Non-perturbative primordial relics and its cosmological implications

极早期宇宙的非微扰现象学研究

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Plenary Talk @ Conference "Quantum Gravity and Cosmology 2024"

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Articles: 1805.03639, 1902.08187, 1908.03942, 2003.03821, 2009.09833, 2010.03537, 2105.12554, 2112.13826, 2207.11910, 2306.17822

Content

- Primordial black holes: hypothetical phenomena at "small" scales

- Sound speed resonance (SSR) mechanism

  Based on PRL 121 (2018) 081306, 1902.08187, 1908.03942, 2003.03821

- Beating the Lyth bound by resonant heavy field loops

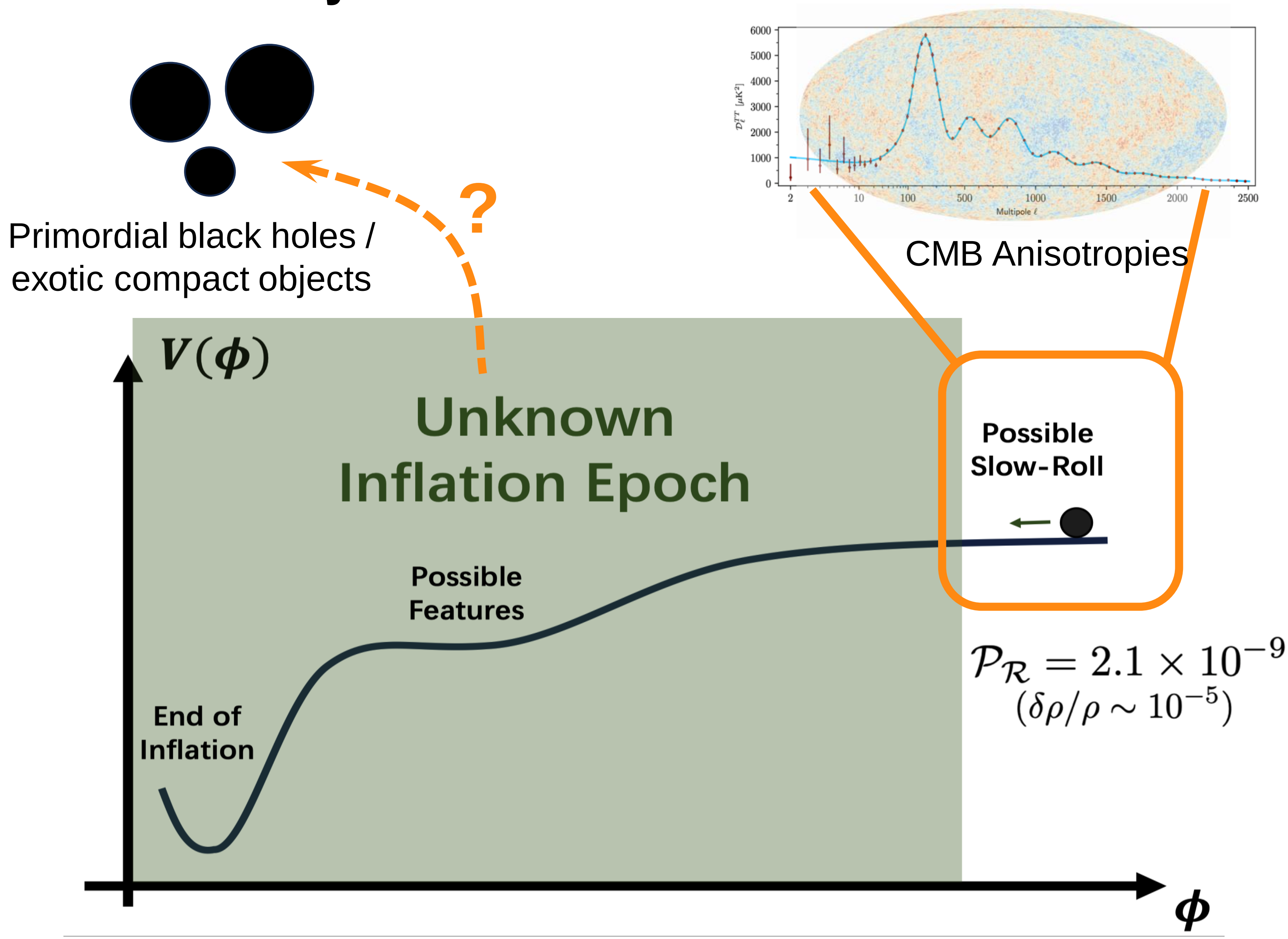
  Based on PRL 127 (2021) 251301

- Limits on Scalar-Induced Gravitational Waves by Pulsar Timing Array Observations

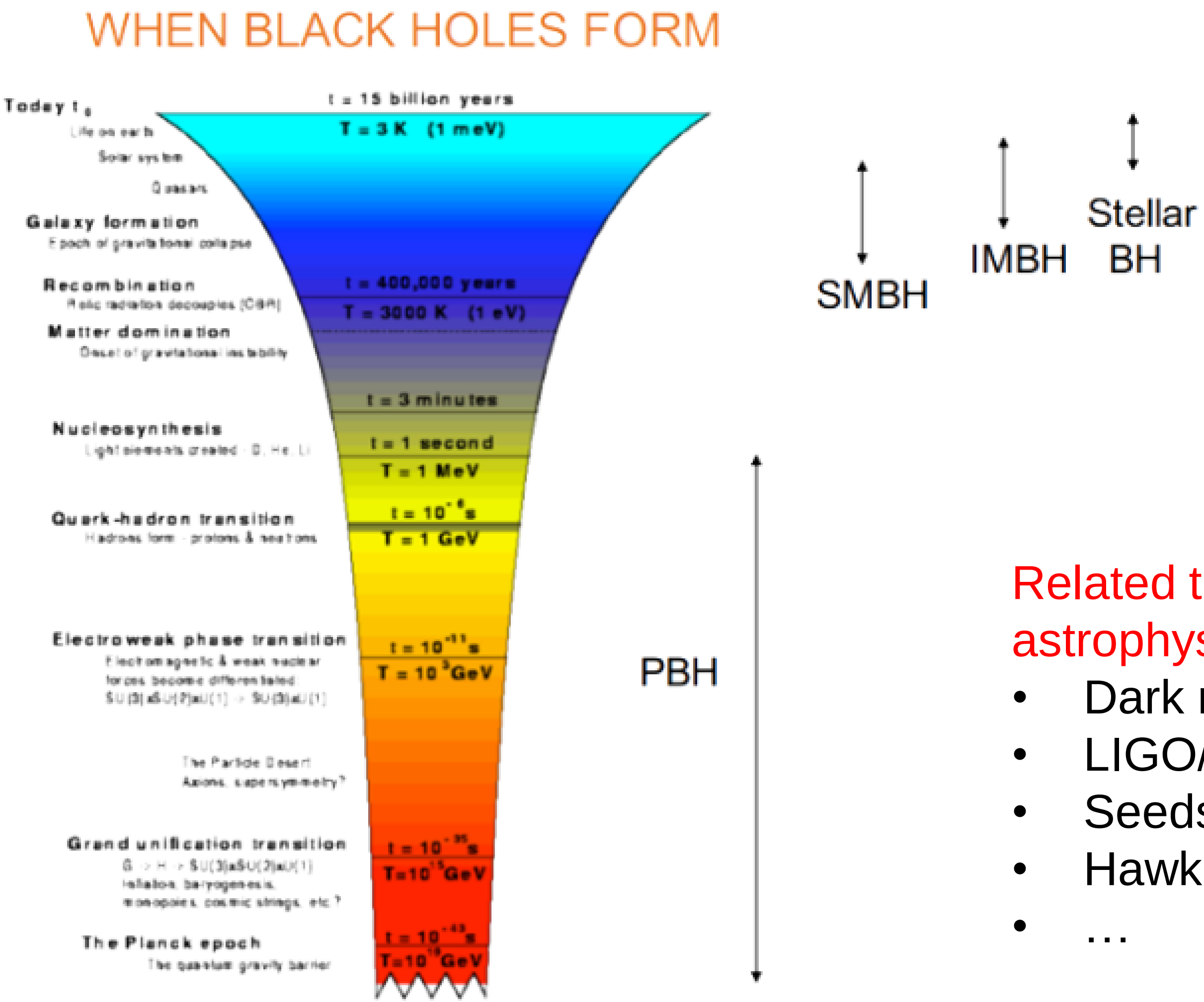
  Based on Science Bulletin 68 (2023) 2487

- Summary

Mysteries at "small" scales



Primordial black holes



But still no definite evidence for PBHs

Nonbaryonic

↓

Dark Matter

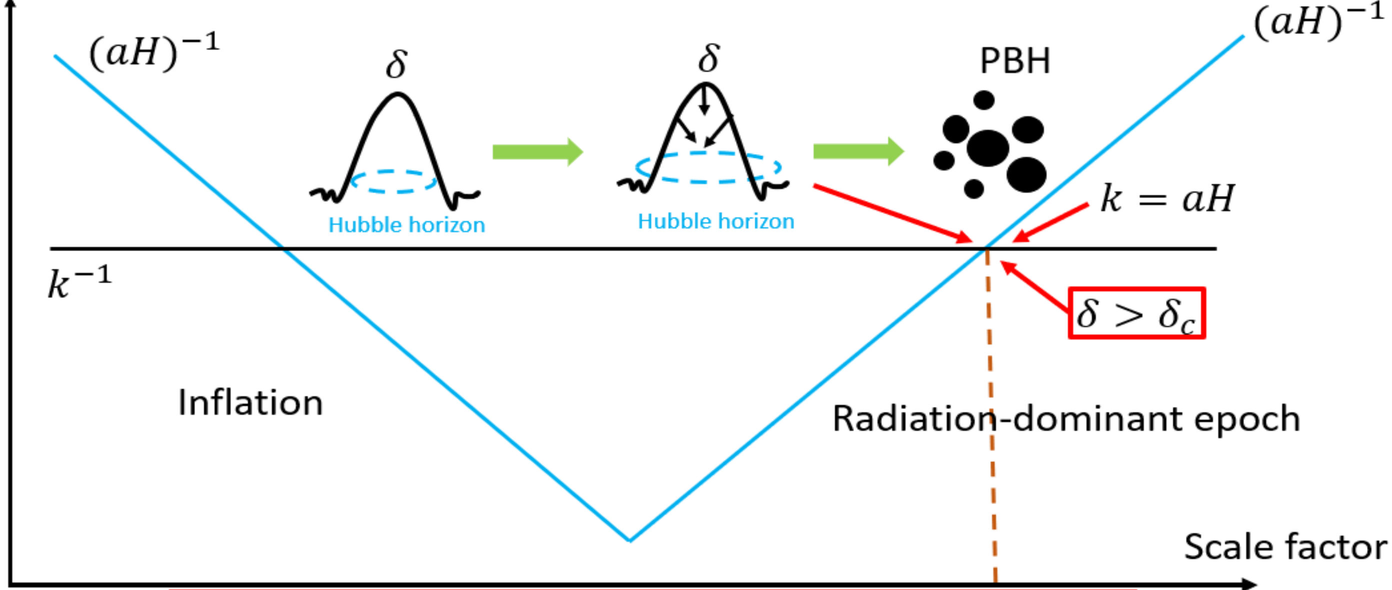
Related to plentiful cosmological and astrophysical phenomena:

- Dark matter;
- LIGO/Virgo event;
- Seeds for SMBHs in galactic nuclei;
- Hawking radiation;
- …

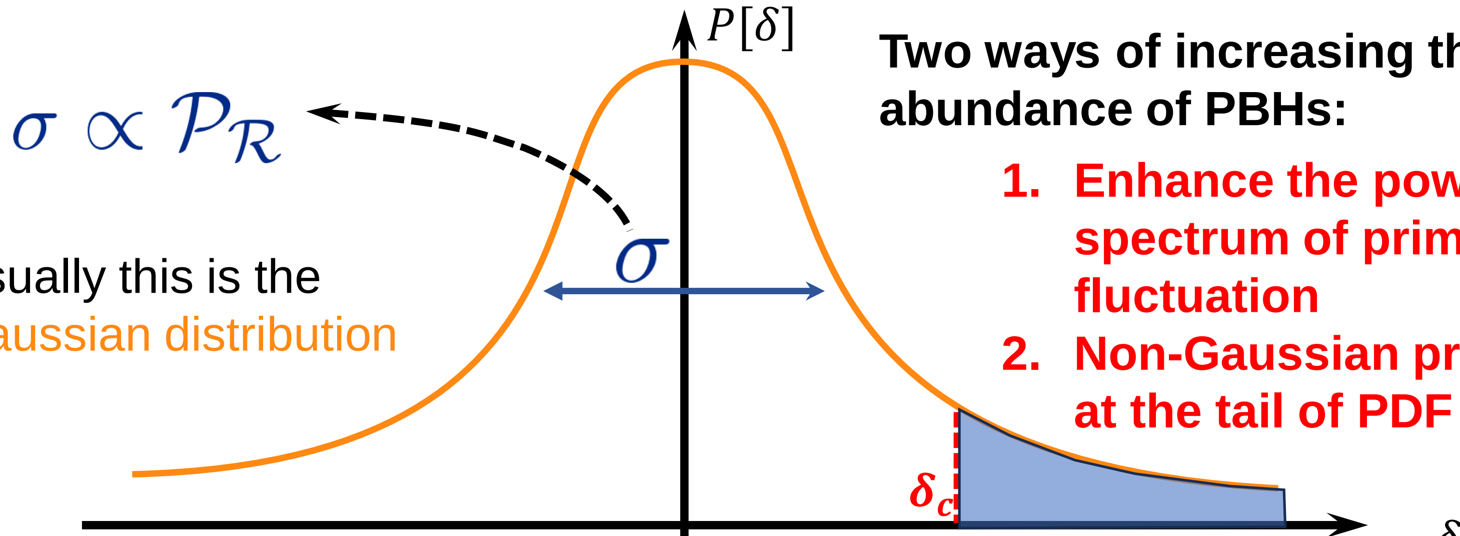
A PBH is a type of black hole which is not formed through the gravitational collapse of a star, but of the sufficiently large density perturbation in the early Universe.

[Carr et al., 2002.12778, 2006.02838]

Primordial black holes



$$\beta_{PBH} \equiv \frac{\rho_{PBH}}{\rho_{tot}} \propto \int_{\delta_c}^{\infty} P[\delta]\, \mathrm{d}\delta$$



Usually this is the
Gaussian distribution

Two ways of increasing the abundance of PBHs:

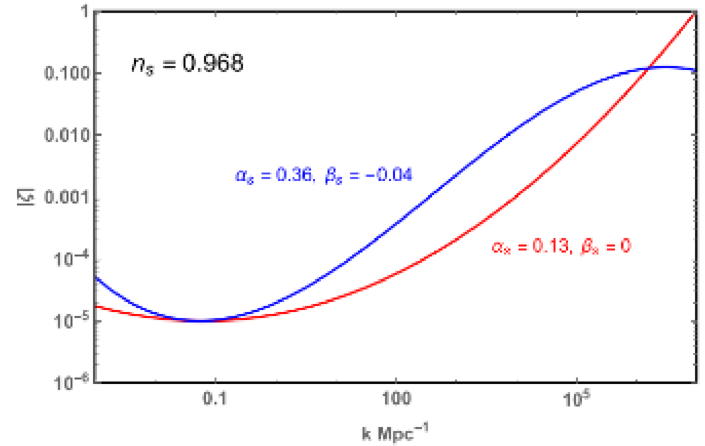
1. **Enhance the power spectrum of primordial fluctuation**
2. **Non-Gaussian production at the tail of PDF**

Enhance power spectrum in inflation scenario

- **Running mass inflation:**

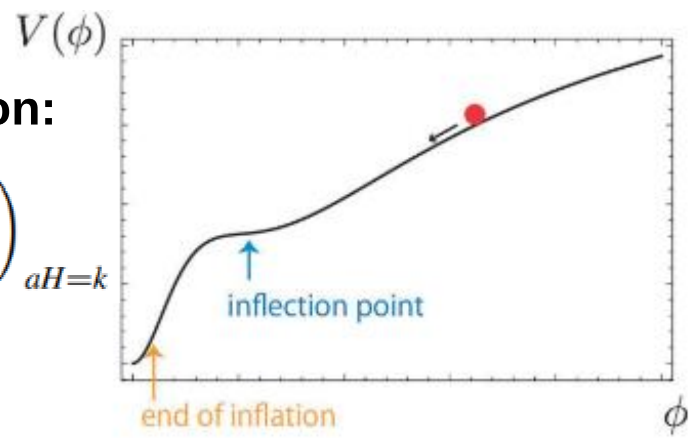
$$\mathcal{P}_{\mathcal{R}_c}(k) := A_{\mathcal{R}_c}\left(\frac{k}{k_*}\right)^{n_s-1+\frac{1}{2!}\alpha_s \ln(k/k_*)+\frac{1}{3!}\beta_s \ln^2(k/k_*)+\cdots}$$

Running parameters



- **Inflection inflation:**

$$\mathcal{P}_{\mathcal{R}_c}(k) = \left(\frac{8\pi G H^2}{\epsilon}\right)_{aH=k}$$

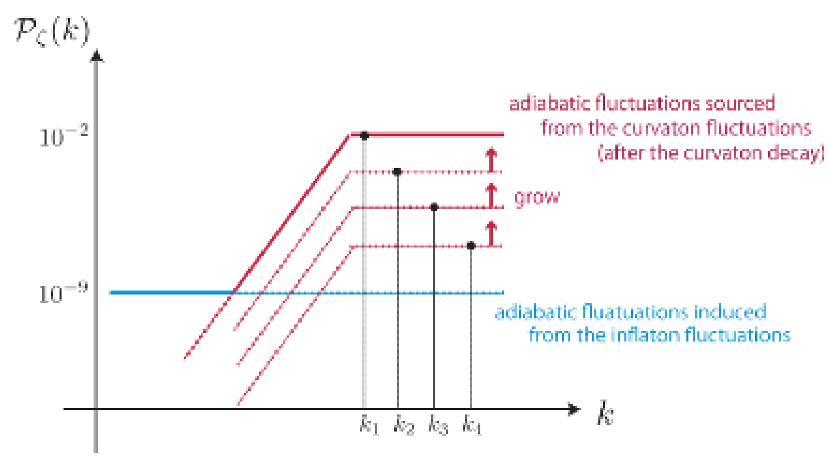


For a comprehensive review, see Misao et al., 1801.05235

- **Axion-like curvaton inflation:**

$$\mathcal{P}_\zeta(k) = \mathcal{P}_{\zeta,\text{inf}}(k) + \mathcal{P}_{\zeta,\text{curv}}(k).$$

$$V_\chi = \Lambda^4\left[1-\cos\left(\frac{\chi}{f}\right)\right] \simeq \frac{1}{2} m_\chi^2 \chi^2$$



A novel method: SSR mechanism

- **Oscillating sound speed:**

$$c_s^2 = 1 - 2\xi[1 - \cos(2k_*\tau)], \text{ with } \tau > \tau_i$$

The amplitude: ξ is small and $\xi < 1/4$, such that c_s^2 is positively definite

The scale: k_* is the oscillation frequency

The beginning of oscillation: $|k_*\tau_i| \gg 1$

PHYSICAL REVIEW LETTERS **121,** 081306 (2018)

[*CYF*, Tong, Wang, **Yan**, PRL 121 (2018) 081306]

Primordial Black Holes from Sound Speed Resonance during Inflation

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We report on a novel phenomenon of the resonance effect of primordial density perturbations arisen from a sound speed parameter with an oscillatory behavior, which can generically lead to the formation of primordial black holes in the early Universe. For a general inflaton field, it can seed primordial density fluctuations, and their propagation is governed by a parameter of sound speed square. Once, if this parameter achieves an oscillatory feature for a while during inflation, a significant nonperturbative resonance effect on the inflaton field fluctuations takes place around a critical length scale, which results in significant peaks in the primordial power spectrum. By virtue of this robust mechanism, primordial black holes with specific mass function can be produced with a sufficient abundance for dark matter in sizable parameter ranges.

DOI: 10.1103/PhysRevLett.121.081306

- **Equation of Motion:** $v_k'' + \left(c_s^2 k^2 - \frac{z''}{z}\right) v_k = 0$,

Mukhanov-Sasaki variable: $v = z\zeta$

$$z = \sqrt{2\epsilon}a/c_s \text{ with } \epsilon \equiv -\dot{H}/H^2$$

- **Mathieu equation:**

$$\frac{d^2 v_k}{dx^2} + (A_k - 2q\cos 2x)v_k = 0$$

where $x = -k_*\tau$, $A_k = \frac{k^2}{k_*^2}(1-2\xi)$ and $q = (2 - \frac{k^2}{k_*^2})\xi$ <<1

narrow resonance

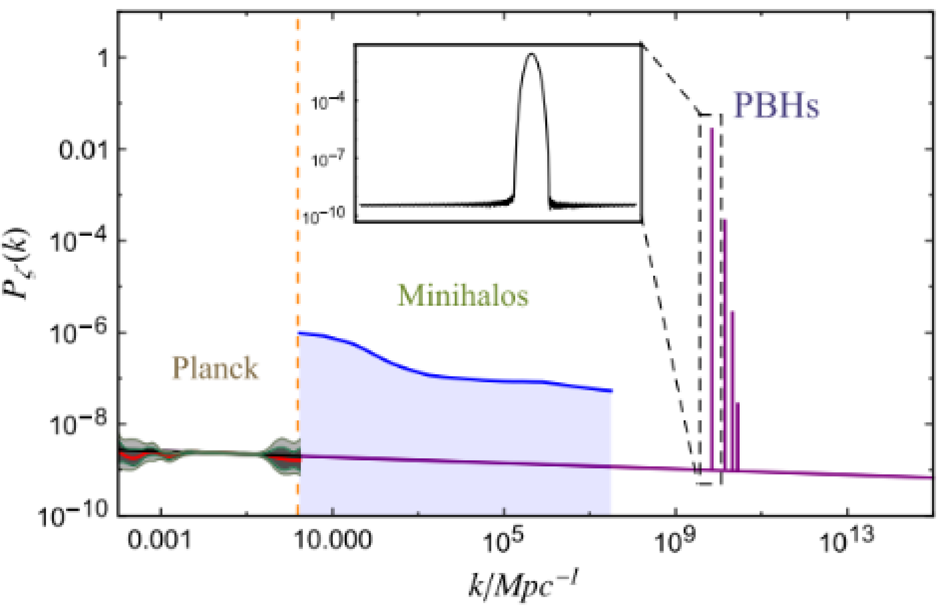
Resonance effect

Generating large inhomogeneities: SSR mechanism

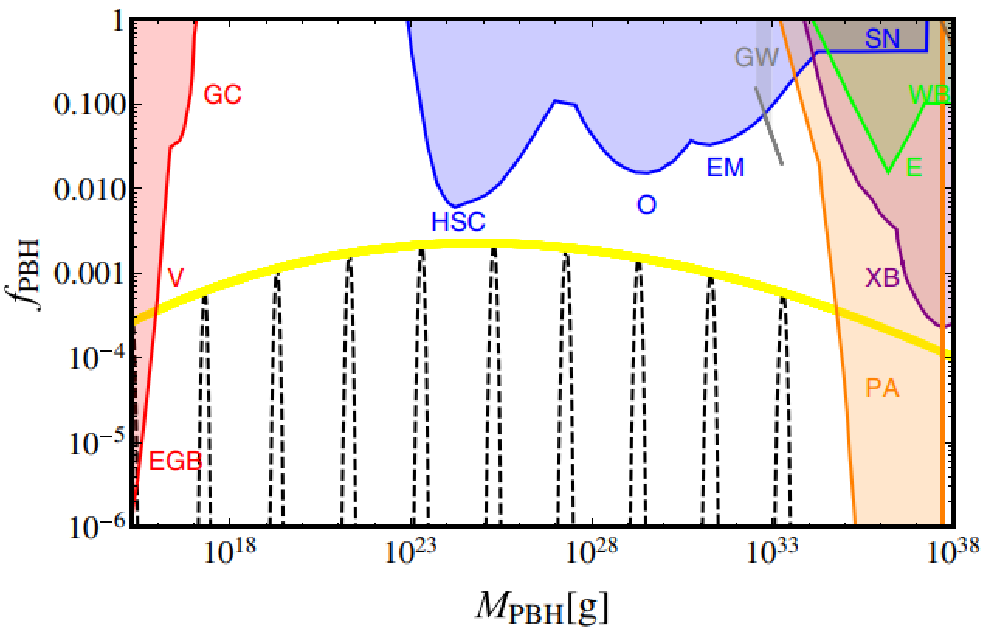
- **Enhanced curvature perturbation:** [*CYF*, Tong, Wang, **Yan**, PRL 121 (2018) 081306]

$$P_\zeta(p) = A_s \left(\frac{p}{k_p}\right)^{n_s-1} \left\{1 + \frac{\xi p_*}{2} \boxed{e^{-\xi p_* \tau_i}} \left[\delta(p - p_*) + \sum_{n=2}^{\infty} a_n \delta(p - n p_*)\right]\right\}$$

Enhancement



$$f_{\rm PBH}(M) \equiv \frac{\Omega_{\rm PBH}}{\Omega_{\rm DM}}$$



Spikes on small scales, scale-invariant on large scales

Inflationary perturbation theory

dimensionless power spectrum

$$P_{\mathscr{R}}(k) = \left(\frac{H}{2\pi}\right)^2 \left(\frac{H}{\dot{\phi}}\right)^2$$

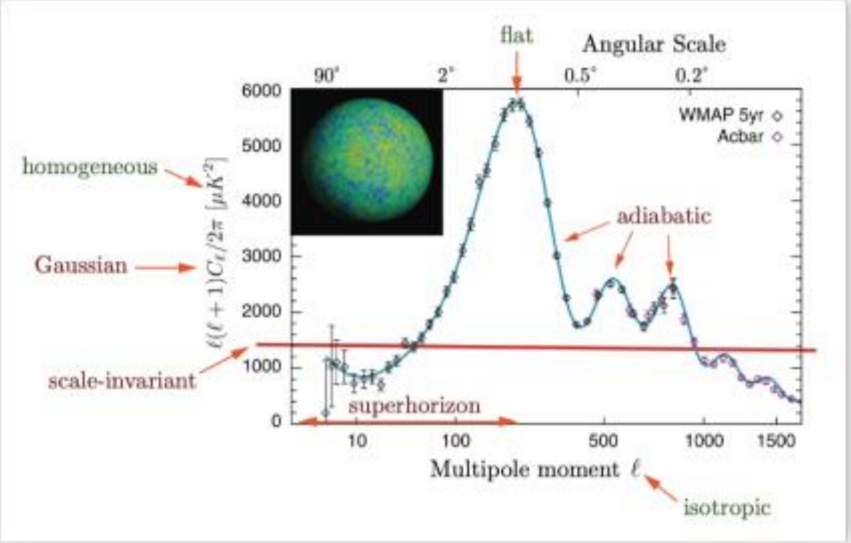
$\delta\phi$ — de Sitter fluctuations

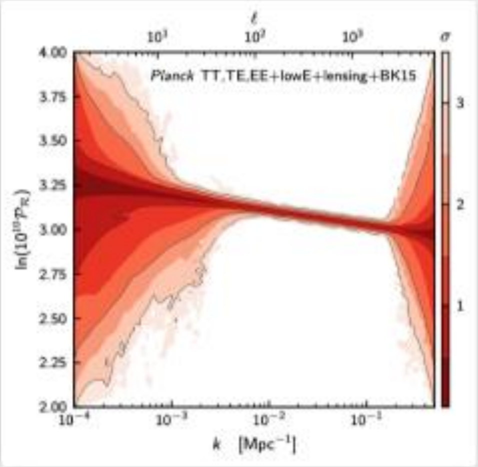
$\delta\phi \to \mathscr{R}$ — conversion

evaluated at horizon crossing

$k = aH$

Scalar Fluctuations observation





Planck 2018 results.

$n_s = 0.9649 \pm 0.0042$

nearly scale-invariant

scale-dependence $\quad P_{\mathscr{R}}(k) = A_s k^{n_s - 1}$

e.g. slow-roll inflation $\quad n_s - 1 = 2\eta - 6\epsilon$

Inflationary perturbation theory

Tensor Fluctuations

Besides scalar fluctuations inflation produces **tensor** fluctuations:

$$ds^2 = dt^2 - a^2(t)(1+h_{ij})dx^i dx^j$$

gravitational waves

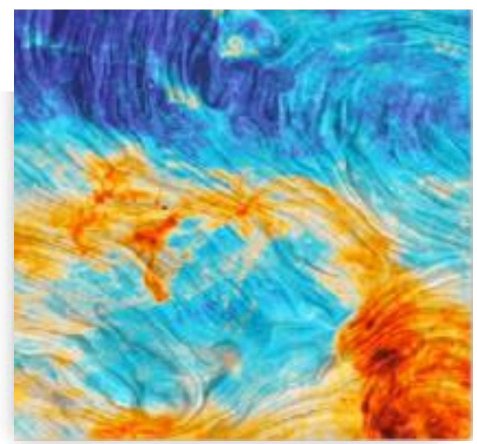
$$P_h(k) = \frac{8}{M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2$$

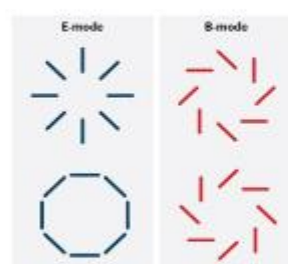
vacuum fluctuations

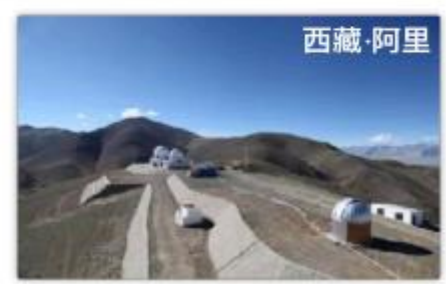
graviton fluctuations

primordial tensor perturbations

CMB B-mode polarization







Lyth bound & effective field theory

tensor-to-scalar ratio

$$r \equiv \frac{\Delta_t^2}{\Delta_s^2} = 8\left(\frac{d\phi}{dN_e}\frac{1}{M_{\rm pl}}\right)^2$$

field evolution over 60 e-folds

tensor-to-scalar ratio r determines the field excursion during inflation

$$\Delta\phi \gtrsim \mathcal{O}(1)\left(\frac{r}{0.01}\right)^{1/2} M_{\rm Pl}$$

$r > 0.01$

Lyth 1996

If we observe tensors it proves that the inflaton field moved over a super-Planckian distance!

current upper limit

$r \lesssim 0.07 \longrightarrow$ r < 0.036

$\Delta\phi \gg M_{\rm Pl}$

But this only consider vacuum contribution!

$$\mathcal{L}_{\rm eff} = \sum_i \frac{\mathcal{O}_i(\phi)}{\Lambda^{\Delta_i - 4}} = \underbrace{\frac{1}{2}(\partial_\mu\phi)^2 - V(\phi)}_{\mathcal{L}_{\rm s.r.}} + \sum_n c_n V(\phi)\frac{\phi^{2n}}{\Lambda^{2n}} + \sum_n d_n \frac{(\partial\phi)^{2n}}{\Lambda^{4n}} + \cdots$$

Beating the Lyth bound by resonant heavy field loops

[*CYF*, Jiang, Sasaki, Vardanyan, Zhou, PRL 127 (2021) 251301]

- $\delta\chi$ is masseless and controls the curvature perturbation;
- $\delta\phi$ is massive, resonant, only gravitationally coupled to $\delta\chi$, and controls isocurvature perturbations.

$$S_3 \supset \frac{1}{2}\int dt \frac{d^3k d^3p}{(2\pi)^6} a^3 \left(-\sqrt{2\epsilon_\chi}\frac{\boldsymbol{k}\cdot\boldsymbol{p}}{k^2}\mathcal{M}^2_{\rm eff}\delta\phi_{\boldsymbol{k}-\boldsymbol{p}}\delta\chi_{-\boldsymbol{k}}\delta\phi_{\boldsymbol{p}} \right)$$
$$-\frac{2}{M^2_{\rm pl}}\int dt \frac{d^3k d^3p}{(2\pi)^6} a^3 \left(e^\lambda_{ij}(\boldsymbol{k})\frac{p_i p_j}{a^2}\right)\delta\phi_{\boldsymbol{p}}\delta\phi_{\boldsymbol{k}-\boldsymbol{p}} h^\lambda_{-\boldsymbol{k}}$$

Condition 1 Resonance must happen inside the Hubble horizon.

e.g. narrow resonance

$$|M_{\rm eff}| \sim O(10)H$$

$$\delta\phi \propto \exp((|M_{\rm eff}|/H)\Delta N)$$
$$\exp(\Delta N) \sim \frac{|M_{\rm eff}|}{H}$$

Condition 2 Resonant heavy field must decay outside the Hubble horizon, in order not to affect curvature perturbations.

$$M_{\rm eff} \gtrsim O(1)H$$

Condition 3 Resonant field mass needs to become small right after Hubble crossing.

A specific model realization

[*CYF*, Jiang, Sasaki, Vardanyan, Zhou, PRL 127 (2021) 251301]

vacuum energy — slope of ϕ — slope of χ → determines the curvature perturbation

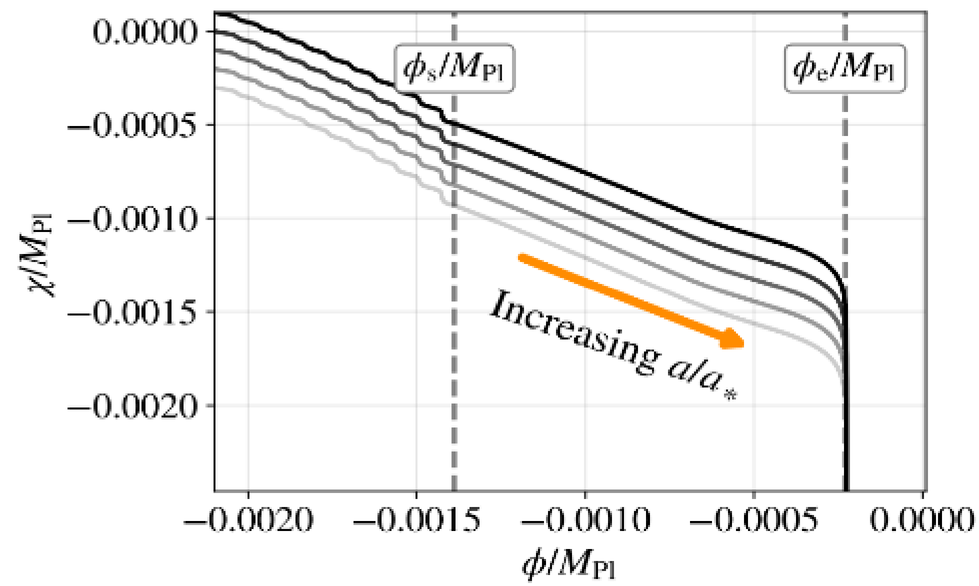
$$V(\phi,\chi) = V_0\left(1 - \sqrt{2\epsilon_\phi}\frac{\phi}{M_{\rm Pl}} + \sqrt{2\epsilon_\chi}\frac{\chi}{M_{\rm Pl}} + \eta_\chi\frac{\chi^2}{2M_{\rm Pl}^2}\right)$$

$$+\Lambda^4(\phi)\cos\left(\frac{\phi}{f_{\rm a}}\right)$$

ϕ oscillation

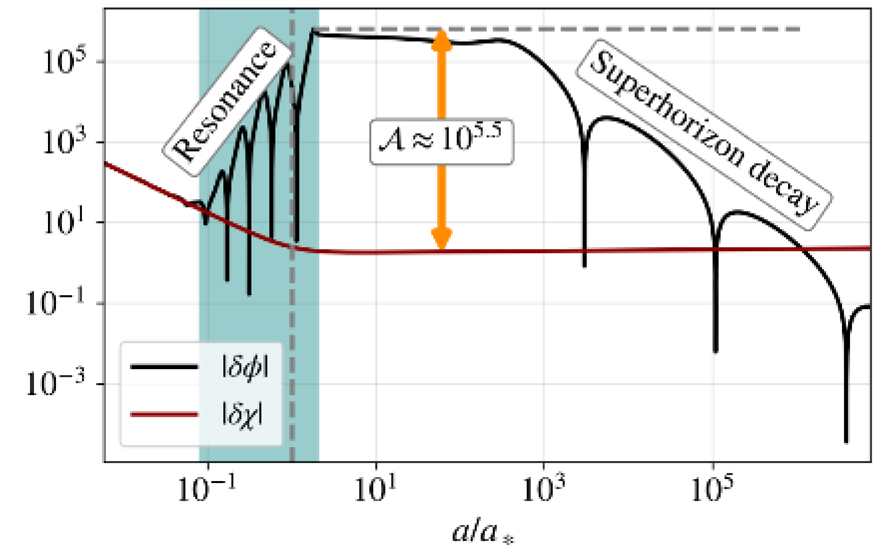
$$+V_{\rm m}(\phi)$$

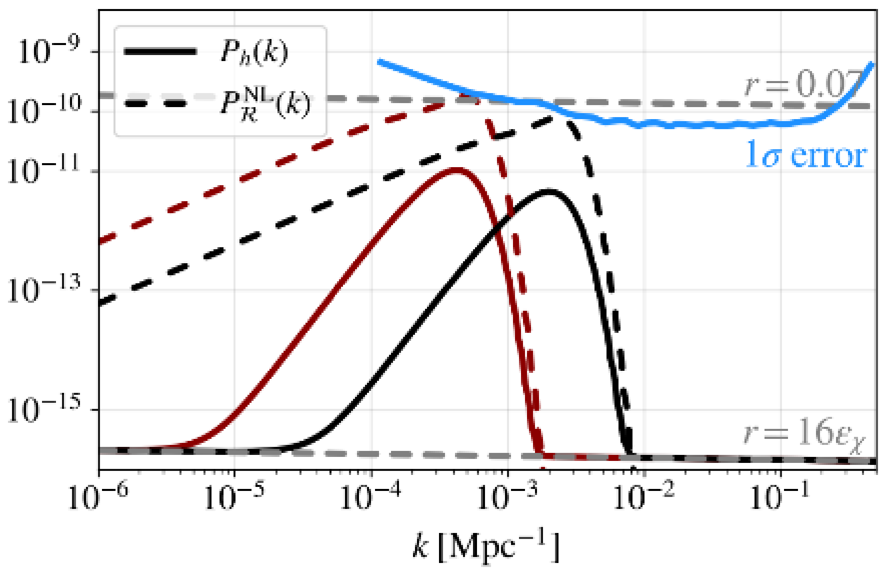
Super-Hubble decay

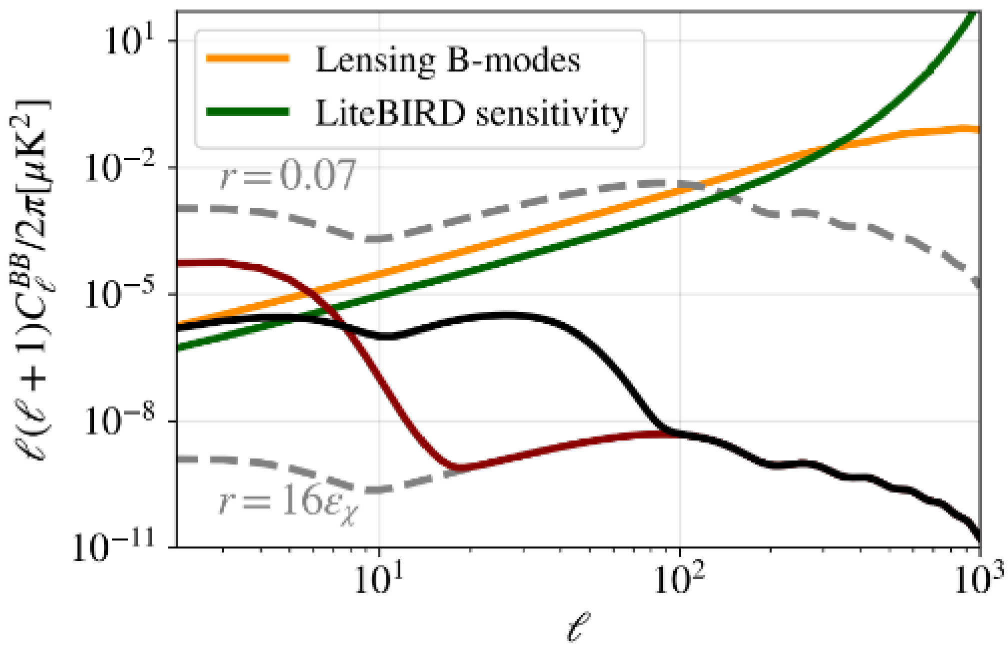


Numerical results & forecast for CMB

[*CYF*, Jiang, Sasaki, Vardanyan, Zhou, PRL 127 (2021) 251301]







Limits on Scalar-Induced Gravitational Waves by Pulsar Timing Array Observations

[*CYF*, He, Ma, Yan, Yuan, 2306.17822, Science Bulletin 68 (2023) 2487]

- For universally explored power spectra, we introduce a broken power-law parameterization to describe the energy spectrum:

$$\Omega_{\rm GW}(f)h_0^2 = A\,\frac{\alpha+\beta}{\beta(f/f_c)^{-\alpha}+\alpha(f/f_c)^{\beta}}$$

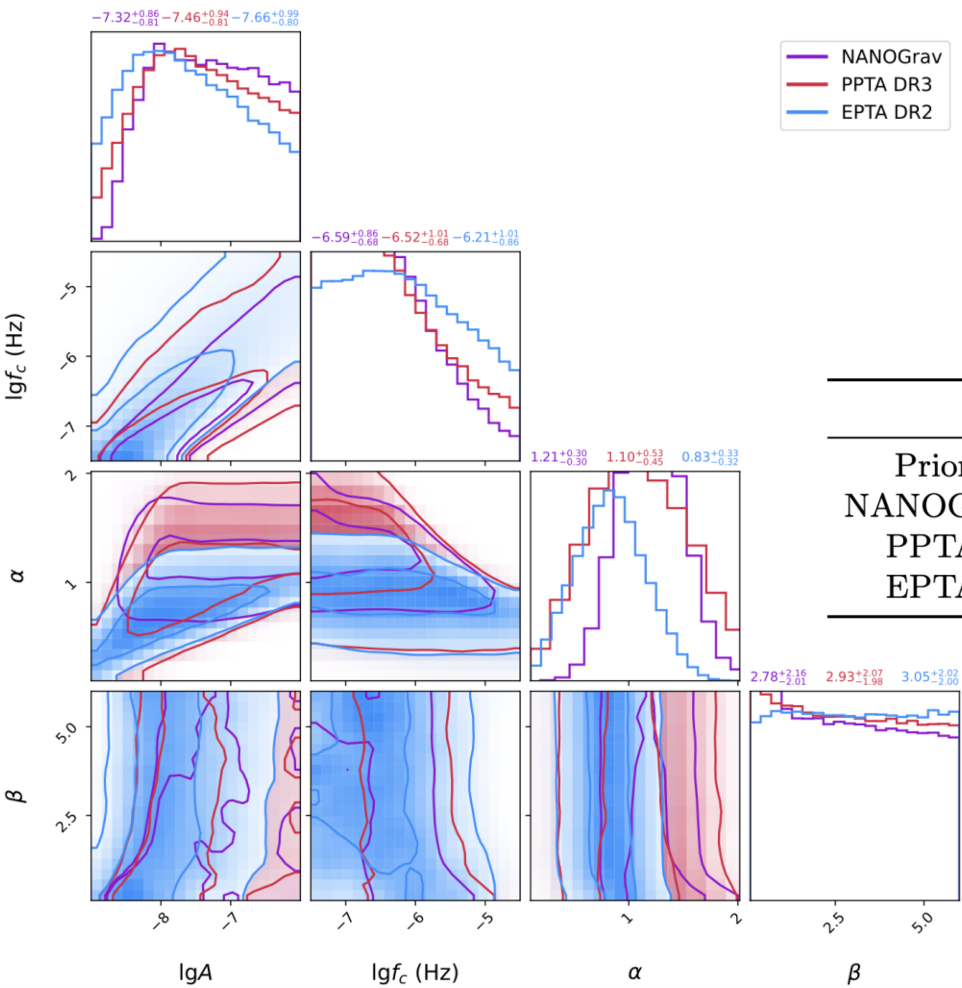
- Data fitting: the maximum likelihood method

$$-2\ln\mathcal{L}(\Theta) = \sum\left[\frac{(\Omega_{\rm GW}h^2)_i - \Omega_{\rm GW}h^2(f_i;\Theta)}{\sigma_i}\right]^2$$

Limits on Scalar-Induced Gravitational Waves by Pulsar Timing Array Observations

[*CYF*, He, Ma, Yan, Yuan, 2306.17822, Science Bulletin 68 (2023) 2487]

- Posterior distributions of key parameters (log A, f_c, α) with 1σ and 2σ



- The best-fit parameter space with NANOGrav, PPTA and EPTA at 2σ:

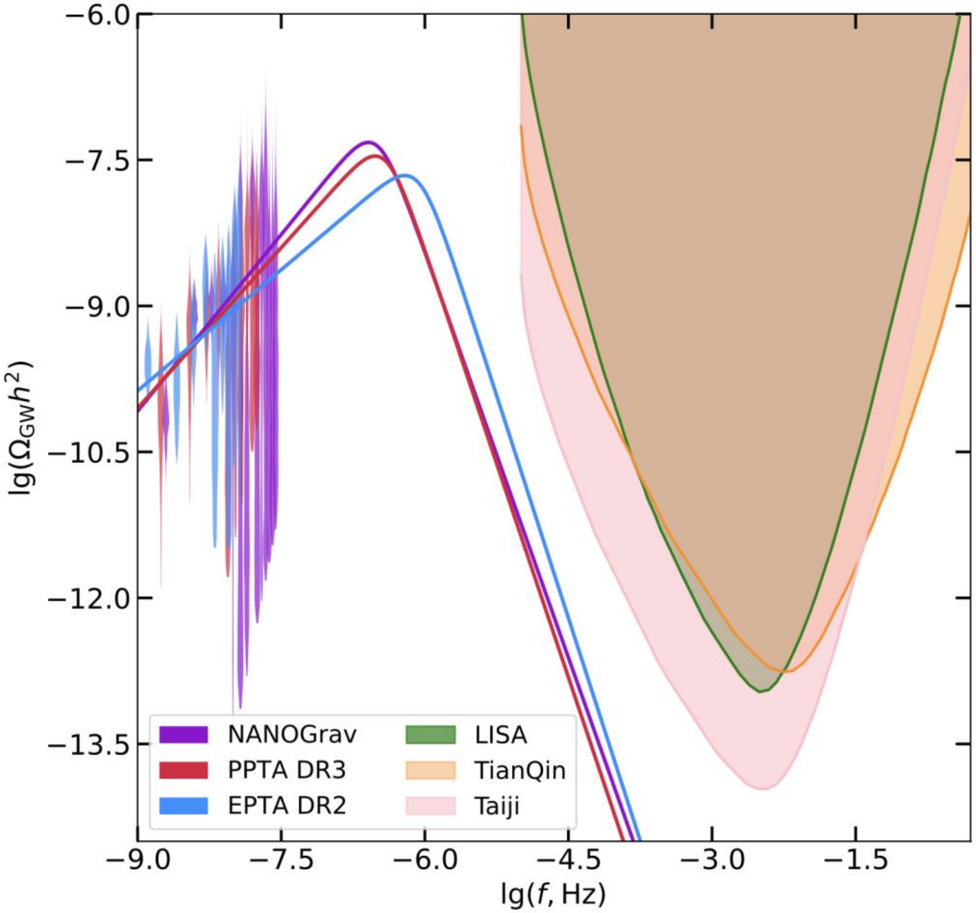
	$\lg A$	$\lg f_c$	α	β
Prior	$\mathcal{U}(-9.0, -6.0)$	$\mathcal{U}(-7.5, -4.5)$	$\mathcal{U}(0.1, 6.0)$	$\mathcal{U}(0.1, 6.0)$
NANOGrav	$-7.32^{+0.86}_{-0.81}$	$-6.59^{+0.86}_{-0.68}$	$1.21^{+0.30}_{-0.30}$	$2.78^{+2.16}_{-2.01}$
PPTA	$-7.46^{+0.94}_{-0.81}$	$-6.52^{+1.01}_{-0.68}$	$1.10^{+0.53}_{-0.45}$	$2.93^{+2.07}_{-1.98}$
EPTA	$-7.66^{+0.99}_{-0.80}$	$-6.21^{+1.01}_{-0.86}$	$0.83^{+0.33}_{-0.32}$	$3.05^{+2.02}_{-2.00}$

- Lack of observations in the UV region $\Rightarrow$ suboptimal convergence for β

Limits on Scalar-Induced Gravitational Waves by Pulsar Timing Array Observations

[*CYF*, He, Ma, Yan, Yuan, 2306.17822, Science Bulletin 68 (2023) 2487]

- Energy spectra of the stochastic GW background



- The signal resides in the near-IR region (α≤2) according to best-fit

- A potential detection of this UV tail by LISA/TianQin/Taiji can yield crucial implications in future

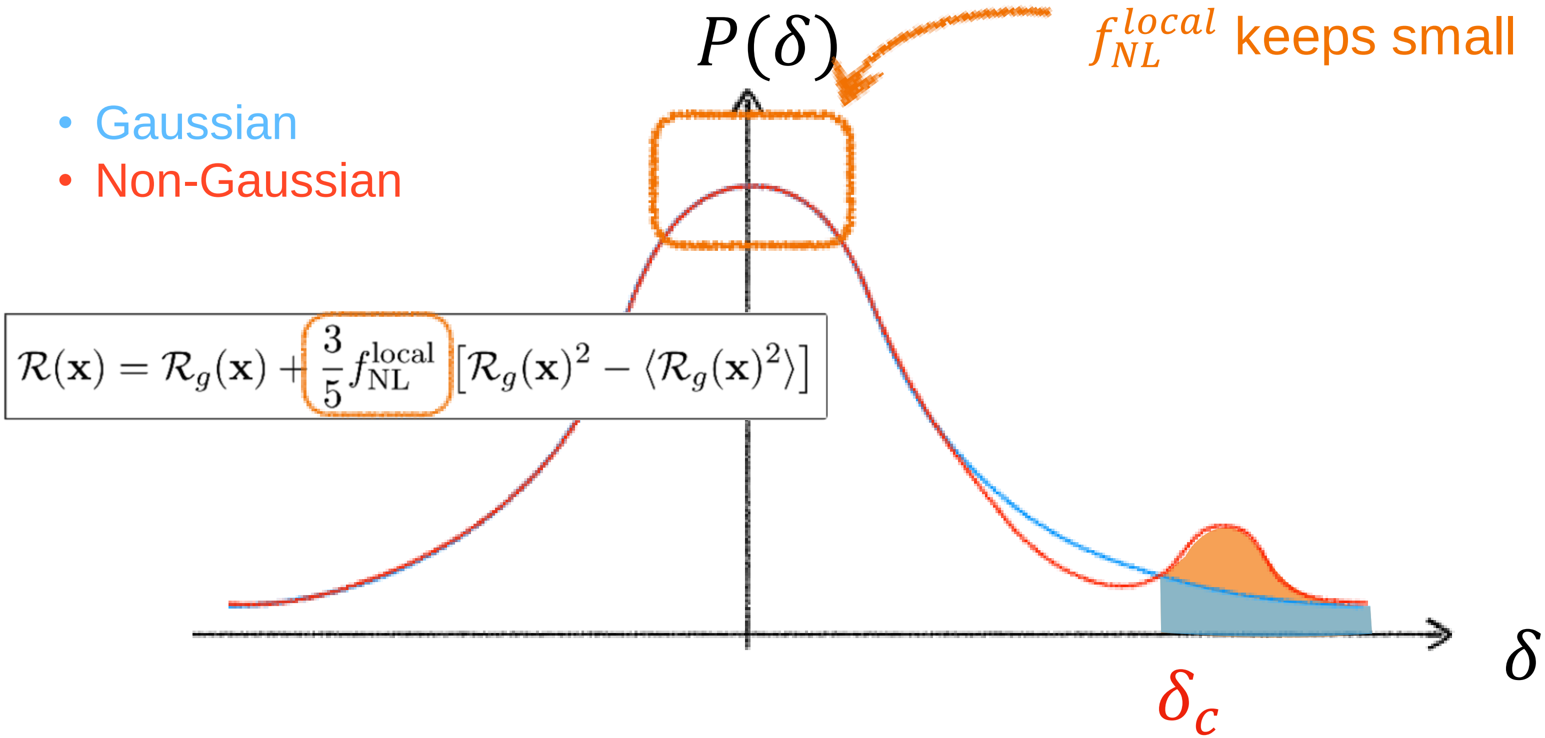
A brief summary

- Sound speed resonance can enhance power spectrum non-perturbatively, and thus produce PBHs efficiently.
- Abundant underlying physics of SSR needs to explore, namely, model realizations.
- The Lyth bound can be beaten by nonlinear corrections from a resonant heavy field.
- A side story: Limits on Scalar-Induced Gravitational Waves by Pulsar Timing Array Observations

Thanks

Non-Gaussian tails and primordial black holes

[*CYF*, Ma, Sasaki, Wang, Zhou, PLB 834 (2022) 137461]



$$\mathcal{R}(\mathbf{x}) = \mathcal{R}_g(\mathbf{x}) + \frac{3}{5} f_{\rm NL}^{\rm local} \left[\mathcal{R}_g(\mathbf{x})^2 - \langle \mathcal{R}_g(\mathbf{x})^2 \rangle\right]$$

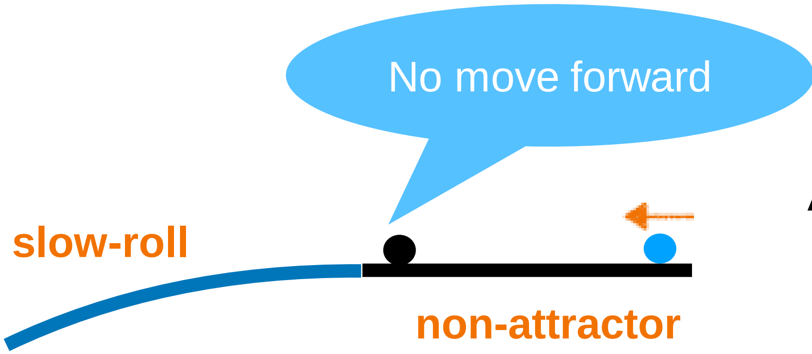
- Perturbative approach becomes invalid when the tail of curvature perturbation is considered

NG tail is a non-perturbative phenomenon!

- $\beta_{\rm PBH}$ needs to be calculated very carefully

Non-attractor inflation revisited

[*CYF*, Chen, Namjoo, Sasaki, Wang, Wang, JCAP 05 (2018) 012]



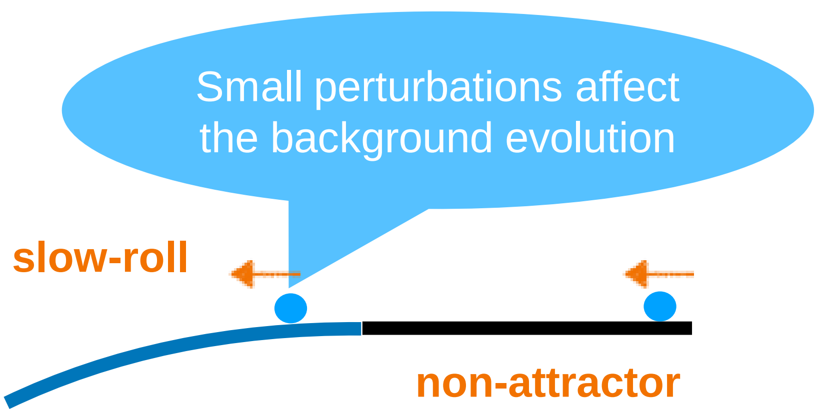
$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2} \propto a^{-6}$$

A flat potential suffers an eternal inflation issue:

$$\boxed{\pi(n) + 3\phi(n) = \pi(n_i) + 3\phi(n_i)}$$

$$y \equiv \frac{\pi(n_i)}{-3\Delta\phi_{USR}}$$

If $y < 1$, inflaton would be trapped in the USR phase without perturbation



"drift limit": $y \gg 1$

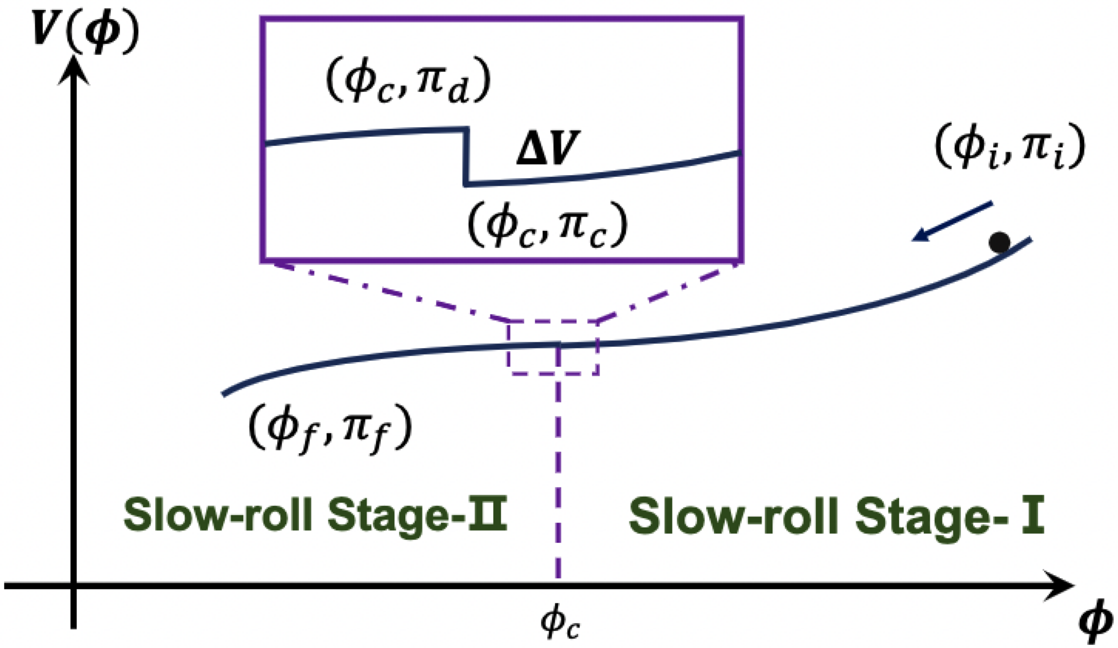
$$P[\mathcal{R}] \sim e^{-3\mathcal{R}}$$

A sketch of non-perturbative phenomenon in non-attractor inflation

Non-attractor inflation revisited: an upward step

[*CYF*, Ma, Sasaki, Wang, Zhou, JCAP 12 (2022) 034]

A concrete example:
$$V(\phi) = \begin{cases} V_0 \left[1 + \sqrt{2\epsilon_I}\,(\phi - \phi_c) + \frac{1}{2}\eta_I\,(\phi - \phi_c)^2\right] , & \phi \geq \phi_c \\ (V_0 + \Delta V) \left[1 + \sqrt{2\epsilon_{II}}\,(\phi - \phi_c) + \frac{1}{2}\eta_{II}\,(\phi - \phi_c)^2\right] , & \phi < \phi_c \end{cases}$$



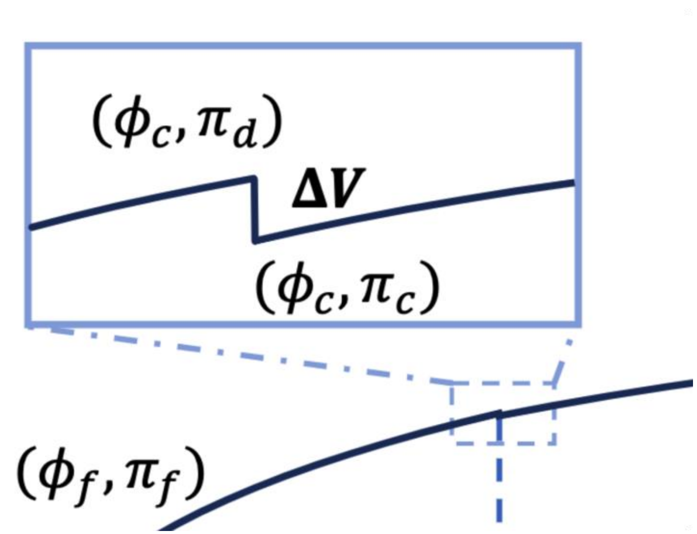
A sketch of the inflaton potential for the SR-SR transition, where two stages of slow-roll inflation are connected by an upward step.

What happens near the step?

[*CYF*, Ma, Sasaki, Wang, Zhou, JCAP 12 (2022) 034]

- At the upward step, $\phi = \phi_c$, the energy conservation leads to the relation:

$$\pi_d = -\sqrt{\pi_c^2 - 6\frac{\Delta V}{V}},$$



- So the height of the step can be governed by the parameter:

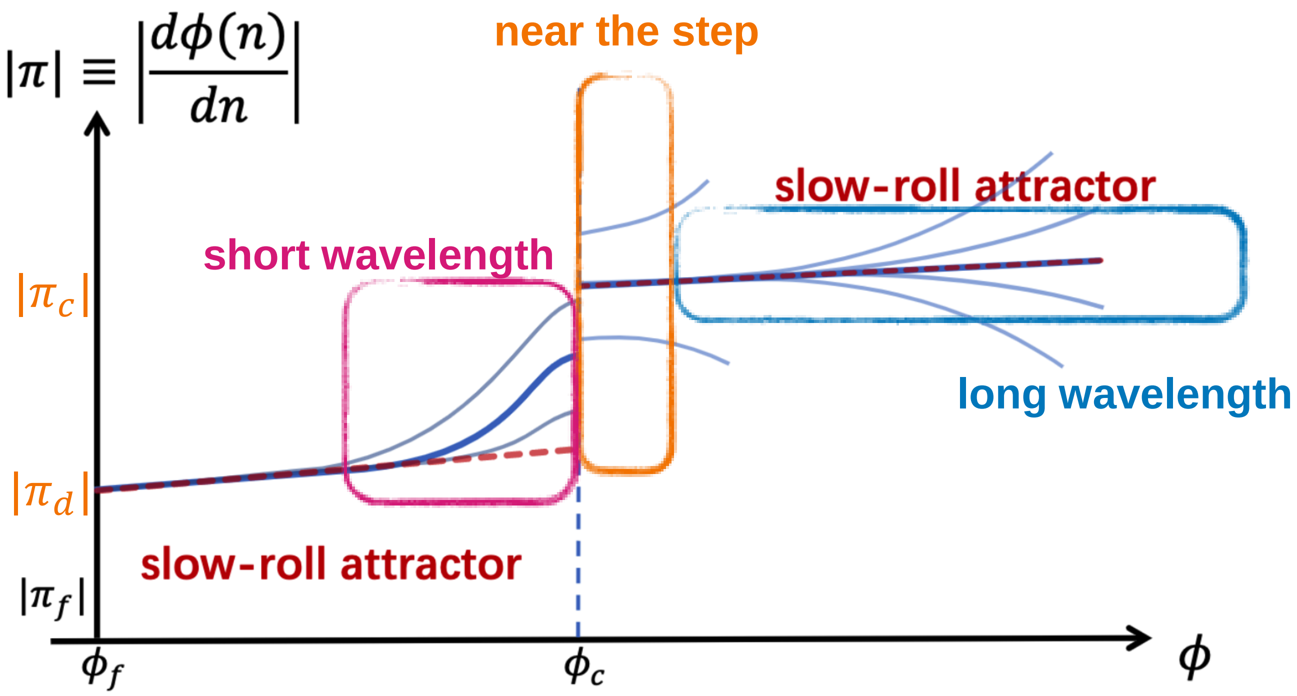
$$0 < \boxed{g \equiv \frac{\pi_d}{\pi_c}} \leq 1$$

For upward case

- Background dynamics of inflaton is simply depicted by a 2-order ODE piecewise:

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}n^2} + 3\frac{\mathrm{d}\phi}{\mathrm{d}n} + 3\sqrt{2\epsilon} + 3\eta(\phi - \phi_c) = 0$$

Off-attractor behavior in slow-roll phase



- **For long wavelength modes:**

  Off-attractor trajectories are converged into slow-roll attractor before reaching the step. The results are agree with slow-roll inflation.

δN formalism

- For near-step modes：

off-attractor trajectories
can not be ignored!!

$$N_{\rm total} = N_I + N_{II}$$
$$\simeq \frac{1}{3}\log\left[\frac{\pi_i}{\pi_i + 3(\phi_i - \phi_c)}\right] + \frac{1}{\eta_{II}}\log\left[-2\eta_{II}\pi_d - 6\sqrt{2\epsilon_{II}}\right] + \text{constant}\ .$$

USR standard result

function of initial conditions

$$\mathcal{R} = N(\phi_i + \delta\phi, \pi_i + \delta\pi) - N(\phi_i, \pi_i)$$
$$\simeq -\frac{1}{3}\log\left[1 + \frac{3\delta\phi}{\pi_c}\right] + \frac{1}{\eta_{II}}\log\left[1 + \frac{2\eta_{II}\delta\pi_d}{6\sqrt{2\epsilon_{II}} + 2\eta_{II}\pi_d}\right]$$

key result in our work

$$\delta\pi_d = \pi_d\left[\sqrt{1 + \frac{6}{g}\frac{\delta\phi}{\pi_d} + 9\left(\frac{\delta\phi}{\pi_d}\right)^2} - 1\right]$$

[*CYF*, Ma, Sasaki, Wang, Zhou, JCAP 12 (2022) 034]

Non-perturbative effects come from this step

[*CYF*, Ma, Sasaki, Wang, Zhou, PLB 834 (2022) 137461]

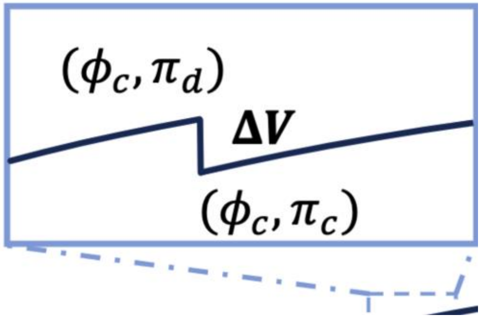
Assuming $g \ll 1$, curvature perturbation can be approximated as follows,

$$\mathcal{R} \simeq \frac{2}{|h|}\left(1-\sqrt{1-|h|\mathcal{R}_G}\right)$$

where we introduced the gaussian part of curvature perturbation:

$$\mathcal{R}_G \equiv (2\eta_{II}/gh)(\delta\phi/\pi_d)$$

$\mathcal{R}$ cannot be large than $2/|h|$



Once the large R is considered, it's highly non-Gaussian. However, the perturbative non-Gaussianities remain small in all orders when |h| <<1

Non-Gaussian coefficient at order $\mathcal{R}_G^{n+1}$ is $O(|h|^n)$

The prediction of a non-Gaussian tail

[*CYF*, Ma, Sasaki, Wang, Zhou, PLB 834 (2022) 137461]

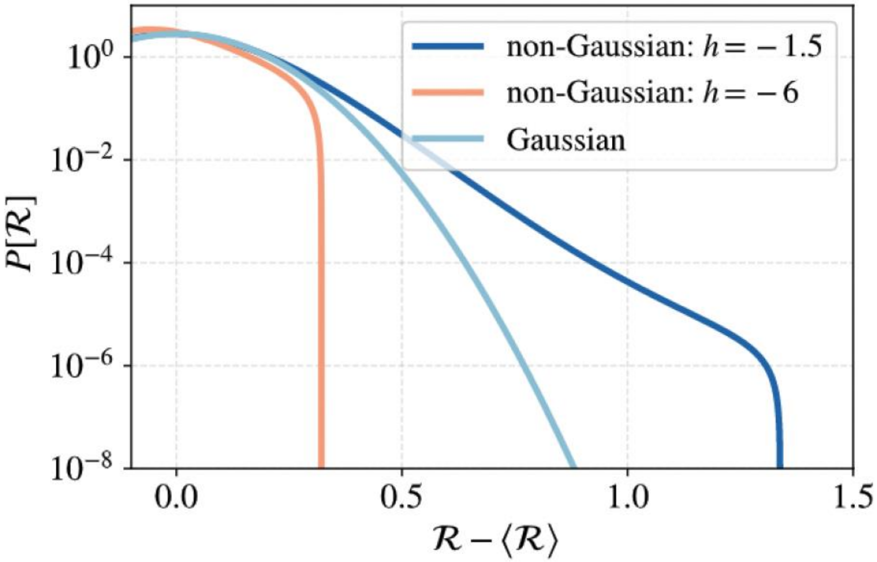
- Probability distribution function (PDF) of curvature perturbation:

$$P[\mathcal{R}]d\mathcal{R} = P[\mathcal{R}_G]d\mathcal{R}_{\rm G}$$

- Since R_G is a Gaussian random variable of which the variance σ_R^2 is given by $\int d\log k\ P_{\mathcal{R}_{\rm G}}(k)$

$$P[\mathcal{R}] = \frac{2-|h|\mathcal{R}}{\Omega}\exp\left[-\frac{\mathcal{R}^2(4-|h|\mathcal{R})^2}{32\sigma_{\mathcal{R}}^2}\right], \text{ for } \mathcal{R} \le 2/|h|$$

where Ω is a normalization coefficient



Comparison among non-Gaussian and Gaussian R with different h.

$$\sigma_{\mathcal{R}}^2 = 0.02$$

The tail of the distribution now becomes highly non-perturbative!

An application of a non-Gaussian tail: PBH formation

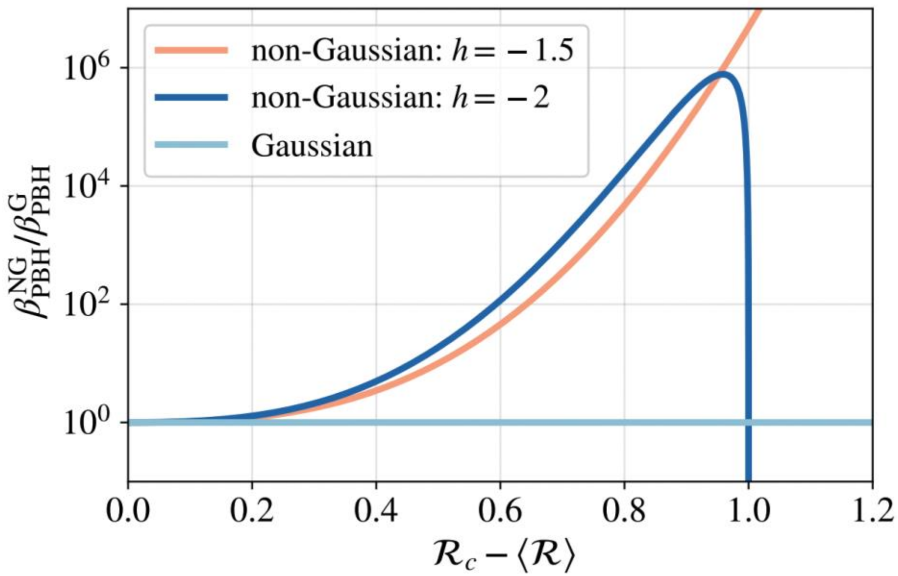
[*CYF*, Ma, Sasaki, Wang, Zhou, JCAP 12 (2022) 034]

- Integrate the PDF we got previously, we get the mass fraction:

$$\beta_{\rm PBH}^{\rm NG} = \frac{\sqrt{2\pi\sigma_{\mathcal{R}}^2}}{\Omega}\left[\mathrm{Erf}\left(\frac{1}{|h|\sqrt{2\sigma_{\mathcal{R}}^2}}\right) - \mathrm{Erf}\left(\frac{\mathcal{R}_c(4-|h|\mathcal{R}_c)}{4\sqrt{2\sigma_{\mathcal{R}}^2}}\right)\right]\Theta\left(\frac{2}{|h|} - \mathcal{R}_c\right) .$$

Here the normalization coefficient is same as before:

$$\Omega \equiv \sqrt{2\pi\sigma_{\mathcal{R}}^2}[1+\mathrm{Erf}(1/(|h|\sqrt{2\sigma_{\mathcal{R}}^2}))].$$



A non-Gaussian tail can either enhance the PBH formation by almost 6 orders or forbid any PBH formation!

For other rigor methods to get β_{PBH} see: Germani et al PRL 122 (2019) 141302; Figueroa et al PRL 127 (2021) 101302; Pi & Sasaki PRL 131 (2023) 011002; ……

Summary on NG tails

- The statistics of primordial fluctuations are measured to be highly Gaussian. Yet it is true, very few attentions were paid on the tail of its probability distribution.
- An upward step along the potential can yield a significantly non-perturbative effect on this tail even it is so tiny.
- Phenomenon 1: the perturbative nonlinearity parameters such as f_{NL}, g_{NL} remain small;
- Phenomenon 2: a small fluctuation can affect the background tendency due to off-attractor trajectories;
- Phenomenon 3: A non-Gaussian tail can either easily enhance the PBH mass fraction by several orders of magnitude, or make it absolutely impossible to form PBHs.

Probing new physics via the GW astronomy with SSR

LHC energy scale $\sim 13\mathrm{TeV}$ No signals!

Propagation speed of GWs: $z < 1$

$$-3 \cdot 10^{-15} \leq c_g/c - 1 \leq 7 \cdot 10^{-16}$$

Hidden New Physics in Gravity Theories

A wide class of MG theories can raise non-trivial GWs speed, for instance, in scalar-tensor theories:

$$\begin{aligned}\mathcal{L} =& G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - \phi^{\mu\nu}\phi_{\mu\nu}\right] \\ &+ G_5(\phi, X)G^{\mu\nu}\phi_{\mu\nu} - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi\phi^{\mu\nu}\phi_{\mu\nu} + 2\phi_{\mu\nu}\phi^{\nu\lambda}\phi^{\mu}_{\lambda}\right]\end{aligned}$$

Horndeski, IJTP (1974)

<u>Perturbative expansion:</u>

$$\begin{aligned}c_g^2 &= \frac{G_4 - X\left(\ddot{\phi}G_{5X} + G_{5\phi}\right)}{G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)} \\ &\simeq 1 - \frac{\phi^2}{M^2} + \dots\end{aligned}$$

Hidden New Physics in Gravity Theories

Another class of MG theory can raise non-trivial GWs speed, namely, when we consider 4-D Einstein-Gauss-Bonnet gravity:

$$S[g_{\mu\nu}] = \int d^D x \sqrt{-g} \left[\frac{M_{\rm P}^2}{2} R - \Lambda_0 + \frac{\alpha}{D-4} \mathcal{G} \right]$$

$$\Downarrow$$

Glavan, Lin, PRL (2019)

Scalar:

$$3M_{\rm P}^2 H^2 + 6\alpha H^4 = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

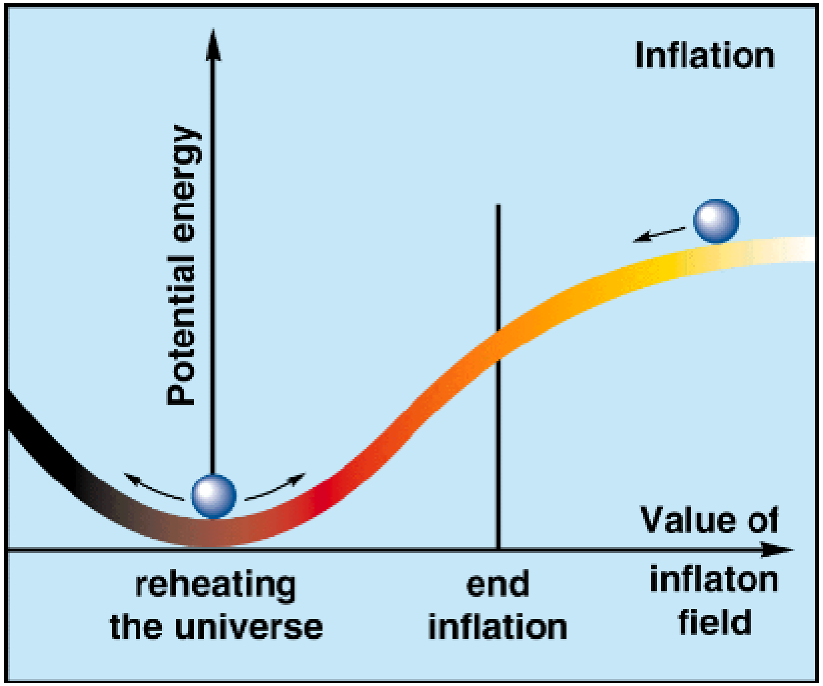
$$-M_{\rm P}^2 \Gamma \dot{H} = \frac{1}{2}\dot{\phi}^2$$

Tensor:

$$\ddot{\gamma}_{ij} + 3H\left(1 - \frac{8\alpha\epsilon H^2}{3M_{\rm P}^2\Gamma}\right)\dot{\gamma}_{ij} - c_{\rm s}^2 \frac{\partial^2 \gamma_{ij}}{a^2} = 0 \qquad c_{\rm s}^2 \equiv 1 - \frac{8\alpha\epsilon H^2}{M_{\rm P}^2\Gamma}$$

Oscillatory Sound Speed

A wide class of MG theories can lead to the non-trivial GWs speed, in particular, oscillation of scalar at reheating triggers on an oscillating sound speed of tensor modes.



$$h_k''(\tau) + 2\mathcal{H}h_k'(\tau) + c_g^2 k^2 h_k(\tau) = 0$$

$$c_g^2 = 1 - \frac{\alpha}{(1+\tau/\tau_0)^2}\cos^2(k_*\tau)$$

In the very late universe,

$$c_g^2 = 1$$

[*CYF*, Lin, Wang, **Yan**, PRL 126 (2021) 071303]

Parametric Resonance

For a general picture of GWs:

$$h_k''(\tau) + 2\mathcal{H}h_k'(\tau) + c_g^2 k^2 h_k(\tau) = 0$$

For sub-Hubble modes

$$k \gg a'/a$$

Friction term can be omitted

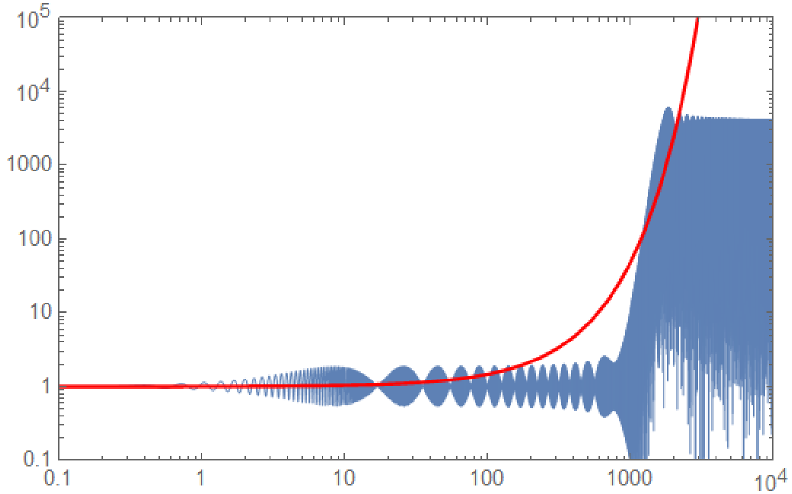
$$\frac{\partial^2 h_k}{\partial x^2} + [A - 2q\cos(2x)]h_k = 0$$

$$x = k_* \tau \qquad A = \frac{k^2}{k_*^2} - 2q \qquad q = \frac{\alpha k^2}{4k_*^2\left(1 + \frac{x}{x_0}\right)^2}$$

[*CYF*, Lin, Wang, **Yan**, PRL 126 (2021) 071303]

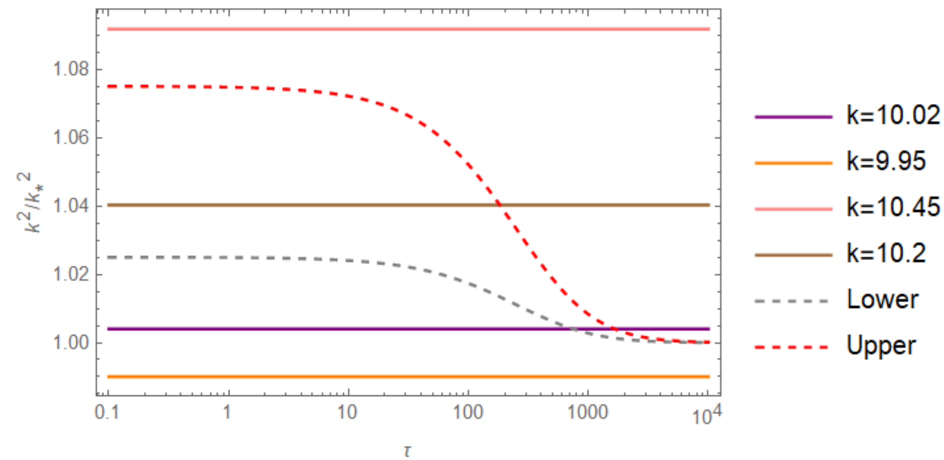
Parametric Resonance

Demo: EoM without Hubble friction term



The resonance started automatically, as well as its exit, due to the evolution of resonance band.

The evolution of resonance band

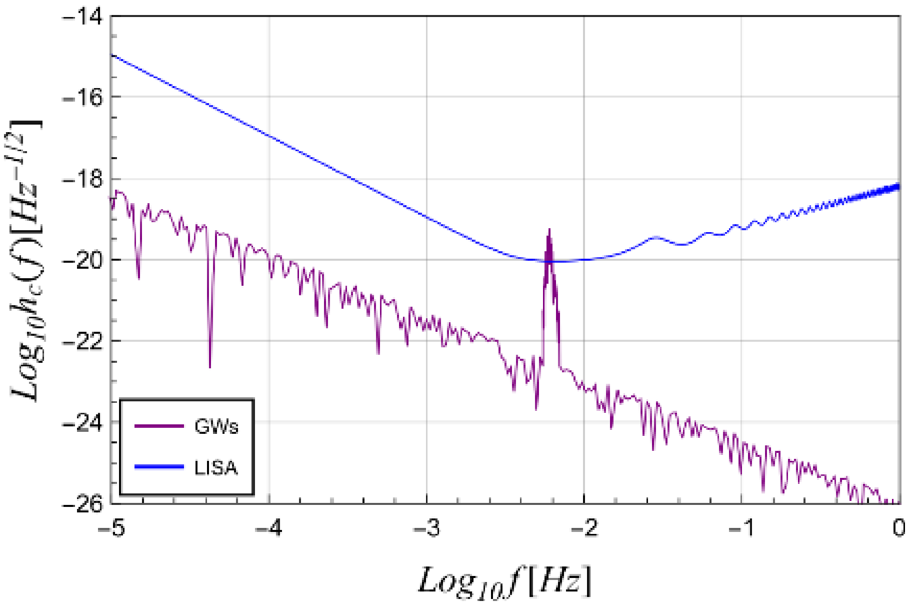


[*CYF*, Lin, Wang, **Yan**, PRL 126 (2021) 071303]

Significant Features

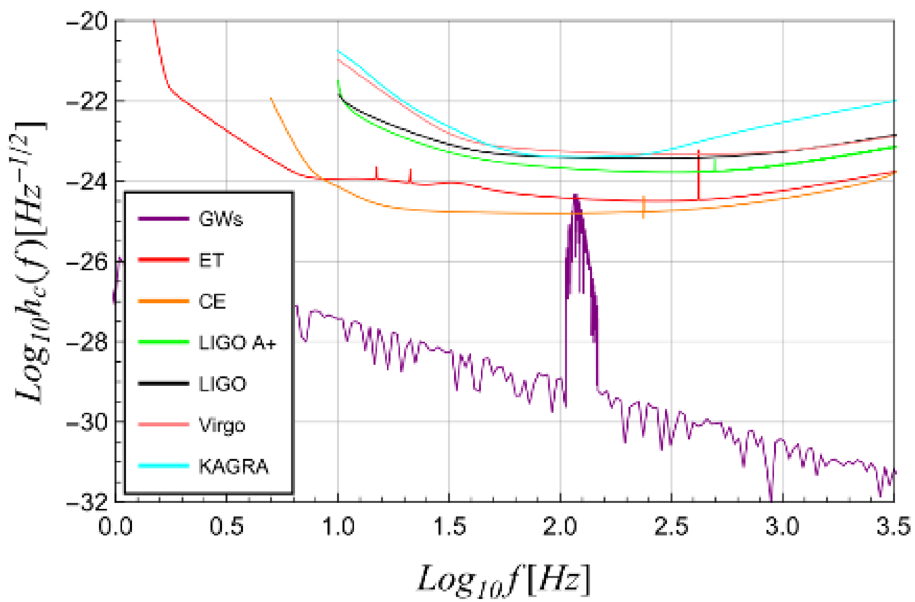
Energy scales:

$\sim \mathrm{TeV}$



Space-based detectors

$\sim 10^4\mathrm{TeV}$



Terrestrial detectors

Bonus:
Non-perturbative enhancement of density perturbations.

[*CYF*, Lin, Wang, **Yan**, PRL 126 (2021) 071303]