UV-extending Higgs inflation in Einstein-Cartan Gravity

Based on: M. He (IBS-CTPU), KK, K. Mukaida (KEK), JHEP01 (2024) 014 arXiv: 2308.14398 [hep-ph];
see also PRL 132 (2024) 191501, arXiv: 2308.15420 [hep-ph]

國科大杭州高等研究院
Hangzhou Institute for Advanced Study,UCAS

Kohei Kamada
(Hangzhou Institute for Advanced Study, UCAS)

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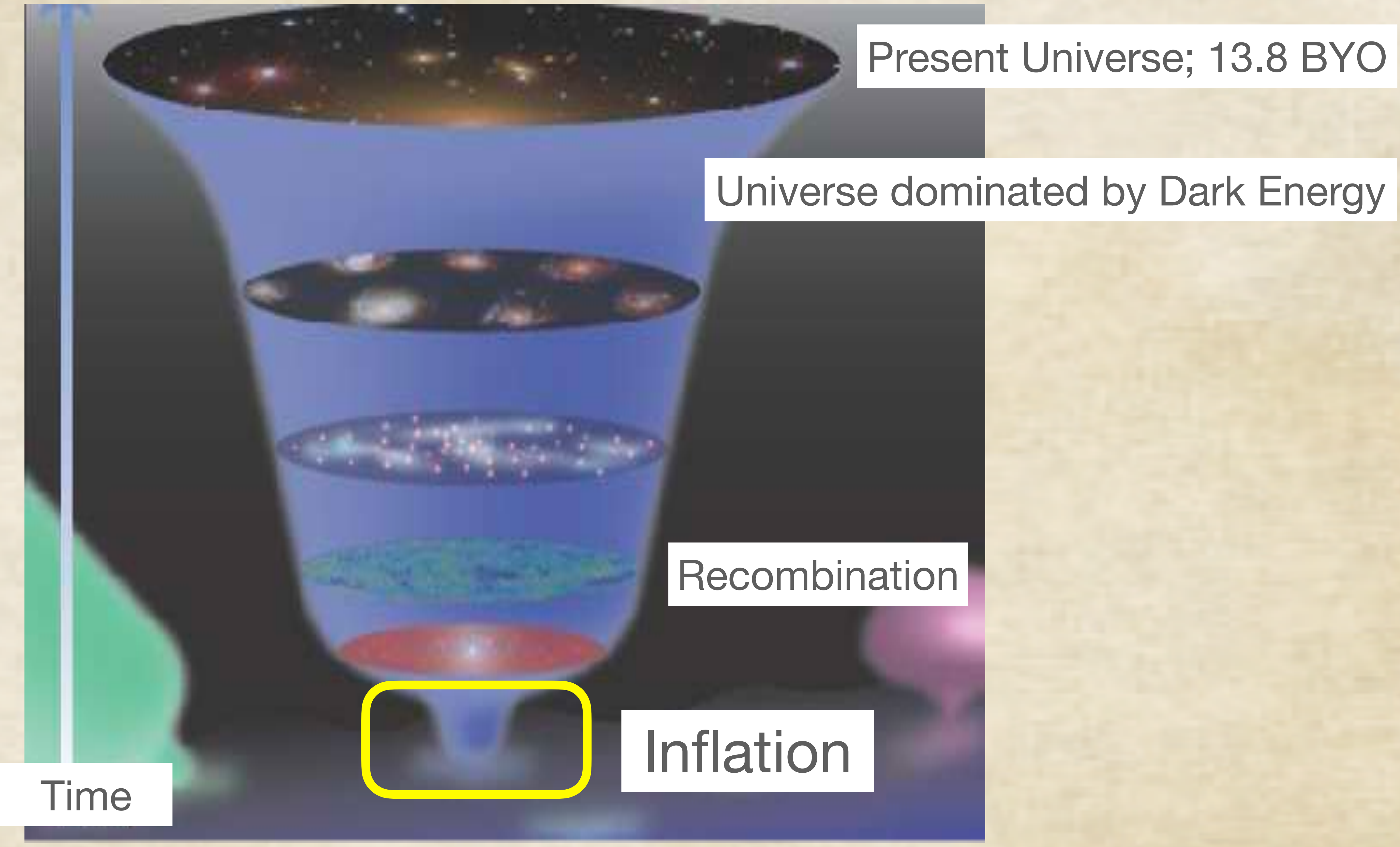
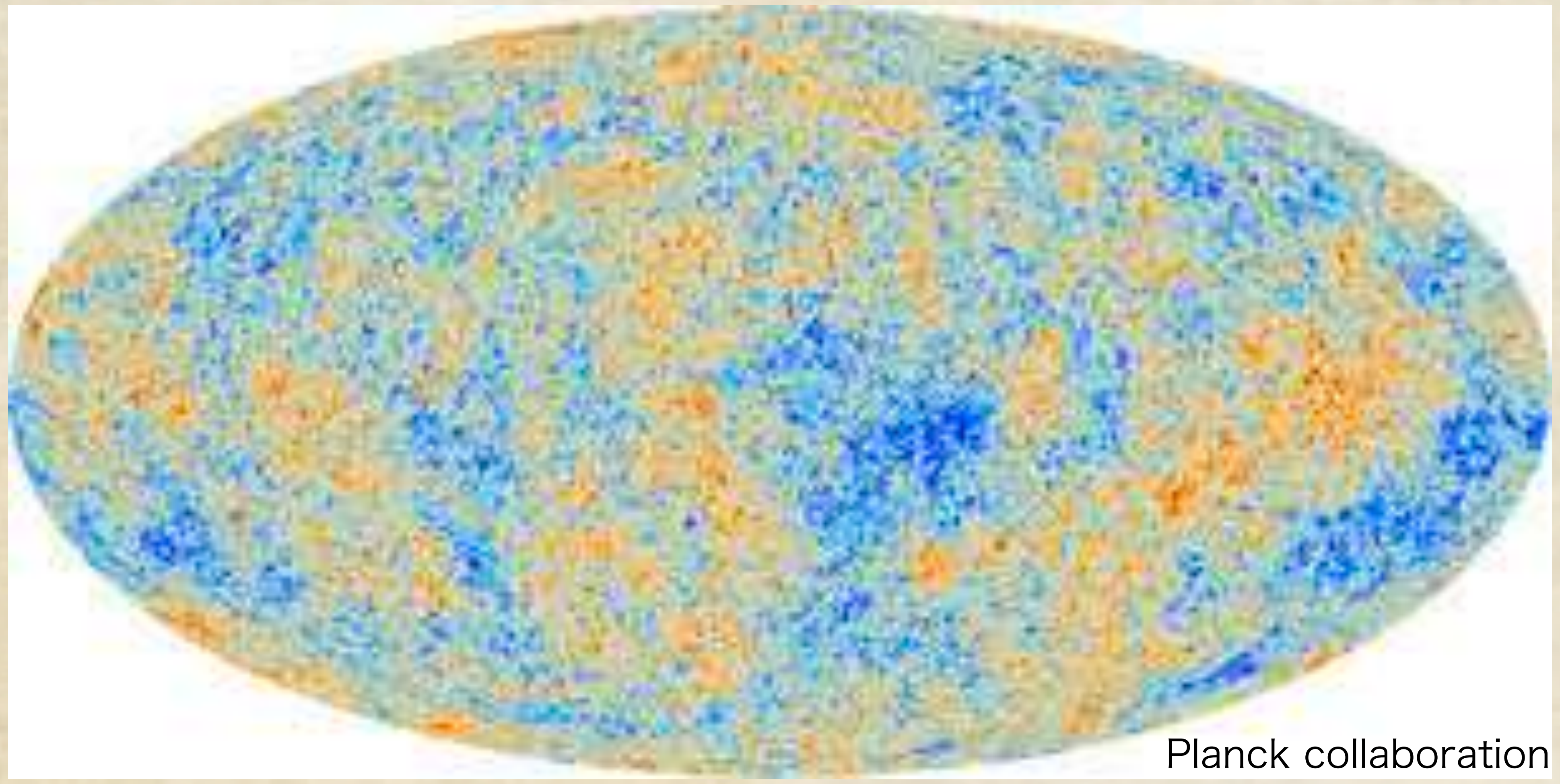
Introduction — Higgs inflation —

Inflation: key ingredient in modern cosmology

- Accelerating expansion of the Universe in the primordial Universe.
- Solution to the horizon, flatness, and monopole problems.
- Origin of the large-scale structure of the present Universe.





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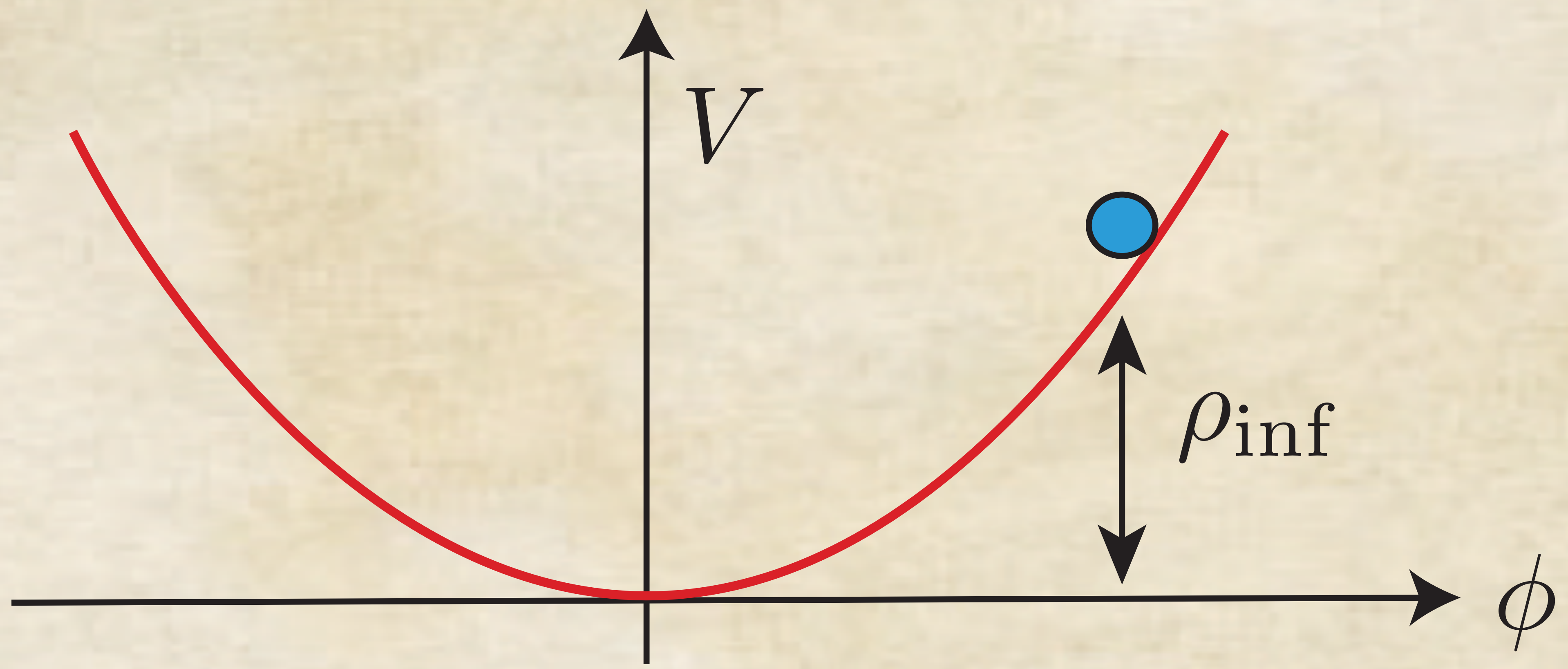

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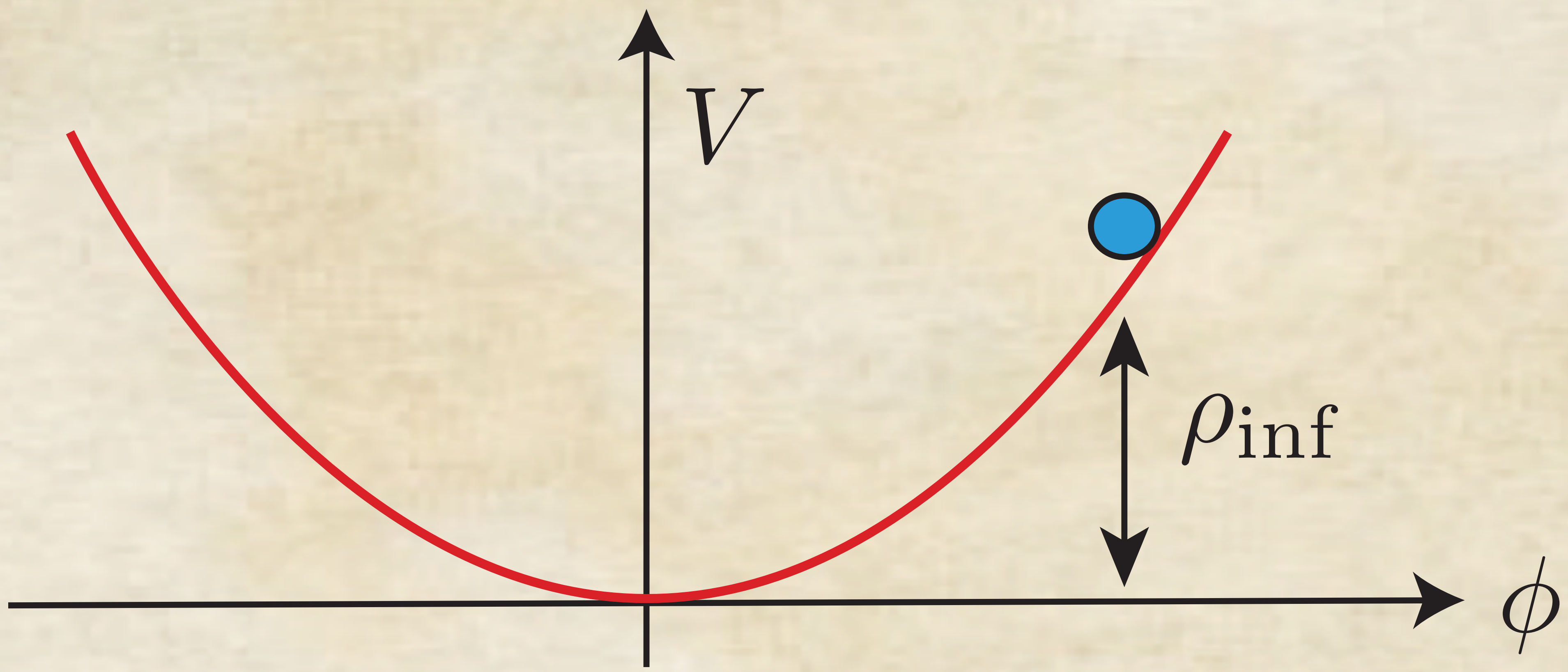

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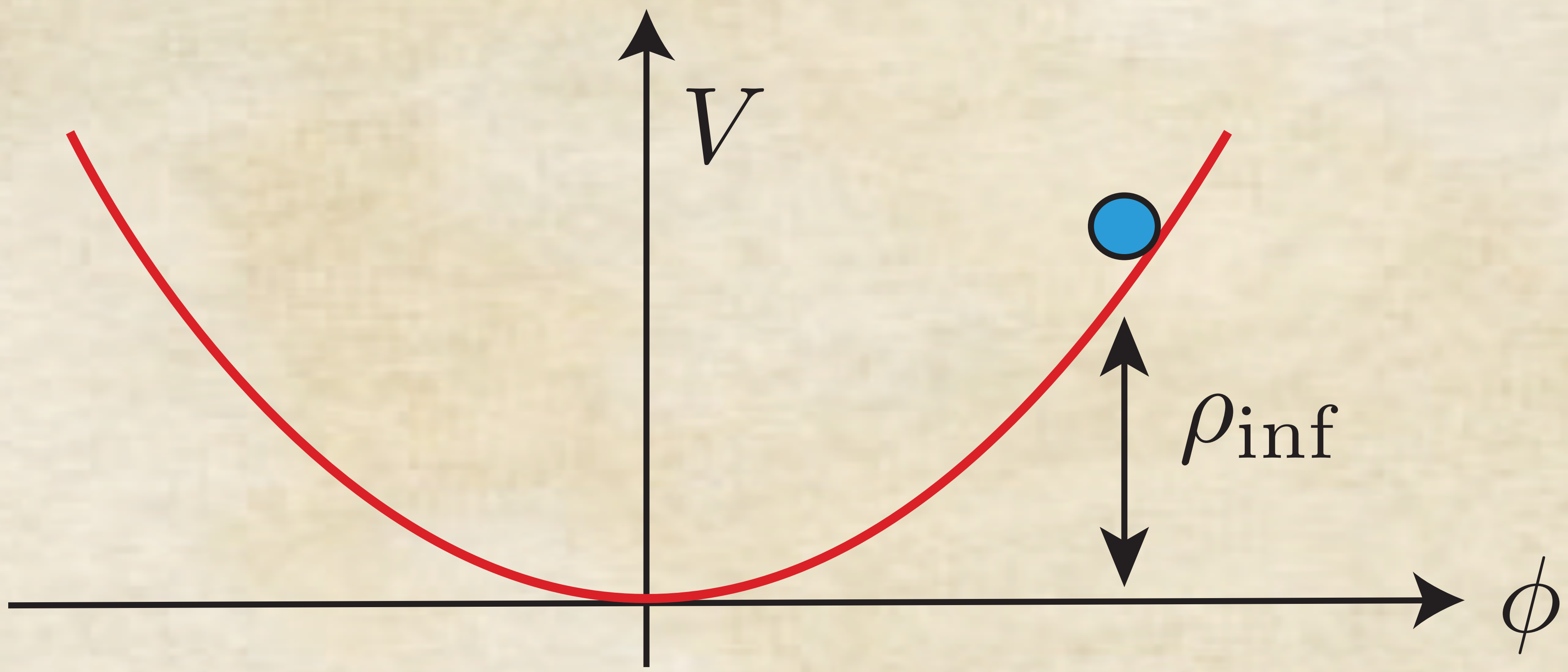

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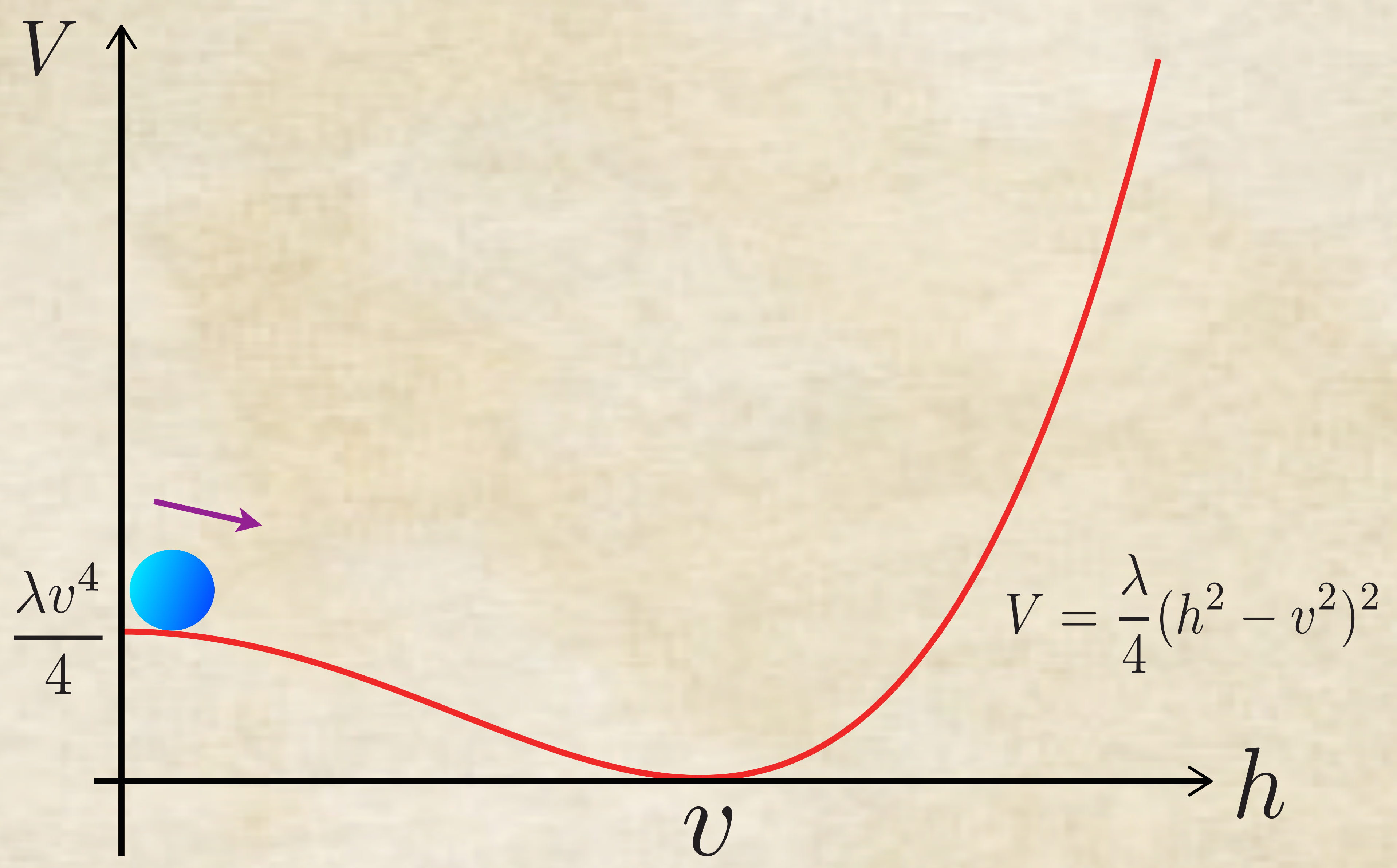
Can the SM Higgs be inflaton?

Can Higgs potential drive accelerating expansion of the Universe 🤔 ?

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - |D_\mu\mathcal{H}|^2 - \lambda(|\mathcal{H}|^2 - v^2)^2\right]$$



New inflation is impossible 😔,

because the slow-roll parameter

$\eta \equiv M_{\rm pl}^2 \frac{V''}{V}$ cannot be small.

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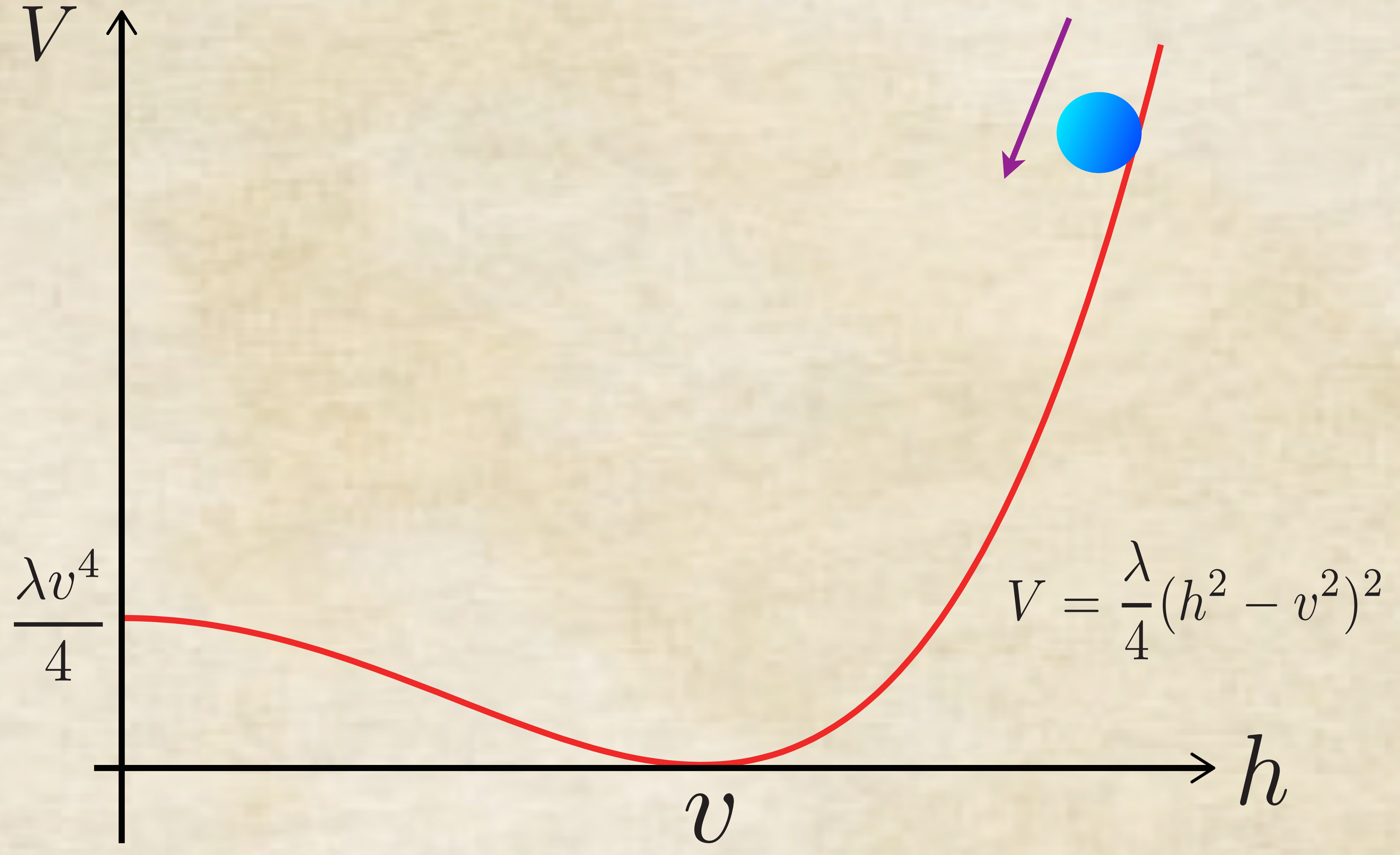

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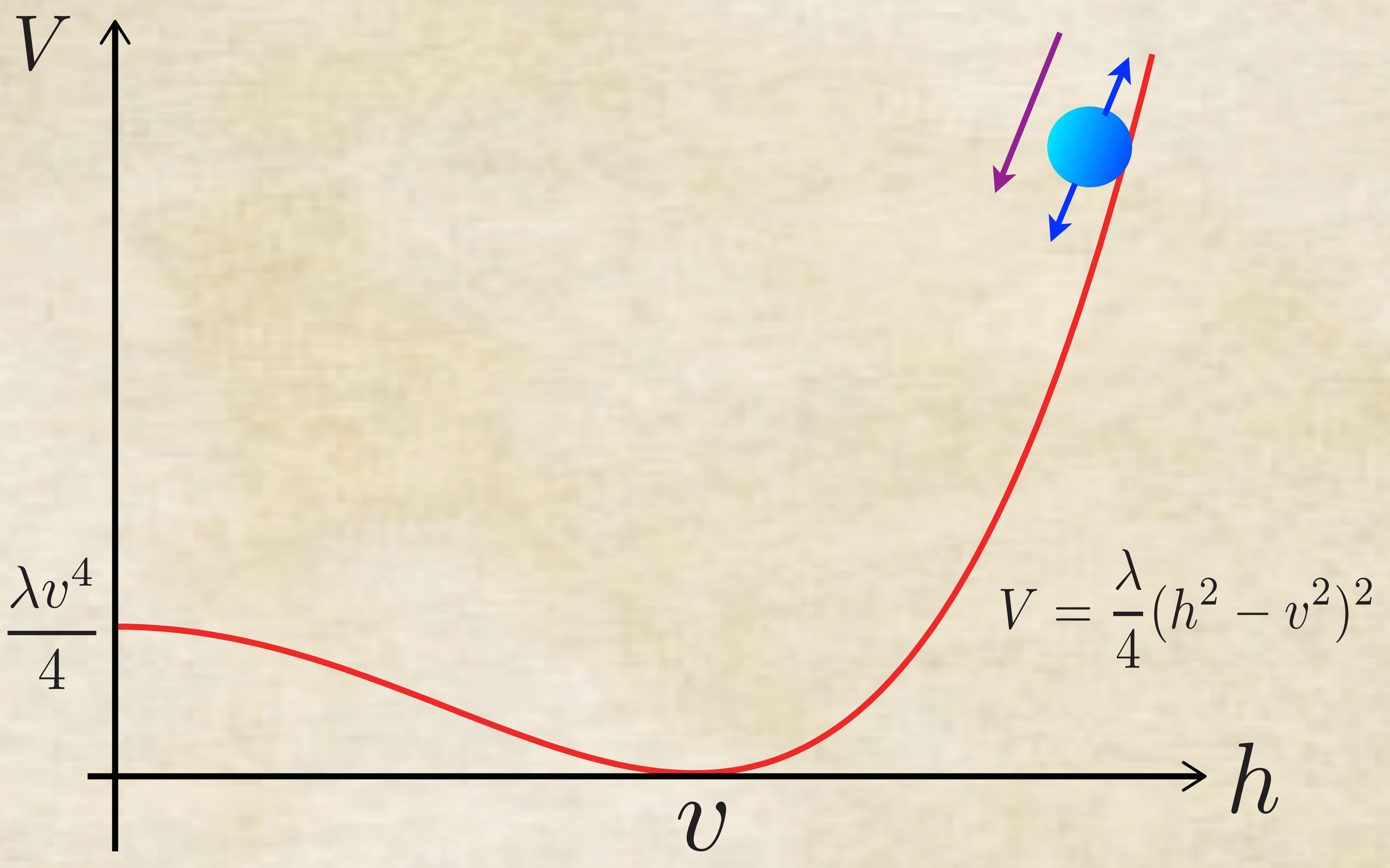

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But the density perturbation generated in this case,

$$\mathcal{P}_\zeta \sim 10^3\lambda$$

is much larger than the one in the real Universe, $\mathcal{P}_\zeta^{\rm obs} \simeq 2.18 \times 10^{-9}$ for $\lambda_{\rm Higgs} \sim \mathcal{O}(1)$ 😔

(Non-minimal) Higgs Inflation (in the metric formalism)

('95 Cerventas-Cota & Dehnen, '08 Bezrukov & Shaposhnikov)

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\left(\frac{M_{\rm pl}^2}{2} + \xi|\mathcal{H}|^2\right)R - |D_\mu\mathcal{H}|^2 - \lambda|\mathcal{H}|^4\right] \qquad \text{w/} \quad \xi \gg 1$$

$$\mathcal{H} = (0, h/\sqrt{2})$$

Conformal transformation

$$g_{\mu\nu}^{\rm E} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \frac{\xi h^2}{M_{\rm pl}^2}$$

$$\mathcal{S} = \int d^4x\sqrt{-g_{\rm E}}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}\frac{\Omega^2 + 6\xi h^2/M_{\rm pl}^2}{\Omega^4}(\partial_\mu h)^2 - \frac{\lambda}{4\Omega(h)^4}h^4\right]$$

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$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2h^2/M_{\rm pl}^2}{\Omega^4}}$$

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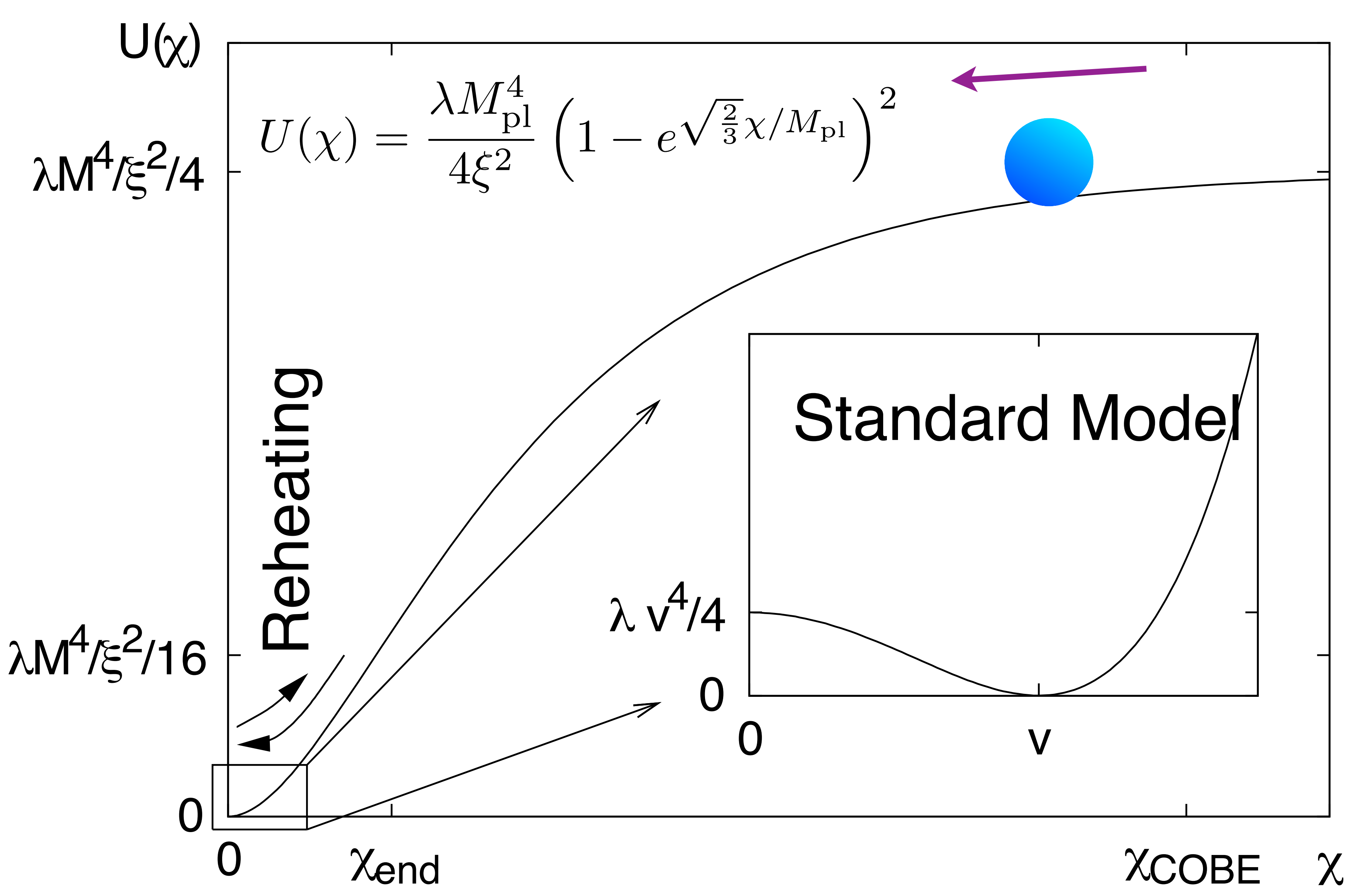

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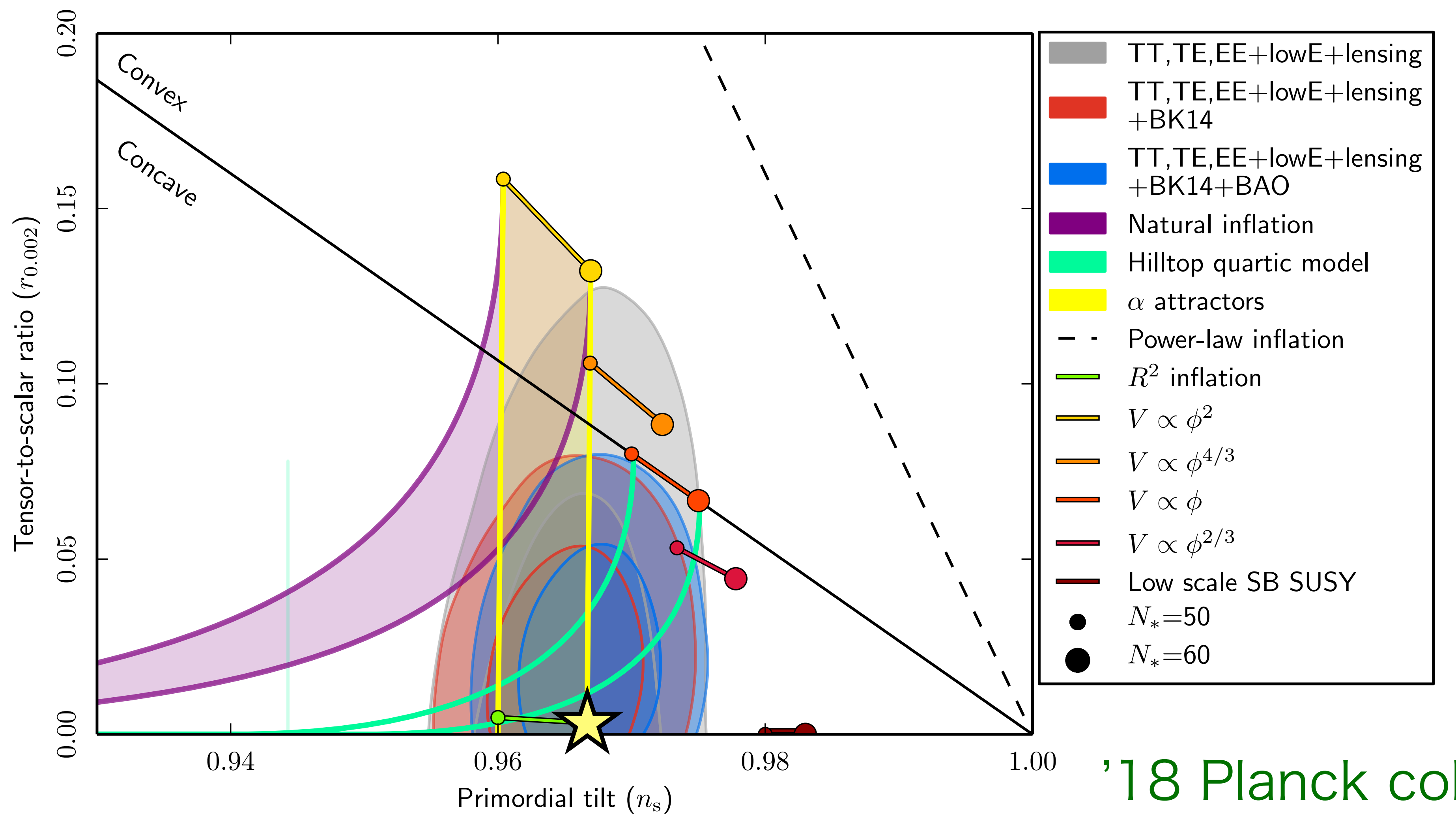

'18 Planck collaboration

Fig. 8. Marginalized joint 68 % and 95 % CL regions for n_s and r at k = 0.002 Mpc^{-1} from *Planck* alone and in combination with BK14 or BK14 plus BAO data, compared to the theoretical predictions of selected inflationary models. Note that the marginalized joint 68 % and 95 % CL regions assume $dn_s/d\ln k = 0$.

$\xi/\lambda^{1/2} \simeq 4 \times 10^4 \gg 1$ $\qquad n_S \simeq 0.97 \quad r \simeq 0.0033$

Simplest model of inflation driven by the SM Higgs, which fits the CMB data very well 🤩.

Slide Background Courtesy: H. Oide

Problems in Higgs inflation and its UV extension

Unitarity bound in Higgs inflation 🧐

Cutoff scale of the theory (at the vacuum): $\Lambda \simeq \frac{M_{\rm pl}}{\xi}$

$$\mathcal{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} \pi_2 + i\pi_3 \\ h + \pi_0 + i\pi_1 \end{pmatrix}$$

('09 Barbon&Espinosa, '10 Burgess, Lee, &Trott, '10 Hertzberg)

Jordan frame: $\xi |\mathcal{H}|^2 R \ni \frac{\xi}{M_{\rm pl}} \pi_i^2 \partial^2 \gamma$

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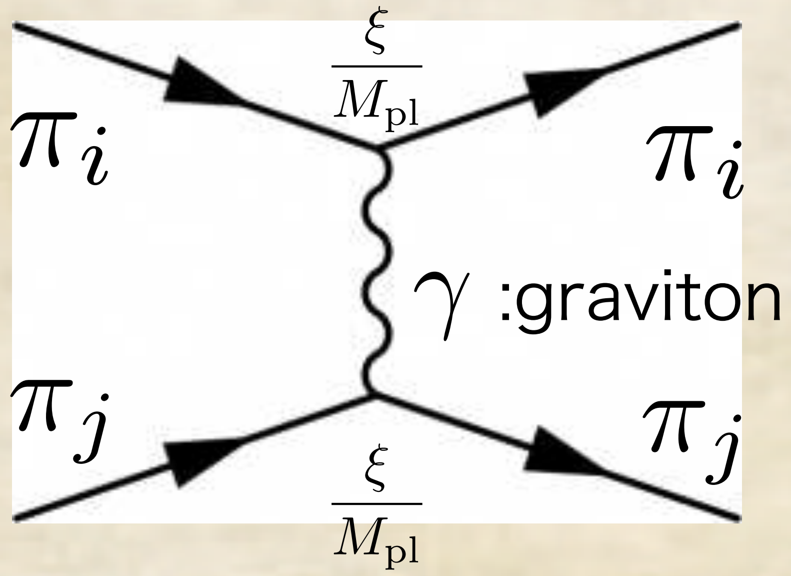

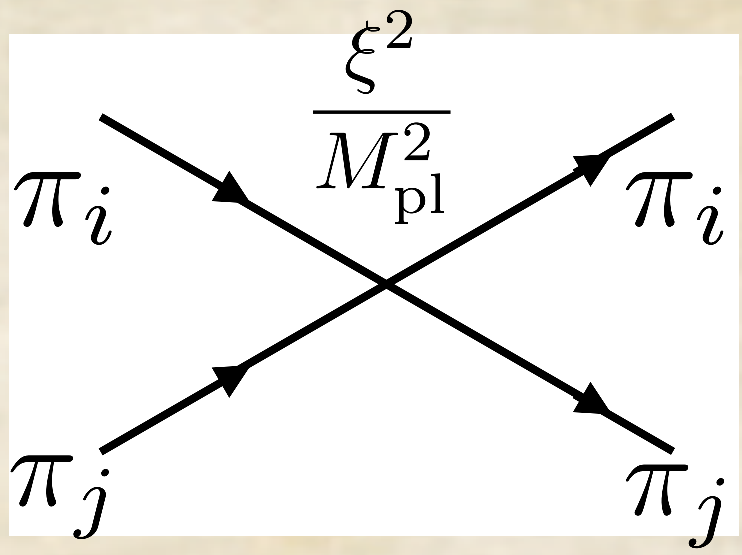

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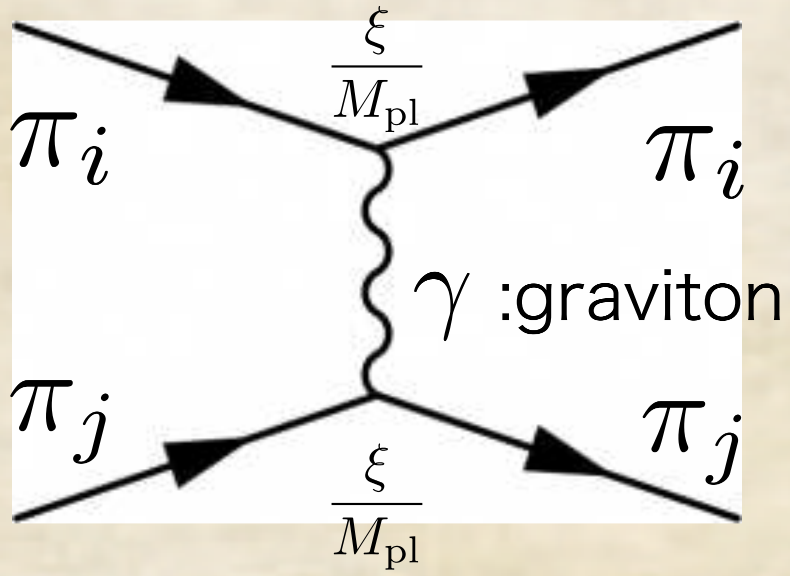

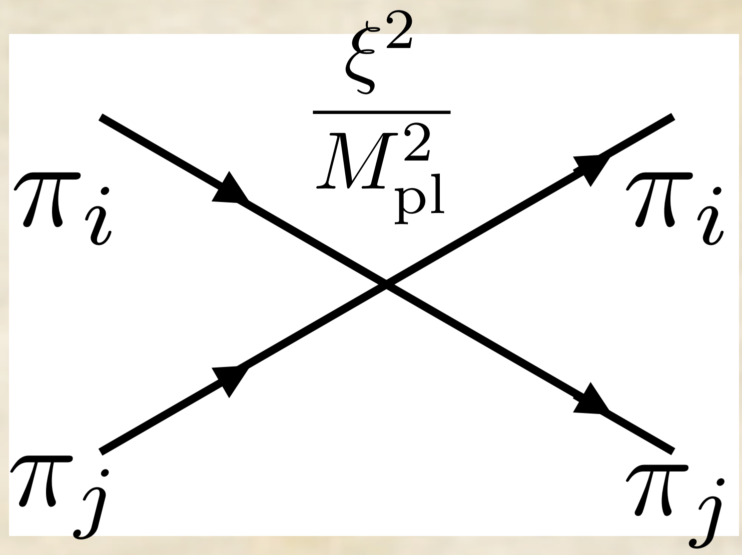

Typical energy scale during inflation : $\rho_{\rm inf}^{1/4} \simeq \dfrac{\lambda^{1/4} M_{\rm pl}}{\sqrt{\xi}} \gg \Lambda \simeq \dfrac{M_{\rm pl}}{\xi}$

Is the predictions in Higgs inflation unreliable?

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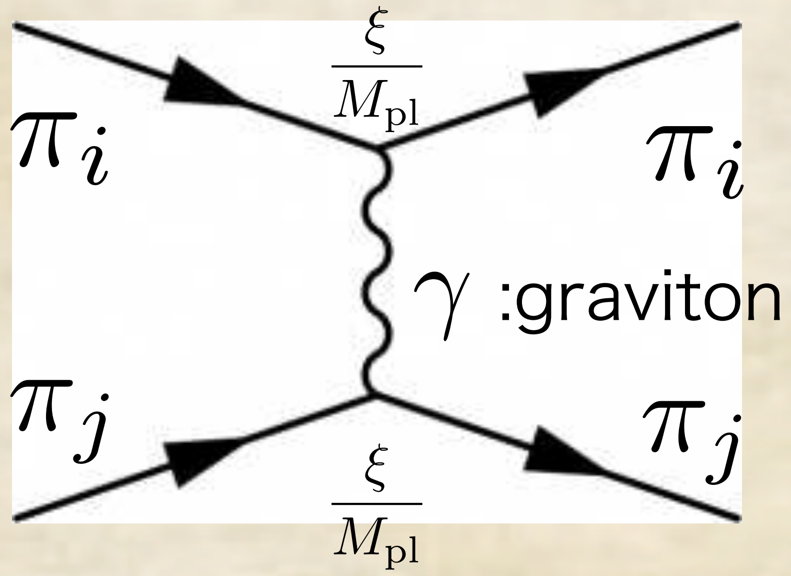

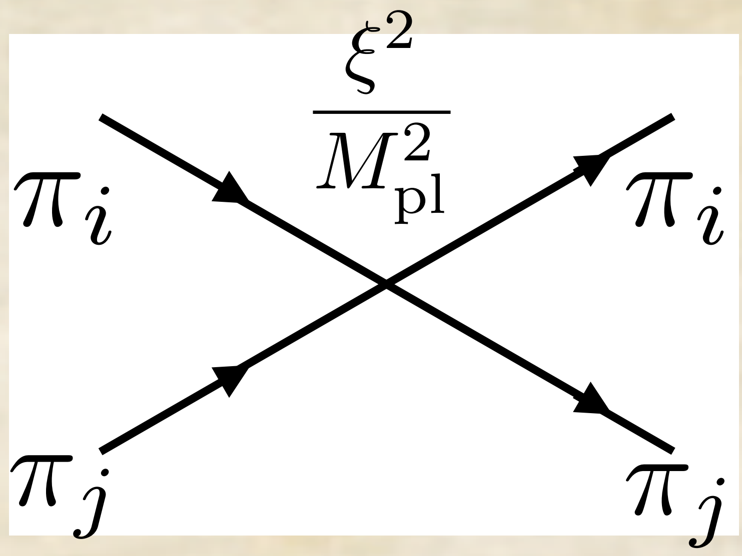

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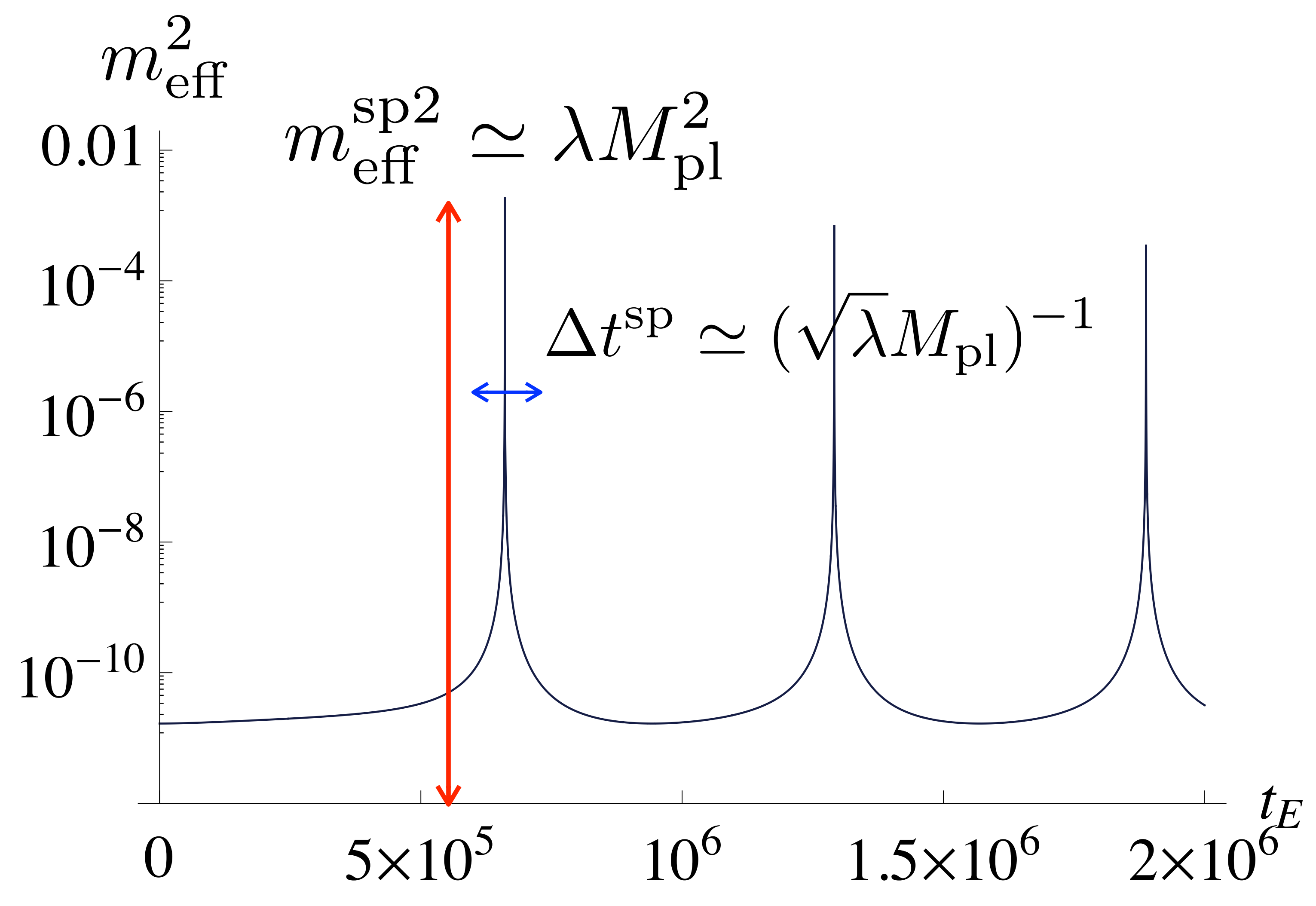
Perhaps OK during inflation, since the cutoff scale during inflation is larger.

('11 Bezrukov, Gorbunov, & Shaposhnikov)

Unitarity violation in this Higgs Inflation 🧐

Longitudinal mode of the weak gauge bosons (or the NG mode) receives mass with spiky feature at the reheating/oscillation phase,

('15 DeCross, Kaiser, Prabhu, Prescod-Weistein, Sfakianakis, '16 Ema, Jinno, Mukaida, Nakayama)



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$$-g_{\rm E}^{\mu\nu}\frac{1}{\Omega^2}\left(1+\frac{12\xi^2}{\Omega^2}\frac{|\mathcal{H}|^2}{M_{\rm pl}^2}\right)\partial_\mu\mathcal{H}\partial_\nu\mathcal{H}^\dagger \qquad \Omega^2 = 1+\frac{2\xi|\mathcal{H}|^2}{M_{\rm pl}^2}$$

$$m^2_{\rm eff} \ni \frac{\xi(1+6\xi)\dot{h}^2}{M^2_{\rm pl}+\xi(1+6\xi)h^2}$$

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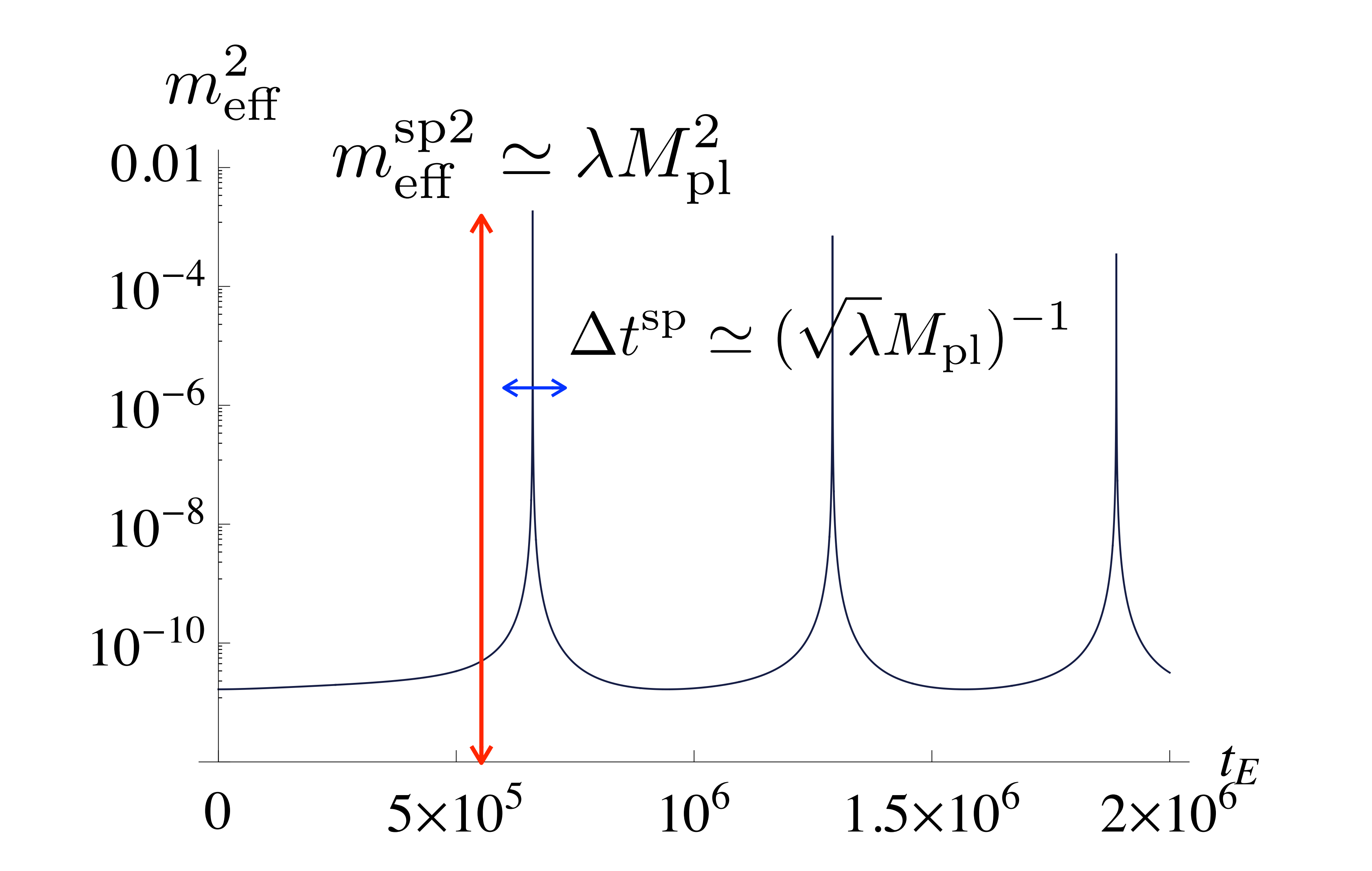

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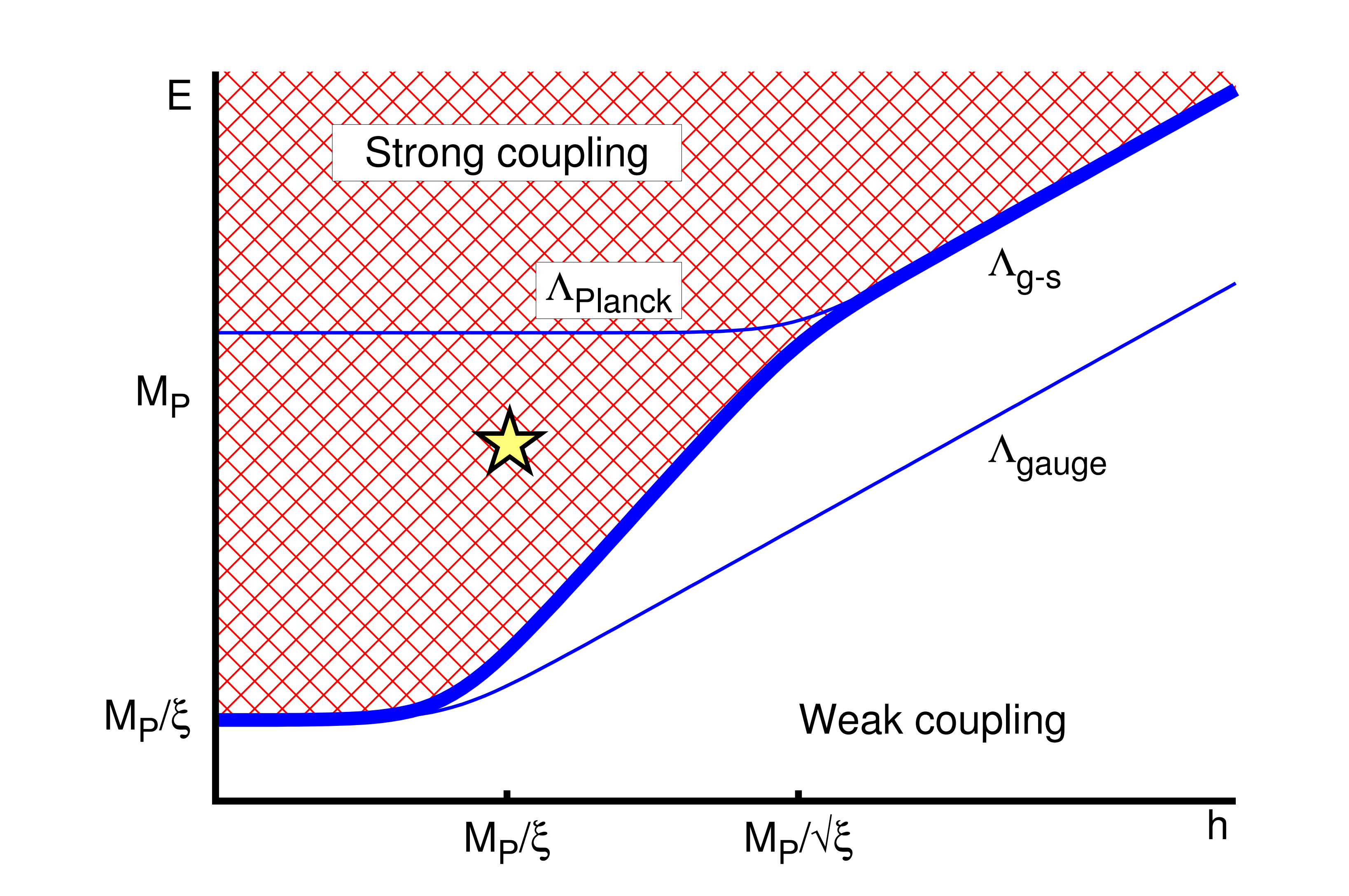

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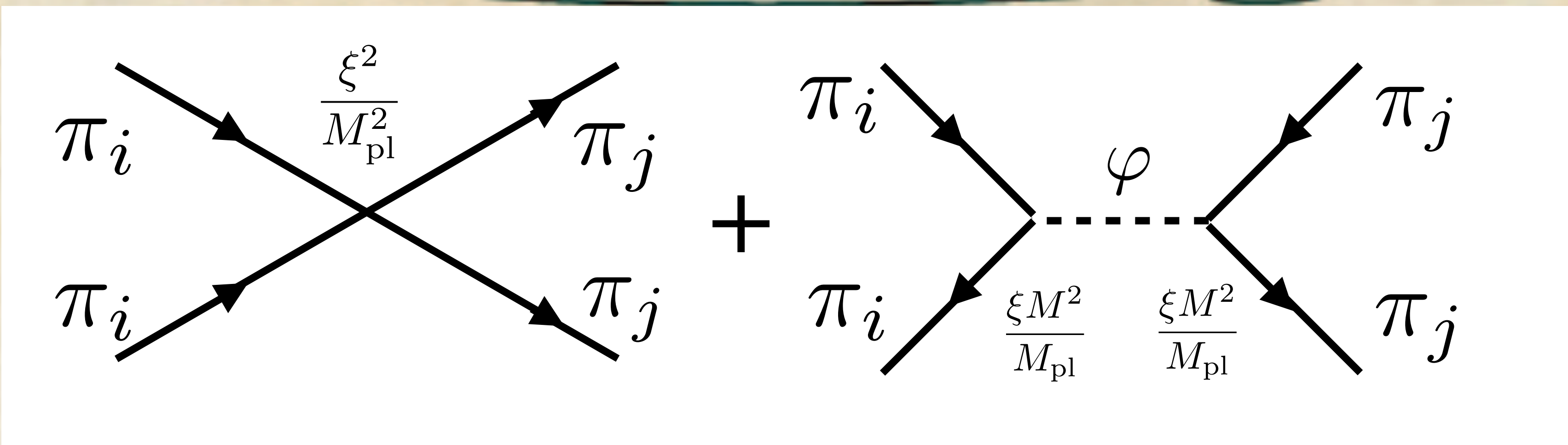
which leads to the violent excitation of gauge boson with $k \simeq \sqrt{\lambda} M_{\rm pl}$

=> Unitarity violation 😫

Slide Background Courtesy: H. Oide

UV-extension by R^2 term



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cutoff scale is pushed up to the Planck scale. ('17 Ema, '18 Gorbunov&Tokareva/ '18 He, Jinno, KK+)

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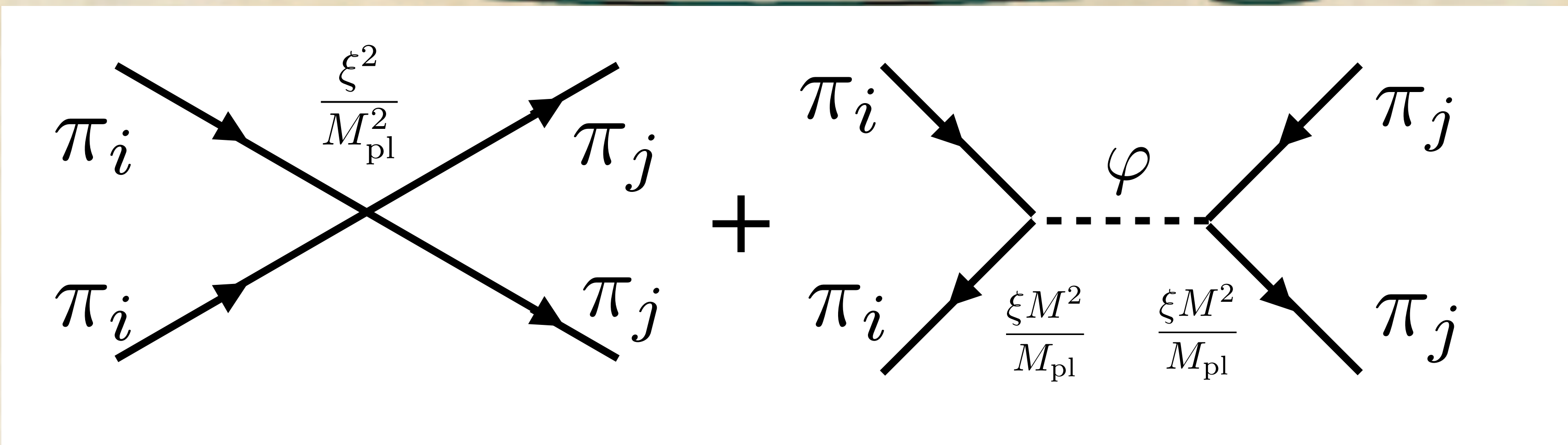

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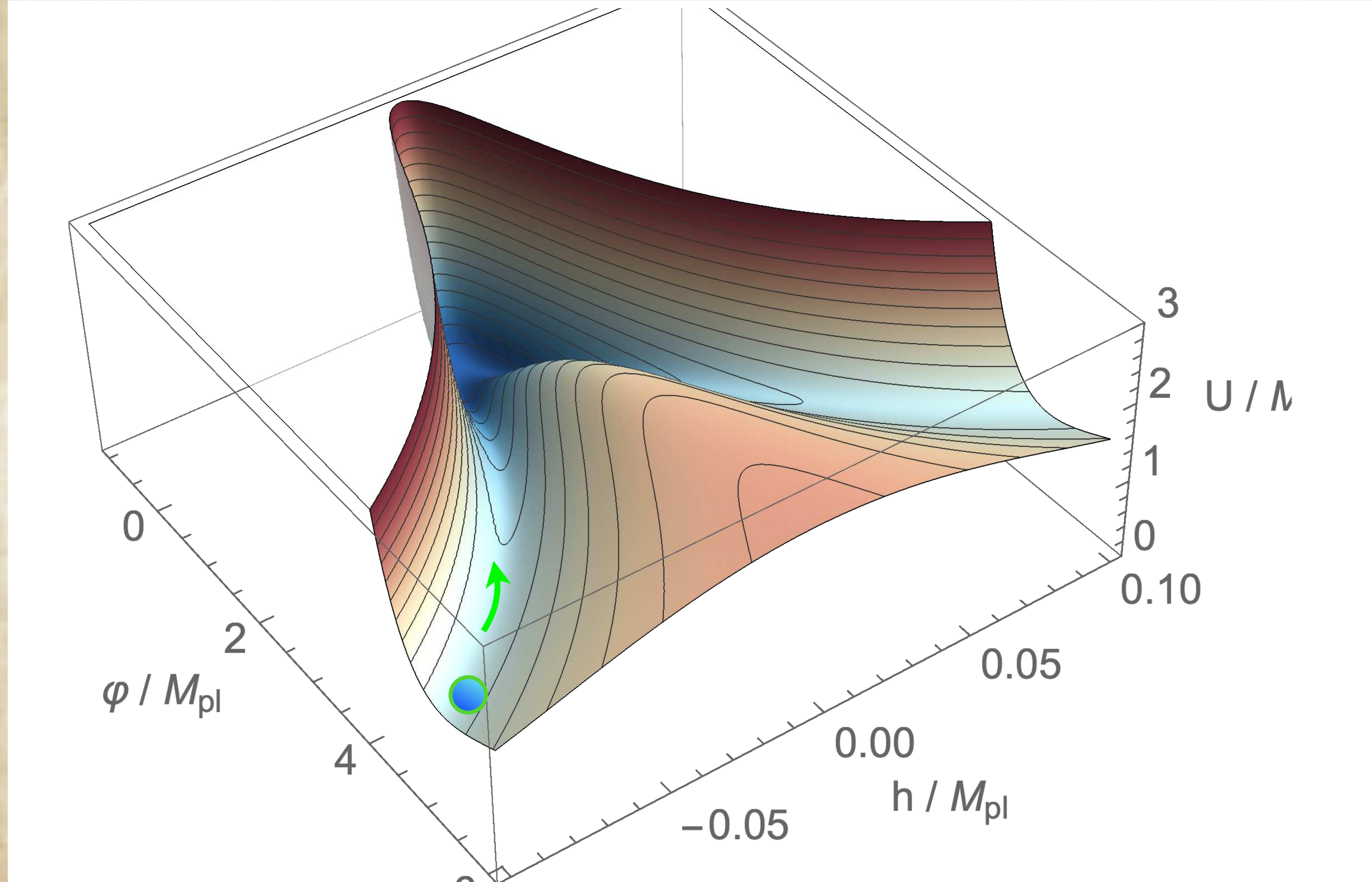

'20 He, Jinno, KK, Starobinsky, Yokoyama

R^2 term gives rise to a new scalar degree of freedom, scalaron, with $m_\sigma = M$ and the inflation becomes a two-field model (in the Einstein frame).

Cosmological predictions are unchanged.

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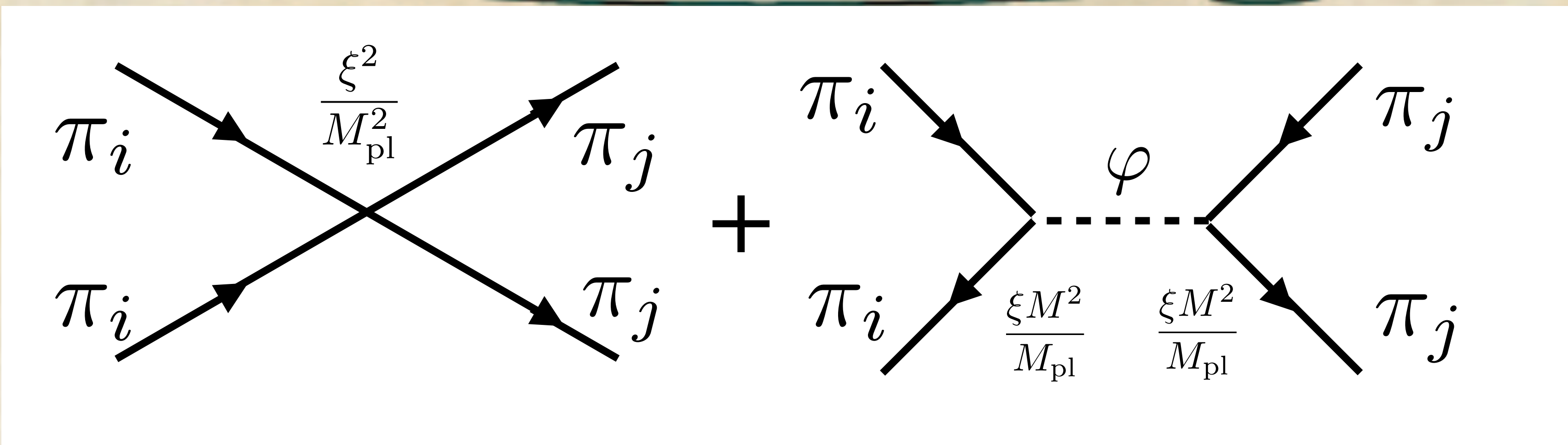

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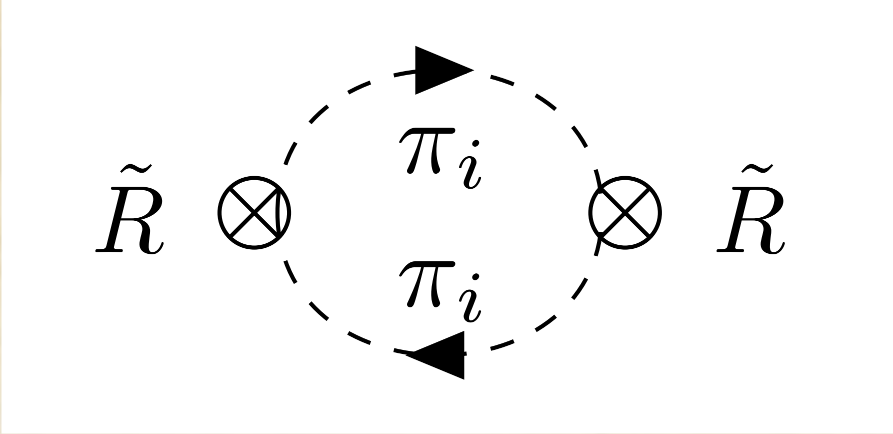
Indeed, this is natural appearance from the quantum correction of the scalar sector at the leading order in the large-N analysis

('15 Salvio & Mazumdar, '19 Ema, '20 Ema, Mukaida, van de Vis)



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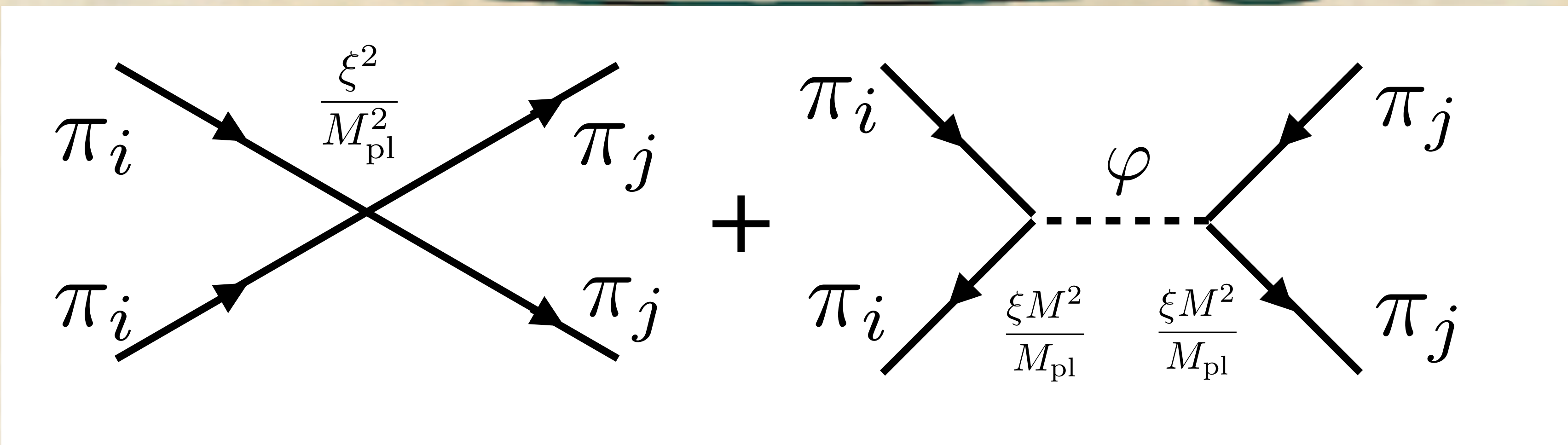

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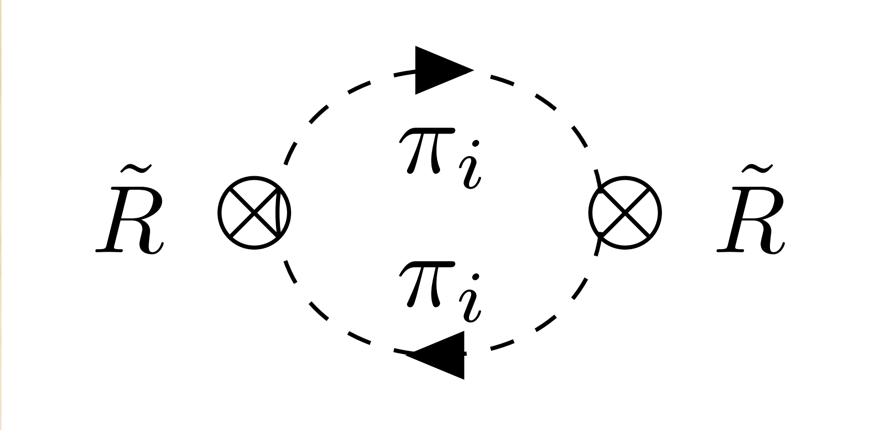

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where the scalaron is non-perturbatively induced from the resummation of the dominant one-loop correction in the theory with conformal mode.

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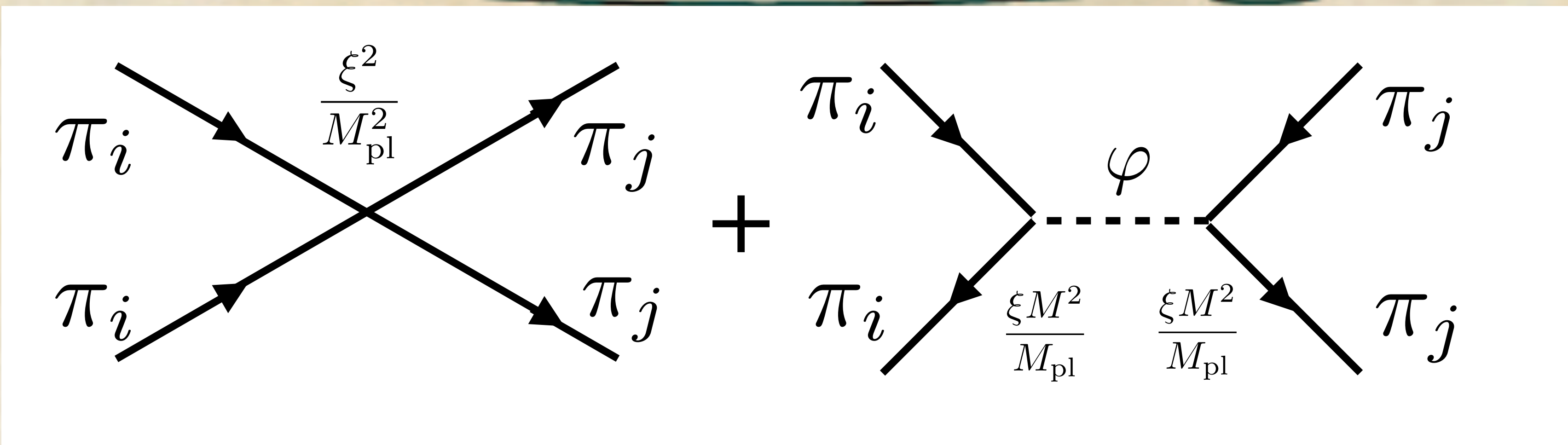

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Practically, an auxiliary field γ in the conformal factor Ω becomes dynamical.

$$S = \int \mathrm{d}^4x\left[\frac{\Phi_{\rm E}^2}{12}\tilde{R} + \frac{1}{2}\tilde{g}^{\mu\nu}\left(\partial_\mu\Phi_{\rm E}\partial_\nu\Phi_{\rm E} - \frac{\Phi_{\rm E}^2/\Omega^2}{6M_{\rm pl}^2}\partial_\mu\phi_i\partial_\nu\phi^i\right) - \left(\frac{\Phi_{\rm E}^2/\Omega^2}{6M_{\rm pl}^2}\right)^2 V(\phi) - \alpha\gamma^2 - \frac{\Phi_{\rm E}^2}{8}\tilde{g}^{\mu\nu}\partial_\mu\ln|\Omega^2|\,\partial_\nu\ln|\Omega^2|\right],$$

$$\Omega^2 \equiv 1 + 12\left(\frac{\xi}{2}\frac{\tilde{\pi}_i^2}{\Phi_{\rm J}^2} + 2\alpha\frac{\gamma}{\Phi_{\rm J}^2}\right)$$

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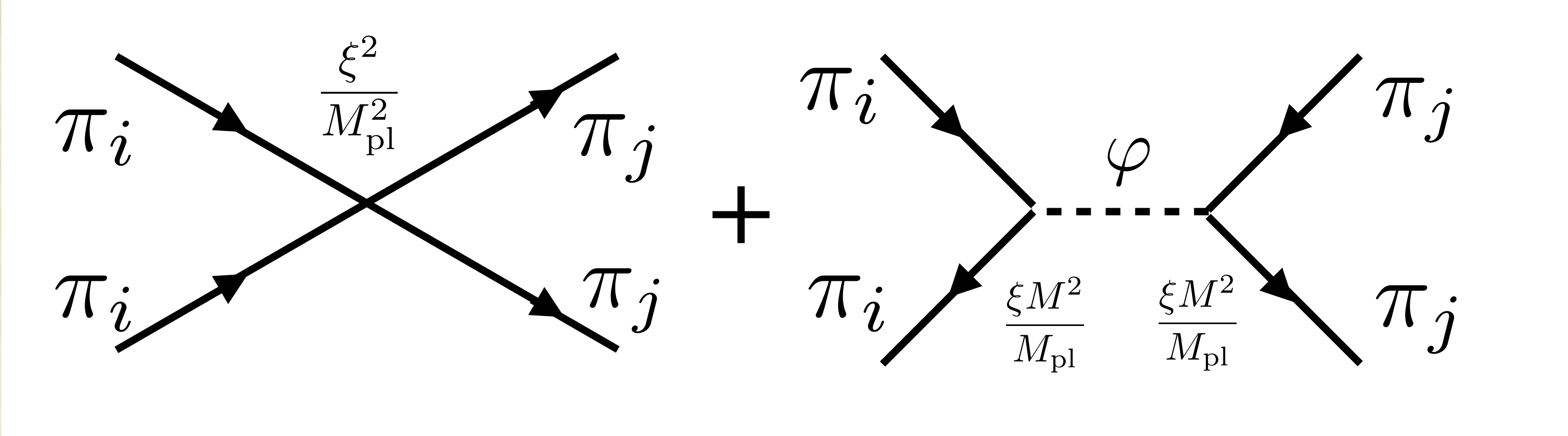

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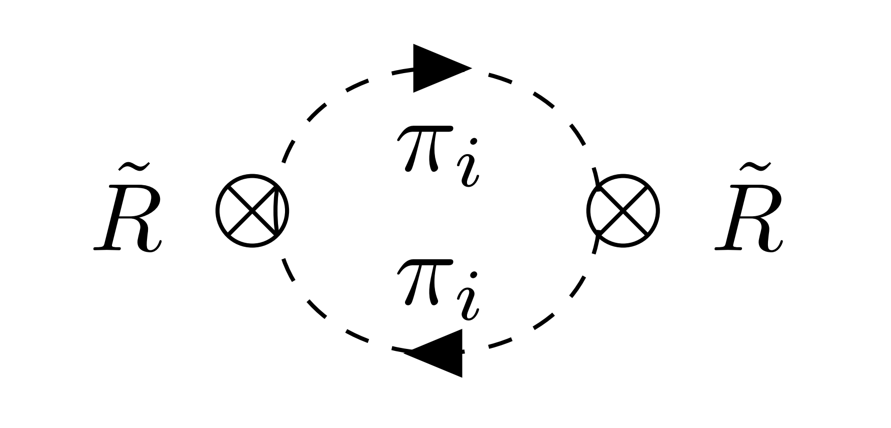

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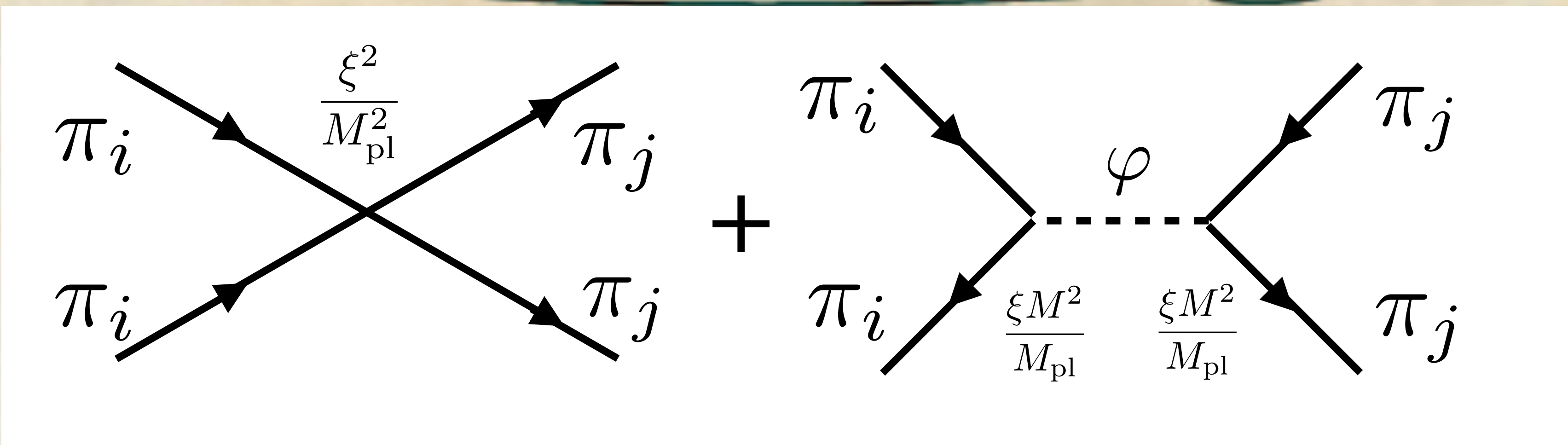

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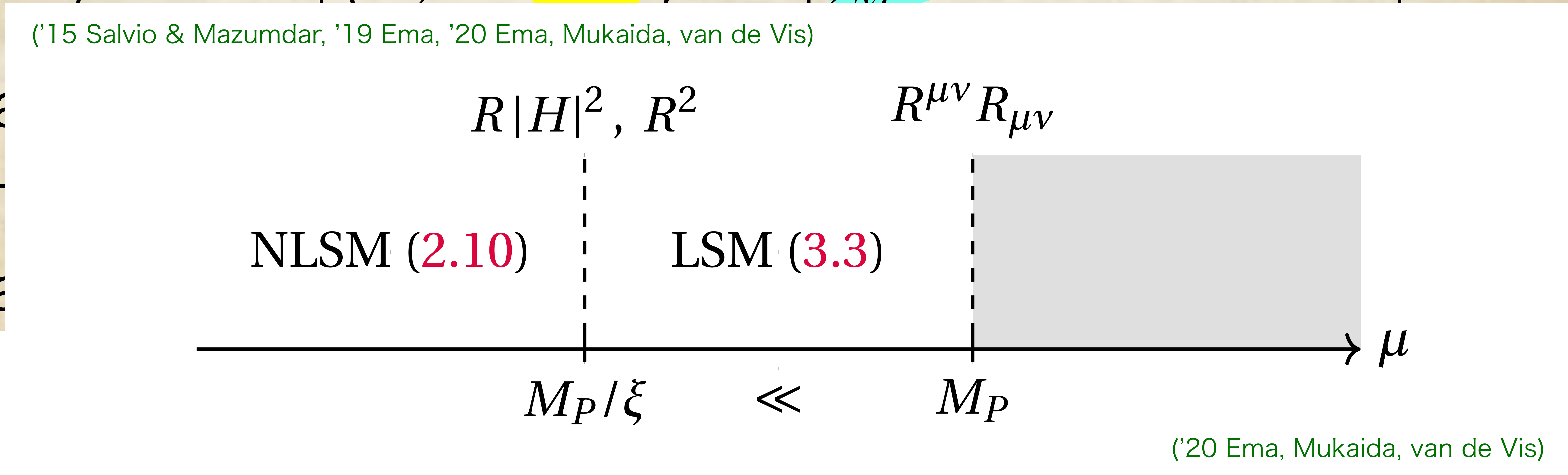

cutoff sca ... '18 He, Jinno, KK+)

Indeed, th ... tion

of the sca ... sis

0 Ema, Mukaida, van de Vis)

pl ... ξ^2 $(\alpha \sim N\xi)$

where the scalaron is non-perturbatively induced from the resummation of the dominant one-loop correction in the theory with conformal mode.

UV-extension by R^2 term

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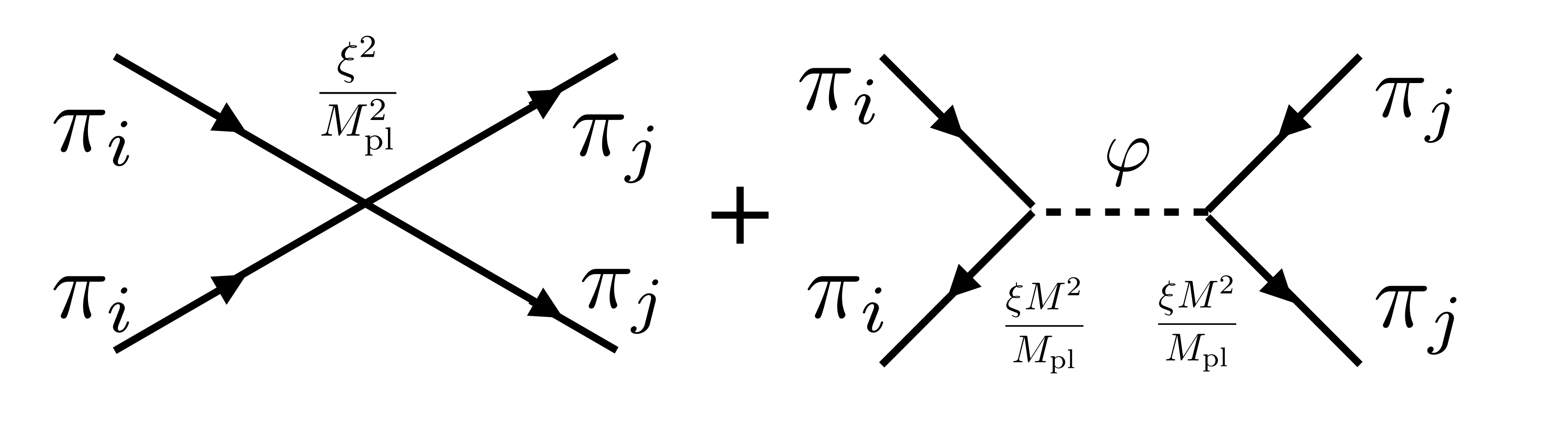
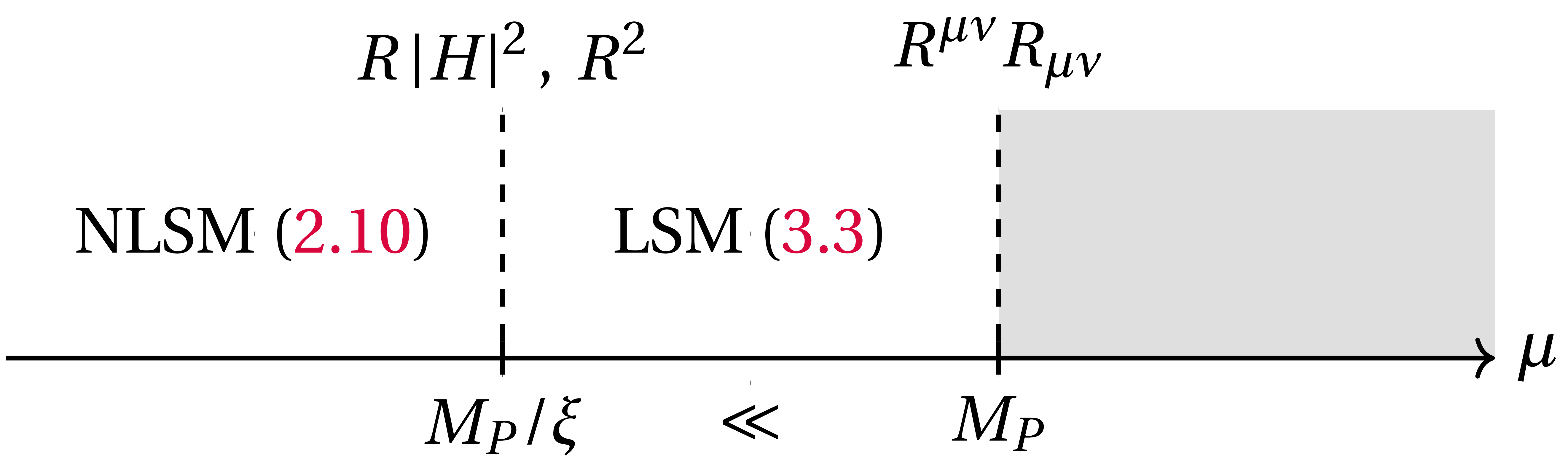


$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\left(\frac{M_{pl}^2}{2} + \xi|\mathcal{H}|^2\right)R + \frac{M_{pl}^2}{12M^2}R^2 - |D_\mu\mathcal{H}|^2 - \lambda|\mathcal{H}|^4\right]$$

('15 Salvio & Mazumdar, '19 Ema, '20 Ema, Mukaida, van de Vis)



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where the scalaron is non-perturbatively induced from the resummation of the dominant one-loop correction in the theory with conformal mode.

But fine-tuning is needed for small Higgs mass and cosmological constant···

Higgs inflation in Palatini and Einstein-Cartan formalism

Higgs inflation with R^2 UV extension is reasonable and free from Unitarity problem.

But it still has a naturalness problem.

How is the situation in its variants 🧐?

Higgs inflation in Palatini formalism

We studied Higgs inflation in metric formalism => connection = Levi-Civita.

$$\bar{\Gamma}^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu} \equiv \frac{1}{2} g^{\rho\sigma} (\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\sigma\mu} - \partial_\rho g_{\mu\nu})$$

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$\bar{\Gamma}^{\rho}_{\mu\nu}$ and $g_{\mu\nu}$ are a priori independent while keeping torsionless.

GR is reproduced by solving the EOM with E-H action.

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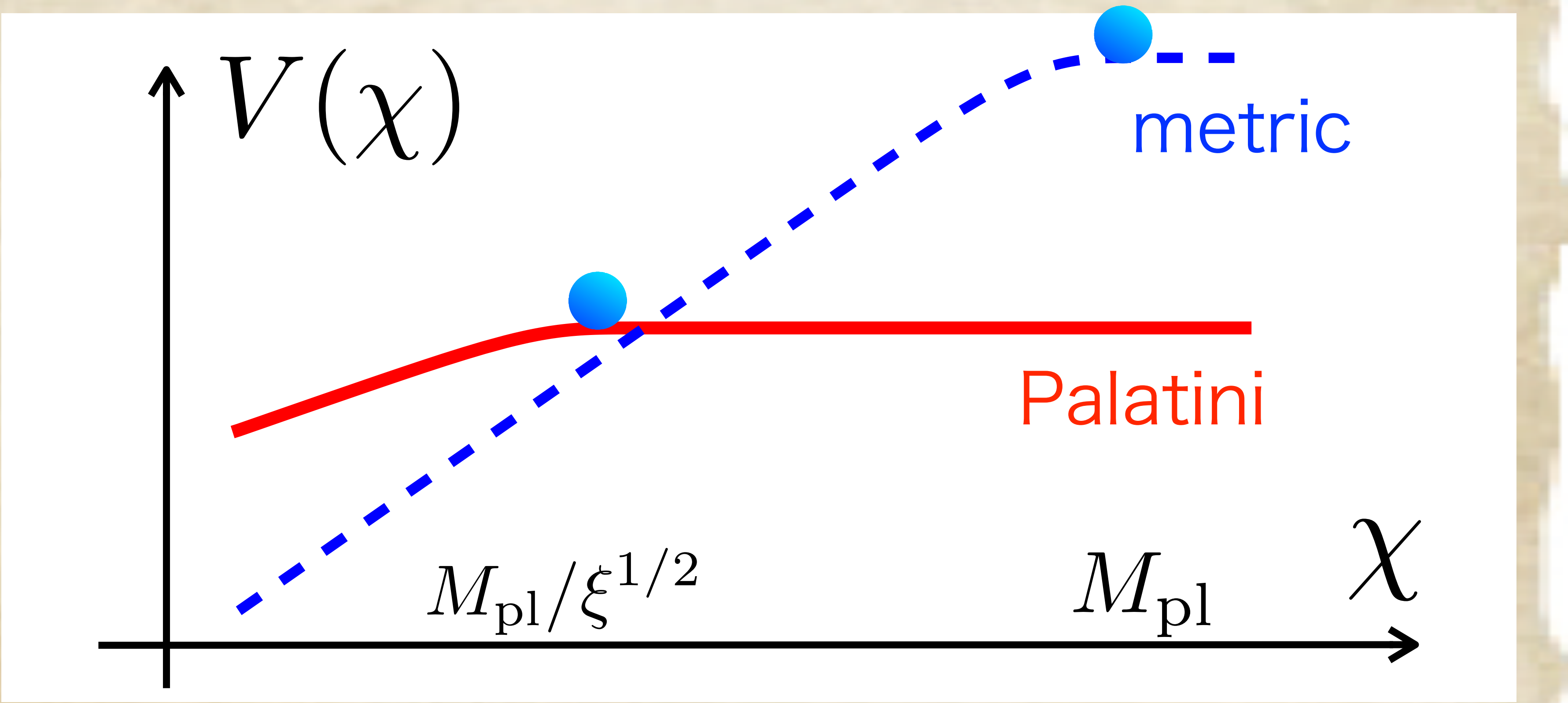
$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\left(\frac{M_{\rm pl}^2}{2} + \xi |\mathcal{H}|^2 \right) R(g, \bar{\Gamma}) - |D_\mu \mathcal{H}|^2 - \lambda |\mathcal{H}|^4 \right]$$

conformal → trans.



Fits CMB data well with $\xi \simeq 10^{10} \lambda$

$n_S \simeq 0.97 \quad r \simeq 10^{-14}$

('08 Bauer & Demir)

Unitarity issues of Higgs inflation in Palatini formalism

=> Much different from that in metric formalism

- Cutoff scale of the theory (at the vacuum): $\Lambda \simeq \frac{M_{\rm pl}}{\sqrt{\xi}} \gg \frac{M_{\rm pl}}{\xi}$ ('11 Bauer & Demir)
- Violent preheating of the NG mode/gauge bosons is absent and no unitarity problem during inflation and reheating. ('19 Rubio & Tomberg)
- Introduction of R^2 term does not induce scalaron but leads to $P(h, (\partial h)^2)$ theory. ('18 Eckell+)
- Extended theory with conformal mode does not lead to UV-extension. ('21 Mikura & Tada)

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How can we understand these differences?

Are there any "natural" models of Higgs inflation?

Einstein-Cartan connects metric to Palatini

$\bar{\Gamma}^{\rho}_{\mu\nu}$ and $g_{\mu\nu}$ are a priori independent with existence of torsion.

GR is reproduced by solving the EOM with E-H action. $T^{\rho}_{\mu\nu} \equiv \bar{\Gamma}^{\rho}_{\mu\nu} - \bar{\Gamma}^{\rho}_{\nu\mu}$

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Higgs inflation in E-C gravity with Nieh-Yan term is a generalization of metric and Palatini Higgs inflation. ('20 Shaposhnikov, Shkerin, Tlmiryasov, & Zell)

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\left(\frac{M_{\rm pl}^2}{2} + \xi|\mathcal{H}|^2\right)R(g,\bar{\Gamma}) - |D_\mu\mathcal{H}|^2 - \lambda|\mathcal{H}|^4\right] - \frac{\xi_\eta}{4}\int d^4x\phi^2\partial_\mu\left(\sqrt{-g}E^{\mu\nu\rho\sigma}T_{\nu\rho\sigma}\right)$$

Integrating out torsion and other non-dynamical DoF

⇒ $\xi_\eta = \xi$ -> metric Higgs inflation

$\xi_\eta = 0$ -> Palatini Higgs inflation

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Higgs i … lization of

metric

$\mathcal{S} = \int d$ … $\rho\sigma T_{\nu\rho\sigma})$

In E-C Higgs inflation,

- How cutoff changes from metric to Palatini limit?
- How quantum correction induces scalaron?
- How R^2 term UV-extends the theory?
- Are there a “natural” value of $r \equiv \xi_\eta / \xi$?

Integrating out torsion and other non-dynamical DoF

⇒ $\xi_\eta = \xi$ -> metric Higgs inflation

$\xi_\eta = 0$ -> Palatini Higgs inflation

Unitarity analysis in Einstein-Cartan Higgs inflation

Extended theory with conformal mode

('23 He, KK, Mukaida)

$$S = \int d^4x \left[\frac{\Phi_J^2}{12}\tilde{R} + \frac{1}{2}\tilde{g}^{\mu\nu}\left(\partial_\mu \Phi_J \partial_\nu \Phi_J - \partial_\mu \tilde{\phi}^i \partial_\nu \tilde{\phi}_i\right) - \left(\frac{\Phi_J^2}{6M_{Pl}^2}\right)^2 V(\phi) - \frac{1}{2}\tilde{\phi}^2 \frac{\tilde{\Box}\Phi_J}{\Phi_J} - \frac{\Phi_J^2}{18}\left(\tilde{T}^2 - \frac{1}{16}\tilde{S}^2\right)\right]$$

$$+ \int d^4x \frac{\xi}{2}\tilde{\phi}^2 \left(\tilde{R} + A_R + r A_{N-Y}\right) ,$$

$$T_\mu \equiv T^\alpha{}_{\mu\alpha} ,$$

$$S^\beta \equiv E^{\mu\nu\alpha\beta} T_{\mu\nu\alpha} ,$$

$$A_R \equiv -\frac{2}{3}\left(\tilde{T}^2 - \frac{1}{16}\tilde{S}^2\right) + 2\tilde{\nabla}_\mu \tilde{T}^\mu + 4\tilde{T}^\mu \partial_\mu \ln|\Phi_J| - 6\frac{\tilde{\Box}\Phi_J}{\Phi_J} , \qquad A_{N-Y} \equiv \frac{1}{2}\tilde{\nabla}_\mu \tilde{S}^\mu + \tilde{S}^\mu \partial_\mu \ln|\Phi_J|$$

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$$\Phi_E \equiv \Omega_0 \Phi_J , \quad \Omega_0^2 \equiv 1 + 6\xi \frac{\tilde{\phi}^2}{\Phi_J^2}$$

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w/ field space metric

$$ds^2 = -d\Phi_E^2 + \frac{1}{6}\frac{\Phi_E^2}{M_{Pl}^2+\xi\phi^2}\left(\delta_{ij} + \frac{6r^2\xi^2}{M_{Pl}^2+\xi\phi^2}\phi_i\phi_j\right)d\phi^i d\phi^j$$

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The 2-to-2 scattering amplitude can be evaluated as

$$\mathcal{M}_{IJ \leftrightarrow KL} = \frac{2}{3} \left[s_{IJ} \bar{R}_{I(KL)J} + s_{IK} \bar{R}_{I(JL)K} + s_{IL} \bar{R}_{I(JK)L} \right] ,$$

('16 Alonso+; '19 Nagai+; 21 Cohen+,⋯)

⇨ $\Lambda \simeq |\bar{R}|^{-1/2}$

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It is frame independent! ⇨ $\Lambda \simeq |\bar{R}|^{-1/2}$

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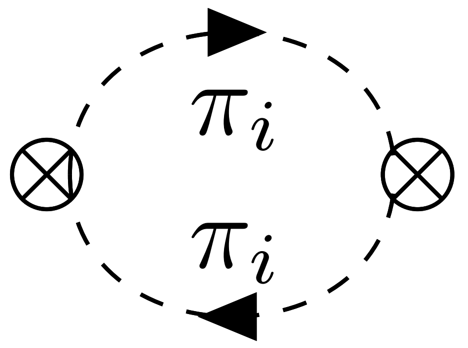
Cutoff scale of (tree-level) E-C Higgs inflation at the vacuum

$$\Lambda_{E\text{-}C}^0 = |R_N|^{-1/2} = \frac{\sqrt{6}M_{Pl}}{\sqrt{1+12\xi\left(1+3r^2\xi\right)}} \sim \begin{cases} \dfrac{M_{Pl}}{r\xi} & \text{for} \quad \frac{1}{\sqrt{\xi}} \lesssim r \lesssim 1\,, \\ \dfrac{M_{Pl}}{\sqrt{\xi}} & \text{for} \quad 0 \leqslant r \lesssim \frac{1}{\sqrt{\xi}}\,, \end{cases}$$

<- "Palatini limit"

UV extension of the Higgs inflation in EC formalism

Quantum correction generates scalaron ('23 He, KK, Mukaida)

$\tilde{R} + A_R + rA_{\mathrm{N-Y}}$ 

$\tilde{R} + A_R + rA_{\mathrm{N-Y}}$ $\Rightarrow$ $\alpha(\tilde{R} + A_R + rA_{\mathrm{N-Y}})^2, \quad \alpha \sim N\xi^2$

leading contribution in the large N and ξ :

R^2 term w/ contributions from torsion and Nieh-Yan term

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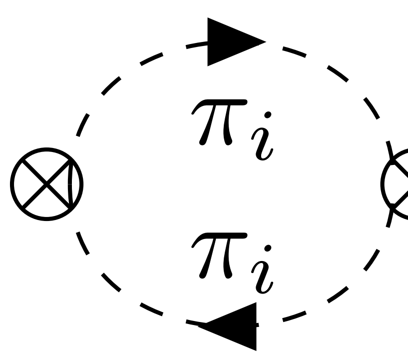
('23 He, KK, Mukaida)

$\tilde{R} + A_R + r A_{\mathrm{N-Y}} \otimes$  $\otimes\ \tilde{R} + A_R + r A_{\mathrm{N-Y}}$ ⇨ $\alpha(\tilde{R} + A_R + r A_{\mathrm{N-Y}})^2, \quad \alpha \sim N\xi^2$

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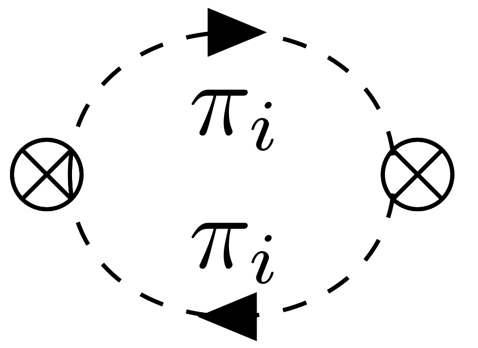
$$S = \int \mathrm{d}^4 x \left[\frac{\Phi_{\mathrm{E}}^2}{12}\tilde{R} + \frac{1}{2}\tilde{g}^{\mu\nu}\left(\partial_\mu \Phi_{\mathrm{E}} \partial_\nu \Phi_{\mathrm{E}} - \frac{\Phi_{\mathrm{E}}^2/\Omega^2}{6M_{\mathrm{pl}}^2}\partial_\mu \phi_i \partial_\nu \phi^i\right) - \left(\frac{\Phi_{\mathrm{E}}^2/\Omega^2}{6M_{\mathrm{pl}}^2}\right)^2 V(\phi) - \alpha\gamma^2 \right.$$
$$\left. - r^2 \frac{\Phi_{\mathrm{E}}^2}{8}\tilde{g}^{\mu\nu}\partial_\mu \ln\left|\Omega^2\right| \partial_\nu \ln\left|\Omega^2\right| \right],$$

$$\Omega^2 \equiv 1 + 12\left(\frac{\xi}{2}\frac{\tilde{\pi}_i^2}{\Phi_{\mathrm{J}}^2} + 2\alpha\frac{\gamma}{\Phi_{\mathrm{J}}^2}\right)$$

Quantum correction generates scalaron ('23 He, KK, Mukaida)

$$\tilde{R} + A_R + rA_{\mathrm{N-Y}} \otimes \quad \otimes \ \tilde{R} + A_R + rA_{\mathrm{N-Y}}$$



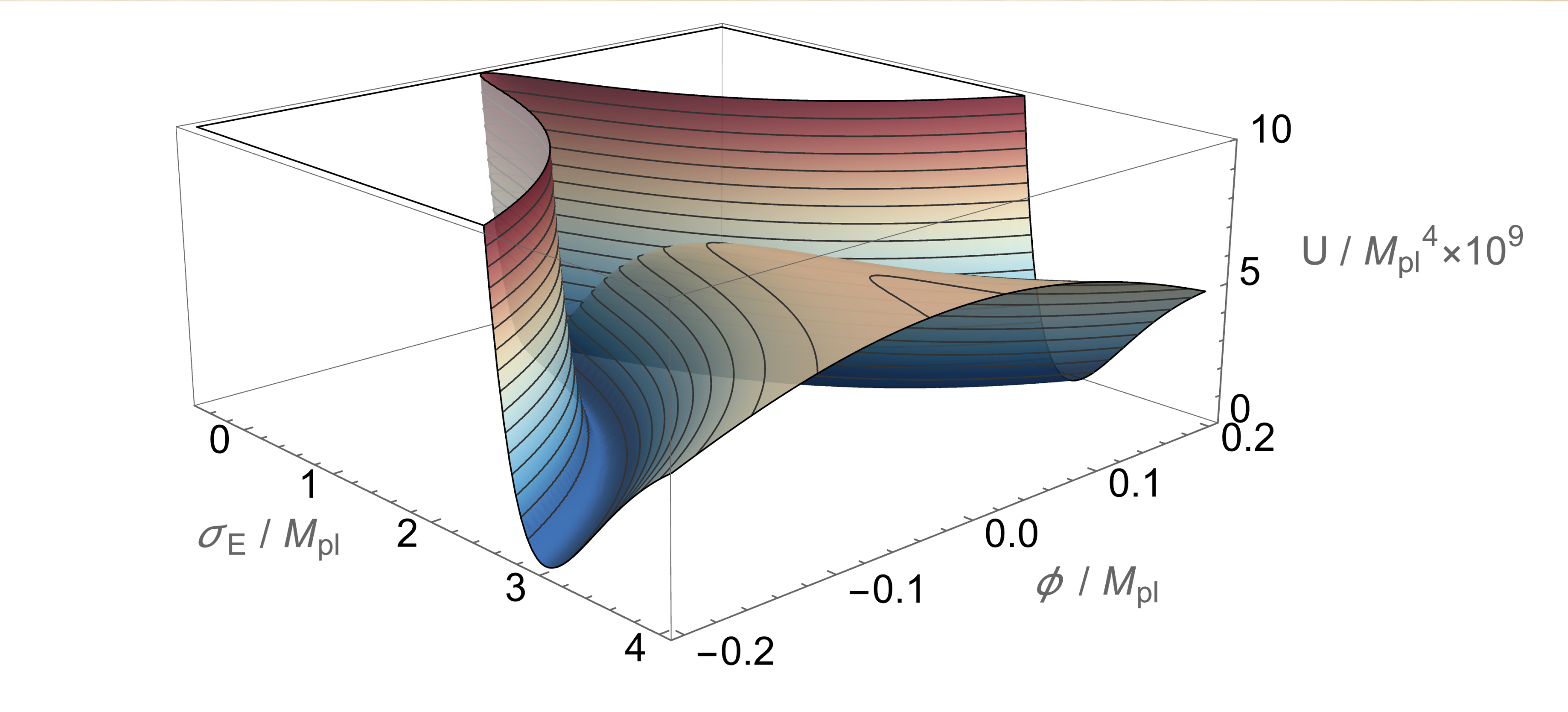
$$\Rightarrow \quad \alpha(\tilde{R} + A_R + rA_{\mathrm{N-Y}})^2, \quad \alpha \sim N\xi^2$$

leading contribution in the large N and ξ :

R^2 term w/ contributions from torsion and Nieh-Yan term

Scalaron is induced with a mass $m_\sigma^2 = \dfrac{M_{\mathrm{Pl}}^2}{12\alpha r^2}$

Cutoff scale $\Lambda_{\mathrm{E\text{-}C}} = |R_{N+s}|^{-1/2} = \dfrac{\sqrt{6}r}{\sqrt{|1-r^2|}} M_{\mathrm{Pl}}$



Mass term and cosmological constants are also induced⋯

Quan

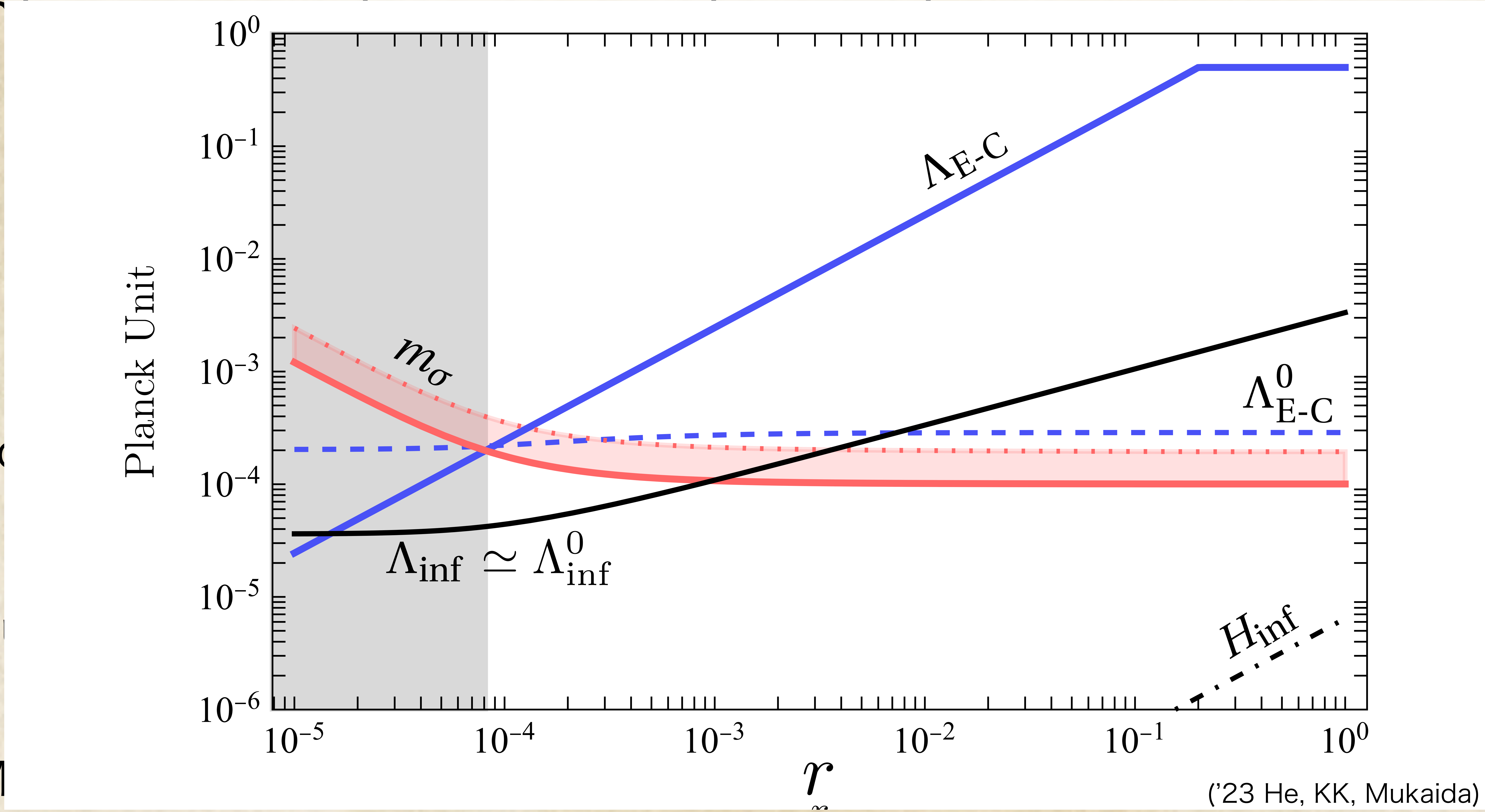
$\tilde{R}$ ξ^2

n term

S

C

M



('23 He, KK, Mukaida)

Quar

$\tilde{R}$ ... ξ^2

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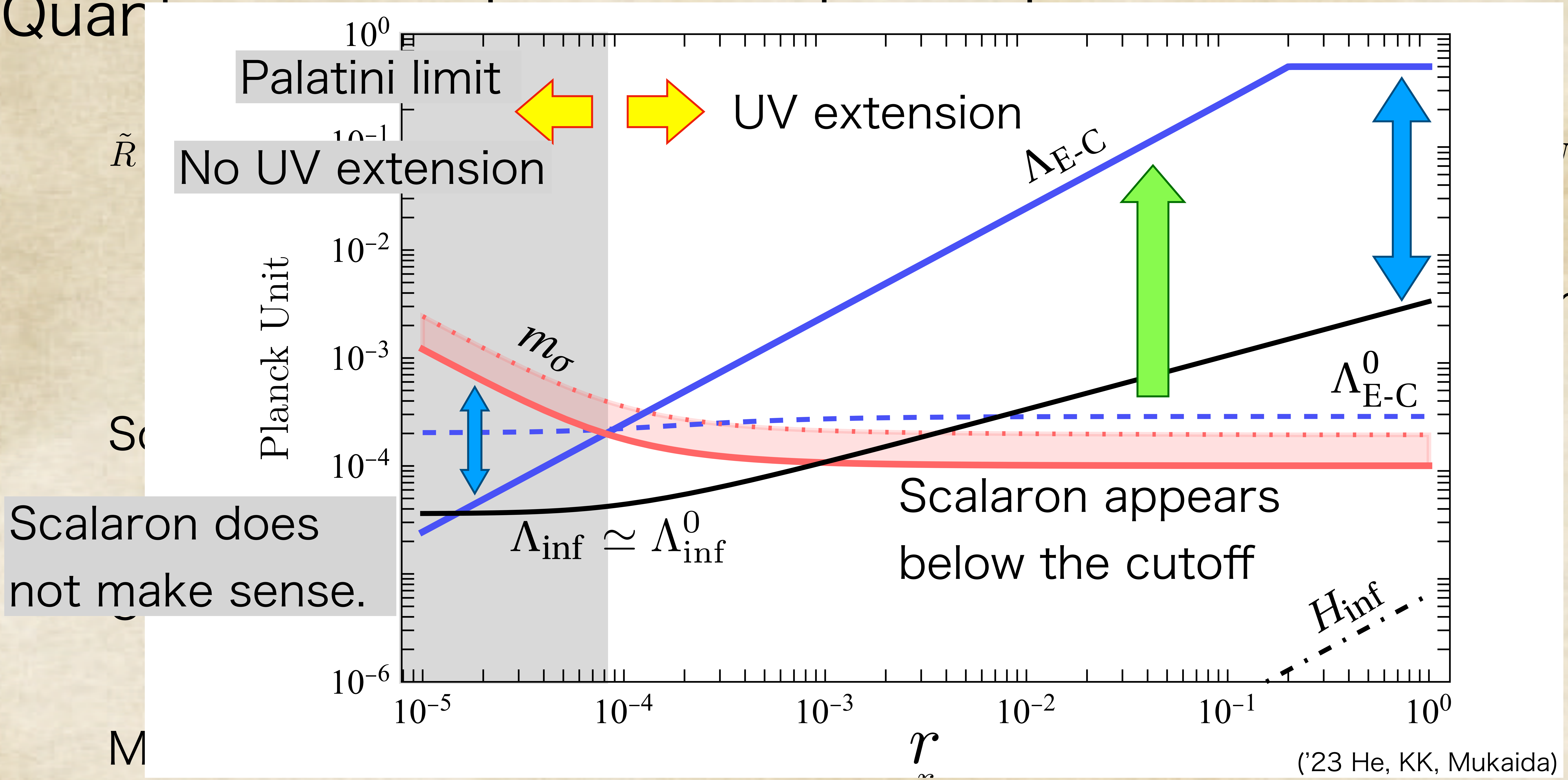

Summary

Summary

- Higgs inflation (HI) in the metric formalism is one of the well-motivated models of inflation, which have some unitarity issues.
- R^2 term can be induced by quantum correction w/ scalaron and UV-extends the theory to resolve the unitairity issues.
- HI in the Palatini formalism is also a good model, with less unitarity issues, but R^2 term does not UV-extends the theory
- HI in the Einstein-Cartan gravity w/ Nieh-Yan term connects HI in the metric and Palatini formalism.
- The theory becomes "Palatini limit" at $r \simeq 1/\sqrt{\xi}$ below which UV-extension by the R^2 term/scalaron does not work.
- Above the region, the model looks fine, but still has the naturalness problem.

Appendix

Comment on the frame independence of the cutoff scale

2308.15420

For the scalar theory with non-trivial kinetic term,

$$\int d^4x \frac{1}{2} g^{\mu\nu} G_{ab}(\pi) \partial_\mu \pi^a \partial_\nu \pi^b$$

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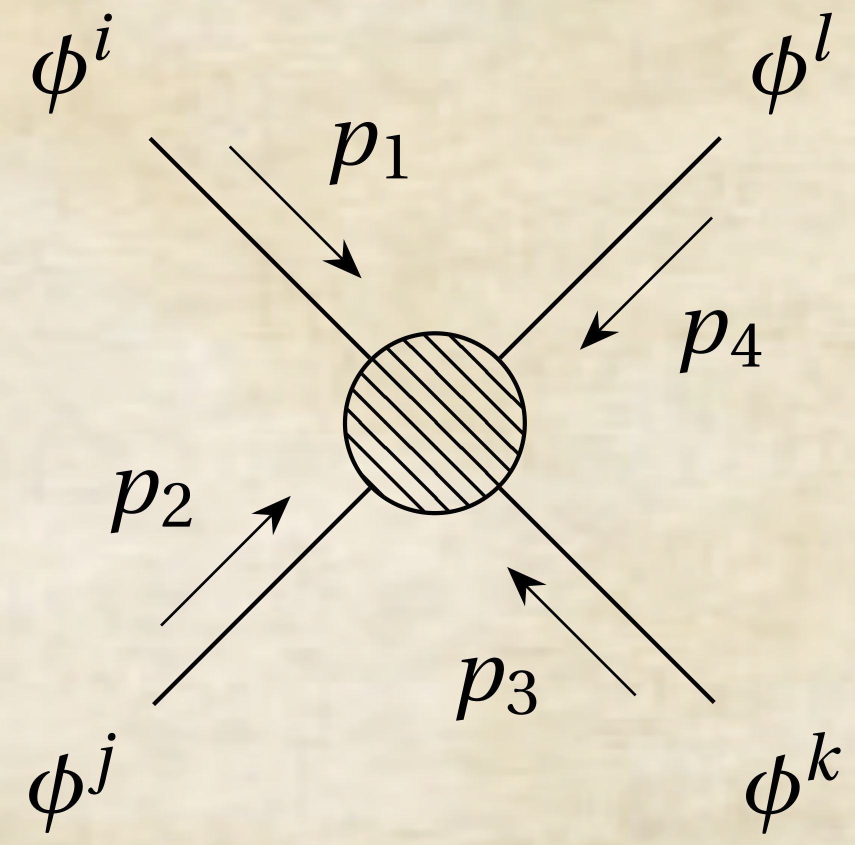
$$G_{IJ} \sim (\partial^2 G_{IJ})\pi^2 \sim \bar{R}\pi^2$$

the 2-to-2 scattering amplitude (with $\pi^I = e^I_a \pi^a$) can be evaluated as

$$\mathcal{M}_{IJ\leftrightarrow KL} = \frac{2}{3}\left[s_{IJ}\bar{R}_{I(KL)J} + s_{IK}\bar{R}_{I(JL)K} + s_{IL}\bar{R}_{I(JK)L}\right],$$

('16 Alonso+; '19 Nagai+; 21 Cohen+,…)



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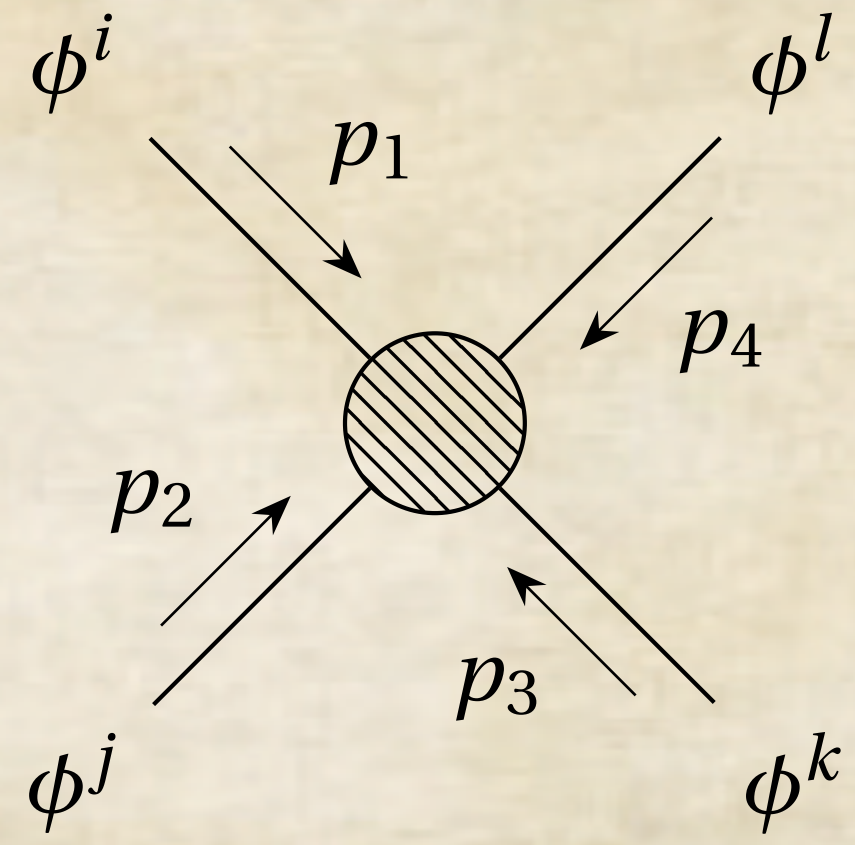

⇨ $\Lambda \simeq |\bar{R}|^{-1/2}$

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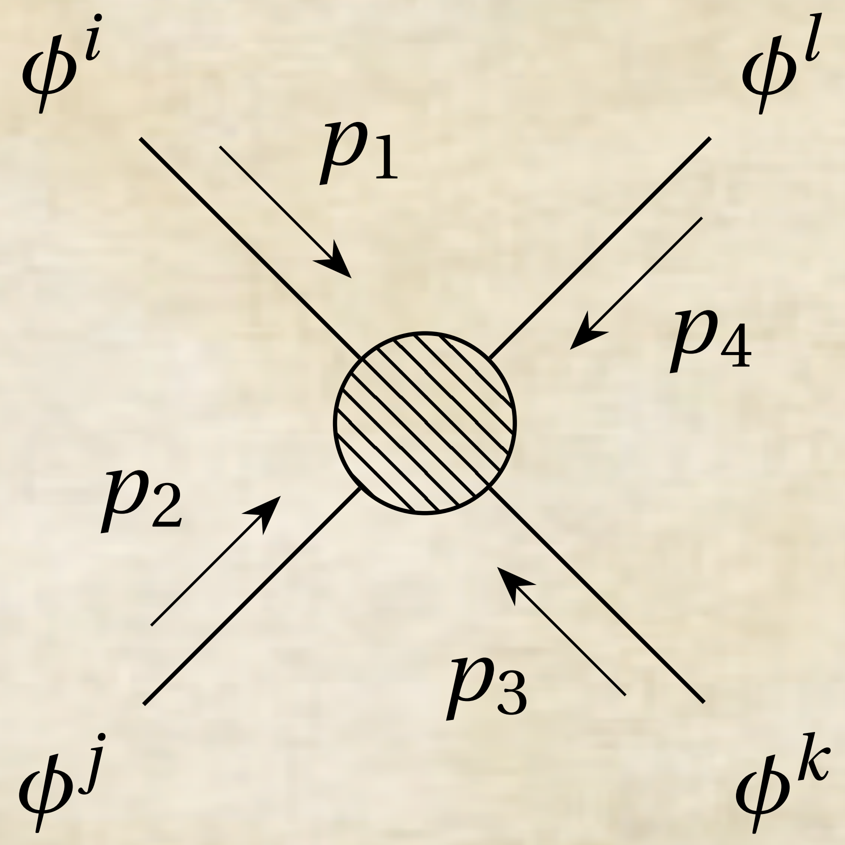

⇨ $\Lambda \simeq |\bar{R}|^{-1/2}$

It is unchanged under field redefinition 🥳

$$\pi^a \to \tilde{\pi}^A(\pi^a)$$

Comment on the frame independence of the cutoff scale

2308.15420

For the scalar theory with non-minimal coupling to gravity,

$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\left(\frac{M_{\rm pl}^2 + \xi \pi_i^2}{2} \right) R - \frac{1}{2} \partial_\mu \pi_i^2 \right]$$

no scattering?

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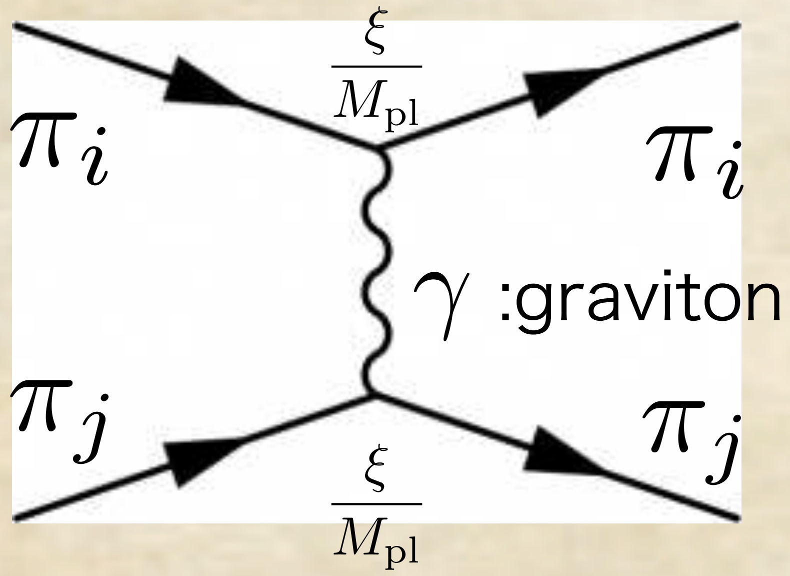
no scattering?



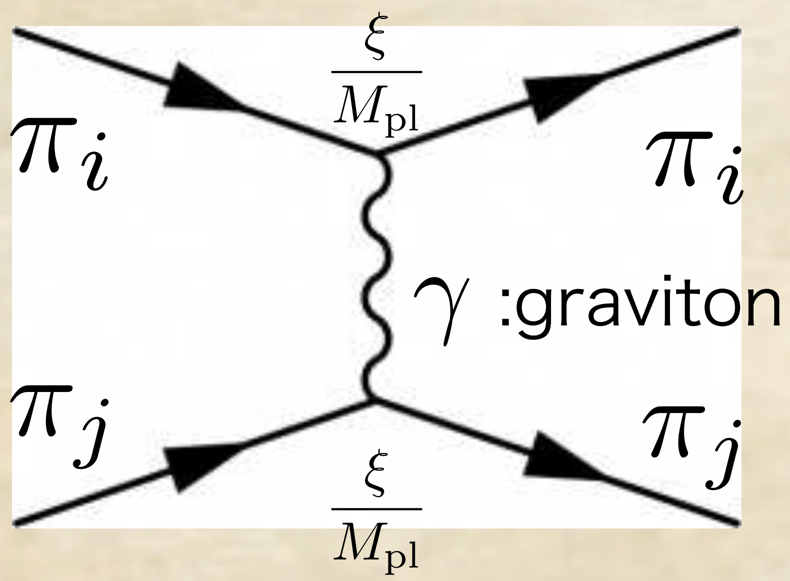
We have scattering
from the non-minimal coupling.

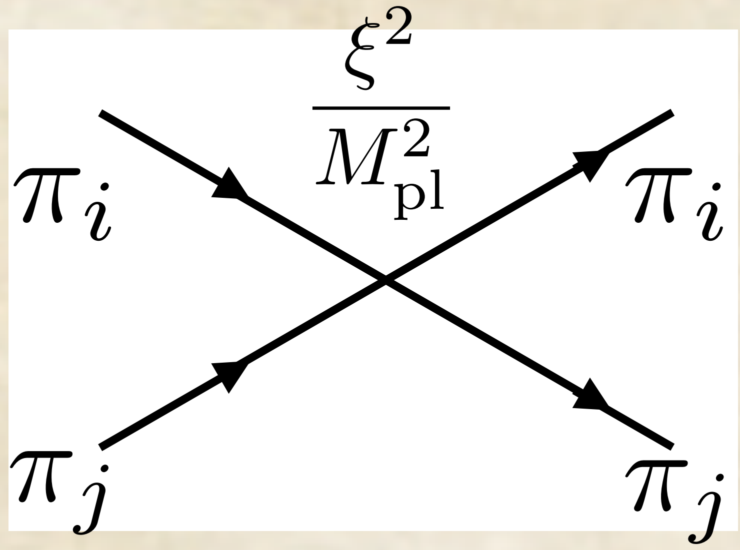
Comment on the frame independence of the cutoff scale

2308.15420

For the scalar theory with non-minimal coupling to gravity,

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\left(\frac{M_{\rm pl}^2 + \xi\pi_i^2}{2}\right)R - \frac{1}{2}\partial_\mu\pi_i^2\right] \longleftrightarrow \mathcal{S} = \int d^4x\sqrt{-g}\left[\left(\frac{M_{\rm pl}^2}{2}\right)R - \frac{1}{\Omega^2}\left(\delta_{ij} + \frac{6\xi^2}{\Omega^2}\frac{\pi_i\pi_j}{M_{\rm pl}^2}\right)\partial_\mu\pi^i\partial^\mu\pi^j\right]$$





scattering appears as the contact term in the Einstein frame,
but the frame independence is not clear even if we use
the geometrical technique (?) 🤔

Comment on the frame independence of the cutoff scale

2308.15420

Extract the conformal mode from graviton,

$$g_{\mu\nu} = \frac{\Phi^2}{6M_{\rm pl}^2}\tilde{g}_{\mu\nu}, \quad \det[\tilde{g}_{\mu\nu}] = -1$$

$$\varphi_{\rm J}^a = (\Phi_J, \pi_J^i)$$

$$\Rightarrow \quad S = \int \mathrm{d}^4x \left(\frac{\Phi_{\rm J}^2}{12}\Omega^2 \tilde{R} - \frac{\tilde{g}^{\mu\nu}}{2} G_{ab}^{\rm J}(\varphi_J)\partial_\mu \varphi_{\rm J}^a \partial_\nu \varphi_{\rm J}^b \right),$$

$$(G_{ab}^{\rm J}) \equiv \begin{pmatrix} -\Omega^2 & -\xi\Phi_{\rm J}\pi_j/M_{\rm pl}^2 \\ -\xi\Phi_{\rm J}\pi_i/M_{\rm pl}^2 & \frac{\Phi_{\rm J}^2}{6M_{\rm pl}^2}\delta_{ij} \end{pmatrix}.$$

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Geometrical technique gives

- $\Lambda \simeq |\bar{R}|^{-1/2} \simeq \dfrac{M_{\rm pl}}{\xi}$ for $\pi\pi \to \pi\pi$
- No scattering amplitude that involve the conformal mode Φ
- Conformal transformation is now the field redefinition for Φ and frame independence is now a manifest 😊.

Comment on the frame independence of the cutoff scale

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Extraction of the conformal mode is also useful to see the quantum effect.