Quantum Gravity and Cosmology
2024
ShanghaiTech University,
Shanghai, China, July 1-5, 2024

Conundrum of higher derivative quantum gravity: Hořava model, renormalization group and asymptotic freedom

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The research was supported by the Russian Science Foundation grant No. 23-12-00051
https://rscf.ru/en/project/23-12-00051/

Plan

Horava gravity:

1) Renormalizable Hořava gravity: projectable models

2) Asymptotic freedom in the (2+1)-dimensional model

3) Beta functions and RG fixed points in (3+1)-dimensions

4) RG flows and AF in (3+1)-dimensions

5) Riddles of higher derivative gravity models:

complexity of operator dimensions;
renormalizability and AF of nonprojectable HG?;
tadpoles, IR divergences and modification of beta functions in quadratic gravity (Donoghue, Percacci et al);
no running of G and Λ *--- metamorphosis of the running scale*

Renormalization of Horava gravity

Saving unitarity in renormalizable QG

Einstein GR $$S_{EH} = \frac{M_P^2}{2} \int dt d^d x \; R$$

nonrenormalizable

$$\Rightarrow \frac{M_P^2}{2} \int dt d^d x \; (h_{ij} \Box h_{ij} + h^2 \Box h + \dots)$$

Higher derivative gravity

Stelle (1977)

$$\int (M_P^2 R + R_{\mu\nu} R^{\mu\nu} + R^2)$$

$$\Rightarrow \int (M_P^2 h_{ij} \Box h_{ij} + h_{ij} \Box^2 h_{ij} + \dots)$$

dominates at $k \gg M_P$

The theory is renormalizable and asymptotically free !

Fradkin, Tseytlin (1981)
Avramidy & A.B. (1985)

But has ghost poles $\Rightarrow$ no unitary interpretation

Horava (2009)

$$\underbrace{\int dt\, d^d x}_{\propto\, b^{-(z+d)}} \left(\dot h_{ij} \dot h_{ij} - h_{ij} (-\Delta)^z h_{ij} + \dots \right)$$

$$\mathbf{x} \mapsto b^{-1}\mathbf{x}\,, \qquad t \mapsto b^{-z} t \qquad \Rightarrow \qquad h_{ij} \mapsto b^{(d-z)/2} h_{ij}$$

Critical theory in $z = d$

LI is necessarily broken. We want to preserve as many symmetries, as possible

$$x^i \mapsto \tilde{x}^i(\mathbf{x}, t) \qquad \Rightarrow \qquad \gamma_{ij} \quad N^i, \qquad i = 1, \dots, d$$

$$t \mapsto \tilde{t}(t) \qquad \Rightarrow \qquad N$$

Foliation preserving diffeomorphisms $x^i \mapsto \tilde{x}^i(\mathbf{x}, t) \ , \quad t \mapsto \tilde{t}(t)$

ADM metric decomposition

$$ds^2 = N^2 dt^2 + \gamma_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \ , \quad i,j = 1, \ldots, d$$

space dimensionality

Anisotropic scaling transformations and scaling dimensions

$$x^i \to \lambda^{-1} x^i, \quad t \to \lambda^{-z} t, \quad N^i \to \lambda^{z-1} N^i, \quad \gamma_{ij} \to \gamma_{ij},$$

$$[x] = -1, \ [t] = -z, \ [N^i] = z-1, \ [\gamma_{ij}] = 0, \quad [K_{ij}] = z.$$

extrinsic curvature

$$K_{ij} = \frac{1}{2N}(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

Basic versions of Horava gravity: ``projectable" theory* $(N = \text{const} = 1)$ *vs ``non-projectable" theory $(N(x,t) \neq \text{const})$

kinetic term -- unitarity

Projectable Horava gravity action

$$S = \frac{1}{2G}\int dt\, d^d x \sqrt{\gamma} N \Big(\underbrace{K_{ij}K^{ij} - \lambda K^2}_{\text{kinetic term -- unitarity}} - \mathcal{V}(\gamma) \Big)$$

$$K_{ij} = \frac{1}{2N}(\dot{\gamma}_{ij} - \nabla_i N_j - \nabla_j N_i)$$

relevant

Potential term in (3+1) dimensions

$$\mathcal{V}(\gamma) = \underbrace{2\Lambda - \eta R + \mu_1 R^2 + \mu_2 R_{ij}R^{ij}}_{\text{relevant}} + \nu_1 R^3 + \nu_2 R R_{ij} R^{ij}$$

$$+\nu_3 R^i_j R^j_k R^k_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk} + \ldots$$

Extra structures in non-projectable theory

$$N(x,t) \neq \text{const} \quad \Rightarrow \quad a_i = \nabla_i \ln N, \ldots$$

Physical spectrum in d+1=4: TT-graviton and scalar

Unitarity domain (no ghosts) $$\frac{1-\lambda}{1-3\lambda} > 0$$

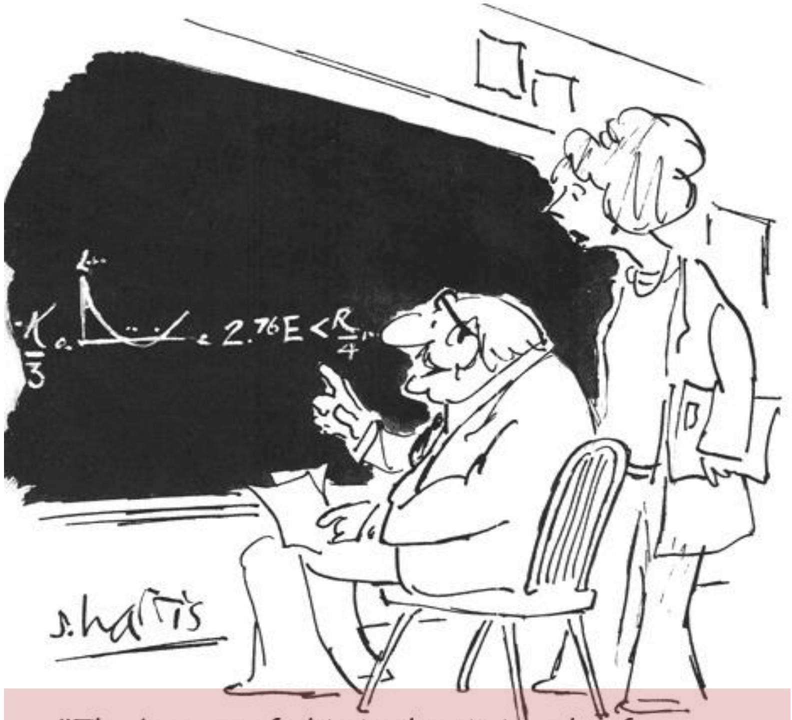
$$\omega_{TT}^2 = \eta k^2 + \mu_2 k^4 + \nu_5 k^6 \, ,$$
$$\omega_s^2 = \frac{1-\lambda}{1-3\lambda}\Big(- \eta k^2 + (8\mu_1 + 3\mu_2)k^4 + (8\nu_4 + 3\nu_5)k^6 \Big)$$

tachyon in IR (whichever sign of η)

No general relativistic IR limit!

Phenomenologically useless model in contrast to nonprojectable HG which has a healthy IR limit fitting GR [D. Blas, O. Pujolas, S.Sibiryakov, JHEP04(2011)018].

GR: active lapse, $\lambda = 1, \eta = 1, \{\nu, \mu\} = 0$



"The beauty of this is that it is only of theoretical importance, and there is no way it can be of any practical use whatsoever."

Single example of local, unitary, renormalizable and asymptotically free (consistent in UV limit) quantum gravity – projectable Hořava gravity

Consider UV limit dominated by marginal operators, disregard relevant cosmological and Einstein terms and check AF.

Long list of problems to be solved that have been solved:

Renormalizability -- projectable HG is renormalizable in any d (nonprojectable?)

D. Blas, M. Herrero-Valea, S. Sibiryakov C. & A.B., PRD 93, 064022 (2016), arXiv:1512.02250

Gauge invariance of counterterms: preserving BRST structure of renormalization

D. Blas, M. Herrero-Valea, S. Sibiryakov C. Steinwachs & A.B., JHEP07(2018)035, arXiv:1705.03480,

Asymptotic freedom of (2+1)-dimensional model

D. Blas, M. Herrero-Valea, S. Sibiryakov C. Steinwachs & A.B., PRL 119, 211301 (2017), arXiv:1706.06809

Beta-functions of (3+1)-dimensional model

A.Kurov, S.Sibiryakov & A.B., PRD 105 (2022) 4, 044009 arXiv: 2110.14688

RG flows of (3+1)-dimensional model and asymptotic freedom

A.Kurov, S.Sibiryakov & A.B. PRD 108 (2023) 12, L121503, arXiv:2310.07841

Asymptotic freedom in (2+1)-dimensions

$$S = \frac{1}{2G} \int dt\, d^2x\; N\sqrt{\gamma}\; \left(K_{ij}K^{ij} - \lambda K^2 + \mu R^2\right)$$

Off-shell extension is not unique:

$$\Gamma_{1-\mathrm{loop}} \to \Gamma_{1-\mathrm{loop}} + \int dt\, d^dx\, \Omega_{ij} \frac{\delta S}{\delta \gamma_{ij}}$$

Essential coupling constants:

$$\lambda, \quad \mathcal{G} \equiv \frac{G}{\sqrt{\mu}}$$

background covariant gauge-fixing term

σ, ξ – *free parameters*

$$S_{\mathrm{gf}} = \frac{\sigma}{2G} \int dt\, d^2x\, \sqrt{\gamma}\, F_i\, \mathcal{O}^{ij} F_i$$

$$F_i = \partial_t n_i + \frac{1}{2\sigma} \mathcal{O}^{-1}_{ij} (\nabla^k h^j_k - \lambda \nabla^j h)$$

$$\mathcal{O}^{ij} = -[\gamma_{ij} \Delta + \xi \nabla_i \nabla_j]^{-1}$$

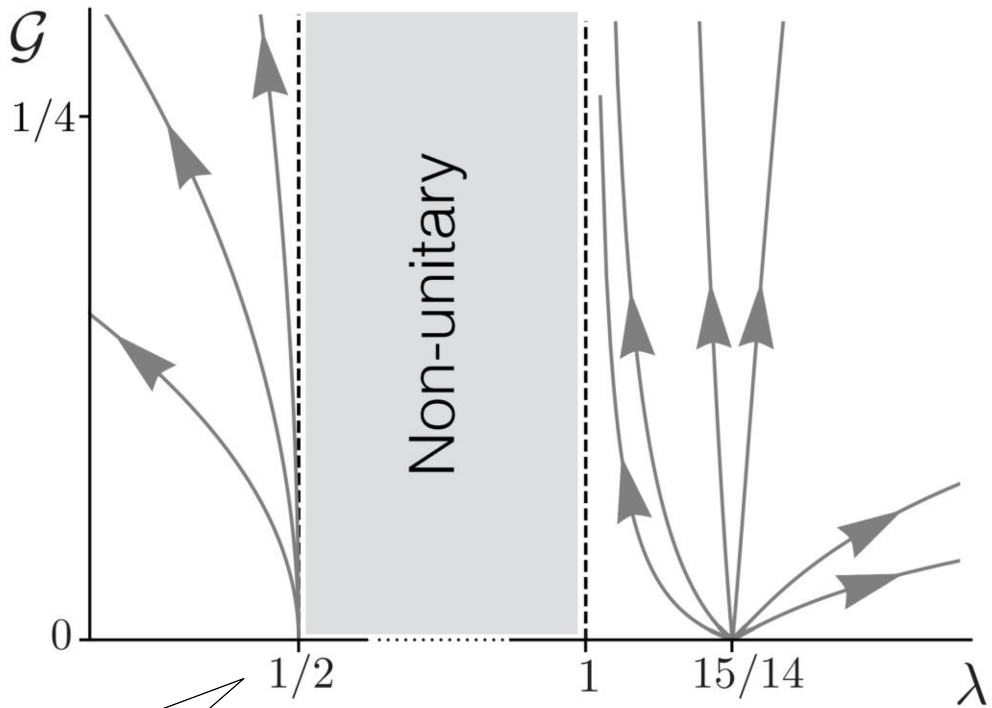
↓

Mathematica package xAct

$$\beta_\lambda = \frac{15 - 14\lambda}{64\pi} \sqrt{\frac{1 - 2\lambda}{1 - \lambda}}\, \mathcal{G}$$

$$\beta_{\mathcal{G}} = -\frac{(16 - 33\lambda + 18\lambda^2)}{64\pi(1 - \lambda)^2} \sqrt{\frac{1 - \lambda}{1 - 2\lambda}}\, \mathcal{G}^2$$

Renormalization flows:



strongly coupled fixed point

$$\mathcal{G} \to \tilde{\mathcal{G}} = \frac{G}{\sqrt{1-2\lambda}}$$

$$\beta_{\tilde{\mathcal{G}}} = -\frac{(1-2\lambda)^2}{64\pi(1-\lambda)^{3/2}}\,\tilde{\mathcal{G}}^2$$

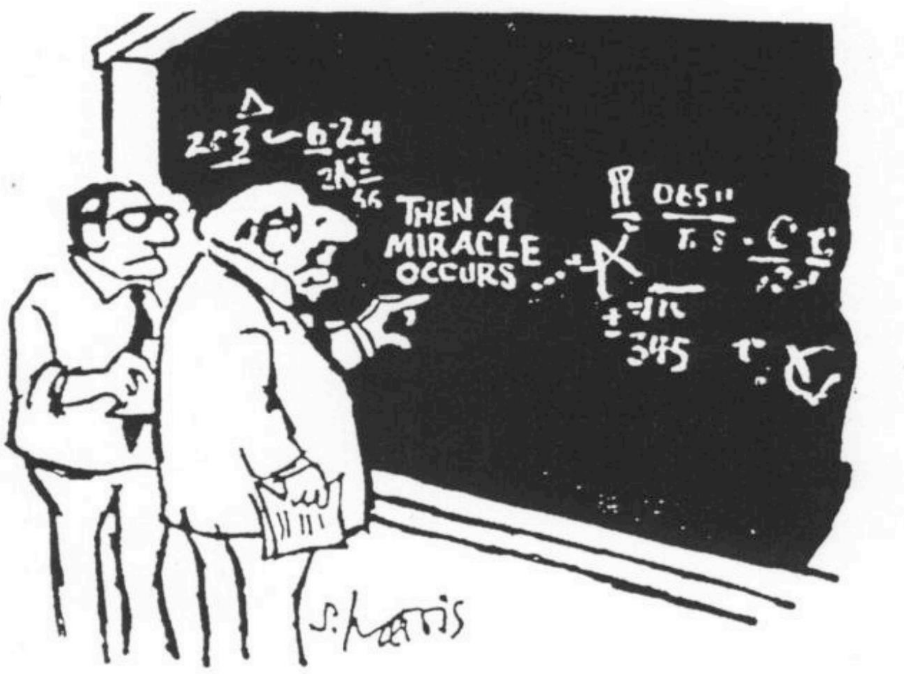
AF UV fixed point

(3+1)-dimensional Horava gravity

$$S = \frac{1}{2G}\int dt\, d^d x \sqrt{\gamma}\Big(K_{ij}K^{ij} - \lambda K^2 - \mathcal{V}(\gamma)\Big)$$

$$\mathcal{V}(\gamma) = \nu_1 R^3 + \nu_2 R R_{ij} R^{ij} + \nu_3 R^i_j R^j_k R^k_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk}$$

Six essential coupling constants $\mathcal{G},\ \lambda$ and $\chi = (u_s, v_1, v_2, v_3)$



I think you should be a little more specific, here in Step 2

$$\beta_G, \quad \beta_\lambda$$

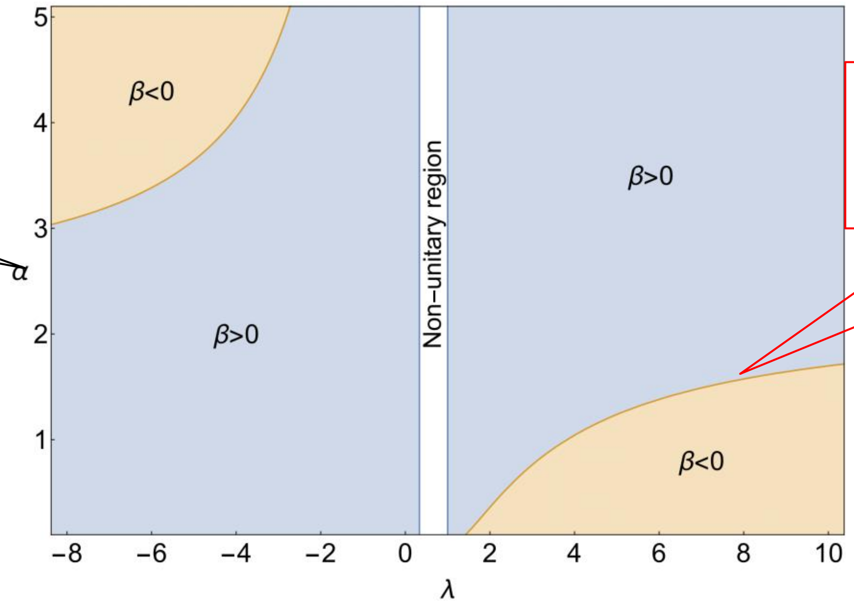
obtained by usual Feynman diagrams

M. Herrero-Valea, S. Sibiryakov & A.B., PRD100 (2019) 026012

⇧

For β_χ *this is impossible !*

$$u_s^2 =\equiv \frac{1}{\nu_5}\frac{(1-\lambda)(8\nu_4+3\nu_5)}{1-3\lambda}$$



$$\lambda(u_s) = \frac{9+7u_s-2u_s^2+2\sqrt{10(u_s^2+u_s^3)}}{3(3+u_s-2u_s^2)}$$

The curves correspond to potential location of fixed points of the full RG flow. The region $\lambda \in [1/3,1]$ is excluded by the requirement of unitarity.

Background field method + heat kernel method + dimensional reduction

One-loop effective action

$$\varGamma_{\rm one-loop}=\frac12{\rm Tr}_4\ln\hat F(\nabla)=-\frac12\int_0^\infty\frac{ds}{s}\,{\rm Tr}_4\,e^{-s\hat F(\nabla)}$$

Action Hessian $\hat F(\nabla)=F^A_B(\nabla)$ *acting in the space of fields* $\varphi=\varphi^A(x)$

Heat kernel (Schwinger-DeWitt) expansion for minimal second order operators

$$\hat F(\nabla)=\Box+\hat P-\frac{\hat 1}{6}R,\qquad \Box=g^{\mu\nu}\nabla_\mu\nabla_\nu$$

$$e^{-s\hat F(\nabla)}\delta(x,y)=\frac{\mathcal D^{1/2}(x,y)}{(4\pi s)^{d/2}}\,g^{1/2}(y)\,e^{-\frac{\sigma(x,y)}{2s}}\sum_{n=0}^\infty s^n\,\hat a_n(x,y)$$

Schwinger-DeWitt (Gilkey-Seely) coefficients

$$\hat a_0\Big|_{y=x}=\hat 1,\quad \hat a_1\Big|_{y=x}=\hat P,$$

$$\hat a_2\Big|_{y=x}=\frac{1}{180}\,(R^2_{\alpha\beta\gamma\delta}-R^2_{\mu\nu}+\Box R)\,\hat 1+\frac{1}{12}\hat{\boldsymbol R}^2_{\mu\nu}+\frac12\hat P^2+\frac16\Box\hat P,\dots$$

One-loop divergences $$\varGamma^{\rm div}_{\rm one-loop}=-\frac{1}{32\pi^2\varepsilon}\int dx\,g^{1/2}{\rm tr}\,\hat a_2(x,x),\qquad \varepsilon=2-\frac{d}{2}\to 0$$

However, in Horava gravity operators are nonminimal

Set of quantum fields $\varphi(x) = h_{ij}(x), n^i(x) \;+\; FP\;\; ghosts$

Structure of operators on a static 3-metric background with generic 3-metric $\gamma_{ij}(\mathbf{x})$

$$\hat{F}(\nabla) = -\hat{1}\,\partial_\tau^2 + \hat{\mathbb{F}}(\nabla) \qquad\qquad \hat{\mathbb{F}} = \left\{\mathbb{F}_{ij}{}^{kl}, \mathbb{F}^k_i\right\} \sim \nabla^6 + R\nabla^4 + R^2\nabla^2 + R^3$$

space parts of metric and vector (shifts and ghosts) operators:

Example – for the ghost operator in σ, ξ -family of gauges:

$$\mathbb{F}^i{}_j(\nabla) = -\frac{1}{2\sigma}\delta^i_j\Delta^3 - \frac{1}{2\sigma}\Delta^2\nabla_j\nabla^i - \frac{\xi}{2\sigma}\nabla^i\Delta\nabla^k\nabla_j\nabla_k$$
$$-\frac{\xi}{2\sigma}\nabla^i\Delta\nabla_j\Delta + \frac{\lambda}{\sigma}\Delta^2\nabla^i\nabla_j + \frac{\lambda\xi}{\sigma}\nabla^i\Delta^2\nabla_j, \qquad \Delta = \gamma^{ij}\nabla_i\nabla_j$$

Extension to ***non-minimal and higher-derivative operators*** *-- the method of universal functional traces (I. Jack and H. Osborn (1984), G.A.Vilkovisky & A.B., Phys. Rept. 119 (1985) 1)*

Idea:

$$\mathrm{Tr}\,\ln\left(\Box^N + P(\nabla)\right) = N\,\mathrm{Tr}\,\ln\Box + \mathrm{Tr}\,\ln\left(1 + P(\nabla)\frac{1}{\Box^N}\right)$$

$$= N\,\mathrm{Tr}\,\ln\Box + \mathrm{Tr}\,P(\nabla)\frac{1}{\Box^N} + \cdots$$

⇓

$$\varGamma^{\mathrm{div}} = \sum_{m,n}\int d^4x\,\mathcal{R}_n^{\mu_1\ldots\mu_m}\nabla_{\mu_1}\ldots\nabla_{\mu_m}\frac{\hat{1}}{\Box^n}\delta(x,y)\,\Big|_{y=x}^{\mathrm{div}}$$

↑ ***universal functional traces***

$$\nabla\ldots\nabla\frac{\hat{1}}{\Box^n}\delta(x,y)\,\Big|_{y=x}^{\mathrm{div}} = \frac{(-1)^n}{\varGamma(n)}\nabla\ldots\nabla\int_0^\infty ds\,s^{n-1}\,e^{s\Box}\,\hat{\delta}(x,y)\,\Big|_{y=x}^{\mathrm{div}}$$

↑ *Schwinger-DeWitt expansion*

Dimensional reduction *method on a static background with generic 3-metric*

$$\mathrm{Tr}_4 \ln(-\partial_\tau^2 + \mathbb{F}) = -\int_0^\infty \frac{ds}{s}\,\mathrm{Tr}_4 e^{-s(-\partial_\tau^2+\mathbb{F})} = -\int d\tau\;\mathrm{Tr}_3\sqrt{\mathbb{F}}$$

square root

How to proceed with the square root of the 6-th order differential operator?

$$\mathbb{F} = \sum_{a=0}^{6} \mathcal{R}_{(a)} \sum_{6\geq 2k\geq a} \alpha_{a,k} \nabla_1 ... \nabla_{2k-a} (-\Delta)^{3-k}, \qquad \mathcal{R}_{(a)} = O\Big(\frac{1}{l^a}\Big)$$

⇓

Pseudodifferential operator – infinite series in curvature invariants $\mathcal{R}_{(a)}$

$$\sqrt{\mathbb{F}} = \sum_{a=0}^{\infty} \mathcal{R}_{(a)} \sum_{k\geq a/2}^{K_a} \tilde{\alpha}_{a,k} \nabla_1 ... \nabla_{2k-a} \frac{1}{(-\Delta)^{k-3/2}}$$

⇓

3D universal Functional traces

$$\mathrm{Tr}_3\sqrt{\mathbb{F}}\,\Big|^{\mathrm{div}} = \sum_{a=2}^{6} \sum_k \tilde{\alpha}_{a,k} \int d^3x\, \mathcal{R}_{(a)}(\mathbf{x}) \nabla_1 ... \nabla_{2k-a} \frac{1}{(-\Delta)^{k-3/2}} \delta(\mathbf{x},\mathbf{x}')\,\Big|^{\mathrm{div}}_{\mathbf{x}=\mathbf{x}'}$$

Results for beta functions of (3+1)-dimensional Horava gravity

$$S = \frac{1}{2G}\int dt\, d^dx \sqrt{\gamma} N \Big(K_{ij}K^{ij} - \lambda K^2 - \mathcal{V}(\gamma) \Big)$$

$$\mathcal{V}(\gamma) = \nu_1 R^3 + \nu_2 R R_{ij} R^{ij} + \nu_3 R^i_j R^j_k R^k_i + \nu_4 \nabla_i R \nabla^i R + \nu_5 \nabla_i R_{jk} \nabla^i R^{jk}$$

Six essential coupling constants $\mathcal{G}$, λ and $\chi = (u_s, v_1, v_2, v_3)$

$$\mathcal{G} = \frac{G}{\sqrt{\nu_5}}, \quad \lambda, \quad u_s = \sqrt{\frac{(1-\lambda)(8\nu_4 + 3\nu_5)}{(1-3\lambda)\nu_5}}, \quad v_a = \frac{\nu_a}{\nu_5}, \quad a = 1,2,3$$

$$\beta_{\mathcal{G}} = \frac{\mathcal{G}^2}{26880\pi^2(1-\lambda)^2(1-3\lambda)^2(1+u_s)^3 u_s^3} \sum_{n=0}^{7} u_s^n \mathcal{P}_n^{\mathcal{G}}[l, v_1, v_2, v_3]$$

$$\beta_{\lambda} = \frac{\mathcal{G}}{120\pi^2(1-\lambda)(1+u_s)u_s}\Big[27(1-\lambda)^2 + 3u_s(11-3\lambda)(1-\lambda) - 2u_s^2(1-3\lambda)^2\Big]$$

$$\beta_{\chi} = \frac{A_\chi \mathcal{G}}{26880\pi^2(1-\lambda)^3(1-3\lambda)^3(1+u_s)^3 u_s^5} \sum_{n=0}^{9} u_s^n \mathcal{P}_n^{\chi}[l, v_1, v_2, v_3]$$

$$A_{u_s} = u_s(1-\lambda), \quad A_{v_1} = 1, \quad A_{v_2} = A_{v_3} = 2$$

$\mathcal{P}_n^{\chi}[l, v_1, v_2, v_3,]$ *are polynomials in λ and v_a,*

Use of Mathematica package xAct

Example (one of the longest ones):

$$
\begin{aligned}
\mathcal{P}_5^{v_1} = & -2(1-\lambda)^2(1-3\lambda)\Big\{168v_2^3(51\lambda^3-149\lambda^2+125\lambda-27)-108v_3^3(9\lambda^3+9\lambda^2\\
& -25\lambda+7)-4v_2^2(1-\lambda)\Big[18v_3(117\lambda^2-366\lambda+109)-284\lambda^2-7265\lambda+5425\Big]\\
& +40320v_1^2(1-\lambda)^2(\lambda+1)-9v_3^2(3467\lambda^3-8839\lambda^2+6237\lambda-865)\\
& +v_1\Big[64v_2^2(1-\lambda)^2(1717\lambda-581)-16v_2(1-\lambda)\Big(3v_3(2741\lambda^2-3690\lambda+949)\\
& +25940\lambda^2-40662\lambda+12022\Big)+27v_3^2(961\lambda^3-2395\lambda^2+1835\lambda-401)\\
& +6v_3(52267\lambda^3-148963\lambda^2+129881\lambda-33185)-288353\lambda^3+542255\lambda^2\\
& -333355\lambda+83485\Big]-2v_2\Big[162v_3^2(3\lambda^3+35\lambda^2-51\lambda+13)+24v_3(1265\lambda^3\\
& -2191\lambda^2+691\lambda+235)+30971\lambda^3-40323\lambda^2+13167\lambda-4451\Big]-12v_3(6551\lambda^3\\
& -11593\lambda^2+6124\lambda-1112)+109519\lambda^3-252396\lambda^2+177357\lambda-34396\Big\}
\end{aligned}
$$

Check of the results: independence of essential beta functions on the choice of gauge (σ, ξ - family of gauge conditions) and spectral sum method in dimensional and zeta-functional regularization.

RG analysis

$\mathcal{G} \to 0$ *asymptotic freedom*

$\mathcal{G} \to \infty$ *Landau pole*

Fixed points equations:

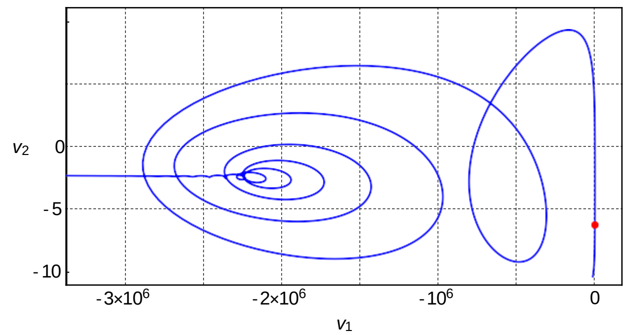
$$\beta_\lambda/\mathcal{G} = 0 \ ,$$

$$\beta_\chi/\mathcal{G} = 0 \ , \qquad \chi = u_s, v_1, v_2, v_3$$

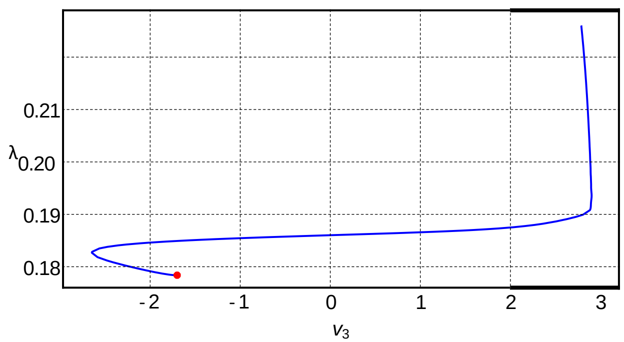
i

λ	u_s	v_1	v_2	v_3	$\beta_{\mathcal{G}}/\mathcal{G}^2$	AF?	UV attractive along λ?
0.1787	60.57	-928.4	-6.206	-1.711	-0.1416	yes	no
0.2773	390.6	-19.88	-12.45	2.341	-0.2180	yes	no
0.3288	54533	3.798×10^8	-48.66	4.736	-0.8484	yes	no
0.3289	57317	-4.125×10^8	-49.17	4.734	-0.8784	yes	no

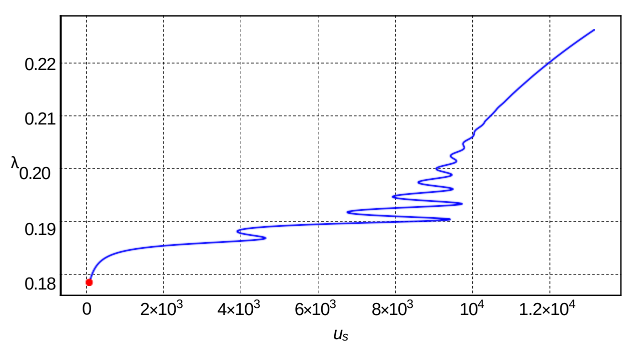
Fixed points at finite λ. All of them are asymptotically free but they RG run into Landau poles.



v_1 **vs** v_2



λ **vs** v_2



λ **vs** u_s

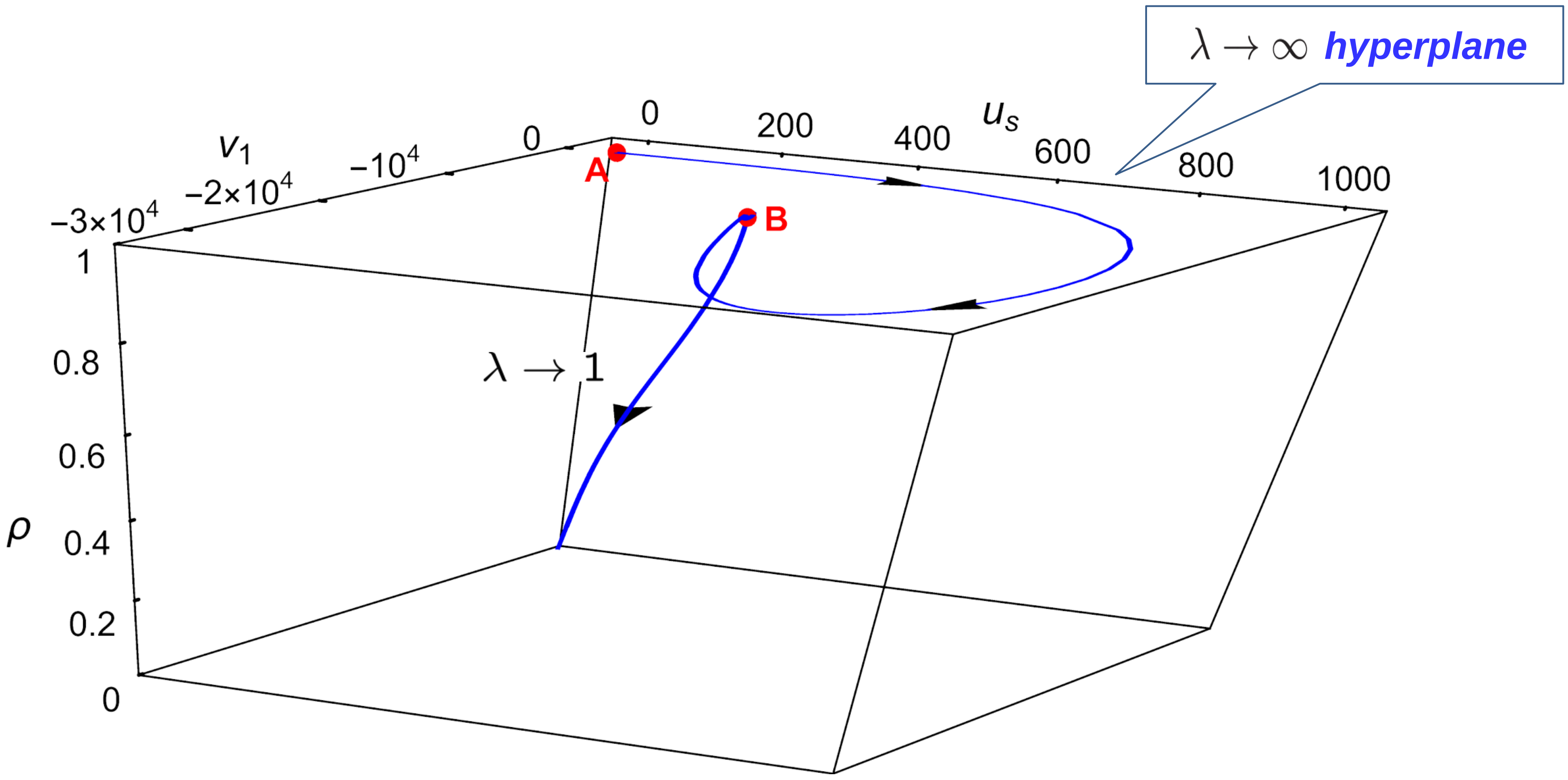
Special limit: $\lambda \to \infty$ *(cosmology implication, A.E. Gumrukcuoglu, S. Mukohyama, 1104.2087)*

Fixed points at $\lambda \to \infty$ *:*

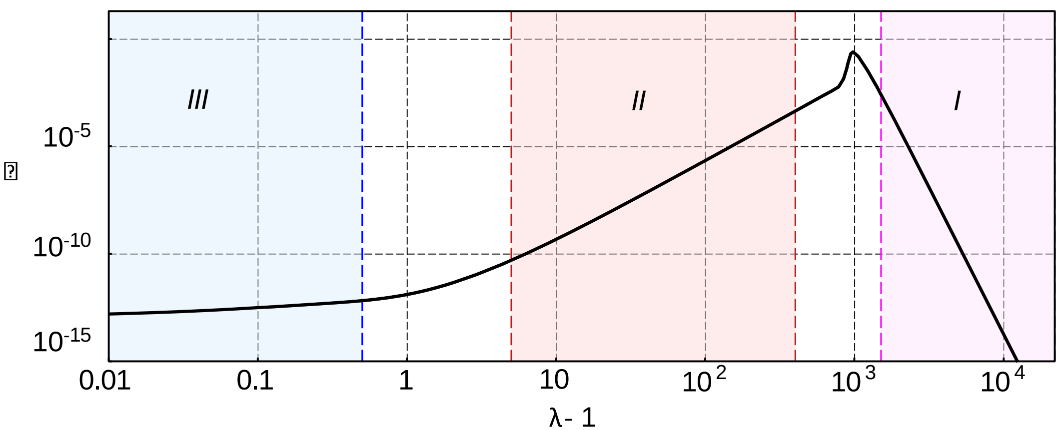
	u_s	v_1	v_2	v_3	$\beta_{\mathcal{G}}/\mathcal{G}^2$	asymptotically free?	UV attractive along λ?
	0.01950	0.4994	-2.498	2.999	-0.2004	yes	no
	0.04180	-0.01237	-0.4204	1.321	-1.144	yes	no
	0.05530	-0.2266	0.4136	0.7177	-1.079	yes	no
	12.28	-215.1	-6.007	-2.210	-0.1267	yes	yes
A	21.60	-17.22	-11.43	1.855	-0.1936	yes	yes
B	440.4	-13566	-2.467	2.967	0.05822	no	yes
	571.9	-9.401	13.50	-18.25	-0.07454	yes	yes
	950.6	-61.35	11.86	3.064	0.4237	no	yes

Special fixed points:* A *is AF,* B *is not AF

Fixed point B as a transient of RG flow from AF UV point A to IR limit with $\lambda \to 1$



Behavior of G on the flow: $G \to 0 \;\; in \;\; UV \;\; and \;\; IR$



Behavior of **G** *as a function of ($\lambda - 1$) along an RG trajectory connecting the point A to $\lambda \to 1$*

In regions I, II and III the dependence is well described by the power law $G \sim (\lambda - 1)^k$

with $k_I = -13.69$, $k_{II} = 3.84$, $k_{III} \approx 0.37$.

Riddles of higher derivative gravity models

1. Complexity of operator dimensions – eigenvalues of stability matrices.

№	θ^1	θ^2	θ^3	θ^4	θ^5
1	1.154	-1.235	0.9825	$-0.2734 \pm 0.2828\, i$	
2	0.5302	12.35	-0.3207	$-71.95 \pm 5.134\, i$	
3	0.3970	10.77	0.3012	$-64.72 \pm 0.6149\, i$	
4	-0.01334	-0.3436	-0.09353	$0.2200 \pm 0.1806\, i$	
5 (A)	-0.01414	-0.06998	0.2565	0.3204	0.06569
6 (B)	-0.01515	0.6032	0.3079	$0.0924 \pm 0.2890\, i$	
7	-0.01516	-1.722	0.1328	$-0.3324 \pm 0.3289\, i$	
8	-0.01517	-0.3657	1.326	$0.4340 \pm 0.4849\, i$	

Violation of LI or lack of gauge invariant operators ?

2. Renormalizability and AF of nonprojectable HG?

It has healthy IR limit fitting GR [D. Blas, O. Pujolas, S.Sibiryakov, JHEP04(2011)018]. There are indications that it is renormalizable.

J. Bellorın, C. Borquez, B. Droguett,
Phys. Rev D 106, 044055 (2022),
2207.08938 **arXiv:2405.04708**

3. Tadpoles, IR divergences and modification of beta functions in quadratic gravity (Donoghue, Percacchi et al)

The problem of running Λ *and* **G**

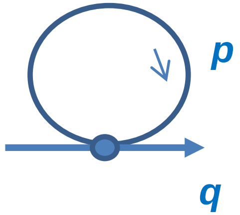
Running coupling constant = ***nonlocal form factor*** *in effective action*

$$g \Rightarrow g(\mu), \quad \mu\frac{d}{d\mu}g(\mu) = \beta\big(g(\mu)\big),$$

$$-\frac{1}{4g^2(\mu)}\int d^4x\,F_{\mu\nu}^2 \Rightarrow -\int d^4x\,F_{\mu\nu}(x)\frac{1}{4g^2\big(\sqrt{-\Box}\big)}F^{\mu\nu}(x) = -\int d^4p\,\hat{F}_{\mu\nu}(-p)\frac{1}{4g^2(p)}\hat{F}^{\mu\nu}(p)$$

running scale

Tadpole (total derivative) nature problem for running Λ *and* **G**



$$-\frac{\Lambda}{16\pi G}\int d^4x\,g^{1/2} \Rightarrow -\frac{1}{16\pi}\int d^4x\,g^{1/2}\frac{\Lambda(\Box)}{G(\Box)}1 = -\frac{1}{16\pi}\int d^4x\,g^{1/2}\frac{\Lambda(0)}{G(0)}$$

$$-\frac{1}{16\pi G}\int d^4x\,g^{1/2}R \Rightarrow -\frac{1}{16\pi}\int d^4x\,g^{1/2}\frac{1}{G(\Box)}R = -\frac{1}{16\pi}\int d^4x\,g^{1/2}\frac{1}{G(0)}R$$

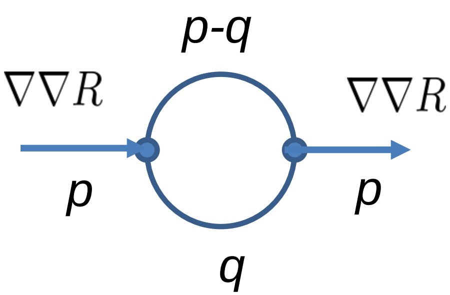
no running

Quadratic gravity action and propagator

$$S \sim \int d^4x \sqrt{g}(-M^2 R + R^2) \sim \int d^4q\, h(q)(M^2q^2 + q^4)h(-q) + \ldots \Rightarrow \ \textit{propagator} \ \sim \frac{1}{M^2q^2 + q^4}$$

UV finite and IR divergent at M=0

$$\int \frac{d^4q}{(q^4 + M^2q^2)((p-q)^4 + M^2(p-q)^2)} \sim \frac{1}{p^4} \log \frac{p^2}{M^2}$$



⇓

$$\int d^4p\, \nabla\nabla R(p) \frac{1}{p^4} \log \frac{p^2}{M^2} \nabla\nabla R(-p) \sim \int d^4x\, R \log\left(\frac{-\Box}{M^2}\right) R$$

running constant of R^2-- action

⇓

Modification of beta-functions by UV finite terms!

D.Buccio,J.Donoghue, G.Menezes, R.Percacci, arXiv:2403.02397

Renormalization group and metamorphosis of the running scale

$$S[g_{\mu\nu}] = \sum_{m,N} \Lambda_N^{(m)} \int d^dx \sqrt{g}\, \Re_N^{(d+m)} \qquad \Re_N^{(m)} = \underbrace{\nabla...\nabla}_{m-2N} \overbrace{\Re...\Re}^{N}, , \qquad \dim \Re_N^{(m)} = m$$

curvature invariants of dimension m

Covariant perturbation theory $g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu}, \quad \tilde{R}^{\mu}{}_{\nu\alpha\beta} = 0$

G.A.Vilkovisky & A.B. (1986-1990)

$$I_M^{(m)}(h) \propto \underbrace{\tilde{\nabla}...\tilde{\nabla}}_{m} \overbrace{h(x)...h(x)}^{M} \equiv I_M^{(m)}(h_1, h_2, ...h_M)\Big|_{x_1=x_2=...x_M=x}$$

RG scale μ $\qquad \Lambda = \mu^{d-\dim I}\lambda(\mu)$ $\qquad$ dimensionless couplings $\{\lambda(\mu)\}$

Effective action

$$\Gamma[g_{\mu\nu}] = \sum_I \mu^{d-\dim I} \int d^dx \sqrt{\tilde{g}}\, \gamma_I\Big(\{\lambda(\mu)\}, \frac{\tilde{\nabla}_1}{\mu}, ...\frac{\tilde{\nabla}_M}{\mu}\Big) I(h_1, ...h_M)\Big|_{\{x\}=x}$$

nonlocal form factors

Employing RG:

$$\mu\frac{d\Gamma}{d\mu}=0 \;\rightarrow\; \mu\frac{d}{d\mu}\lambda(\mu)=\beta(\mu)\Big(\{\lambda(\mu)\}\Big)$$

Choice of scale

$\mu \rightarrow ?$

$$\tilde{D}\equiv\Big(-\sum_{N=1}^{\infty}\tilde{\Box}_N\Big)^{1/2},\qquad \tilde{\Box}_N\equiv\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\nabla}_\nu,$$

$$\tilde{D}I_N=\tilde{D}_N I_N,\qquad \tilde{D}_N\equiv\Big(-\sum_{M=1}^{N}\tilde{\Box}_M\Big)^{1/2}$$

Replacement $\mu\rightarrow\tilde{D}$ **and UV limit** $\tilde{\nabla}\rightarrow\infty,\quad \frac{\tilde{\nabla}}{\tilde{D}_N}\rightarrow O(1)$

$$\mu^{4-\dim I}\,\gamma_I\Big(\lambda(\mu)\,\Big|\,\frac{\tilde{\nabla}_1}{\mu},\ldots\frac{\tilde{\nabla}_N}{\mu}\Big)\Big|_{\mu\rightarrow\tilde{D}_N}\;\Longrightarrow$$

$$(\tilde{D}_N)^{4-\dim I}\,\gamma_I\Big(\lambda(\tilde{D}_N)\,\Big|\,O(1)\Big)\equiv(\tilde{D}_N)^{4-\dim I}\lambda_I\Big(\tilde{D}_N\Big)$$

Covariantization:

$$h_{\mu\nu} = -\frac{2}{\Box} R_{\mu\nu} + O[\Re^2], \qquad \tilde{\nabla}_\mu = \nabla_\mu + O[\Re],$$

$$I_N^{(m)}(h_1, h_2, \dots h_N) \to \frac{1}{\Box_1 \dots \Box_N} \underbrace{\nabla \dots \nabla}_{m} \Re_1 \dots \Re_N + O[\Re^{N+1}]$$

⇩

RG form factor

$$\Gamma[g_{\mu\nu}] \to \int d^4x \sqrt{g} \sum_{m,N \geq 1}^{\infty} \frac{(D_N)^{4-m}}{\Box_1 \dots \Box_N} \lambda_N^{(m)}(D_N) \underbrace{\nabla \dots \nabla}_{m} \Re_1 \dots \Re_N \Big|_{\{x\}=x}$$

dimensionful factors

Linear in curvature terms:

Does not contribute – total derivative:
no running

$$D_1 = \sqrt{-\Box}$$

$$\int d^4x \sqrt{g} \sum_m^{\infty} \frac{(D_1)^{4-2m}}{\Box} \lambda_1^{(m)}(D_1) \underbrace{\nabla \dots \nabla}_{2m} \Re(x) \sim \int d^4x \sqrt{g}\, \Box \left(\sum_m^{\infty} c_m \lambda_1^{(m)}(\sqrt{-\Box}) R(x) \right)$$

Quadratic in curvature terms:

$$D_2 = \sqrt{-\Box_1 - \Box_2},$$

Integration by parts

$$\int d^4x \sqrt{g}\, F(\Box_1, \Box_2) \Re_1 \Re_2 = \int d^4x \sqrt{g}\, \Re_1 F(\Box, \Box) \Re_2$$

$$\int d^4x \sqrt{g} \sum_m \lambda_2^{(m)}(D_2) \frac{(D_2)^{4-m}}{\Box_1 \Box_2} \underbrace{\nabla \ldots \nabla}_{m} \Re_1 \Re_2 \Big|_{x_1 = x_2}$$

all factors in $\frac{(\mathrm{D}_2)^{4-\mathrm{m}}}{\Box_1 \Box_2} \underbrace{\nabla \ldots \nabla}_{\mathrm{m}}$ *completely cancel out due to integration by parts and Bianchi identities!*

$$= \int d^4x \sqrt{g} \Big(R_{\mu\nu} F_1(\Box) R^{\mu\nu} + R F_2(\Box) R \Big) + O[\Re^3]$$

dimensionless RG form factors

Running couplings of quadratic curvature terms:

Metamorphosis to high-energy partners of the cosmological constant [J. F. Donoghue, Phys. Rev. D 105, 105025 (2022), 2201.12217]

Conclusions

Single, known now, example of local, unitary, renormalizable and asymptotically free (consistent in UV limit) quantum gravity – projectable Hořava gravity

Salvation of unitarity in local renormalizable QG via LI violation

Asymptotic freedom in (2+1)-dimensional theory

Beta functions and AF RG flows of (3+1)-dimensional HG

Enigma of higher-derivative gravity models

THANK YOU!