{(\mathcal{R}m)^3} & , \quad r \geq \mathcal{R} \end{cases}$$

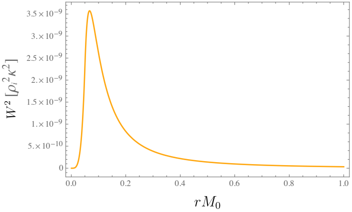
$$U_{\text{ball}}(r;m) \approx 0, \qquad P(r) = \begin{cases} \rho\left(\sqrt{a(\mathcal{R})/a(r)} - 1\right) & , \quad r < \mathcal{R} \\ 0 & , \quad r \geq \mathcal{R} \end{cases}$$

Setting $M_2 = M_0/2$, $f_2 = 10^{-8}$, $\mathcal{R} = 0.05/M_0$ and $r_h = 0.1/M_0 > \mathcal{R}$ (in GR is a BH)









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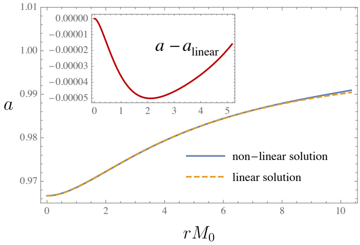
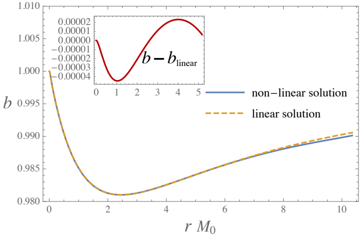
Checking the validity of the linearization mechanism:

comparison between the linear and non-linear solutions (same parameters)





Ostrogradsky stability and metastability

$$\frac{r_h}{2G_N} = M = \frac{4\pi}{3}\mathcal{R}^3\bar{\rho}, \qquad \left(\bar{\rho} \equiv \mathcal{V}^{-1}\int d^3x\,\rho(\vec{x})\right)$$

using $r_h \ll 1/\Lambda_G$ and $\mathcal{R} < r_h$ (for BHs in Einstein gravity)

$$\bar{\rho} = \frac{3r_h}{8\pi G_N \mathcal{R}^3} \equiv \frac{3r_h \bar{M}_{\rm Pl}^2}{\mathcal{R}^3} \gg \frac{3r_h}{\mathcal{R}} M_2^2 \bar{M}_{\rm Pl}^2$$

We saw Ostrogradsky instabilities are avoided for $\rho \ll M_2 \bar{M}_{\rm Pl}^3$. These conditions can be both satisfied for

$$\boxed{M_2 \ll \frac{\mathcal{R}}{3r_h}\bar{M}_{\rm Pl}}$$

which is easily satisfied for values of M_2 that correspond to a natural Higgs mass

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Another plus: the absence of microscopic BHs is good if the Higgs vacuum is metastable (as suggested by data):

a BH with $r_h < 1/h_{\rm max}$ (where $h_{\rm max}$ is the value of the Higgs field for which the effective Higgs potential acquires its maximum) is very dangerous for the Standard Model as it can induce EW vacuum decay *[Burda, Gregory, Moss (2015/2016)]*

When UCOs are stable, they can be DM *[MacGibbon (1987)]; Salvio, Veermäe (2019); [Aydemir, Holdom, Ren (2020)]*

Further slides

Quasi-Conformal Models and the Early Universe

Are there viable models of the early universe that are quasi conformal, i.e.

$$1/f_0 \approx 0, \qquad \xi_{ab} \approx -\delta_{ab}/6 \quad ?$$

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Are there viable models of the early universe that are quasi conformal, i.e.

$$1/f_0 \approx 0, \qquad \xi_{ab} \approx -\delta_{ab}/6 \quad ?$$

Two options were found:

- The inflaton is a fundamental scalar with a quasi-conformal non-minimal coupling to the Ricci scalar. In this case the field excursion must not exceed the Planck mass by far. An example is hilltop inflation.
- The inflaton is a pseudo-Goldstone boson (natural inflation). In this case one can elegantly obtain a UV completion within an asymptotically free QCD-like theory.

$$V_N(\phi) = \Lambda^4 \left(1 + \cos\left(\frac{\phi}{f}\right)\right)$$

The conditions to avoid the Ostrogradsky instability are satisfied

Natural quasi-conformal inflation

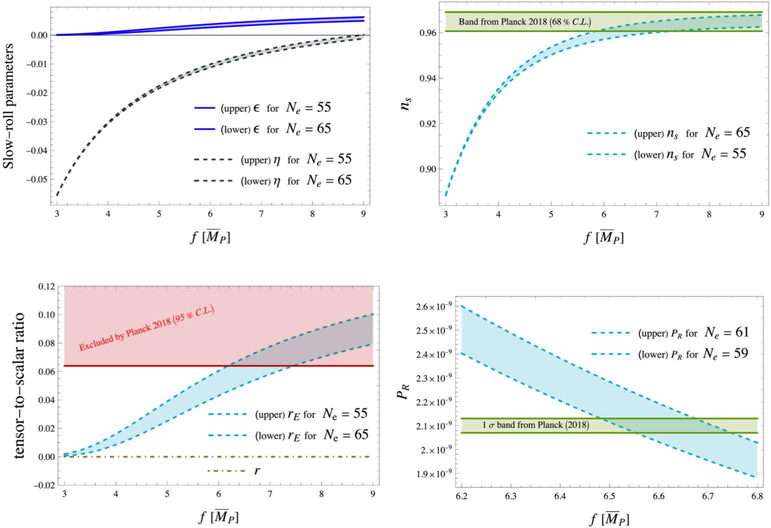

Figure: *We have set $\Lambda \approx 6 \times 10^{-3} \bar{M}_{\rm Pl}$ to fit the observed value of the curvature power spectrum P_R and chosen $N = 1$ and $f_2 = 10^{-8}$ (the value of f_2 influences the plot of r only). The bounds from the latest Planck analysis from 2018 are also shown.*

A possible way to avoid the ghost

Weinberg (1979) proposed to add *all (infinite) terms*:

$$\mathscr{L}_{\mathrm{W}} = -\Lambda - \frac{\bar{M}_{\mathrm{Pl}}^2}{2} R + \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} + \alpha_3 R^3 + \beta_3 R_{\mu\nu} R^{\mu\nu} R + \ldots,$$

including all the (perturbatively) renormalizable and nonrenormalizable terms in the matter sector.

- IF all couplings $g = \{\alpha_i, \beta_i, \ldots\}$ flow to a *finite dimensional* surface
- IF all these *infinite* terms do not develop ghost d.o.f. (possible because Ostrogradsky theorem applies to a finite number of higher derivatives)

Then we would have a UV-complete relativistic field theory of all forces without ghosts

(So far), however, this possibility is unfortunately uncomputable. Requiring computability we go back to quadratic gravity

Trading negative energies with negative norm

Diagonalization of the Hamiltonian

For $V = 0$ the Hamiltonian is

$$H = \omega_1 \left(-\tilde{a}_1^\dagger \tilde{a}_1 + \frac{1}{2} \right) + \omega_2 \left(\tilde{a}_2^\dagger \tilde{a}_2 + \frac{1}{2} \right)$$

We have $\quad [\tilde{a}_1, \tilde{a}_1^\dagger] = -1, \qquad [\tilde{a}_2, \tilde{a}_2^\dagger] = 1, \qquad$ (all other commutators vanish)

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Onset of "negative norms"

As usual $[a_1, N_1] = a_1$ and $[a_2, N_2] = a_2$ by defining $N_1 \equiv -\tilde{a}_1^\dagger \tilde{a}_1$ and $N_2 \equiv \tilde{a}_2^\dagger \tilde{a}_2$
The spectrum of N_1 is bounded from below if you introduce an indefinite metric:

$$-\nu_n n = \langle n | a_1^\dagger a_1 | n \rangle$$

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$$-\nu_n n = \langle n | a_1^\dagger a_1 | n \rangle = |c|^2 \langle n-1 | n-1 \rangle = |c|^2 \nu_{n-1}$$

Classical dynamics: a simple scalar field example

To simplify consider

$$\mathscr{L} = -\frac{1}{2}\phi\Box\phi - \frac{c_4}{2}\phi\Box^2\phi - V(\phi)$$

It is a toy version of out theory:

- $-\frac{1}{2}\phi\Box\phi$ represents the Einstein-Hilbert part
- $-\frac{c_4}{2}\phi\Box^2\phi$ represents the quadratic terms
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Two-derivative form

Add $\frac{c_4}{2}\left(\Box\phi - \frac{A-\phi/2}{c_4}\right)^2$ (vanishes when the EOM of the auxiliary field A are used)

$$\implies \mathscr{L} = -\frac{1}{2}\phi_+\Box\phi_+ + \frac{1}{2}\phi_-\Box\phi_- + \frac{m^2}{2}\phi_-^2 - V(\phi_+ + \phi_-)$$

where $m^2 \equiv 1/c_4$ has to be taken positive to avoid tachyonics.

Classical dynamics: a simple scalar field example

The EOMs are

$$\Box\phi_+ = -V'(\phi_+ + \phi_-), \qquad \Box\phi_- = -m^2\phi_-^2 + V'(\phi_+ + \phi_-).$$

For definiteness take $V(\phi) = \lambda\phi^4/4$, where $\lambda > 0$, which stabilizes ϕ_+, while ϕ_- feels

$$v(\varphi) = \frac{m^2}{2}\varphi^2 - \frac{\lambda}{4}\varphi^4, \qquad \varphi = \text{typical order of magnitude of field values}$$

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Ghost metastability

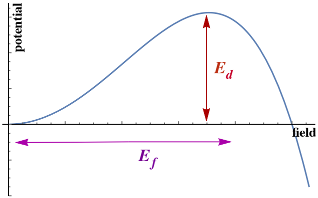
For

$$\varphi \ll E_f \equiv \frac{m}{\sqrt{\lambda/2}}$$

and

$$E \ll E_d \equiv \frac{m}{(4\lambda)^{1/4}}$$

(where E is the energy associated with the field derivatives)
the runaway solutions are avoided



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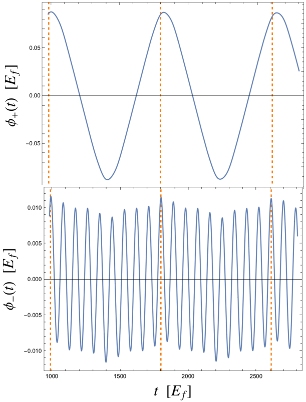
Example in the figure: $\lambda = 10^{-2}$, $\phi_+(0) = 10^{-2}E_f$, $\phi_-(0) = 10^{-2}E_f$, $\dot\phi_+(0) = (1.5 \cdot 10^{-1}E_d)^2$ and $\dot\phi_-(0) = -(10^{-2}E_d)^2$.



Explicit nonlinear calculations in Starobinsky inflation

$$ds^2 = dt^2 - a(t)^2 \sum_{i=1}^{3} e^{2\alpha_i(t)} dx^i dx^i$$

$$\alpha_1 \equiv \beta_+ + \sqrt{3}\beta_-, \quad \alpha_2 \equiv \beta_+ - \sqrt{3}\beta_-, \quad \alpha_3 = -2\beta_+.$$

One can reduce the system to first-order equations through the definitions

$$\gamma_\pm = \dot{\beta}_\pm, \quad \delta_\pm = \dot{\gamma}_\pm, \quad \epsilon_\pm = \dot{\delta}_\pm$$

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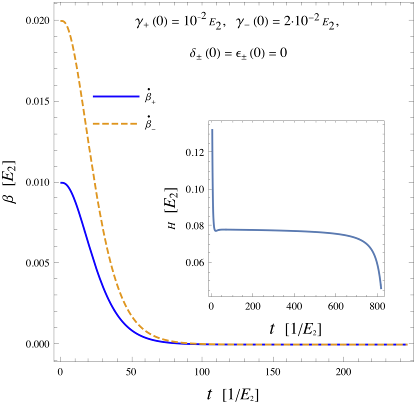
Small initial values for the anisotropy

$(|\gamma_\pm(0)| \ll E_2,\ \sqrt{|\delta_\pm(0)|} \ll E_2,\ \sqrt[3]{|\epsilon_\pm(0)|} \ll E_2$ and $\sqrt{\bar{M}_{\rm Pl} H} \ll E_m)$

do not create problems: the anisotropy quickly goes to zero and one recovers the GR behavior

Example in the figure: Starobinsky inflation with $f_2 = 10^{-8}$, $f_0 \approx 1.6 \cdot 10^{-5}$, $\phi(0) \approx 5.5 \bar{M}_{\rm Pl}$ and $\sqrt{\pi_\phi(0)} \approx 7.1 \cdot 10^{-6} \bar{M}_{\rm Pl}$

Possible to illustrate the argument in simple models



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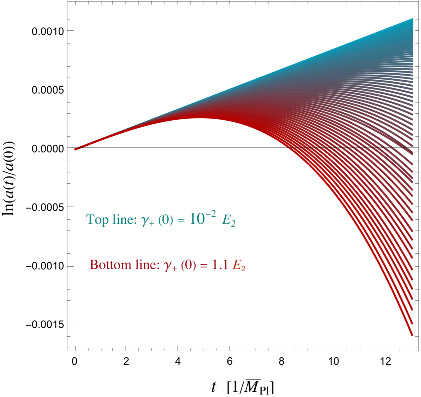
The patches where those conditions are *not* satisfied quickly collapse:

The scale factor in the Jordan frame shrinks as shown in the figure $\longrightarrow$

Example in the figure: Starobinsky inflation with $\gamma_-(0) = 10^{-1} E_2$, $\delta_\pm(0) = 0$, $\epsilon_\pm(0) = 0$, $f_2 = 10^{-8}$, $f_0 \approx 1.6 \cdot 10^{-5}$, $R(0) \approx 1.3 \cdot 10^2 f_0^2 \bar{M}_{\rm Pl}^2$ and $H(0) = 1.2 E_2$.



General check of the ghost metastability: linear analysis

Check that ***all*** linear modes around dS are bounded (for a fixed initial condition) for any wave number q

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Scalar modes: they are like in GR plus an gravity-isocurvature mode:

$$g_B(\eta, q) \equiv \frac{H}{\sqrt{2q}} \left(\frac{3}{q^2} + \frac{3i\eta}{q} - \eta^2 \right) e^{-iq\eta} + \mathcal{R} - \text{term}$$

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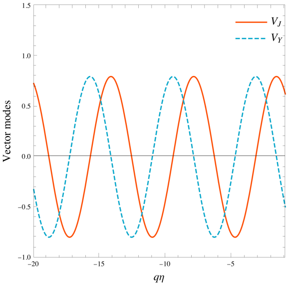
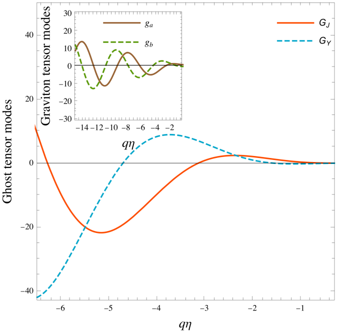
Vector and tensor modes:





UCO dark matter

Different possibilities:

- If UCOs are formed with a horizon, then they evaporate, but have to stop doing so after they lose most of their mass and enter the softened gravity regime, leaving remnant Dark Matter (DM) *[MacGibbon (1987)]*.

- When UCOs are formed without a horizon, e.g., via the collapse of primordial fluctuations, which can, for example, be produced during inflation.

  Horizonless UCOs can account for DM

  - in the linear regime (light objects) *[Salvio, Veermäe (2019)]*
  - or non-linear regime (heavy objects) *[Aydemir, Holdom, Ren (2020)]*

Implications for BSM phenomenology

Given that gravity is now UV complete it makes sense to look for relativistic field theoretic Standard Model extensions that hold up to infinite energies.

Two options

- All couplings flow to zero at infinite energy: total asymptotic freedom
- (Some of) the couplings flow to an interacting UV fixed point (while the other ones flow to zero). This option typically requires non perturbative methods (lattice?)

Totally asymptotically free (TAF) phenomenology

TAF achieves total unification: all couplings flow to a common value in the UV (zero)

In order to eliminate the Landau poles or run into a non-perturbative regime so far we needed to avoid U(1) gauge factors

$\Longrightarrow$ explanation of the electric charge quantization

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We find 2 options: $\mathrm{SU}(4)_{\mathrm{PS}} \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$

that, unlike SU(5) and SO(10), are not severely constrained by proton decay

$$\Longrightarrow \quad \text{one can have} \quad M_{\mathrm{NP}} \sim \mathrm{TeV}$$

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- TAF models that predict new physics have been found

$$W_R\,, \quad Z'\,, \quad H'\,, \quad \text{etc}$$

[Giudice, Isidori, Salvio, Strumia (2014); Pelaggi, Strumia, Vignali (2015)]

TAF Pati-Salam

	Fields	spin	generations	$\mathrm{SU}(2)_L$	$\mathrm{SU}(2)_R$	$\mathrm{SU}(4)_{\mathrm{PS}}$
skeleton model	$\psi_L = \begin{pmatrix} \nu_L & e_L \\ u_L & d_L \end{pmatrix}$	$1/2$	3	$\bar{2}$	1	4
	$\psi_R = \begin{pmatrix} \nu_R & u_R \\ e_R & d_R \end{pmatrix}$	$1/2$	3	1	2	$\bar{4}$
	ϕ_R	0	1	1	2	$\bar{4}$
	$\phi = \begin{pmatrix} H_U^0 & H_D^+ \\ H_U^- & H_D^0 \end{pmatrix}$	0	1	2	$\bar{2}$	1
extra fields	ψ	$1/2$	$N_\psi \leq 3$	2	$\bar{2}$	1
	Q_L	$1/2$	2	1	1	10
	Q_R	$1/2$	2	1	1	$\overline{10}$
	Σ	0	1	1	1	15

TAF Pati-Salam

	Fields	spin	generations	$\mathrm{SU}(2)_L$	$\mathrm{SU}(2)_R$	$\mathrm{SU}(4)_{\mathrm{PS}}$
skeleton model	$\psi_L = \begin{pmatrix} \nu_L & e_L \\ u_L & d_L \end{pmatrix}$	1/2	3	$\bar{2}$	1	4
	$\psi_R = \begin{pmatrix} \nu_R & u_R \\ e_R & d_R \end{pmatrix}$	1/2	3	1	2	$\bar{4}$
	ϕ_R	0	1	1	2	$\bar{4}$
	$\phi = \begin{pmatrix} H_U^0 & H_D^+ \\ H_U^- & H_D^0 \end{pmatrix}$	0	1	2	$\bar{2}$	1
extra fields	ψ	1/2	$N_\psi \leq 3$	2	$\bar{2}$	1
	Q_L	1/2	2	1	1	10
	Q_R	1/2	2	1	1	$\overline{10}$
	Σ	0	1	1	1	15

Because of quark-lepton unification in the Pati-Salam model, flavor bounds force the masses of the vectors in $SU(4)_{\mathrm{PS}}/SU(3)_c$ to be larger than 100 TeV

TAF Trinification

Another TAF model can be built by extending the minimal trinification

Field	spin	generations	$\mathrm{SU}(3)_L$	$\mathrm{SU}(3)_R$	$\mathrm{SU}(3)_c$	Δb_L	Δb_R	Δb_c
$Q_R = \begin{pmatrix} u_R^1 & u_R^2 & u_R^3 \\ d_R^1 & d_R^2 & d_R^3 \\ d_R'^1 & d_R'^2 & d_R'^3 \end{pmatrix}$	$1/2$	3	1	3	$\bar{3}$	0	1	1
$Q_L = \begin{pmatrix} u_L^1 & d_L^1 & \bar{d}_R'^1 \\ u_L^2 & d_L^2 & \bar{d}_R'^1 \\ u_L^3 & d_L^3 & \bar{d}_R'^3 \end{pmatrix}$	$1/2$	3	$\bar{3}$	1	3	1	0	1
$L = \begin{pmatrix} \bar{\nu}_L' & e_L' & e_L \\ \bar{e}_L' & \nu_L' & \nu_L \\ e_R & \nu_R & \nu' \end{pmatrix}$	$1/2$	3	3	$\bar{3}$	1	1	1	0
H	0	3	3	$\bar{3}$	1	$\frac{1}{2}$	$\frac{1}{2}$	0

Trinification does not predict quark-lepton unification and thereby is safer than Pati-Salam from the point of view of flavour bounds.

Higgs naturalness demands $M_{W_R} \leq 2$ TeV $\sqrt{\Delta}$ $\Delta \equiv$ fine-tuning factor

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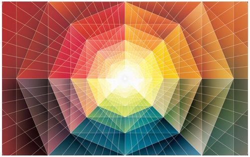
Agravity

It is also possible to generate scales dynamically

The dimensionful terms (the Planck mass, the electroweak scale and the cosmological constant) can all be dynamically generated through dimensional transmutation (**Agravity**) *[Salvio, Strumia (2014)]*



Quanta Magazine (Simons Foundation)