A hypothesis on the death of the universe

D. Anselmi

In a scenario where the expansion of the universe is accelerating and the event horizon is located at a finite comoving distance, i.e., $\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\mathrm{d}\mathbf{r}^2 = 0$

$$a(t) = \mathrm{e}^{Ht}$$

$$d(t) = \int_{r(t)}^{r(\infty)} \mathrm{d}r = \int_t^\infty \frac{\mathrm{d}t'}{a(t')} < \infty \qquad d(t) = \frac{1}{Ha(t)}$$

$a(t)$ being the scale factor, an admissible state is the one where all the unstable particles have decayed, and the stable ones are so distant from one another that they will be unable to exchange physical signals for eternity. We label such a state "total dilution". In a hypothetical situation of this type, the particles are not actually "particles", but just wave functions that do not have the chance to be brought to reality again by means of wave-function collapses, since they can no longer interact with macroscopic bodies, like a detector, let alone meet an "observer". Thus, the state of total dilution is also a state of "cosmic virtuality".

Hypothesis

total dilution is the final state for a generic set of initial conditions, i.e., there exists a time t_{dis}, such that, for every $t > t_{\mathrm{dis}}$, every pair of particles is separated by a comoving distance larger than $d(t)$. Is the expansion ultimately going to expand everything, including the planetary systems, the celestial bodies, maybe even the atoms?

Motivation: the role of virtuality in the theory of quanta

In quantum mechanics, virtuality plays a key role, by mathematically filling the gap between two subsequent measurements on a system. Entanglement is a striking manifestation of the virtual nature of quantum states. The "predominance" of virtuality over reality is also apparent in quantum field theory, where a propagator is almost everywhere virtual, the sole exception being its (relatively tiny) on-shell contribution. Another place where virtuality plays a crucial role is quantum gravity, where it is possible to introduce "purely virtual" particles by tweaking the usual diagrammatics in a certain way . The concept leads to a unitary and renormalizable theory of quantum gravity , whose main prediction (in the realm of current or planned observations) is a very constrained window for the value of the tensor-to-scalar ratio r of primordial fluctuations. It is delimited from above by the prediction of the Starobinsky $R + R^2$ model, and from below by the properties of the purely virtual particles themselves

Prediction for r: $0.4 < 1000\, r < 3$

PVP bound

Staro

D. Anselmi, E. Bianchi and M. Piva, Predictions of quantum gravity in inflationary cosmology: effects of the Weyl-squared term, J. High Energy Phys. 07 (2020) 211,

Theory of quantum gravity emerging from PVPs

Start from the "interim" classical action

$$S_{\mathrm{QG}} = -\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left(R+\frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}-\frac{R^2}{6m_\phi^2}\right)$$

Starobinsky: same with m_χ equal infinity

$$-\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left(R-\frac{R^2}{6m_\phi^2}\right)$$

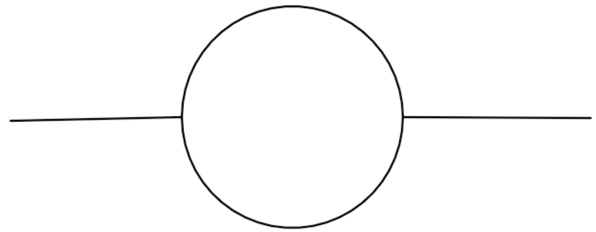
Deal with the unitarity issue by defining new (non time ordered) correlation functions and diagrams, then slice out the unphysical sector. This switches to a different theory by projection, and defines a physical (unitary plus renormalizable) theory. The projection (integrating out the PVPs) also changes the classical action

Price to pay: violation of microcausality (chronological order – past, present, future – makes no sense for intervals below 1/ m_χ), but unitarity is ok

D. Anselmi, Diagrammar of physical and fake particles and spectral optical theorem, J. High Energy Phys. 11 (2021) 030, and arXiv: 2109.06889 [hep-th]

D. Anselmi, A new quantization principle from a minimally non time-ordered product, J. High Energy Phys. 12 (2022) 088, and arXiv:2210.14240 [hep-th]

Bubble diagram



$$\mathrm{i}\,\mathcal{M} = \frac{\lambda^2}{2}\int \frac{\mathrm{d}^D k}{(2\pi)^D}\frac{1}{-k^2-m^2+\mathrm{i}\epsilon}\,\frac{1}{-(k-p)^2-m^2+\mathrm{i}\epsilon}$$

$$= -\frac{\lambda^2}{2}\int \frac{\mathrm{d}^{D-1}\boldsymbol{k}}{(2\pi)^{D-1}}\frac{1}{2\omega_k}\frac{1}{2\omega_{k-p}}\left(\frac{\mathrm{i}}{-p^0-\omega_k-\omega_{k-p}+\mathrm{i}\epsilon}+\frac{\mathrm{i}}{p^0-\omega_k-\omega_{k-p}+\mathrm{i}\epsilon}\right)$$

$$= -\frac{\lambda^2}{2}\int \frac{\mathrm{d}^{D-1}\boldsymbol{k}}{(2\pi)^{D-1}}\frac{1}{2\omega_k}\frac{1}{2\omega_{k-p}}\left[\mathrm{PV}\frac{\mathrm{i}}{-p^0-\omega_k-\omega_{k-p}}+\mathrm{PV}\frac{\mathrm{i}}{p^0-\omega_k-\omega_k}\right.$$

$$\left.+\pi\delta(-p^0-\omega_k-\omega_{k-p})+\pi\delta(p^0-\omega_k-\omega_{k-p})\right],$$

Integrate on energy using residues

Use $\dfrac{1}{x+i\epsilon} = \mathrm{PV}\dfrac{1}{x} - i\pi\delta(x)$

PVP

AP prescription: drop any delta with the frequency of a fakeon particle:

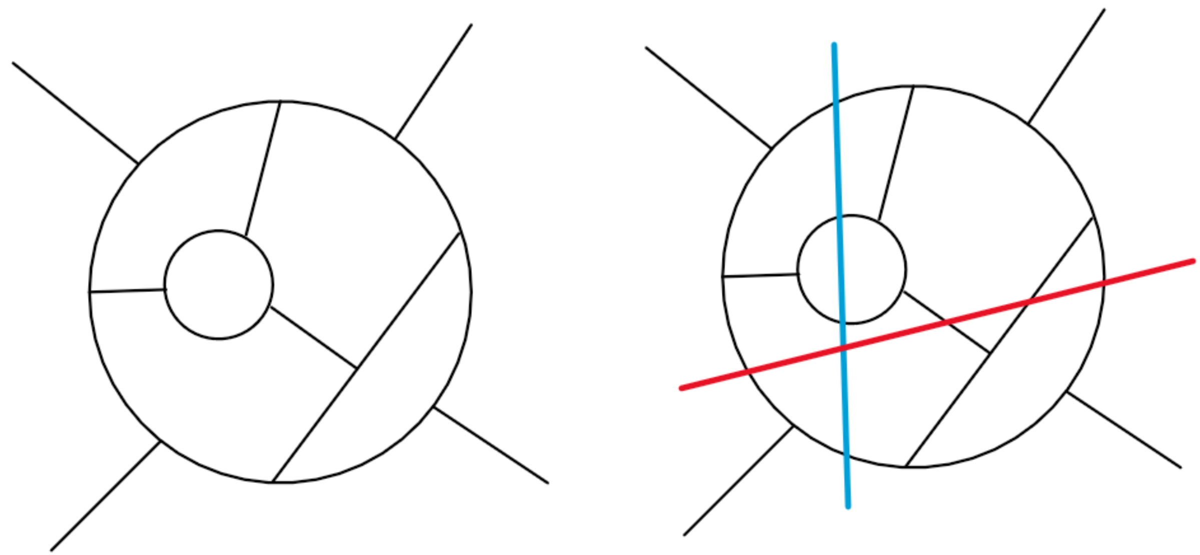
$$\mathrm{i}\,\mathcal{M} = -\frac{\lambda^2}{2}\int \frac{\mathrm{d}^{D-1}\boldsymbol{k}}{(2\pi)^{D-1}}\frac{1}{2\omega_k}\frac{1}{2\omega_{k-p}}\left(\mathrm{PV}\frac{\mathrm{i}}{-p^0-\omega_k-\omega_{k-p}}+\mathrm{PV}\frac{\mathrm{i}}{p^0-\omega_k-\omega_{k-p}}\right).$$

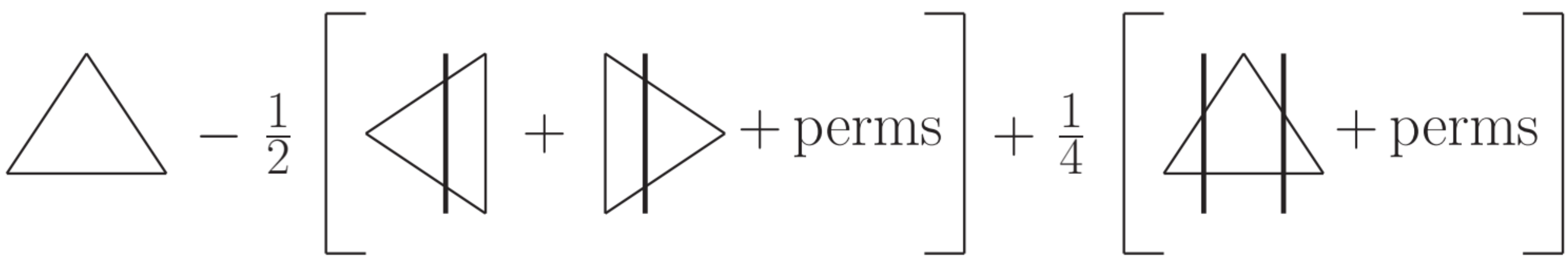
$$\mathcal{M} = -\frac{\lambda^2}{32\pi^2}\ln\frac{-p^2}{\tilde{\Lambda}^2_{\mathrm{UV}}} = -\frac{\lambda^2}{32\pi^2}\ln\frac{|p^2|}{\tilde{\Lambda}^2_{\mathrm{UV}}}.$$

Courtesy L. Modesto,
(from his talk)

Non time-ordered diagrams

Feynman diagram | non T ordered diagram





PVPs can also be consistently introduced in finite time QFT, where nothing is on shell

Correction factor to Starobinsky prediction, subject to the PVP bound

$$\frac{1}{9} < \boxed{\frac{1}{1+\frac{m_\phi^2}{2m_\chi^2}}} < 1 \qquad\qquad \boxed{\frac{m_\phi}{4} < m_\chi} < \infty$$

Inflaton mass

PVP mass

The correction is less than 1% as soon as the PVP mass is just one order of magnitude larger than the inflaton mass.

Probably the observational results will just confirm the Starobinsky prediction

$$r \approx \frac{3}{1000}$$

and the window will remain buried under the observational errors for long

Comparison with asymptotically local quantum field theories (ALQFT) (talks by Modesto, Calcagni, Rachwal)

Propagators and vertices of a nonlocal field theory which tend to propagators and vertices of a local theory at momenta larger than some scale Λ

Same degrees of freedom/same on-shell part (graviton + inflaton, or graviton only), but different virtual sectors, hence quantitatively different physical predictions

Renormalizability and unitarity ok,
Microcausality violations also ok (also quantitatively different)

Asymptotic locality leaves room for a large family of theories. In this approach, one needs a selection principle to identify the ALQG good for describing nature

The significance of virtuality in quantum physics suggests that maybe the entire universe will one day become purely virtual, *de facto*. As far as we can tell today, the only possibility to make "virtuality win over reality" is to envision scenarios where mechanisms make the accelerated expansion of the universe prevail. Then, at some point, the relatively tiny on-shell contributions to the particle propagators will become devoid of practical consequences.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = 0.$$

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a(t)^2 (\mathrm{d}r^2 + r^2 \mathrm{d}\theta^2 + r^2 \sin^2\theta \, \mathrm{d}\varphi^2),$$

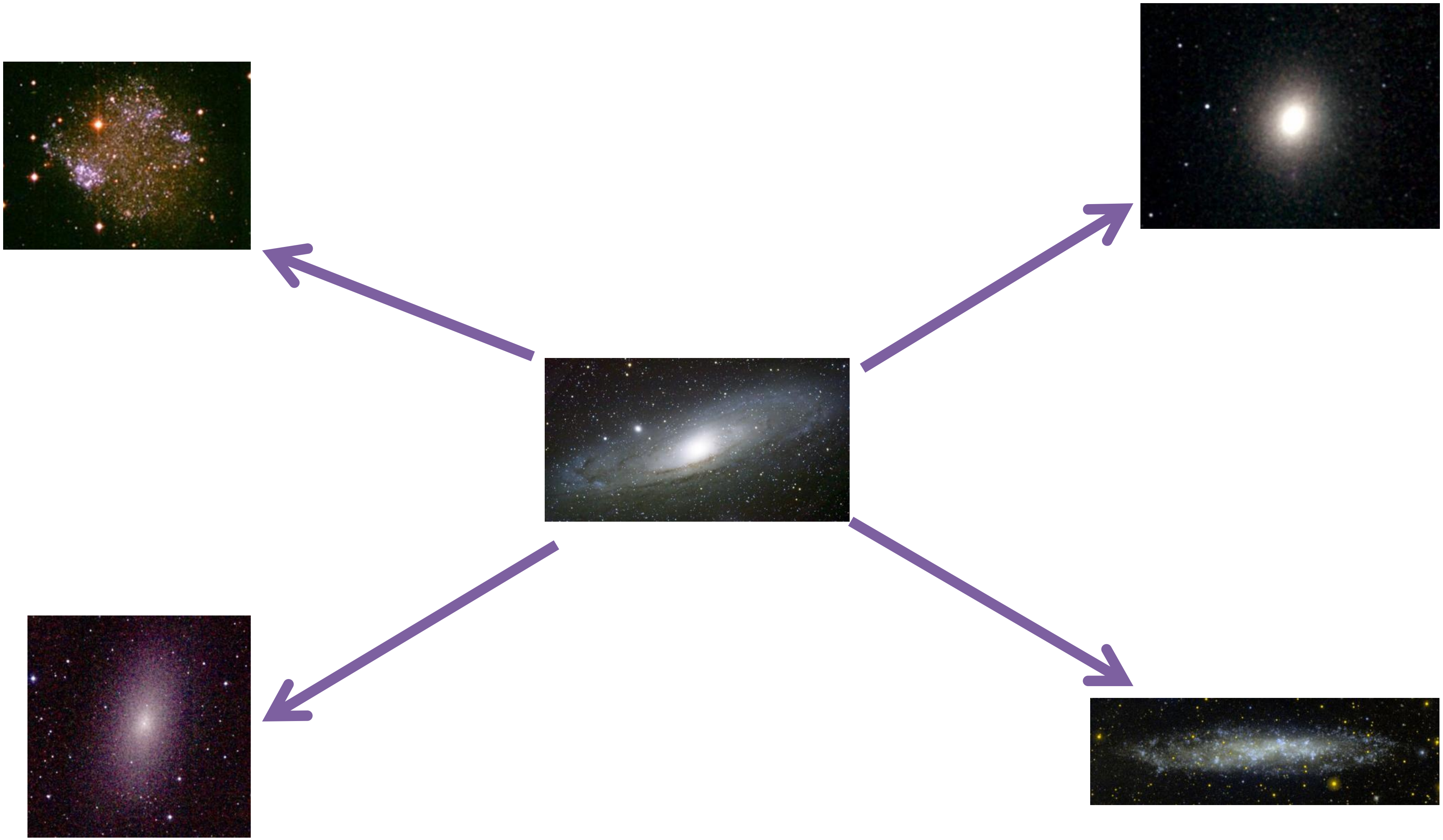
$$a(t) = a_0 \mathrm{e}^{Ht}$$

$$d(t) = \int_t^\infty \frac{\mathrm{d}t'}{a(t')} < \infty,$$

"total dilution"

"cosmic virtuality"

Homogeneity (clusters) make the expansion prevail, hence cluster will totally dilute



Question: is homogeneity necessary? Answer: NO

Dynamic isotropic coordinates

$$\mathrm{d}s^2 = \frac{\left(1-\frac{r_g}{4r}\right)^2}{\left(1+\frac{r_g}{4r}\right)^2}\mathrm{d}t^2 - \left(1+\frac{r_g}{4r}\right)^4(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\varphi^2)$$

$$a(t) = \mathrm{e}^{Ht}$$

$$\mathrm{d}s^2 = \frac{\left(1-\frac{r_g}{4ra(t)}\right)^2}{\left(1+\frac{r_g}{4ra(t)}\right)^2}\mathrm{d}t^2 - \left(1+\frac{r_g}{4ra(t)}\right)^4 a(t)^2(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\varphi^2)$$

Multi black-hole systems

$$\mathrm{d}s^2 = \left(1-\frac{\Phi(x,y,z)}{a(t)}\right)\mathrm{d}t^2 - \left(1+\frac{\Phi(x,y,z)}{a(t)}\right)a(t)^2(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2) + \mathcal{O}(\Phi^2)$$

$$\Phi(x,y,z) = \sum_i \frac{r_{ig}}{|\mathbf{r}-\mathbf{r}_i|};$$

Majumdar-Papapetrou system

$$\mathrm{d}s^2 = \left(1+\frac{\Phi}{2}\right)^{-2}\mathrm{d}t^2 - \left(1+\frac{\Phi}{2}\right)^{2}(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2)$$

$$A_\mu = \frac{1}{\sqrt{4\pi G}}\frac{\Phi}{2+\Phi}(1,0,0,0)$$

Kastor and Traschen

$$\mathrm{d}s^2 = \left(1+\frac{\Phi}{2a(t)}\right)^{-2}\mathrm{d}t^2 - \left(1+\frac{\Phi}{2a(t)}\right)^{2}a(t)^2(\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2)$$

$$A_\mu = \frac{1}{\sqrt{4\pi G}}\frac{\Phi}{2a(t)}\left(1+\frac{\Phi}{2a(t)}\right)^{-1}(1,0,0,0).$$

$$a(t) = \mathrm{e}^{Ht}$$

geodesics

$$\frac{\mathrm{d}U^\mu}{\mathrm{d}s} + \Gamma^\mu_{\nu\rho} U^\nu U^\rho = \frac{q}{m}\sqrt{4\pi} F^\mu_\nu U^\nu$$

solution

$$U^\mu = (1, 0, 0, 0)/\sqrt{g_{00}} = (\mathrm{d}t/\mathrm{d}s, 0, 0, 0)$$

$$-\frac{\partial_i g_{00}}{2g_{00}} = -\frac{q\partial_i\sqrt{g_{00}}}{m\sqrt{G}\sqrt{g_{00}}}$$

$$q = m\sqrt{G}$$

the expansion prevails and the system ultimately evolves into the state of total dilution

Maybe expansion prevails any time the gravitational force is compensated for by an opposing one?

In about five billion years, the sun will expand into a red giant. Over the course of millions of years after that, it will shed its outer layers and transform into a white dwarf. At that point, the gravitational force will be balanced by the electron degeneracy pressure, preventing further gravitational collapse. In the rest of time, the white dwarf will slowly lose its heat by radiating. There is theoretical speculation that, in a hugely extended timeframe, the white dwarf will eventually cool down completely and become a "black dwarf". In that process, the balance between gravity and the fermion degeneracy pressure will remain in place.

The black dwarf is considered to be stable. The results of the previous section confirm that it is stable even when we take into account the expansion of the universe.

The fate of neutron stars and white dwarfs

$$\mathrm{d}s^2 = g_{00}(u)\mathrm{d}t^2 - a(t)^2 g_r(u)(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)$$

$$u = ra(t) \quad a(t) = \mathrm{e}^{Ht}$$

$$L = -m\sqrt{g_{00} - g_r a^2 v^2}, \qquad \mathbf{p} = \frac{\partial L}{\partial \mathbf{v}} = -m^2 a^2 g_r \frac{\mathbf{v}}{L}$$

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = \frac{\partial L}{\partial \mathbf{r}} = \frac{am^2\mathbf{r}}{2Lr}(g_{00}' - g_r' a^2 v^2) \equiv \mathbf{f}_G$$

fluid

$$T^{\mu\nu} = (\varepsilon + P)U^\mu U^\nu - P g^{\mu\nu}$$

$$(\varepsilon + P)U^\nu D_\nu p^\mu = m(g^{\mu\nu} - U^\nu U^\mu)D_\nu P$$

$$\frac{\partial \mathbf{p}}{\partial t} + (\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{p} = \mathbf{f}_G + \frac{L}{\varepsilon + P}\boldsymbol{\nabla}P - \frac{\mathbf{p}}{\varepsilon + P}\left[\frac{\partial P}{\partial t} + (\mathbf{v}\cdot\boldsymbol{\nabla})P\right] \equiv \mathbf{f}_G + \mathbf{f}_P$$

$$d(t) = \frac{1}{Ha(t)}$$

Static coordinates $\mathbf{u} = \mathbf{r}a$

$$L = -m\sqrt{g_{00} - g_r(\dot{\mathbf{u}} - H\mathbf{u})^2}, \qquad \mathbf{P} = \frac{\partial L}{\partial \dot{\mathbf{u}}} = \frac{\mathbf{p}}{a} = -\frac{m^2}{L} g_r(\dot{\mathbf{u}} - H\mathbf{u})$$

$$\frac{\mathrm{d}\mathbf{P}}{\mathrm{d}t} = \frac{\partial L}{\partial \mathbf{u}} = -H\mathbf{P} + \frac{m^2\mathbf{u}}{2uL}(g'_{00} - g'_r(\dot{\mathbf{u}} - H\mathbf{u})^2) \equiv -H\mathbf{P} + \mathbf{F}_G.$$

circular orbits $\quad u = \text{constant}, \; \dot{\mathbf{u}} \cdot \mathbf{u} = 0$

$$\ddot{\mathbf{u}} = -\omega^2 \mathbf{u} \qquad |\dot{\mathbf{u}}| = \omega u$$

$$u(2g_r + ug'_r)(\omega^2 + H^2) = g'_{00}$$

Newton approx

$$g_{00} \simeq 1 - \frac{2MG}{u}, \qquad g_r \simeq 1 + \frac{2MG}{u} \qquad \omega^2 + H^2 \simeq \frac{MG}{u^3}$$

$$\frac{\partial \mathbf{P}}{\partial t} + (\dot{\mathbf{u}} \cdot \boldsymbol{\nabla})\mathbf{P} = -H\mathbf{P} + \frac{m^2\mathbf{u}}{2uL}(g'_{00} - g'_r(\dot{\mathbf{u}} - H\mathbf{u})^2)$$
$$+ \frac{L}{\bar{\varepsilon} + \bar{P}}\boldsymbol{\nabla}\bar{P} - \frac{\mathbf{P}}{\bar{\varepsilon} + \bar{P}}\left[\frac{\partial \bar{P}}{\partial t} + (\dot{\mathbf{u}} \cdot \boldsymbol{\nabla})\bar{P}\right] \equiv \mathbf{F}_c + \mathbf{F}_G + \mathbf{F}_P$$

$$\mathbf{F}_c = -H\mathbf{P} \qquad \text{“centrifugal force”}$$

Solution

$$\rho_* \equiv \frac{8\pi m^4}{3h^3}$$

$$\begin{cases} \rho(u) = \rho_* \left(\dfrac{g_{00}(u_{\max}) - g_r(u_{\max})u_{\max}^2 H^2}{g_{00}(u) - g_r(u)u^2 H^2} - 1 \right)^{3/2} & \text{for } u \leqslant u_{\max}, \\ \rho(u) = 0 & \text{for } u > u_{\max}. \end{cases}$$

$$g_{00}(u) = 1 - \frac{4GH^2}{u}\mu_2(\nu_1, u) - \frac{2G}{u}\nu_2(u) + \frac{2G}{3}\left(3\nu_1(u) - 2G\nu_1(u_{\max})\right)$$
$$g_r(u) = 2 - g_{00}(u) - 4GH^2\mu_1(\nu_1, u),$$

$$\nu_k(u) = 4\pi \int_0^u \frac{w^k \tilde{\varepsilon}(w)\mathrm{d}w}{1 - w^2H^2}, \qquad \mu_k(f, u) = \int_0^u w^k f(w)\mathrm{d}w,$$

When the Newton constant G is switched off, the Einstein equations (B.5) are solved by the FLRW metric ($g_{00} = g_r = 1$), so (5.20) gives

$$\rho(u) = \rho_* H^3 \left(\frac{u^2 - u_{\max}^2}{1 - u^2H^2}\right)^{3/2}$$

renounce $\rho(u_{\max}) = 0$

$$\rho(u) = \left(\frac{\hat{\rho}^{2/3}}{1 - u^2H^2} - \rho_*^{2/3}\right)^{3/2}$$

nonrelativistic limit at $H = 0$

$$g_{00}(r) \simeq 1 + 2G\int_0^r \frac{\mathrm{d}w}{w^2}M(w) - \frac{2G}{3}\int_0^{r_{\max}} \frac{\mathrm{d}w}{w}\frac{\mathrm{d}M(w)}{\mathrm{d}w}, \qquad g_r \simeq 2 - g_{00}$$

$$P(r) \simeq G\int_r^{r_{\max}} \frac{\mathrm{d}w}{w^2}\rho_m(w)M(w) \qquad (r < r_{\max})$$

$$M(r) = 4\pi\int_0^r w^2\rho_m(w)\mathrm{d}w$$

$$\left(\frac{\rho_m(r)}{\rho_*}\right)^{2/3} \simeq 8\pi G\int_r^{r_{\max}} \frac{\mathrm{d}w}{w^2}\int_0^w z^2\rho_m(z)\mathrm{d}z$$

$$\rho_m(r) \simeq \rho_*\left[\frac{2GM}{r_{\max}}\left(1 - \frac{r}{r_{\max}}\right)\right]^{3/2} \qquad (r \lesssim r_{\max})$$

$$G \neq 0,\ H = 0 \qquad u_{\max}^2 H^2 \lesssim \frac{GM}{u_{\max}} \ll 1$$

$$\mathbf{F}_c \simeq mH^2\mathbf{u} \text{ at } G = 0 \qquad |\mathbf{F}_c| \sim mH^2 u_{\max}$$

$$|\mathbf{F}_G| \sim GmM/u_{\max}^2$$

present value of the Hubble parameter $\quad r \simeq 10^{-75}$

$r \equiv GM/(u_{\max}^3 H^2) \simeq 10^{34}$ sun $\quad 10^{-66}$

$r \simeq 10^{43}$ proton $\quad$ inflation

The fate of the universe

The event horizon is $u = ra =$ constant. The Schwartzschild radius is also given by $u = ra =$ constant, as emphasized by (2.18), (2.19) and (2.20). To simplify as much as possible, we face three possibilities: a) if a celestial body expands in the $\mathbf{u}$ variables, its constituents eventually reach the state of total dilution; b) if it stays in equilibrium, it resists the veer towards that fate; c) if it contracts, it collapses to form a black hole.

Eventually, the black hole evaporates, emitting particles and radiation. The emitted particles are expected to move away and disperse into space, traveling indefinitely and becoming increasingly diluted. This way, more and more particles eventually reach the state of total dilution. So, the cases a) and c) lead to the same outcome. Only the intermediate situation of equilibrium b) can resist the drift towards the ultimate fate.

In some sense, black-hole evaporation is a way to counteract the gravitational attraction and make the expansion of the universe prevail. In addition, an event horizon may not be necessary to have emission of radiation [8], and evaporation. A system could produce pairs and lose energy through a gravitational analogue [6, 7, 8] of the Schwinger pair production mechanism [9]. If so, the equilibrium *b*) would just be temporary. Nature would host a more "democratic" process (in the sense that it would not be just the prerogative of a privileged class, such as black holes) to counteract gravitational attraction and sustain expansion over time, towards the final state of total dilution. This scenario aligns with the idea that quanta do not favor stability but uncertainty, providing avenues for escape.

Once the state of total dilution is reached, say at time t_{dis}, the isolated particles are not actually particles, but wave functions, bound to remain so forever, because they cannot collapse at later times. Particles may receive signals from other particles after t_{dis}, coming from their past histories. However, those signals are just radiation. Even if particle A starts a journey towards particle B before t_{dis}, it is unable to reach B at $t > t_{\text{dis}}$, because it would have to overcome the dilution occurred at t_{dis}. We do not know of wave-function collapses triggered by radiation only.