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Quantum Gravity and Cosmology 2024

Bounce Cosmology: Problems and its Solutions

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Outline

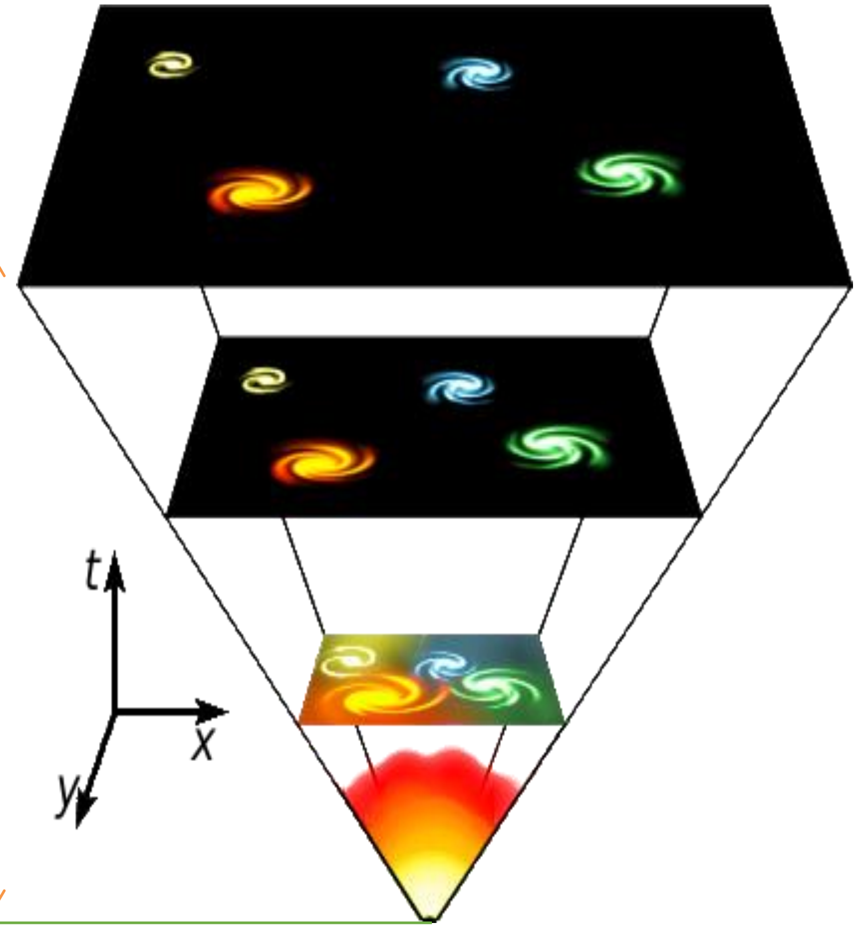
- The Big-Bang and inflationary scenarios
- Bounce cosmology
  - Why bounce?
  - Problems and its solutions

Question: What will our universe be like in its early stage?

The Big Bang theory

George Gamow
(1904-1968)

13,800,000,000 years



SINGULARITY!

(1948, Alpher, Bethe, Gamow)

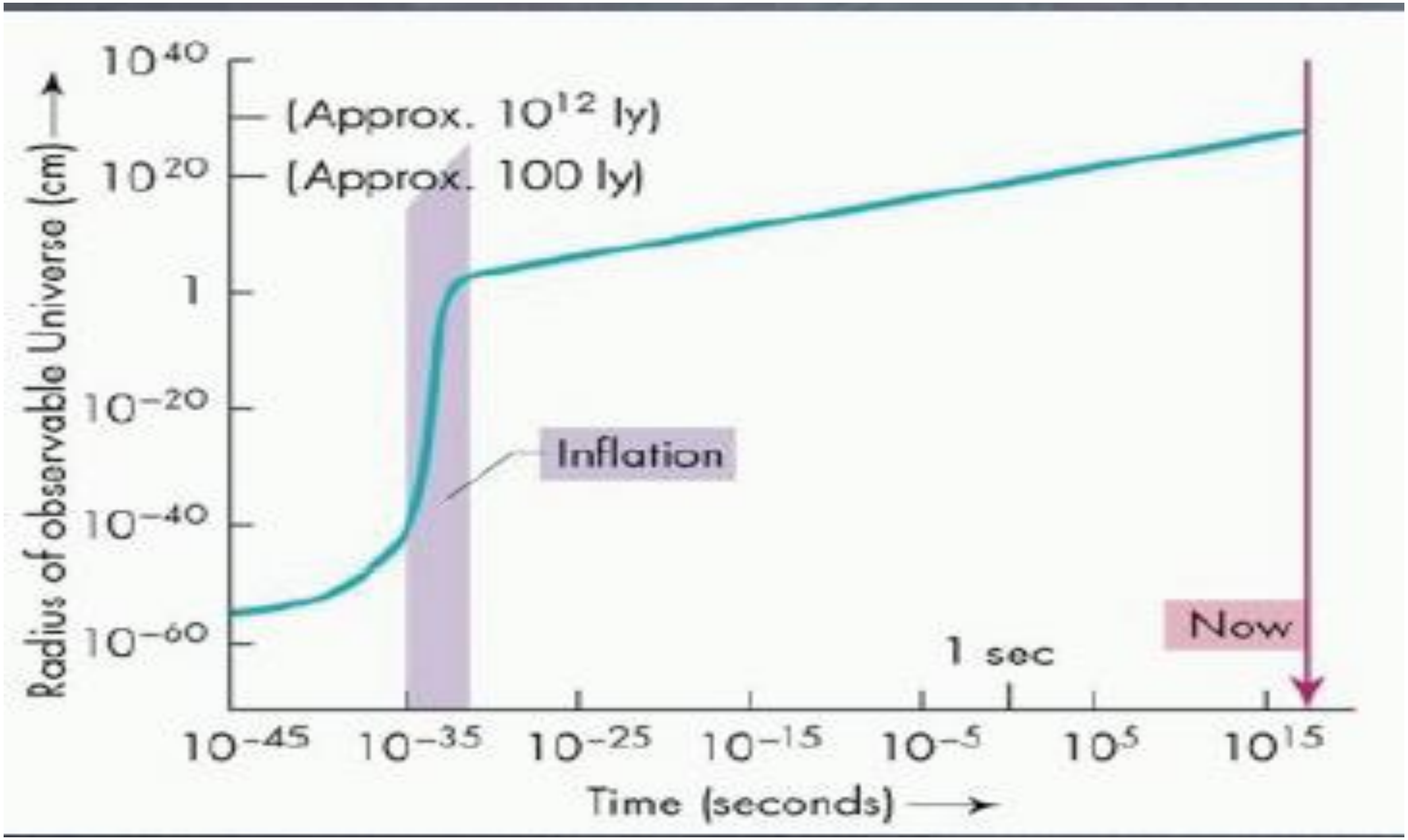
The Big Bang theory

The Big-Bang theory is both successful and non-successful!

- ✓ The age of galaxies
- ✓ The redshift of the galactic spectrum
- ✓ The He abundance
- ✓ The prediction of CMB temperature

- ? Flatness problem
- ? Horizon problem
- ? Unwanted relics problem
- ? Singularity problem

The Inflationary Scenario



(1980, Alan Guth et al.)

The Inflationary Scenario

Inflation assume a rapid expansion that can:

- ✓ stretch the space-time to a large volume
- ✓ dissipate all the matter and curvature.
- ✓ decoherent the fluctuations to form galaxies and large scale structures.

Horizon problem

Flatness problem

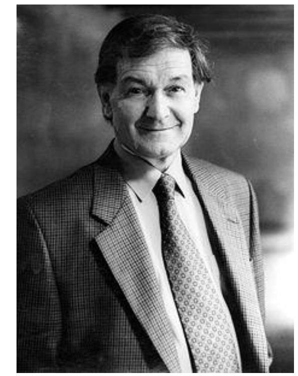
Unwanted relics problem

Singularity problem

The Singularity Problem

SINGULARITY THEOREM

S. Hawking



R. Penrose

The universe will meet a singularity when

(1) it is described by General Relativity;

$$S = \int d^4x \sqrt{-g}\left[\frac{R}{16\pi G} + L_m\right]$$

(2) it satisfies Null Energy Condition;

$$T_{mn} n^{m} n^{n} = (r + P)\ ^{3}\ 0$$

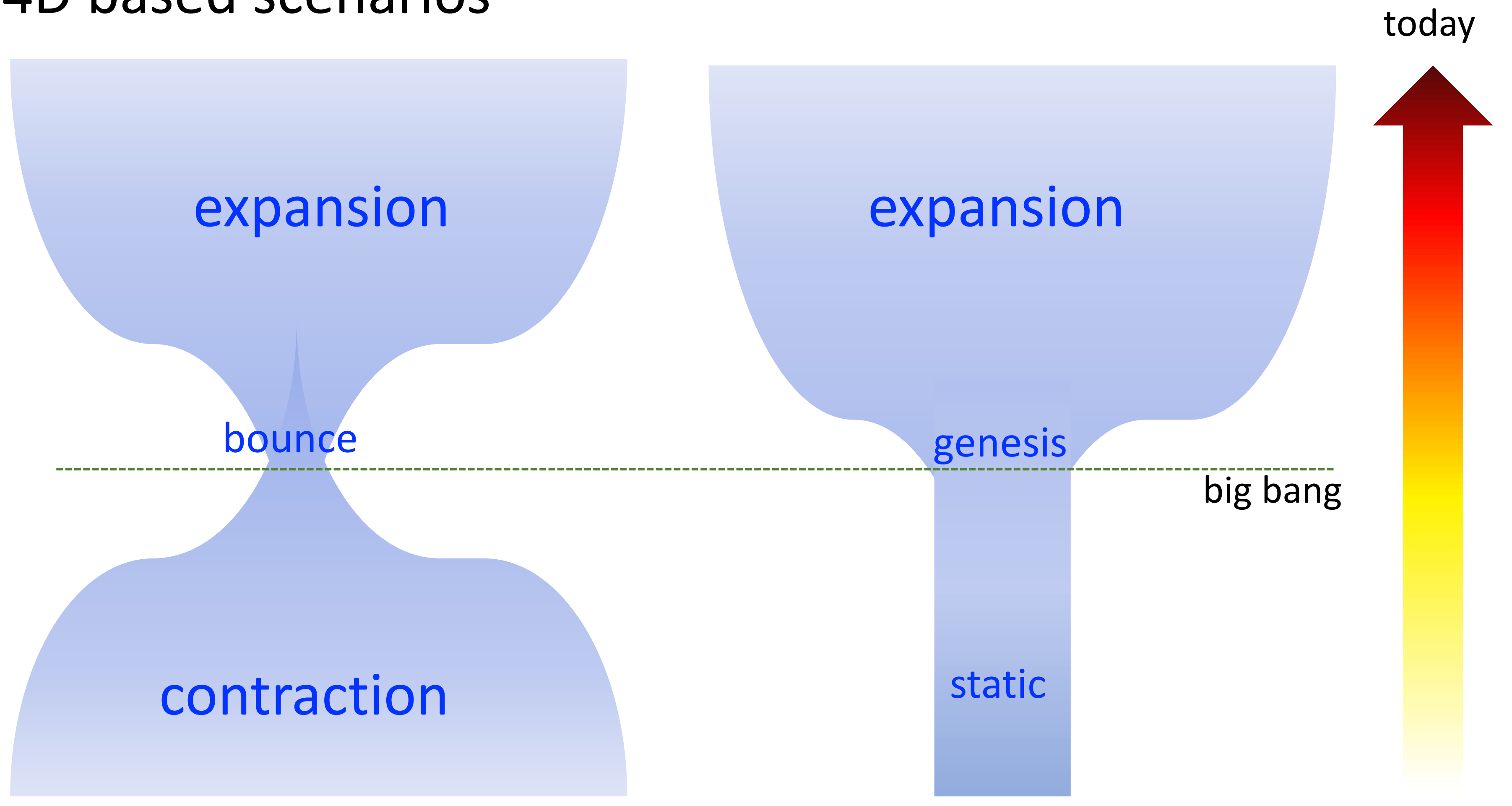
where at finite time point

$$a_u(\mathrm{t}) \to 0, \quad \rho_u(\mathrm{t}) \to \infty$$ for any null vector n^{μ} : $n_{\mu} n^{\mu} = 0$

(1973, Hawking et al., 1994, Borde et al.)

Non-Singular Cosmology

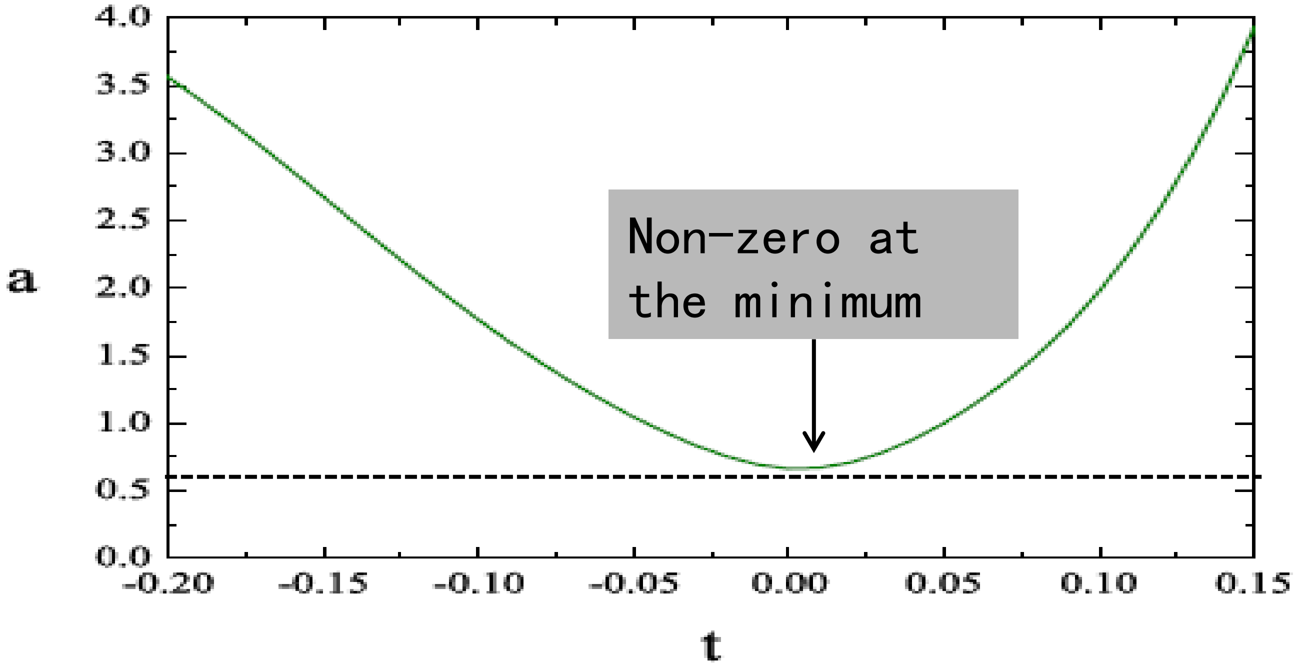
4D based scenarios



Bounce Scenario

Emergent Scenario

Bounce Cosmology



Contraction: $H < 0$ *Expansion:* $H > 0$

Bouncing Point: $\dot{H} > 0$ $\rho + p < 0$

Violating the Null Energy Condition (NEC)!

(Y. F. Cai, **TQ**, Y. S. Piao, M. Z. Li, X. M. Zhang, JHEP 2004.)

However, there may be new problems……

Problems in background

Anisotropy

Problems in perturbations

Scale Invariant Spectrum

↓

Ghost Instability

↓

Gradient Instability

What are these problems?
How do we get rid of them?

1. Cosmic Anisotropy

If the initial metric is not exact isotropic:

$$ds^2 = -dt^2 + a^2(t)\sum_{i=1}^{3} e^{2\beta_i(t)} dx^{i2}$$

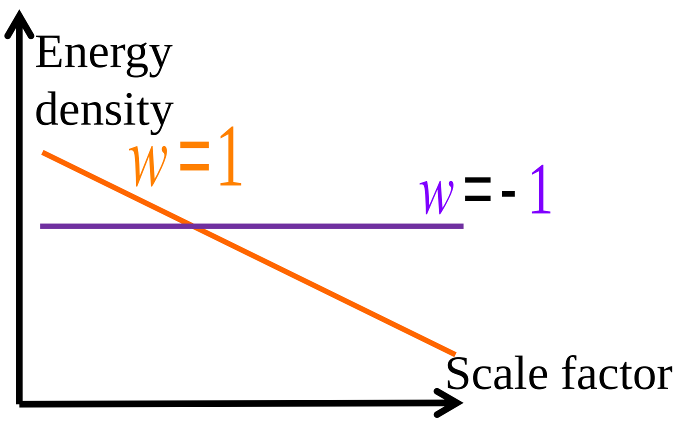
<u>*Friedmann Equation:*</u>

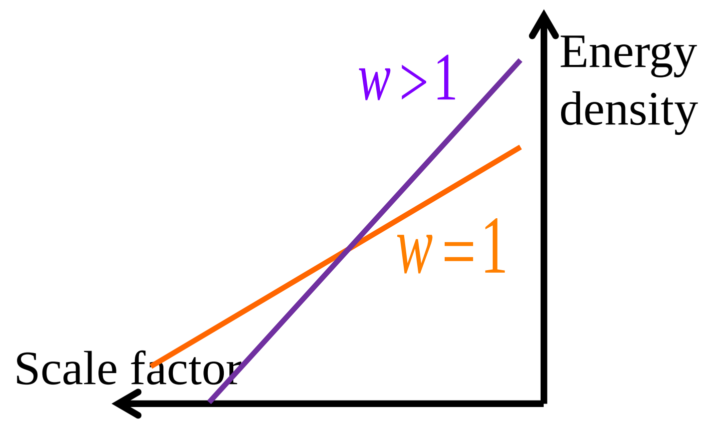
$$3H^2 = \rho_{bg} + \frac{1}{2}\sum_{i=1}^{3}\dot{\beta}_i^2$$

↑ *Matter* ↑ *Anisotropy*

<u>*EoM for anisotropy:*</u>

$$\ddot{\beta}_i + 3H\dot{\beta}_i = 0 \quad \Rightarrow \quad w_{ani} = 1, \quad \rho_{ani} \propto a^{-3(1+w)} = a^{-6}$$

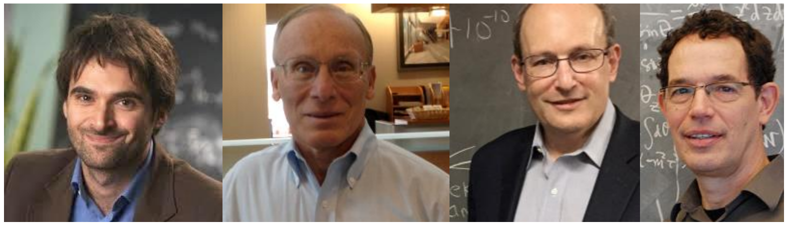




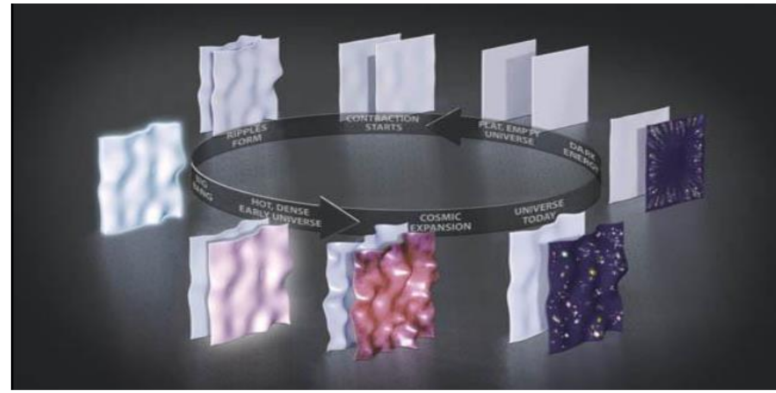
So we need contracting phase with w>1!

(J. Erickson, D. Wesley, P. Steinhardt, N. Turok, 2004.)

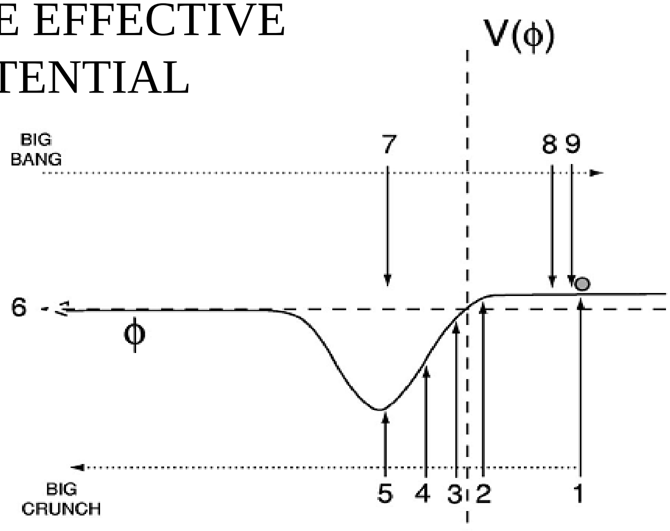
One of the Solutions: Ekpyrosis



(Khoury, Ovrut, Steinhardt & Turok, 2001)



THE EFFECTIVE POTENTIAL



The effective potential (Ekpyrotic phase):

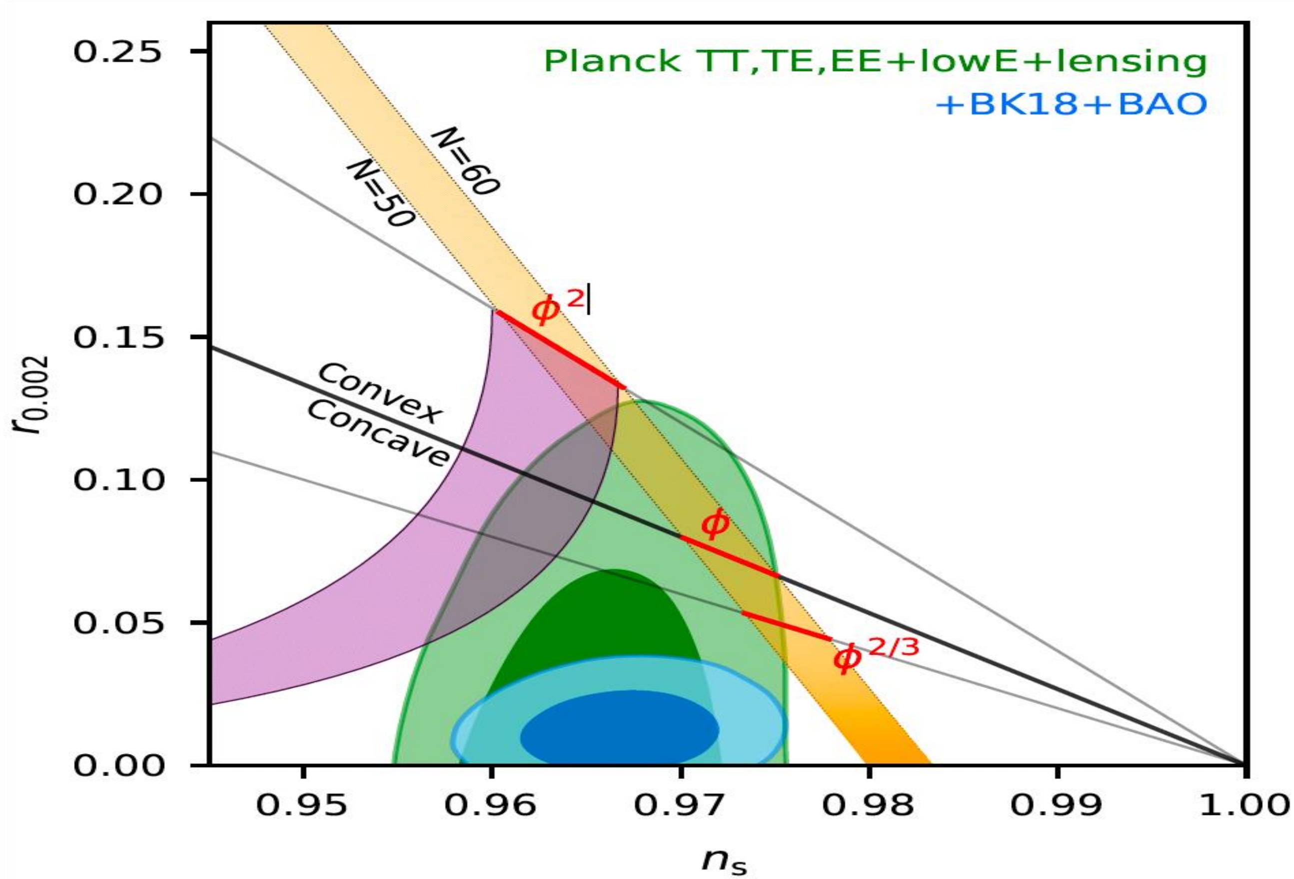
$$V(f) = -V_0 \exp\left(-\sqrt{\frac{2}{p}}\frac{f}{M_p}\right) < 0$$

In order to make $w_{eff} = \frac{\dot{\phi}^2 - 2V}{\dot{\phi}^2 + 2V} > 1$

1 DE domination; 2 decelerated expansion; 3 turnaround; 4 Ekpyrotic contraction; 5 before big crunch; 6 a singular bounce in 4D; 7 after big bang; 8 radiation domination; 9 matter domination

2. Scale Invariance of Power Spectrum

Observations from PLANCK+BICEP/KECK 2018



(BICEP/Keck collaboration, 2021)

2. Scale Invariance of Power Spectrum

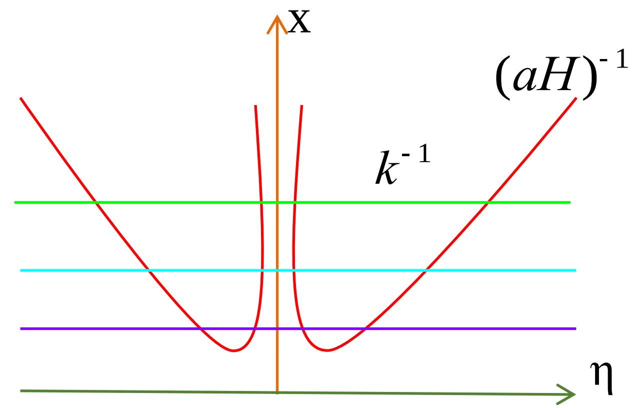
The perturbation equation:

$$(\zeta z)'' + \left(c_s^2 k^2 - \frac{z''}{z}\right)(\zeta z) = 0$$

Solution:

$$\zeta \sim (c_s k)^{\frac{3}{2}\frac{1-w}{1+3w}}\eta^0, \quad (c_s k)^{-\frac{3}{2}\frac{1-w}{1+3w}}\eta^{-\frac{3}{2}\frac{1-w}{1+3w}}$$

constant — *growing for viable bounce models*



sketch plot of perturbations during the bounce

Power spectrum:

$$P_\zeta \equiv \frac{k^3}{2\pi^2}|\zeta|^2 \sim k^{3-\frac{3(1-w)}{1+3w}}\eta^{-\frac{1-w}{1+3w}}$$

$$n_s - 1 = 3 - \frac{3(1-w)}{1+3w} \approx 0 \quad \Rightarrow \quad w \approx 0$$

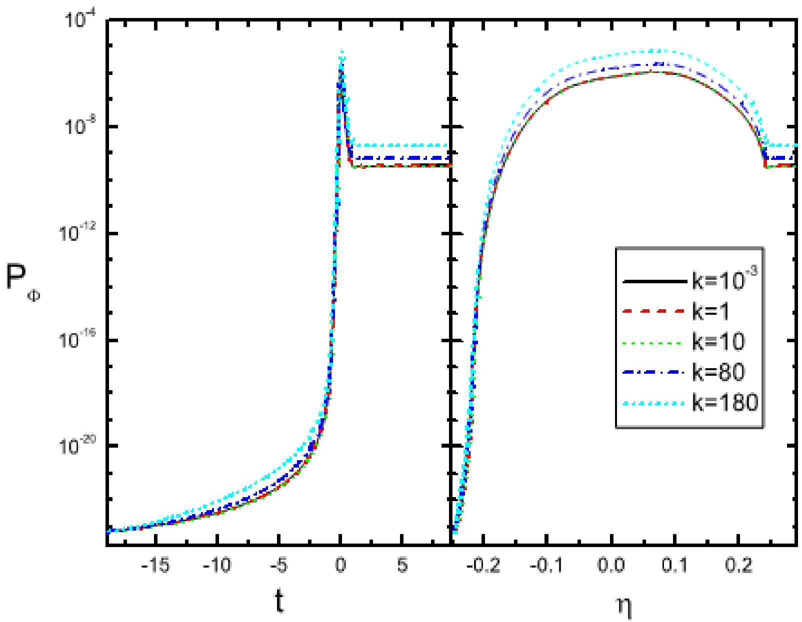
(D. Wands, 1999;
F. Finelli and R. Brandenberger, 2002.)

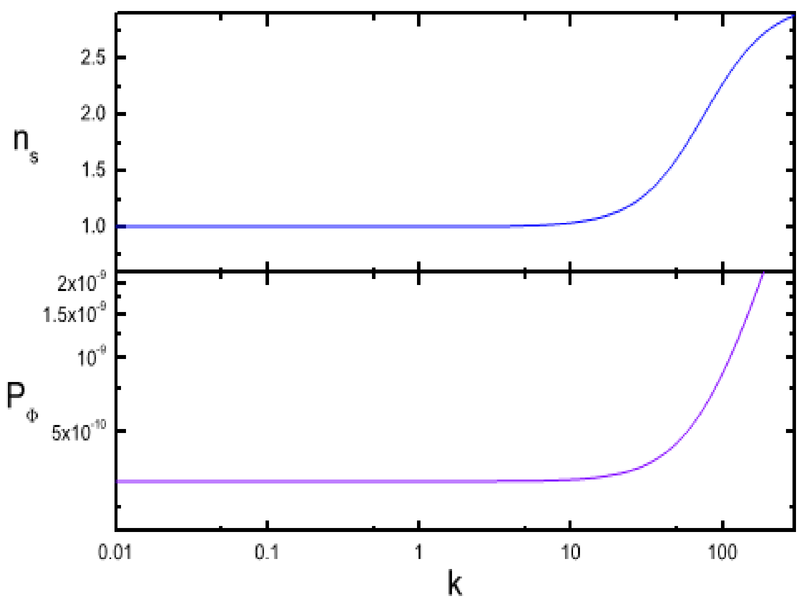
David Wands

Robert Brandenberger

2. Scale Invariance of Power Spectrum

The numerical plots of the power spectrum and spectral index:





(Just for illustration, from another concrete model)

(Y. F. Cai, **TQ,** R. Brandenberger, X. M. Zhang, 2009)

To Be Large or Not To Be Large? Is it a Problem?

Isotropy: $w>1$

Scale Invariance: $w=0$

Possible Solutions:

1) *To have another field in contracting phase to generate power spectrum (Entropic Mechanism);*

F. Finelli, PLB 2002;

K. Koyama and D. Wands, JCAP 2007;

K. Koyama, S. Mizuno, D. Wands, CQG 2007;

TQ, X. Gao and E. N. Saridakis, PRD 2014… …

2) *To have inflationary period following the bounce (Bounce Inflation).*

Y. S. Piao, B. Feng and X. M. Zhang, PRD 2004;

Z. G. Liu, Z. Guo and Y. S. Piao, PRD 2013;

J. Q. Xia, Y. F. Cai, H. Li and X. M. Zhang, PRL 2014;

TQ and Y. T. Wang, JHEP 2015……

3. Ghost Instability

NEC violation will generally cause ghost mode!

Example: "Phantom" field!

Lagrangian:

$$L_{phantom} = -\frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi)$$

↑ Ghost mode!

Hamiltonian (density):

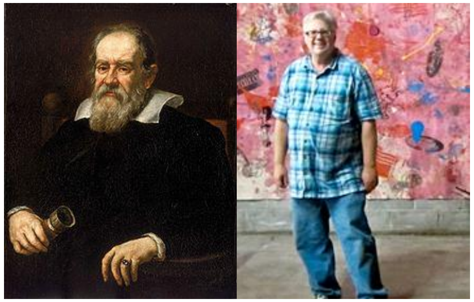
$$H_{phantom} = \Pi\dot{\phi} - L = -\omega \int d^3k (a_k^\dagger a_k + \frac{1}{2})$$

unbounded energy!

S. Carroll, M. Hoffman, M. Trodden, 2003; J. Cline, S. Jeon, G. Moore, 2004.

Solution: Galileon Theories

Galileon/Horndeski theories (2008/1974)



$$L_2 = K(\phi, X)$$

$$L_3 = -G_3(\phi, X)\Box\phi$$

$$L_4 = G_4(\phi, X)R + G_{4,X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2]$$

$$L_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - G_{5,X}[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3]/6$$

1) Higher derivative in lagrangian but 2nd order in equation of motion

2) Multi-degrees of freedom but only one is dynamical

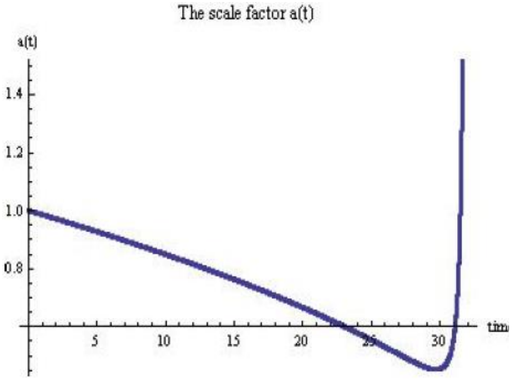
3) violating NEC free of ghosts.

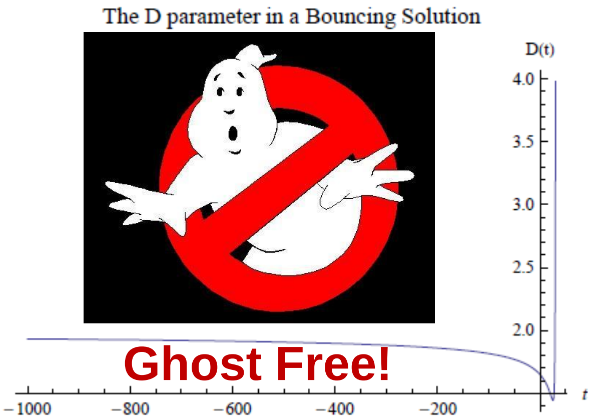
e. g. (**TQ** et al., 2011)

$$L = F^2 e^{2\phi}(\partial\phi)^2 + \frac{F^3}{2M^3}(\partial\phi)^4 + \frac{F^3}{M^3}(\partial\phi)^2\Box\phi$$

Perturbation action:

$$\delta^{(2)}S = 3\int dtd^3xDM_P^2\left[\dot{\zeta}^2 - \frac{c_s^2}{a^2}(\partial\zeta)^2\right]$$





Solution: Galileon Theories

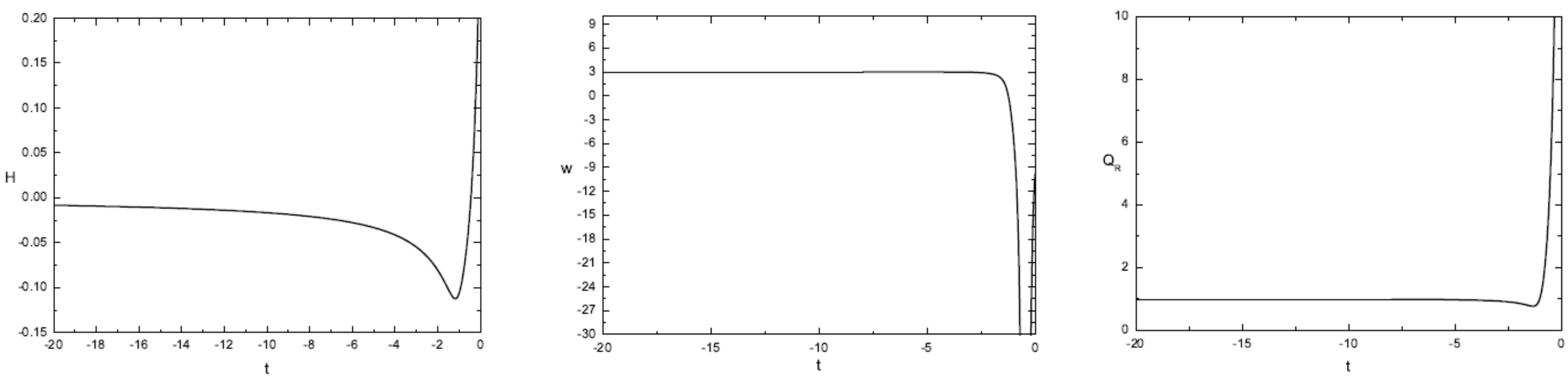
Taking into account the anisotropic and scale invariance issues, one can get more improved models:

1) multi-field bounce model ***(TQ**, X. Gao and E. N. Saridakis, 2013):*

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2}R-\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi-V(\phi)+\frac{g}{2}\nabla_\mu\phi\nabla^\mu\phi\blacksquare\phi\right]\quad \text{with } V(\phi)=-V_0e^{c\phi}$$

Perturbed action: $S^{(2)}=\int d\eta d^3x\, a^2Q_{\mathcal{R}}c_s^{-2}[\mathcal{R}'^2-c_s^2(\partial\mathcal{R})^2]$

Curvaton action: $S_\sigma=\int d^4x\sqrt{-g}\left[-\frac{1}{2}e^{c\phi}\nabla_\mu\sigma\nabla^\mu\sigma\right]$ ➡ $P_{\delta\sigma}\sim k^0|\eta_*-\eta|^{\frac{2}{p-1}}$



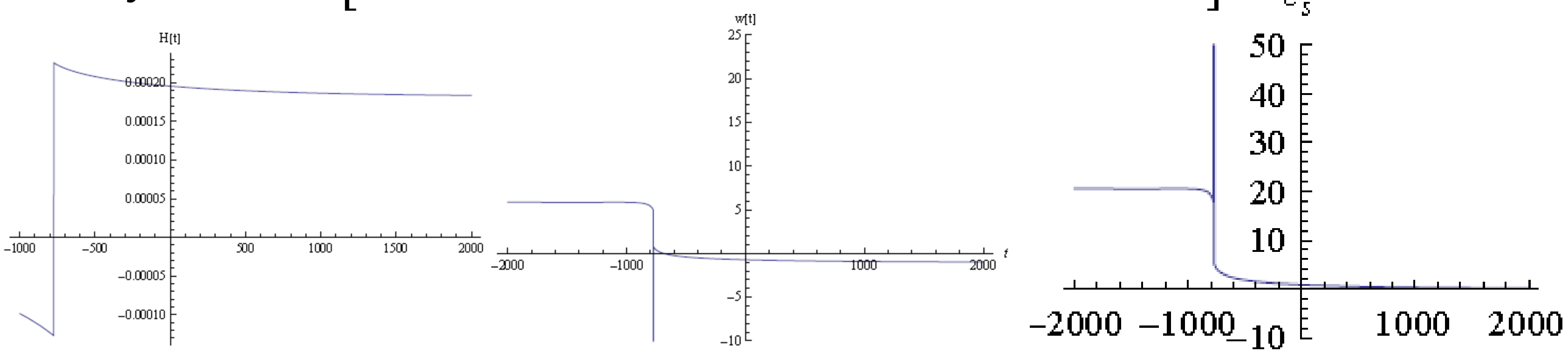
Solution: Galileon Theories

Taking into account the anisotropic and scale invariance issues, one can get more improved models:

2) bounce inflation model ***(TQ,** Y. T. Wang, 2015):*

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + k(\phi)X + t(\phi)X^2 - V(\phi) + G(\phi, X)\blacksquare\phi\right]$$

$$\frac{Q}{c_s^2}$$

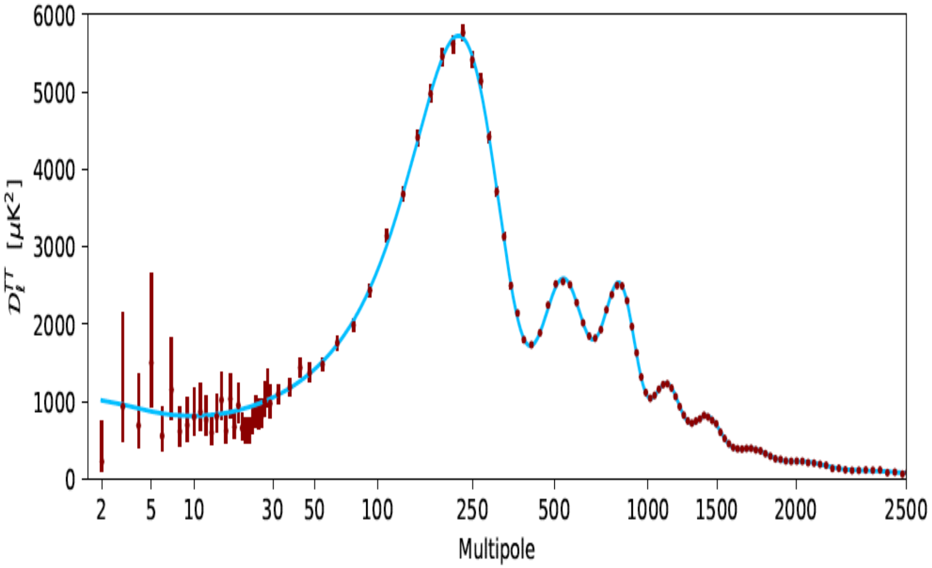


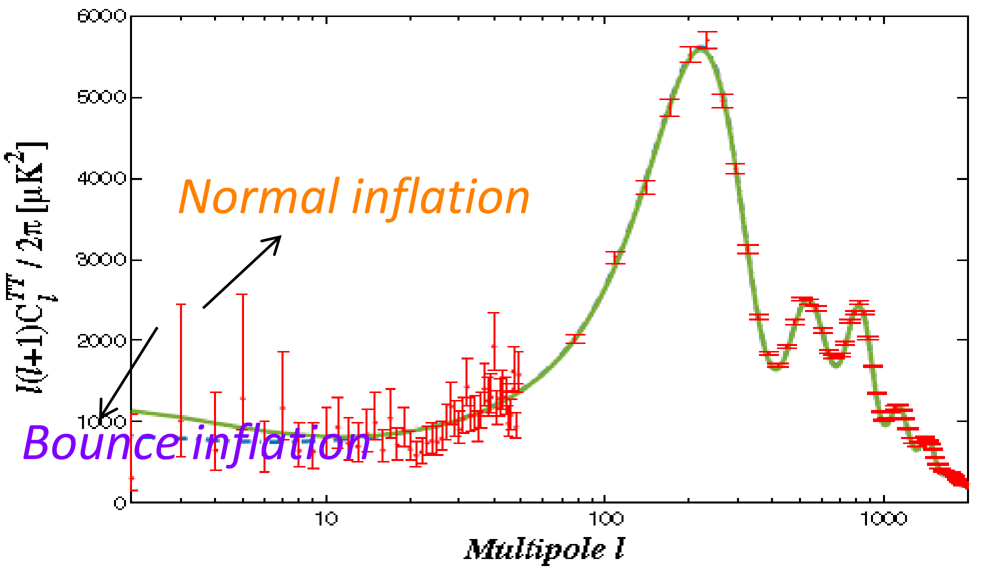
$$P_\zeta = \begin{cases} (k/aH)^{\frac{2\varepsilon_e}{\varepsilon_e - 1}}, & |\varepsilon_e| \ll 1 \\ (k/aH)^{\frac{2\varepsilon_c}{\varepsilon_c - 1}}, & \varepsilon_c \gg 1 \end{cases}$$

$$n_\zeta = \begin{cases} 1 + \frac{2\varepsilon_e}{\varepsilon_e - 1} \approx 1 \text{ (SI)} & \text{For large k;} \\ 1 + \frac{2\varepsilon_c}{\varepsilon_c - 1} > 1 \text{ (blue)} & \text{For small k.} \end{cases}$$

Bonus: Small-l suppression

For single field bounce inflation model, the C_l^{TT} can be suppressed in small-l (k) region!





Observations from PLANCK 2018

TQ, Y. T. Wang, JHEP 2015

4. Gradient Instability

In our model building, we found that it is difficult to also have the sound speed squared to be positive all the time.

The perturbation EoM:

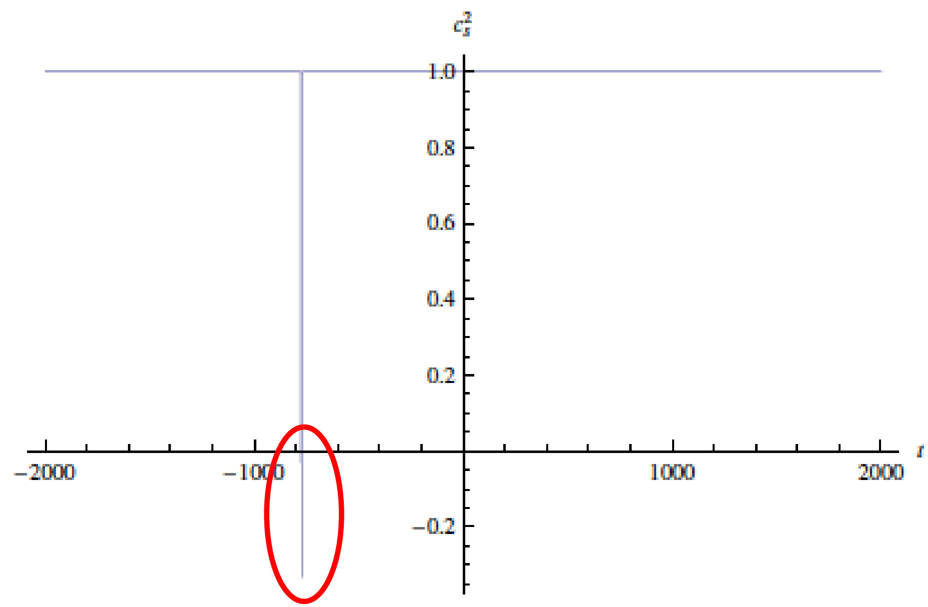
$$u_k'' + \left(c_s^2 k^2 - \frac{z''}{z} \right) u_k = 0$$

↑ sound speed squared

For $c_s^2 < 0$

$$u_k \sim e^{i\sqrt{c_s^2}kh} \sim e^{\pm|c_s|kh}$$

for large k modes.



Gradient Instability

(**TQ,** J. Evslin, Y. F. Cai, M. Z. Li, X. M. Zhang, 2011/Y. F. Cai, D. A. Easson, R. Brandenberger, 2012/**TQ,** X. Gao, E. Saridakis, 2013/M. Koehn, J. L. Lehners and B. A. Ovrut, 2014/L. Battara, M. Koehn, J. L. Lehners and B. A. Ovrut, 2014/**TQ** and Y. T. Wang, 2015……)

No-go Theorem

It has been proved that gradient instability is inevitable in cubic Galileon theories!

Abstract. We study spatially flat bouncing cosmologies and models with the early-time Genesis epoch in a popular class of generalized Galileon theories. We ask whether there exist solutions of these types which are free of gradient and ghost instabilities. We find that irrespectively of the forms of the Lagrangian functions, the bouncing models either are plagued with these instabilities or have singularities. The same result holds for the original Genesis model and its variants in which the scale factor tends to a constant as $t \to -\infty$. The result remains valid in theories with additional matter that obeys the Null Energy Condition and interacts with the Galileon only gravitationally. We propose a modified Genesis model which evades our no-go argument and give an explicit example of healthy cosmology that connects the modified Genesis epoch with kination (the epoch still driven by the Galileon field, which is a conventional massless scalar field at that stage).

It is this set of issues we address in this paper. We consider the simplest and best studied generalized Galileon theory interacting with gravity. The Lagrangian is (mostly negative signature; $\kappa = 8\pi G$)

$$L = -\frac{1}{2\kappa}R + F(\pi, X) + K(\pi, X)\Box\pi \ , \tag{1.1}$$

Contents

Although linear perturbation theory suggests that, for some constructions, cubic Galileon theories can avoid pathologies during a period of NEC violation, it has been unclear until now whether this is possible when the NEC violating period includes a non-singular bounce. In fact, the recent arguments suggest that either the speed of sound of co-moving curvature modes becomes imaginary (*i.e.*, ghost or gradient instability) for some wavelengths during the NEC violating phase [6, 7] or the evolution must reach a singularity [8].

M. Libanov, S. Mironov, V. Rubakov, 2016; Anna Ijjas, Paul J. Steinhardt, 2016.

Solution: one must consider theories beyond cubic Galileon!

Solution: Effective Field Theory Description

The Effective field theory lagrangian (Cheung et al., 2007; Gleyzes et al., 2013; Kase et al., 2014)

$$S = \int d^4x\sqrt{-g}\Big[\frac{M_p^2}{2}f(t)R - \Lambda(t) - c(t)g^{00} + \frac{M_2^4(t)}{2}(\delta g^{00})^2 - \frac{m_3^3(t)}{2}\delta K\delta g^{00} - m_4^2(t)\left(\delta K^2 - \delta K_{\mu\nu}\delta K^{\mu\nu}\right) + \frac{\tilde{m}_4^2(t)}{2}R^{(3)}\delta g^{00} - \bar{m}_4^2(t)\delta K^2 + \frac{\bar{m}_5(t)}{2}R^{(3)}\delta K + \frac{\bar{\lambda}(t)}{2}(R^{(3)})^2 + ... - \frac{\tilde{\lambda}(t)}{M_p^2}\nabla_i R^{(3)}\nabla^i R^{(3)} + ...\Big],$$

__Cubic Galileon:__ $f = 1 \qquad m_4^2 = \tilde{m}_4^2 = \bar{m}_4^2 = \bar{m}_5 = \bar{\lambda} = \tilde{\lambda} = 0$

__Horndeski:__ $m_4^2 = \tilde{m}_4^2 \quad \bar{m}_4^2 = \bar{m}_5 = \bar{\lambda} = \tilde{\lambda} = 0$

__Beyond Horndeski (high order space derivative)__: every coefficient can be non-zero!

Eliminating The Gradient Instability

According to the No-Go Theorem proved using EFT approach (Y. Cai, Y. Wan, H. Li, **TQ,** Y. S. Piao, JHEP (2017); Y. Cai, H. Li, **TQ,** Y. S. Piao, EPJC (2017))

	$g_i < 0$	$g_i > 0$
Cubic Galileon	No way	No way
Beyond cubic galileon (in EFT language)	$Q_T = 0$: $g \sim (t - t_g)^p, Q_T \sim (t - t_g)^n$, $n \geq 2p$	$Q_T = 0$: $Q_T \sim (-t)^{-p}, g \sim (-t)^{-n}$, $p > n > 1$
	$Q_{\tilde{m}_4} = 0$	$Q_{\tilde{m}_4} = 0$

where $\gamma = HQ_T - \frac{m_3^3}{2M_p^2} + \frac{1}{2}\dot{f}$ $\quad Q_{\tilde{m}_4} = f + \frac{2\tilde{m}_4^2}{M_p^2}$

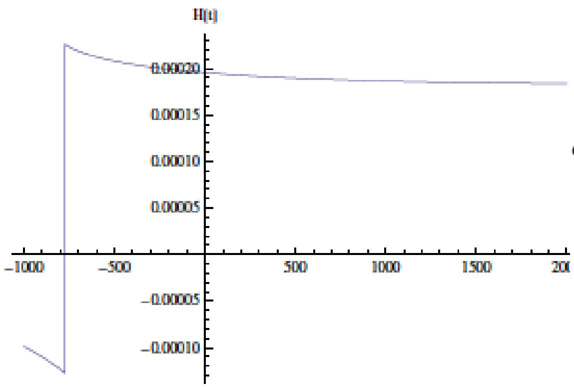
Q_T is the coefficient in front of kinetic term of tensor perturbation action.

Model Construction

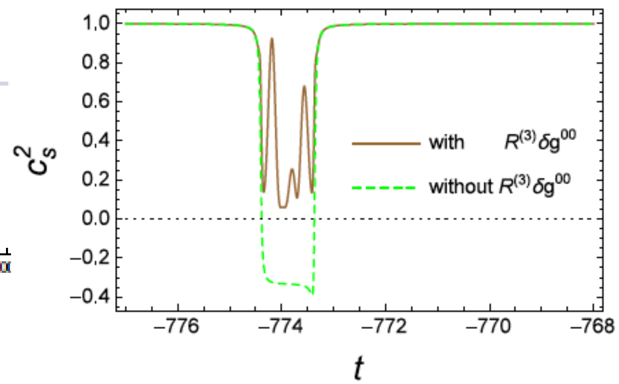
According to this conclusion, we can construct models free of gradient instability!

Action of a New Bounce Inflation Model:

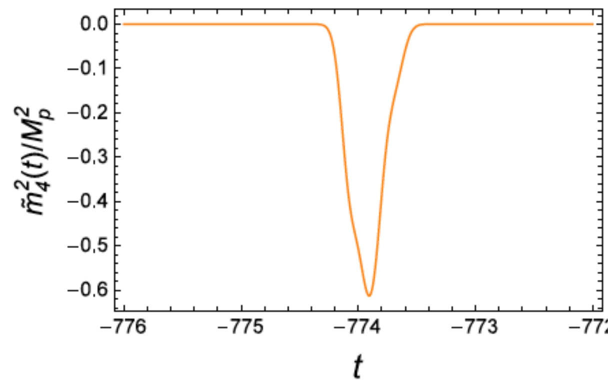
$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R + \mathcal{K}(\phi)X - G_3(\phi,X)\Box\phi + T(\phi)X^2 - V(\phi) + \frac{\tilde{m}_4^2(t)}{2}R^{(3)}\delta g^{00}\right]$$



Background



c_s^2 with/without the $\frac{\tilde{m}_4^2(t)}{2}R^{(3)}\delta g^{00}$ term (green/brown)



Evolution of $\tilde{m}_4^2(t)$

For covariant models, see Y. Cai and Y. S. Piao, JHEP 2017;
Y. Cai, Y. T. Wang, J. Y. Zhao and Y. S. Piao, 1709.07464; …………

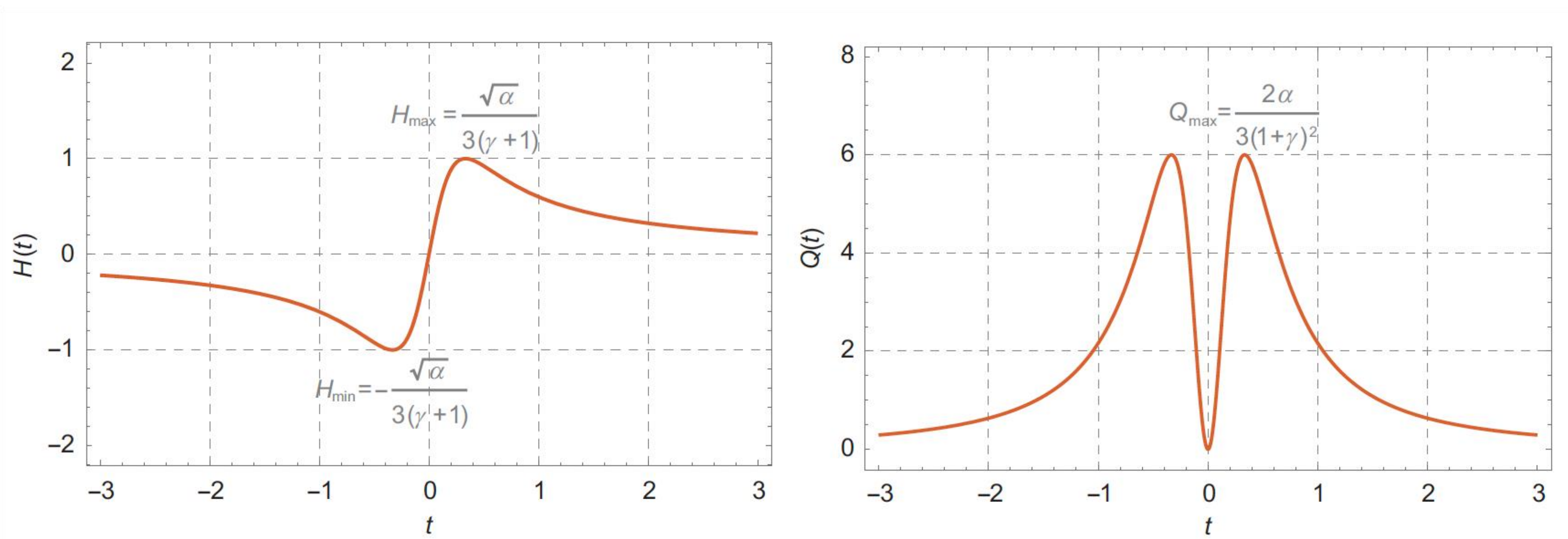
Bounce Model from Modified Gravity

One can also construct bounce models from modified gravity theories.

An example: $f(Q)$ theory   J. B. Jimenez, L. Heisenberg, T. Koivisto, PRD, 2018

$$S = \int d^4x \sqrt{-g}[f(Q) + L_m] \qquad \text{where} \qquad Q \sim \nabla g_{\mu\nu}$$

Moreover, $Q = R + tot.dev.$, therefore when $f(Q) = Q, S = \int Q \sim \int R$ (GR)!



K. Hu, T. Paul, **TQ**, Sci.China Phys.Mech.Astron. 2024

Conclusions

- Big-Bang/inflation scenarios suffers from singularity problem.
- Singularity problem can be solved by a bounce scenario, but there are new problems.
    - Anisotropy problem---Ekpyrotic phase contraction;
    - Scale invariance---Matter contraction/Multifield contraction/Bounce Inflation;
    - Ghost instability---Galileon/Horndeski/Beyond H;
    - Gradient instability---EFT approach;
- Avoiding singularity in Modified Gravity: still developing.

Thanks For Attention!